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On some spaces of almost lacunary convergent sequences derived by Riesz mean and weighted almost lacunary statistical convergence in a real n-normed space

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Abstract

In this paper, we introduce some new spaces of almost convergent sequences derived by Riesz mean and the lacunary sequence in a real n-normed space. By combining the definitions of lacunary sequence and Riesz mean, we obtain a new concept of statistical convergence which will be called weighted almost lacunary statistical convergence in a real n-normed space. We examine some connections between this notion with the concept of almost lacunary statistical convergence and weighted almost statistical convergence, where the base space is a real n-normed space.

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Keywords: Riesz mean; weighted lacunary statistical convergence; almost convergence; lacunary sequence; n-norm

1 Introduction

The concept of 2-normed space has been initially introduced by Gähler [1]. Later, this concept was generalized to the concept of n-normed spaces by Misiak [2]. Since then, many others have studied these concepts and obtained various results [3–10].

The idea of statistical convergence was given by Zygmund [11] in 1935, in order to extend the convergence of sequences. The concept was formally introduced by Fast [12] and Steinhaus [13] and later on by Schoenberg [14], and also independently by Buck [15]. Many years later, it has been discussed in the theory of Fourier analysis, ergodic theory, and number theory under different names. In 1993, Fridy and Orhan [16] introduced the concept of lacunary statistical convergence. Statistical convergence has been generalized to the concept of a 2-normed space by Gürdal and Pehlivan [3] and to the concept of an n-normed space by Reddy [9].

Moricz and Orhan [17] have defined the concept of statistical summability (R, p_r). Later on, Karakaya and Chishti [18] have used (R, p_r)-summability to generalize the concept of statistical convergence and have called this new method weighted statistical convergence. Mursaleen *et al.* [19] have altered the definition of weighted statistical convergence and have found its relation with the concept of statistical (R, p_r)-summability. In general, the statistical convergence of weighted mean is studied as a regular matrix transformation. In

[18] and [19], the concept of statistical convergence is generalized by using a Riesz summability method and it is called weighted statistical convergence. For more details related to this topic, we may refer to [5, 20–23].

In this paper, we introduce some new spaces of almost convergent sequences derived by Riesz mean and lacunary sequence in a real n-normed space. By combining the definitions of lacunary sequence and Riesz mean, we obtain a new concept of statistical convergence, which will be called weighted almost lacunary statistical convergence in a real n-normed space. We examine some connections between this notion with the concept of almost lacunary statistical convergence and weighted almost statistical convergence, where the base space is a real n-normed space.

2 Definitions and preliminaries

Let K be a subset of natural numbers $\mathbb{N}$ and we denote the set $K_n = \{j \in K : j \leq n\}$. The cardinality of K_n is denoted by $|K_n|$. The natural density of K is given by $\delta(K) := \lim_r \frac{1}{r}|K_r|$, if it exists. The sequence $x = (x_j)$ is statistically convergent to ξ provided that, for every $\varepsilon > 0$, the set $K = K(\varepsilon) := \{j \in \mathbb{N} : |x_j - \xi| \geq \varepsilon\}$ has natural density zero.

Let (p_k) be a sequence of non-negative real numbers and $P_r = p_1 + p_2 + \cdots + p_r$ for $r \in \mathbb{N}$. Then the Riesz transformation of $x = (x_k)$ is defined as

$$t_r := \frac{1}{P_r}\sum_{k=1}^{r} p_k x_k. \tag{2.1}$$

If the sequence t_r has a finite limit ξ, then the sequence x is said to be (R, p_r)-convergent to ξ. Let us note that if $P_r \to \infty$ as $r \to \infty$ then the Riesz transformation is a regular summability method, that is, it transforms every convergent sequence to convergent sequence and preserves the limit.

If $p_k = 1$ for all $k \in \mathbb{N}$ in (2.1), then the Riesz mean reduces to the Cesaro mean C_1 of order one.

By a lacunary sequence $\theta = (k_r)$, where $k_0 = 0$, we will mean an increasing sequence of non-negative integers with $k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$. We write $h_r = k_r - k_{r-1}$. The ratio $\frac{k_r}{k_{r-1}}$ will be denoted by q_r.

Throughout the paper, we will use the following notations, which have been defined in [24].

Let $\theta = (k_r)$ be a lacunary sequence, (p_k) be a sequence of positive real numbers such that $H_r := \sum_{k \in I_r} p_k$, $P_{k_r} := \sum_{k \in (0,k_r]} p_k$, $P_{k_{r-1}} := \sum_{k \in (0,k_{r-1}]} p_k$, $Q_r := \frac{P_{k_r}}{P_{k_{r-1}}}$, $P_0 = 0$ and the intervals determined by θ and (p_k) are denoted by $I'_r = (P_{k_{r-1}}, P_{k_r}]$, $H_r = P_{k_r} - P_{k_{r-1}}$. If $p_k = 1$ for all $k \in \mathbb{N}$, then H_r, P_{k_r}, $P_{k_{r-1}}$, Q_r and I'_r reduce to h_r, k_r, k_{r-1}, q_r and I_r, respectively.

If $\theta = (k_r)$ is a lacunary sequence and $P_r \to \infty$ as $r \to \infty$, then $\theta' = (P_{k_r})$ is a lacunary sequence, that is, $P_0 = 0$, $0 < P_{k_{r-1}} < P_{k_r}$ and $H_r = P_{k_r} - P_{k_{r-1}} \to \infty$ as $r \to \infty$.

Throughout the paper, we will take $P_r \to \infty$ as $r \to \infty$, unless otherwise stated.

Lorentz [25] has proved that a sequence x is almost convergent to a number ξ if and only if $t_{km}(x) \to \xi$ as $k \to \infty$, uniformly in m, where

$$t_{km}(x) = \frac{x_m + x_{m+1} + \cdots + x_{m+k-1}}{k}, \quad k \in \mathbb{N}, m \geq 0. \tag{2.2}$$

We write $f - \lim x = \xi$ if x is almost convergent to ξ. Maddox [26] has defined $x = (x_j)$ to be strongly almost convergent to a number ξ if and only if $t_{km}(|x - \xi e|) \to 0$ as $k \to \infty$, uniformly in m, where $x - \xi e = (x_j - \xi)$ for all j and $e = (1, 1, \dots)$.

Let $n \in \mathbb{N}$ and X be a real vector space of dimension $d \geq n \geq 2$. A real-valued function $\|\cdot, \dots, \cdot\| : X^n \to \mathbb{R}$ satisfying the following conditions is called an n-norm on X and the pair $(X, \|\cdot, \dots, \cdot\|)$ is called a linear n-normed space:

(1) $\|x_1, \dots, x_n\| = 0$ if and only if $x_1, \dots, x_n$ are linearly dependent,
(2) $\|x_1, \dots, x_n\|$ is invariant under permutation,
(3) $\|\alpha x_1, \dots, x_{n-1}, x_n\| = |\alpha| \|x_1, \dots, x_{n-1}, x_n\|$ for any $\alpha \in \mathbb{R}$,
(4) $\|x_1, \dots, x_{n-1}, y + z\| \leq \|x_1, \dots, x_{n-1}, y\| + \|x_1, \dots, x_{n-1}, z\|$, for all $y, z, x_1, \dots, x_{n-1} \in X$.

A sequence $x = (x_j)$ in an n-normed space $(X, \|\cdot, \dots, \cdot\|)$ is said to be convergent to some $\xi \in X$ in the n-norm if for each $\varepsilon > 0$ there exists a positive integer $j_0 = j_0(\varepsilon)$ such that $\|x_j - \xi, z_1, \dots, z_{n-1}\| < \varepsilon$ for all $j \geq j_0$ and for every nonzero $z_1, \dots, z_{n-1} \in X$.

A sequence $x = (x_j)$ is said to be statistically convergent to ξ if for every $\varepsilon > 0$ the set $K := \{j \in \mathbb{N} : \|x_j - \xi, z_1, \dots, z_{n-1}\| \geq \varepsilon\}$ has natural density zero for every nonzero $z_1, \dots, z_{n-1} \in X$, in other words, $x = (x_j)$ is statistically convergent to ξ in n-normed space $(X, \|\cdot, \dots, \cdot\|)$ if $\lim_{j\to\infty} \frac{1}{j} |\{j \in \mathbb{N} : \|x_j - \xi, z_1, \dots, z_{n-1}\| \geq \varepsilon\}| = 0$, for every nonzero $z_1, \dots, z_{n-1} \in X$. For $\xi = 0$, we say this is statistically null.

3 Main results

Throughout the paper $w(X)$, $l_\infty(X)$ denote the spaces of all and bounded X valued sequence spaces, respectively, where $(X, \|\cdot, \dots, \cdot\|)$ is a real n-normed space.

The set of all almost convergent sequences and strongly almost convergent sequences with respect to the n-norm $\|\cdot, \cdot\|$ are denoted by F and $[F]$, respectively, as follows:

$$F = \left\{ \begin{array}{l} x \in l_\infty(X) : \lim_{k\to\infty} \|t_{km}(x - \xi e), z_1, \dots, z_{n-1}\| = 0, \text{ uniformly in } m, \\ \text{for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\},$$

and

$$[F] = \left\{ \begin{array}{l} x \in l_\infty(X) : \lim_{k\to\infty} t_{km}(\|x - \xi e, z_1, \dots, z_{n-1}\|) = 0, \text{ uniformly in } m, \\ \text{for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\},$$

where $t_{km}(x)$ is defined as in (2.2). We write $F - \lim x = \xi$ if x is almost convergent to ξ with respect to the n-norm and $[F] - \lim x = \xi$ if x is strongly almost convergent to ξ with respect to the n-norm. It is easy to see that the inclusions $[F] \subset F \subset l_\infty(X)$ hold.

Now, we define some new sequence spaces in a real n-normed space as follows:

$$[\tilde{R}, p_r, \theta]_n = \left\{ \begin{array}{l} x : \lim_{r\to\infty} \|\frac{1}{H_r} \sum_{k \in I_r} p_k t_{km}(x - \xi e), z_1, \dots, z_{n-1}\| = 0, \text{ uniformly in } m, \\ \text{for some } \xi \text{ and for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\},$$

$$(\tilde{R}, p_r, \theta)_n = \left\{ \begin{array}{l} x : \lim_{r\to\infty} \frac{1}{H_r} \sum_{k \in I_r} p_k \|t_{km}(x - \xi e), z_1, \dots, z_{n-1}\| = 0, \text{ uniformly in } m, \\ \text{for some } \xi \text{ and for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\},$$

$$|\tilde{R}, p_r, \theta|_n = \left\{ \begin{array}{l} x : \lim_{r\to\infty} \frac{1}{H_r} \sum_{k \in I_r} p_k t_{km}(\|x - \xi e, z_1, \dots, z_{n-1}\|) = 0, \text{ uniformly in } m, \\ \text{for some } \xi \text{ and for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\}.$$

The following results are obtained for some special cases:

(1) If we take $m = 0$ then the sequence spaces above are reduced to the sequence spaces $[C_1,\theta]_n$, $(C_1,\theta)_n$, $|C_1,\theta|_n$, respectively as follows:

$$[C_1,\theta]_n = \left\{ \begin{array}{l} x : \lim_{r\to\infty} \| \frac{1}{H_r} \sum_{k\in I_r} p_k t_{k0}(x - \xi e), z_1, \dots, z_{n-1} \| = 0, \\ \text{for some } \xi \text{ and for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\},$$

$$(C_1,\theta)_n = \left\{ \begin{array}{l} x : \lim_{r\to\infty} \frac{1}{H_r} \sum_{k\in I_r} p_k \| t_{k0}(x - \xi e), z_1, \dots, z_{n-1} \| = 0 \\ \text{for some } \xi \text{ and for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\},$$

$$|C_1,\theta|_n = \left\{ \begin{array}{l} x : \lim_{r\to\infty} \frac{1}{H_r} \sum_{k\in I_r} p_k t_{k0} \| x - \xi e, z_1, \dots, z_{n-1} \| = 0, \\ \text{for some } \xi \text{ and for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\}.$$

(2) If we take $p_k = 1$ for all $k \in \mathbb{N}$, then the sequence spaces above are reduced to the following spaces:

$$[w_\theta]_n = \left\{ \begin{array}{l} x : \lim_{r\to\infty} \| \frac{1}{h_r} \sum_{k\in I_r} t_{km}(x - \xi e), z_1, \dots, z_{n-1} \| = 0, \text{ uniformly in } m, \\ \text{for some } \xi \text{ and for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\},$$

$$(w_\theta)_n = \left\{ \begin{array}{l} x : \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} \| t_{km}(x - \xi e), z_1, \dots, z_{n-1} \| = 0, \text{ uniformly in } m, \\ \text{for some } \xi \text{ and for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\},$$

$$|w_\theta|_n = \left\{ \begin{array}{l} x : \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} t_{km}(\| x - \xi e, z_1, \dots, z_{n-1} \|) = 0, \text{ uniformly in } m, \\ \text{for some } \xi \text{ and for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\}.$$

(3) Let us choose $\theta = (k_r) = 2^r$ for $r > 0$, then these sequence spaces above are reduced to the following spaces:

$$[\tilde{R},p_r]_n = \left\{ \begin{array}{l} x : \lim_{r\to\infty} \| \frac{1}{P_r} \sum_{k=1}^{r} p_k t_{km}(x - \xi e), z_1, \dots, z_{n-1} \| = 0, \text{ uniformly} \\ \text{in } m, \text{ for some } \xi \text{ and for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\},$$

$$(\tilde{R},p_r)_n = \left\{ \begin{array}{l} x : \lim_{r\to\infty} \frac{1}{P_r} \sum_{k=1}^{r} p_k \| t_{km}(x - \xi e), z_1, \dots, z_{n-1} \| = 0, \text{ uniformly} \\ \text{in } m, \text{ for some } \xi \text{ and for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\},$$

$$|\tilde{R},p_r|_n = \left\{ \begin{array}{l} x : \lim_{r\to\infty} \frac{1}{P_r} \sum_{k=1}^{r} p_k t_{km}(\| x - \xi e, z_1, \dots, z_{n-1} \|) = 0, \text{ uniformly} \\ \text{in } m, \text{ for some } \xi \text{ and for every nonzero } z_1, \dots, z_{n-1} \in X \end{array} \right\}.$$

(4) If we select $\theta = (k_r) = 2^r$ for $r > 0$ and the base space as $(X, \|\cdot,\cdot\|)$ then these sequence spaces above are reduced to the sequence spaces which can be seen in [5].
(5) If we choose $p_k = 1$ for all $k \in \mathbb{N}$ and $\theta = (k_r) = 2^r$ for $r > 0$, then these sequence spaces above are reduced to the sequence spaces $[C_1]_n$, $(C_1)_n$, $|C_1|_n$, respectively.

Now, we give the following theorem to demonstrate some inclusion relations among the sequence spaces $|\tilde{R},p_r,\theta|_n$, $(\tilde{R},p_r,\theta)_n$, $[\tilde{R},p_r,\theta]_n$, $|C_1,\theta|_n$, $(C_1,\theta)_n$, $[C_1,\theta]_n$ with the spaces F and $[F]$.

Theorem 3.1 *The following statements are true*:
(1) $[F] \subset F \subset (\tilde{R},p_r,\theta)_n \subset [\tilde{R},p_r,\theta]_n \subset [C_1,\theta]_n$.
(2) $[F] \subset |\tilde{R},p_r,\theta|_n \subset (\tilde{R},p_r,\theta)_n \subset [\tilde{R},p_r,\theta]_n \subset [C_1,\theta]_n$.
(3) $[F] \subset |\tilde{R},p_r,\theta|_n \subset |C_1,\theta| \subset (C_1,\theta)_n \subset [C_1,\theta]_n$.

Proof We give the proof only for (2). The proofs of (1) and (3) can be done, similarly. So we omit them. Let $x \in [F]$ and $[F] - \lim x = \xi$. Then $t_{km}(\|x - \xi e, z_1, \dots, z_{n-1}\|) \to 0$ as $k \to \infty$, uniformly in m, for every nonzero $z_1, \dots, z_{n-1} \in X$. Since $H_r \to \infty$ as $r \to \infty$, then its weighted lacunary mean also converges to ξ as $r \to \infty$ uniformly in m. This proves that $x \in |\tilde{R}, p_r, \theta|_n$ and $[F] - \lim x = |\tilde{R}, p_r, \theta|_n - \lim x = \xi$. Also since

$$\left\| \frac{1}{H_r} \sum_{k \in I_r} p_k t_{km}(x - \xi e), z_1, \dots, z_{n-1} \right\| \le \frac{1}{H_r} \sum_{k \in I_r} p_k \left\| t_{km}(x - \xi e), z_1, \dots, z_{n-1} \right\| \le \frac{1}{H_r} \sum_{k \in I_r} p_k t_{km} \bigl(\|x - \xi e, z_1, \dots, z_{n-1}\| \bigr),$$

then it follows that $[F] \subset |\tilde{R}, p_r, \theta|_n \subset (\tilde{R}, p_r, \theta)_n \subset [\tilde{R}, p_r, \theta]_n$ and $[F] - \lim x = |\tilde{R}, p_r, \theta|_n - \lim x = (\tilde{R}, p_r, \theta)_n - \lim x = [\tilde{R}, p_r, \theta]_n - \lim x = \xi$. Since uniform convergence of $\| \frac{1}{H_r} \times \sum_{k \in I_r} p_k t_{km}(x - \xi e), z_1, \dots, z_{n-1} \|$ with respect to m, as $r \to \infty$, implies convergence for $m = 0$ and for every nonzero $z_1, \dots, z_{n-1} \in X$. It follows that $[\tilde{R}, p_r, \theta]_n \subset [C_1, \theta]_n$ and $[\tilde{R}, p_r, \theta]_n - \lim x = [C_1, \theta]_n - \lim x = \xi$. This completes the proof. □

Theorem 3.2 *Let* $\theta = (k_r)$ *be a lacunary sequence and* $\liminf_r Q_r > 1$. *Then* $(\tilde{R}, p_r)_n \subseteq (\tilde{R}, p_r, \theta)_n$ *with* $(\tilde{R}, p_r)_n - \lim x = (\tilde{R}, p_r, \theta)_n - \lim x = \xi$.

Proof Suppose that $\liminf_r Q_r > 1$, then there exists a $\delta > 0$ such that $Q_r \ge 1 + \delta$ for sufficiently large values of r, which implies that $\frac{H_r}{P_{k_r}} \ge \frac{\delta}{1+\delta}$. If $x \in (\tilde{R}, p_r)_n$ with $(\tilde{R}, p_r)_n - \lim x = \xi$, then for sufficiently large values of r, we have

$$\begin{aligned} & \frac{1}{P_{k_r}} \sum_{k=1}^{k_r} p_k \left\| t_{km}(x - \xi e), z_1, \dots, z_{n-1} \right\| \\ &= \frac{1}{P_{k_r}} \left(\sum_{k=1}^{k_{r-1}} p_k \left\| t_{km}(x - \xi e), z_1, \dots, z_{n-1} \right\| + \sum_{k=k_{r-1}+1}^{k_r} p_k \left\| t_{km}(x - \xi e), z_1, \dots, z_{n-1} \right\| \right) \\ &\ge \frac{H_r}{P_{k_r}} \left(\frac{1}{H_r} \sum_{k \in I_r} p_k \left\| t_{km}(x - \xi e), z_1, \dots, z_{n-1} \right\| \right) \\ &\ge \frac{\delta}{1 + \delta} . \frac{1}{H_r} \sum_{k \in I_r} p_k \left\| t_{km}(x - \xi e), z_1, \dots, z_{n-1} \right\|, \end{aligned}$$

for each $m \ge 0$ and for every nonzero $z_1, \dots, z_{n-1} \in X$. Then, it follows that $x \in (\tilde{R}, p_r, \theta)_n$ with $(\tilde{R}, p_r, \theta)_n - \lim x = \xi$ by taking the limit as $r \to \infty$. This completes the proof. □

Theorem 3.3 *Let* $\theta = (k_r)$ *be a lacunary sequence with* $\limsup_r Q_r < \infty$. *Then* $(\tilde{R}, p_r, \theta)_n \subseteq (\tilde{R}, p_r)_n$ *with* $(\tilde{R}, p_r, \theta)_n - \lim x = (\tilde{R}, p_r)_n - \lim x = \xi$.

Proof Let $x \in (\tilde{R}, p_r, \theta)_n$ with $(\tilde{R}, p_r, \theta)_n - \lim x = \xi$. Then for $\varepsilon > 0$, there exists q_0 such that for every $q > q_0$

$$L_q = \frac{1}{H_q} \sum_{k \in I_q} p_k \left\| t_{km}(x - \xi e), z_1, \dots, z_{n-1} \right\| < \varepsilon, \tag{3.1}$$

for each $m \geq 0$ and for every nonzero $z_1, \ldots, z_{n-1} \in X$, that is, we can find some positive constant M such that

$$L_q \leq M \quad \text{for all } q. \tag{3.2}$$

$\limsup_r Q_r < \infty$ implies that there exists some positive number K such that

$$Q_r \leq K \quad \text{for all } r \geq 1. \tag{3.3}$$

Therefore for $k_{r-1} < r \leq k_r$, we have by (3.1), (3.2), and (3.3)

$$\begin{aligned}
&\frac{1}{P_r} \sum_{k=1}^{r} p_k \| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \| \\
&\quad \leq \frac{1}{P_{k_{r-1}}} \sum_{k=1}^{k_r} p_k \| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \| \\
&\quad = \frac{1}{P_{k_{r-1}}} \biggl(\sum_{k \in I_1} p_k \| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \| + \sum_{k \in I_2} p_k \| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \| + \cdots \\
&\qquad + \sum_{k \in I_{q_0}} p_k \| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \| + \cdots + \sum_{k \in I_r} p_k \| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \| \biggr) \\
&\quad = \frac{1}{P_{k_{r-1}}} (L_1 H_1 + L_2 H_2 + \cdots + L_{q_0} H_{q_0} + L_{q_0+1} H_{q_0+1} + \cdots + L_r H_r) \\
&\quad \leq \frac{M}{P_{k_{r-1}}} (H_1 + H_2 + \cdots + H_{q_0}) + \frac{\varepsilon}{P_{k_{r-1}}} (H_{q_0+1} + \cdots + H_r) \\
&\quad = \frac{M}{P_{k_{r-1}}} (P_{k_1} - P_{k_0} + \cdots + P_{k_{q_0}} - P_{k_{q_0-1}}) + \frac{\varepsilon}{P_{k_{r-1}}} (P_{k_{q_0}} - P_{k_{q_0-1}} + \cdots + P_{k_r} - P_{k_{r-1}}) \\
&\quad = M \frac{P_{k_{q_0}}}{P_{k_{r-1}}} + \varepsilon \frac{P_{k_r} - P_{k_{q_0}}}{P_{k_{r-1}}} \\
&\quad \leq M \frac{P_{k_{q_0}}}{P_{k_{r-1}}} + \varepsilon K,
\end{aligned}$$

for each $m \geq 0$ and for every nonzero $z_1, \ldots, z_{n-1} \in X$. Since $P_{k_{r-1}} \to \infty$ as $r \to \infty$, we get $x \in (\tilde{R}, p_r)_n$ with $(\tilde{R}, p_r)_n - \lim x = \xi$. This completes the proof. □

Corollary 3.4 *Let* $1 < \liminf_r Q_r \leq \limsup_r Q_r < \infty$. *Then* $(\tilde{R}, p_r, \theta)_n = (\tilde{R}, p_r)_n$ *and* $(\tilde{R}, p_r, \theta)_n - \lim x = (\tilde{R}, p_r)_n - \lim x = \xi$.

Proof It follows from Theorem 3.2 and Theorem 3.3. □

In the following theorem, we give the relations between the sequence spaces $(w_\theta)_n$ and $(\tilde{R}, p_r)_n$.

Theorem 3.5

(1) *If* $p_k < 1$ *for all* $k \in \mathbb{N}$, *then* $(w_\theta)_n \subseteq (\tilde{R}, p_r)_n$ *and* $(w_\theta)_n - \lim x = (\tilde{R}, p_r)_n - \lim x = \xi$.

(2) *If* $p_k > 1$ *for all* $k \in \mathbb{N}$ *and* $(\frac{H_r}{h_r})$ *is upper-bounded, then* $(\tilde{R}, p_r)_n \subseteq (w_\theta)_n$ *and* $(\tilde{R}, p_r)_n - \lim x = (w_\theta)_n - \lim x = \xi$.

Proof

(1) If $p_k < 1$ for all $k \in \mathbb{N}$, then $H_r < h_r$ for all $r \in \mathbb{N}$. So, there exists an M_1, a constant, such that $0 < M_1 \leq \frac{H_r}{h_r} < 1$ for all $r \in \mathbb{N}$. Let $x \in (w_\theta)_n$ with $(w_\theta)_n - \lim x = \xi$, then for an arbitrary $\varepsilon > 0$ we have

$$\frac{1}{H_r}\sum_{k\in I_r} p_k \| t_{km}(x-\xi e), z_1, \ldots, z_{n-1} \| \leq \frac{1}{M_1}\frac{1}{h_r}\sum_{k\in I_r} \| t_{km}(x-\xi e), z_1, \ldots, z_{n-1} \|,$$

for each $m \geq 0$ and for every nonzero $z_1, \ldots, z_{n-1} \in X$. Therefore, we get the result by taking the limit as $r \to \infty$.

(2) Let $p_k > 1$ for all $k \in \mathbb{N}$, then $H_r > h_r$ for all $r \in \mathbb{N}$. Suppose that $(\frac{H_r}{h_r})$ is upper-bounded, then there exists an M_2, a constant, such that $1 < \frac{H_r}{h_r} \leq M_2 < \infty$ for all $r \in \mathbb{N}$. Let $x \in (\tilde{R}, p_r)_n$ and $(\tilde{R}, p_r)_n - \lim x = \xi$. So the result is obtained by taking the limit as $r \to \infty$ for each $m \geq 0$ and for every nonzero $z_1, \ldots, z_{n-1} \in X$, from the following inequality:

$$\frac{1}{h_r}\sum_{k\in I_r} \| t_{km}(x-\xi e), z_1, \ldots, z_{n-1} \| \leq M_2\frac{1}{H_r}\sum_{k\in I_r} p_k \| t_{km}(x-\xi e), z_1, \ldots, z_{n-1} \|. \qquad \square$$

Now, we define a new concept of statistical convergence in n-normed space, which will be called weighted almost lacunary statistical convergence:

Definition 3.6 The weighted almost lacunary density of $K \subseteq \mathbb{N}$ is denoted by $\delta_{(\tilde{R},\theta)}(K) = \lim_{r\to\infty} \frac{1}{H_r}|K_r(\varepsilon)|$ if the limit exists. We say that the sequence $x = (x_j)$ is weighted almost lacunary statistically convergent to ξ if for every $\varepsilon > 0$, the set $K_r(\varepsilon) = \{k \in I'_r : p_k \| t_{km}(x-\xi e), z_1, \ldots, z_{n-1} \| \geq \varepsilon\}$ has weighted lacunary density zero, *i.e.*

$$\lim_{r\to\infty} \frac{1}{H_r}\left|\left\{k \in I'_r : p_k \left\| t_{km}(x-\xi e), z_1, \ldots, z_{n-1} \right\| \geq \varepsilon\right\}\right| = 0 \tag{3.4}$$

uniformly in m, for every nonzero $z_1, \ldots, z_{n-1} \in X$. In this case, we write $(S_{(\tilde{R},\theta)}, n) - \lim_k x_k = \xi$. By $(S_{(\tilde{R},\theta)}, n)$ we denote the set of all weighted almost lacunary statistically convergent sequences in n-normed space.

(1) If we take $p_k = 1$ for all $k \in \mathbb{N}$ in (3.4) then we obtain the definition of almost lacunary statistical convergence in n–normed space, that is, x is called almost lacunary statistically convergent to ξ if for every $\varepsilon > 0$, the set $K_\theta(\varepsilon) = \{k \in I_r : \| t_{km}(x-\xi e), z_1, \ldots, z_{n-1} \| \geq \varepsilon\}$ has lacunary density zero, *i.e.*

$$\lim_{r\to\infty} \frac{1}{h_r}\left|\left\{k \in I_r : \left\| t_{km}(x-\xi e), z_1, \ldots, z_{n-1} \right\| \geq \varepsilon\right\}\right| = 0 \tag{3.5}$$

uniformly in m, for every nonzero $z_1, \ldots, z_{n-1} \in X$. In this case, we write $(S_\theta, n) - \lim_j x_j = \xi$. By (S_θ, n) we denote the set of all weighted almost lacunary statistically convergent sequences in n-normed space.

(2) Let us choose $\theta = (k_r)$ for $r > 0$ then the definition of weighted almost lacunary statistical convergence which is given in (3.4) is reduced to the definition of weighted almost statistically convergence, that is, x is called weighted almost

statistically convergent to ξ if for every $\varepsilon > 0$, the set $K_{P_r}(\varepsilon) = \{k \leq P_r : p_k\|t_{km}(x - \xi e), z_1, \ldots, z_{n-1}\| \geq \varepsilon\}$ has weighted density zero, *i.e.*

$$\lim_{r\to\infty} \frac{1}{P_r}\left|\left\{k \leq P_r : \left\|t_{km}(x - \xi e), z_1, \ldots, z_{n-1}\right\| \geq \varepsilon\right\}\right| = 0 \tag{3.6}$$

uniformly in m, for every nonzero $z_1, \ldots, z_{n-1} \in X$. In this case, we write $(S_{\tilde{R}}, n) - \lim_j x_j = \xi$. By $(S_{\tilde{R}}, n)$ we denote the set of all weighted almost lacunary statistically convergent sequences in n-normed space.

(3) Let us choose $\theta = (k_r)$ for $r > 0$ and $p_k = 1$ for all $k \in \mathbb{N}$, then the definition of weighted almost lacunary statistical convergence, which is given in (3.4), is reduced to the definition of almost statistical convergence.

Theorem 3.7 *If the sequence x is $(\tilde{R}, p_r, \theta)_n$-convergent to ξ then the sequence x is weighted almost lacunary statistically convergent to ξ.*

Proof Let the sequence x be $(\tilde{R}, p_r, \theta)_n$-convergent to ξ and $K_{rm}(\varepsilon) = \{k \in I'_r : p_k\|t_{km}(x - \xi e), z_1, \ldots, z_{n-1}\| \geq \varepsilon\}$. Then for a given $\varepsilon > 0$, we have

$$\begin{aligned}\frac{1}{H_r}\sum_{k\in I_r} p_k\left\|t_{km}(x - \xi e), z_1, \ldots, z_{n-1}\right\| &\geq \frac{1}{H_r}\sum_{\substack{k\in I_r\\ k\in K_{rm}(\varepsilon)}} p_k\left\|t_{km}(x - \xi e), z_1, \ldots, z_{n-1}\right\| \\ &\geq \varepsilon\frac{1}{H_r}\left|K_{rm}(\varepsilon)\right|\end{aligned}$$

for each $m \geq 0$ and for every nonzero $z \in X$. Hence, we see that the sequence x is weighted almost statistically convergent to ξ by taking the limit as $r \to \infty$. □

Theorem 3.8 *Let $p_k\|t_{km}(x - \xi e), z_1, \ldots, z_{n-1}\| \leq M$ for all $k \in \mathbb{N}$, for each $m \geq 0$ and for every nonzero $z_1, \ldots, z_{n-1} \in X$. Then $(S_{(\tilde{R},\theta)}, n) \subset (\tilde{R}, p_r, \theta)_n$ with $(S_{(\tilde{R},\theta)}, n) - \lim x = (\tilde{R}, p_r, \theta)_n - \lim x = \xi$.*

Proof Let x be convergent to ξ in $(S_{(\tilde{R},\theta)}, n)$ and let us take

$$K_{rm}(\varepsilon) = \left\{k \in I'_r : p_k\left\|t_{km}(x - \xi e), z_1, \ldots, z_{n-1}\right\| \geq \varepsilon\right\}.$$

Since $p_k\|t_{km}(x - \xi e), z_1, \ldots, z_{n-1}\| \leq M$ for all $k \in \mathbb{N}$ for each $m \geq 0$, for every nonzero $z_1, \ldots, z_{n-1} \in X$ and $H_r \to \infty$ as $r \to \infty$, then for a given $\varepsilon > 0$ we have

$$\begin{aligned}\frac{1}{H_r}\sum_{k\in I_r} p_k\left\|t_{km}(x - \xi e), z_1, \ldots, z_{n-1}\right\| &= \frac{1}{H_r}\sum_{\substack{k\in I_r\\ k\in K_{rm(\varepsilon)}}} p_k\left\|t_{km}(x - \xi e), z_1, \ldots, z_{n-1}\right\| \\ &\quad + \frac{1}{H_r}\sum_{\substack{k\in I_r\\ k\notin K_{rm(\varepsilon)}}} p_k\left\|t_{km}(x - \xi e), z_1, \ldots, z_{n-1}\right\| \\ &\leq M\frac{1}{H_r}|K_{rm(\varepsilon)}| + \frac{h_r}{H_r}\varepsilon \\ &\leq M\frac{1}{H_r}|K_{rm(\varepsilon)}| + \varepsilon,\end{aligned}$$

for each $m \geq 0$ and for every nonzero $z_1, \ldots, z_{n-1} \in X$. Since ε is arbitrary, we have $x \in (\tilde{R}, p_r, \theta)_n$ by taking the limit as $r \to \infty$. □

Theorem 3.9 *The following statements are true.*

(1) *If $p_k \leq 1$ for all $k \in \mathbb{N}$, then $(S_\theta, n) \subseteq (S_{(\tilde{R},\theta)}, n)$.*
(2) *Let $p_k \geq 1$ for all $k \in \mathbb{N}$ and $(\frac{H_r}{h_r})$ be upper-bounded, then $(S_{(\tilde{R},\theta)}, n) \subseteq (S_\theta, n)$.*

Proof

(1) If $p_k \leq 1$ for all $k \in \mathbb{N}$, then $H_r \leq h_r$ for all $r \in \mathbb{N}$. So, there exist M_1 and M_2, constants, such that $0 < M_1 \leq \frac{H_r}{h_r} \leq M_2 \leq 1$ for all $r \in \mathbb{N}$. Let $x \in (S_\theta, n)$ with $(S_\theta, n) - \lim x = \xi$, then for an arbitrary $\varepsilon > 0$ we have

$$\begin{aligned}
&\frac{1}{H_r}\left|\left\{k \in I_r' : p_k \left\| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \right\| \geq \varepsilon\right\}\right| \\
&\quad = \frac{1}{H_r}\left|\left\{P_{k_{r-1}} < k \leq P_{k_r} : p_k \left\| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \right\| \geq \varepsilon\right\}\right| \\
&\quad \leq \frac{1}{M_1}\frac{1}{h_r}\left|\left\{P_{k_{r-1}} \leq k_{r-1} < k \leq P_{k_r} \leq k_r : \left\| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \right\| \geq \varepsilon\right\}\right| \\
&\quad = \frac{1}{M_1}\frac{1}{h_r}\left|\left\{k_{r-1} < k \leq k_r : \left\| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \right\| \geq \varepsilon\right\}\right| \\
&\quad = \frac{1}{M_1}\frac{1}{h_r}\left|\left\{k \in I_r : \left\| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \right\| \geq \varepsilon\right\}\right|,
\end{aligned}$$

for each $m \geq 0$ and for every nonzero $z_1, \ldots, z_{n-1} \in X$. Hence, we obtain the result by taking the limit as $r \to \infty$.

(2) Let $(\frac{H_r}{h_r})$ be upper-bounded, then there exist M_1 and M_2, constants, such that $1 \leq M_1 \leq \frac{H_r}{h_r} \leq M_2 < \infty$ for all $r \in \mathbb{N}$. Suppose that $p_k \geq 1$ for all $k \in \mathbb{N}$, then $H_r \geq h_r$ for all $r \in \mathbb{N}$. Let $x \in (\tilde{R}, p_r)_n$ and $(\tilde{R}, p_r)_n - \lim x = \xi$, then for an arbitrary $\varepsilon > 0$ we have

$$\begin{aligned}
&\frac{1}{h_r}\left|\left\{k \in I_r : \left\| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \right\| \geq \varepsilon\right\}\right| \\
&\quad = \frac{1}{h_r}\left|\left\{k_{r-1} < k \leq k_r : \left\| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \right\| \geq \varepsilon\right\}\right| \\
&\quad \leq M_2\frac{1}{H_r}\left|\left\{k_{r-1} \leq P_{k_{r-1}} < k \leq k_r \leq P_{k_r} : p_k \left\| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \right\| \geq \varepsilon\right\}\right| \\
&\quad = M_2\frac{1}{H_r}\left|\left\{P_{k_{r-1}} < k \leq P_{k_r} : p_k \left\| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \right\| \geq \varepsilon\right\}\right| \\
&\quad = M_2\frac{1}{H_r}\left|\left\{k \in I_r' : p_k \left\| t_{km}(x - \xi e), z_1, \ldots, z_{n-1} \right\| \geq \varepsilon\right\}\right|,
\end{aligned}$$

for each $m \geq 0$ and for every nonzero $z_1, \ldots, z_{n-1} \in X$. Hence, the result is obtained by taking the limit as $r \to \infty$. □

Theorem 3.10 *For any lacunary sequence θ, if $\liminf_r Q_r > 1$ then $(S_{\tilde{R}}, n) \subseteq (S_{(\tilde{R},\theta)}, n)$ and $(S_{\tilde{R}}, n) - \lim x = (S_{(\tilde{R},\theta)}, n) - \lim x = \xi$.*

Proof Suppose that $\liminf_r Q_r > 1$, then there exists a $\delta > 0$ such that $Q_r \geq 1 + \delta$ for sufficiently large values of r, which implies that $\frac{H_r}{P_{k_r}} \geq \frac{\delta}{1+\delta}$. If $x \in (S_{\tilde{R}}, n)$ with $(S_{\tilde{R}}, n) - \lim x = \xi$, then for every $\varepsilon > 0$ and for sufficiently large values of r, we have

$$\begin{aligned}
&\frac{1}{P_{k_r}}\left|\left\{k \le P_{k_r} : p_k\left\|t_{km}(x-\xi e), z_1, \dots, z_{n-1}\right\| \ge \varepsilon\right\}\right| \\
&\quad \ge \frac{1}{P_{k_r}}\left|\left\{P_{k_{r-1}} < k \le P_{k_r} : p_k\left\|t_{km}(x-\xi e), z_1, \dots, z_{n-1}\right\| \ge \varepsilon\right\}\right| \\
&\quad = \frac{H_r}{P_{k_r}}\left(\frac{1}{H_r}\left|\left\{P_{k_{r-1}} < k \le P_{k_r} : p_k\left\|t_{km}(x-\xi e), z_1, \dots, z_{n-1}\right\| \ge \varepsilon\right\}\right|\right) \\
&\quad \ge \frac{\delta}{1+\delta}\left(\frac{1}{H_r}\left|\left\{k \in I'_r : p_k\left\|t_{km}(x-\xi e), z_1, \dots, z_{n-1}\right\| \ge \varepsilon\right\}\right|\right),
\end{aligned}$$

for each $m \ge 0$ and for every nonzero $z_1, \dots, z_{n-1} \in X$. Hence, we get the result by taking the limit as $r \to \infty$. □

Theorem 3.11 *Let* $\theta = (k_r)$ *be a lacunary sequence with* $\limsup_r Q_r < \infty$, *then* $(S_{(\tilde{R},\theta)}, n) \subseteq (S_{\tilde{R}}, n)$ *and* $(S_{\tilde{R}}, n) - \lim x = (S_{(\tilde{R},\theta)}, n) - \lim x = \xi$.

Proof If $\limsup_r Q_r < \infty$, then there is a $K > 0$ such that $Q_r \le K$ for all $r \in \mathbb{N}$. Suppose that $x \in (S_{(\tilde{R},\theta)}, n)$ with $(S_{(\tilde{R},\theta)}, n) - \lim x = \xi$ and let

$$N_r := \left|\left\{k \in I'_r : p_k\left\|t_{km}(x-\xi e), z_1, \dots, z_{n-1}\right\| \ge \varepsilon\right\}\right|. \tag{3.7}$$

By (3.7), given $\varepsilon > 0$, there is a $r_0 \in \mathbb{N}$ such that $\frac{N_r}{H_r} < \varepsilon$ for all $r > r_0$. Now, let $M := \max\{N_r : 1 \le r \le r_0\}$ and let r be any integer satisfying $k_{r-1} < r \le k_r$, then for each $m \ge 0$ and for every nonzero $z_1, \dots, z_{n-1} \in X$ we can write

$$\begin{aligned}
&\frac{1}{P_r}\left|\left\{k \le P_r : p_k\left\|t_{km}(x-\xi e), z_1, \dots, z_{n-1}\right\| \ge \varepsilon\right\}\right| \\
&\quad \le \frac{1}{P_{k_{r-1}}}\left|\left\{P_{k_{r-1}} < k \le P_{k_r} : p_k\left\|t_{km}(x-\xi e), z_1, \dots, z_{n-1}\right\| \ge \varepsilon\right\}\right| \\
&\quad = \frac{1}{P_{k_{r-1}}}(N_1 + N_2 + \cdots + N_{r_0} + N_{r_0+1} + \cdots + N_r) \\
&\quad \le \frac{M.r_0}{P_{k_{r-1}}} + \frac{1}{P_{k_{r-1}}}\varepsilon(H_{r_0+1} + \cdots + H_r) \\
&\quad = \frac{M.r_0}{P_{k_{r-1}}} + \varepsilon\frac{(P_{k_r} - P_{k_{r_0}})}{P_{k_{r-1}}} \\
&\quad \le \frac{M.r_0}{P_{k_{r-1}}} + \varepsilon Q_r \le \frac{M.r_0}{P_{k_{r-1}}} + \varepsilon K,
\end{aligned}$$

which completes the proof by taking the limit as $r \to \infty$. □

Corollary 3.12 *Let* $1 < \liminf_r Q_r \le \limsup_r Q_r < \infty$. *Then* $(S_{(\tilde{R},\theta)}, n) = (S_{\tilde{R}}, n)$ *and* $(S_{\tilde{R}}, n) - \lim x = (S_{(\tilde{R},\theta)}, n) - \lim x = \xi$.

Proof It follows from Theorem 3.10 and Theorem 3.11. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally in the preparation of this article. Both authors read and approved the final manuscript.

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