The size of the proton and the deuteron

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Abstract. We have recently measured the $2S^{F=1}_{1/2} - 2P^{F=2}_{3/2}$ energy splitting in the muonic hydrogen atom μp to be 49881.88 (76) GHz. Using recent QED calculations of the fine-, hyperfine, QED and finite size contributions we obtain a root-mean-square proton charge radius of $r_{\rm p} = 0.84184\,(67)$ fm. This value is ten times more precise, but 5 standard deviations smaller, than the 2006 CODATA value of $r_{\rm p} = 0.8768\,(69)$ fm. The source of this discrepancy is unknown. Using the precise measurements of the 1S-2S transition in regular hydrogen and deuterium and our value of $r_{\rm p}$ we obtain improved values of the Rydberg constant, $R_\infty = 10973731.568160$ (16) $\rm m^{-1}$, and the rms charge radius of the deuteron $r_{\rm d} = 2.12809$ (31) fm.

1. Introduction

The hydrogen atom (H) is the simplest of all atoms, and this simplicity is beautiful. Quantum electrodynamics (QED), originally motivated by the discovery of the Lamb shift in hydrogen [1],

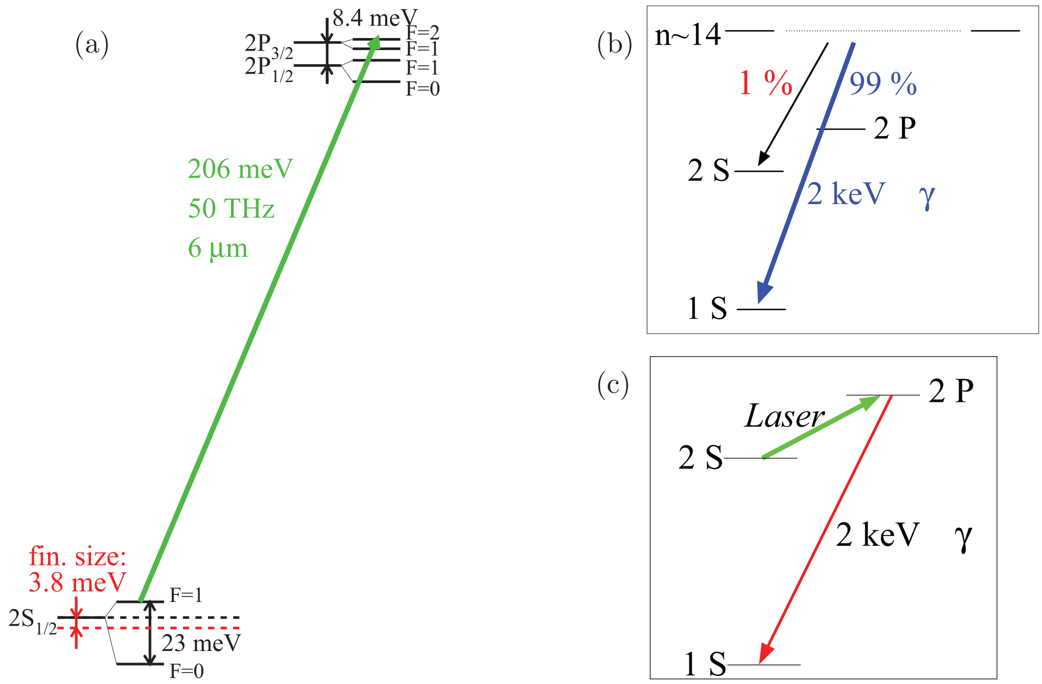

Figure 1. (a) The $n = 2$ levels in muonic hydrogen. Vacuum polarization is the dominant contribution of the 2S Lamb shift of 202 meV. The finite size effect is as large as 2% of the total Lamb shift. The measured $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ transition is indicated in green. The 1S ground state is 1.9 keV below the $n = 2$ states plotted here. (b,c) Experimental principle. (b) 99% of the muons stopped in H_2 gas at 1 mbar pressure proceed directly to the 1S ground state thereby emitting x-rays of the Lyman series around 2 keV. 1% of the muons form long-lived metastable 2S states with a lifetime $\tau_{2S} = 1\,\mu$s at 1 mbar. (c) Laser light of suitable wavelength around $\lambda = 6\,\mu$m drives the 2S-2P transition. The 2P state de-excites within 8 ps to the 1S ground state via emission of a Lyman-α x-ray at 1.9 keV.

can be used to accurately calculate the transition frequencies in the hydrogen atom [2, 3]. High-precision spectroscopy in hydrogen [4–9] can then be used to test the laws of physics, and to deduce accurate fundamental physical constants [10].

For more than a decade, the test of bound-state QED using hydrogen has been hampered by the lack of an accurate value of the proton rms charge radius, r_p. This quantity is required in the calculation of the nuclear size effects on the energy levels (in particular the S-states) in the hydrogen atom. For example, the 1S ground state of H is shifted by as much as 1 MHz.

Traditionally, the proton's rms charge radius has been determined by electron scattering [11–14]. A recent reevaluation of the world e-p scattering data has resulted in $r_\mathrm{p}= 0.897(18)$ fm [15, 16], i.e., with a relative uncertainty $u_r = 1.8\,\%$. Very recently, a preliminary value $r_\mathrm{p} = 0.879(8)$ fm from a new measurement in Mainz has become available [17].

Both spectroscopy and QED calculations of H have reached a truly astonishing accuracy. Although the finite size effect on the S states in H enters only at the 10^{-10} level, the proton charge radius deduced from the hydrogen measurements $r_\mathrm{p} = 0.880(8)$ fm (see Tab. XLV of Ref. [10], adjustment 10), is twice more accurate than the e-p scattering world average [15, 16]. The 2006 CODATA value of $r_\mathrm{p} = 0.8768(69)$ fm [10], is hence completely dominated by H spectroscopy.

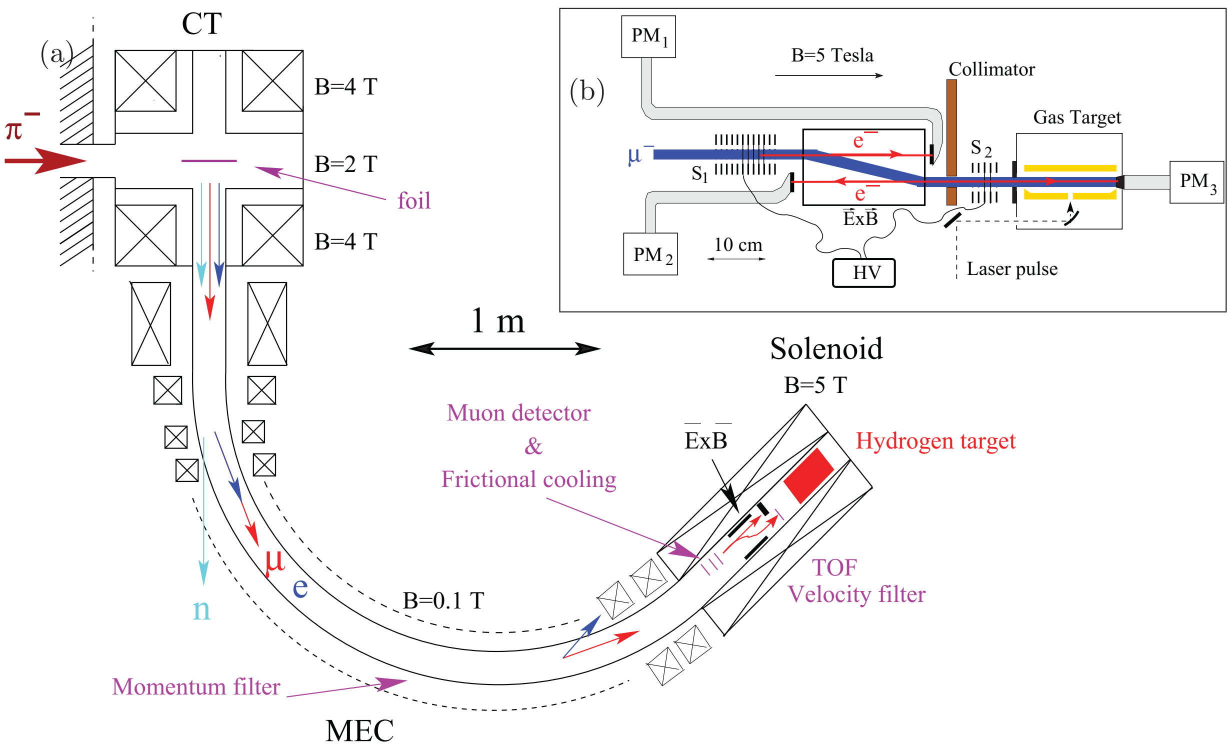

Figure 2. (a) Low-energy negative muon beam line: Negative pions are injected into the cyclotron trap (CT) and decay into muons μ^- which are decelerated in a foil. Slow μ^- leave the CT axially and are separated from background in the muon extraction channel (MEC). The experiment takes place in the 5 T solenoid shown in (b): Slow μ^- pass two stacks of ultra-thin carbon foils ($S_{1,2}$) where a few electrons are ejected. The e^- are detected in plastic scintillators read out by PMTs ($PM_{1..3}$), while the slower μ^- are separated from the e^- by an $\vec{E} \times \vec{B}$ drift in a capacitor. The delayed coincidence [$PM_1 \wedge$ ($PM_2 \vee$ PM_3)] triggers the laser system (Fig. 3). The μ^- stops inside the gas target filled with 1 mbar of H_2 gas. The stop volume is illuminated by the laser pulse injected into a multi-pass cavity, and viewed by 20 LAAPDs (not shown).

2. Muonic hydrogen

The leading order finite size effect on the S states in hydrogen-like systems with low Z is

$$E_{\text{fin.size}} = \frac{2}{3}\left(\frac{m_r}{m_e}\right)^3 \frac{(Z\alpha)^2}{n^3} m_e c^2 \left(\frac{2\pi Z\alpha\, r_N}{\lambda_C}\right)^2, \tag{1}$$

where Z is the nuclear charge, α the fine structure constant, n the principal quantum number, r_N the nuclear radius, λ_C the Compton wavelength of the electron, m_e the electron mass, and

$$m_r = \frac{m_e\, m_N}{m_e + m_N} \tag{2}$$

is the reduced mass of the hydrogen-like system with nuclear mass m_N. One observes immediately the scaling of the finite size effect with the third power of the reduced mass m_r. This scaling reflects the overlap of the electron's wave function with the nucleus.

Muonic hydrogen μp is a hydrogen atom where the proton p is orbited by a negative muon μ^-, instead of an electron. The muon mass $m_\mu = 207\, m_e$, and so the reduced mass m_r is 186 times larger for μp than for H. This results in a 186 times smaller Bohr radius of muonic hydrogen.

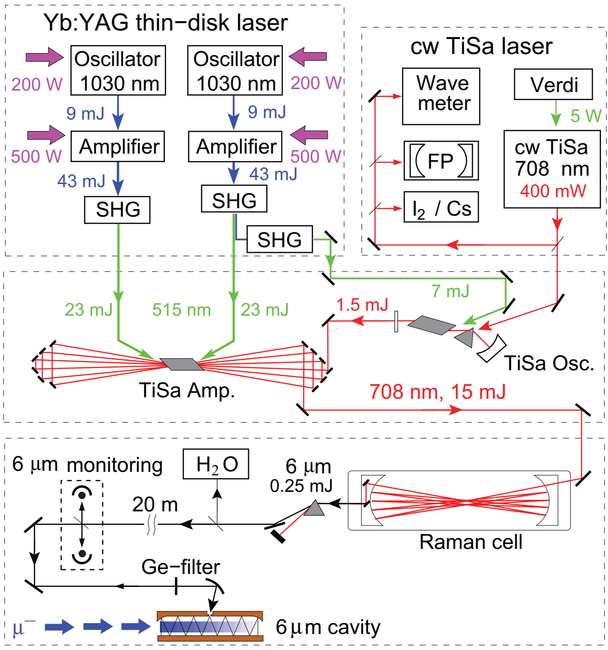

Figure 3. Laser system. A cw pumped Yb:YAG thin disk laser can deliver light within $\sim$ 200 ns of a randomly occurring trigger, given by a single detected muon. After frequency doubling (SHG) the green light is used to pump a pulsed MOPA TiSa laser system. The oscillator of the pulsed TiSa is injection-seeded with cw light from a frequency controlled TiSa ring laser. The TiSa pulses are converted to the required IR light at 6 μm via three sequential Raman Stokes shifts in a high-pressure (16 bar) H_2 Raman cell. Water vapour absorption spectroscopy is used to calibrate the IR light at 6 μm.

The classical Lamb shift (2S-2P energy difference) in electronic hydrogen H is dominated by the self energy (SE) of the electron, and the finite size effect contributes to the Lamb shift only at the $1 \cdot 10^{-4}$ level. In contrast, vacuum polarization (VP) dominates the Lamb shift in muonic hydrogen μp, and the finite size effect is as large as 2 % of the Lamb shift (see Fig. 1). This is the reason why we set out in 1998 to measure the $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ energy difference in μp by means of pulsed laser spectroscopy at $\lambda \approx 6\,\mu$m.

The energy difference $\Delta\widetilde{E}$ between the $2S_{1/2}^{F=1}$ and the $2P_{3/2}^{F=2}$ states in muonic hydrogen is the sum of radiative, recoil, and proton structure contributions, and the fine and hyperfine splittings for this particular transition. It is given by [2, 18–22]

$$\Delta\widetilde{E} = 209.9779\,(49) - 5.2262\,r_\mathrm{p}^2 + \;0.0347\,r_\mathrm{p}^3 \;\mathrm{meV} \qquad (3)$$

where $r_\mathrm{p} = \sqrt{< r_\mathrm{p}^2 >}$ is the rms charge radius of the proton, given in fm. The uncertainty of 0.0049 meV in $\Delta\widetilde{E}$ is dominated by the proton polarizability term [20] of 0.015(4) meV. A detailed derivation of Eq. (3) is given in the Supplementary Information of Ref [23].

3. Experiment

The details of the experiment have been given elsewhere [23–29]. In brief, single low-energy negative muons ($\sim$ 5 keV kinetic energy) from our novel beam line [26] (Fig. 2 (a)) at the Paul-Scherrer-Institute (PSI, Switzerland) enter a 5 T solenoid and are individually detected through secondary electrons emitted in 20 nm thin carbon foils (Fig. 2 (b)). The electrons are detected in plastic scintillators which are read out by photomultipliers ($PM_{1..3}$), a delayed coincidence of which provides the trigger signal for the laser [30, 31] (Fig. 3).

The muons are then stopped in a low-pressure hydrogen gas target filled with 1 mbar H_2 gas at room temperature. About 1 % of the muons form long-lived μp(2S) atoms with a lifetime of $\sim 1\,\mu$s at this gas pressure [32–35] (see Fig. 1(b)).

The pulsed laser system (Fig. 3) basically pulse-amplifies the cw light from a frequency controlled cw Ti:sapphire (TiSa) laser, creating 5 ns long pulses of 15 mJ energy at $\lambda \approx 708$ nm.

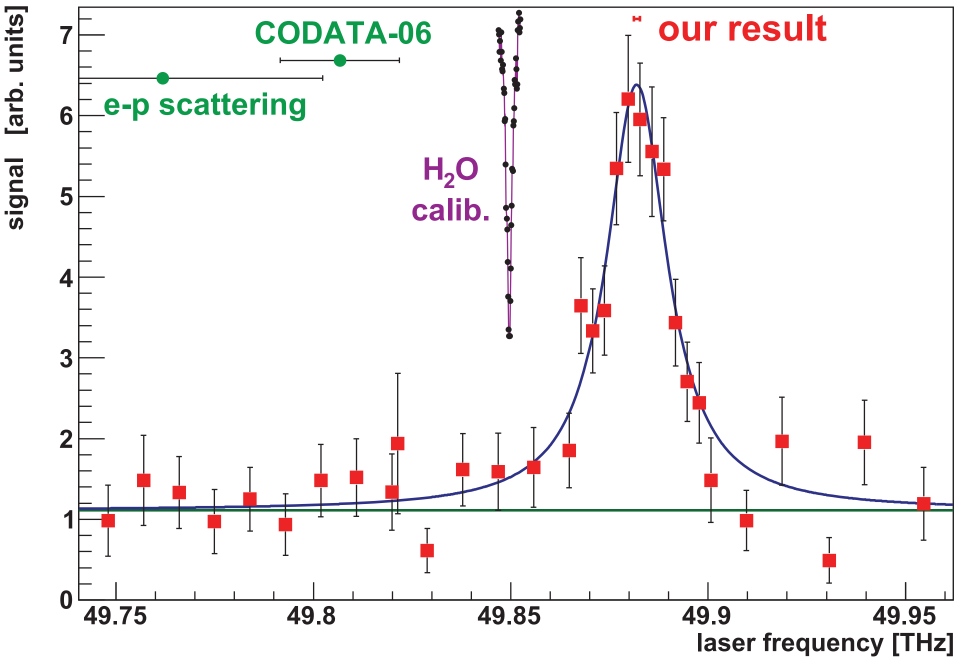

Figure 4. The measured resonance, a water absorption measurement used for calibration, and the predicted line positions using the proton rms charge radius from electron scattering [15, 16] and CODATA [10].

0.2 mJ pulses at $\lambda \approx 6\,\mu$m enter a multi-pass cavity [28] surrounding the muon stop volume. We detect x-rays using large-area avalanche photo diodes (LAPPDs). Our signal (Fig. 4) is the number of K_α x-rays detected at the time when the laser illuminates the muon stop volume (Fig. 1(c)), *versus* the laser frequency.

Tuning the cw TiSa by some frequency offset $\Delta\nu$ changes the IR light at $\lambda \approx 6\,\mu$m by the same $\Delta\nu$. Although the frequency of each laser in the step is well-known, we rely on well-known water vapour absorption lines [37, 38] for the final laser frequency calibration. The absolute water vapour line positions are known to 1 MHz [37, 38], but pulse to pulse instabilities of our laser system limit the frequency determination to 300 MHz.

3.1. Lamb shift in muonic hydrogen

The centre of the $2S_{1/2}^{F=1}$ to $2P_{3/2}^{F=2}$ transition in μp is at 49881.88(76) GHz [23]. The uncertainty of 0.76 GHz (15 ppm) contains 700 MHz statistical uncertainty from the free fit of a Lorentzian resonance line on top of a flat background, and the 300 MHz total systematic uncertainty which is exclusively due to our laser wavelength calibration procedure using H_2O vapour absorption.

Other systematic effects we have considered are Zeeman shift in the 5 T field (< 30 MHz), AC and DC Stark shifts (< 1 MHz), Doppler shift (< 1 MHz) and pressure shift (< 2 MHz). Our measured resonance position is not influenced by molecular effects, because the formed muonic molecules ppμ^+ are known to deexcite quickly [34, 39] and cannot contribute to our signal. Also, the width of our resonance line of 18.0(2.2) GHz agrees with the expected width of 20(1) GHz, whereas molecular lines would be wider.

The free fit gives $\chi^2 = 28.1$ for 28 degrees of freedom (dof). A fit of a flat line, assuming no resonance, gives $\chi^2 = 283$ for 31 dof, making this resonance line 16 σ significant. The frequency of the resonance centre corresponds to an energy of $\Delta\widetilde{E} = 206.2949(32)$ meV (see Eq. (3)).

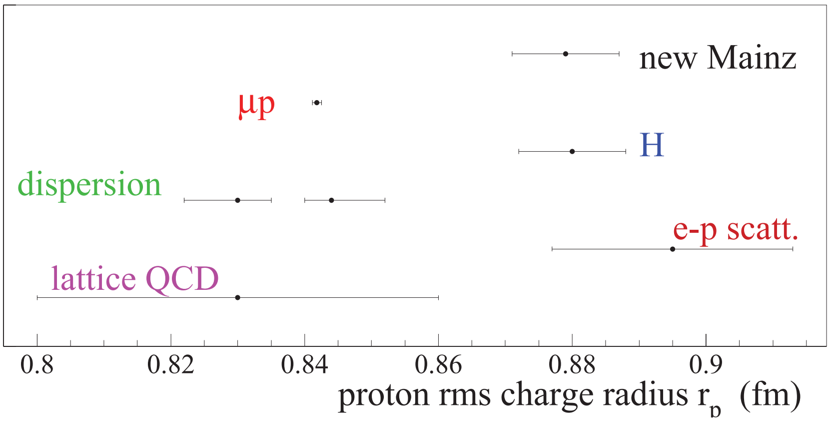

Figure 5. Proton charge radius. Our new value $r_{\rm p}$= 0.84184(67) fm from muonic hydrogen (μp) spectroscopy [23] is in strong disagreement with the values extracted from hydrogen spectroscopy ("H", Ref. [10], see Tab. XLV, adjustment 10), the world average from electron scattering ("e-p scatt.") [15, 16], and the new electron scattering value from Mainz ("new Mainz") [17]. It agrees, however, nicely with "Lattice QCD" from Ref. [40], and "dispersion" (two model-dependent results from Ref. [41]).

3.2. The rms charge radius of the proton

The rms charge radius of the proton we find is

$$r_{\rm p} = 0.84184(36)(56)\,\text{fm}. \tag{4}$$

The first uncertainty is due to the experimental uncertainty of 0.76 GHz (700 MHz statistical and 300 MHz systematic, added in quadrature), and the second uncertainty originates from the first term in Eq. (3). Theory, mainly the proton polarizability, gives the dominant contribution to our total relative uncertainty of 8×10^{-4}. Our experimental precision would allow us to deduce $r_{\rm p}$ two times better, with a relative uncertainty of 4×10^{-4}.

3.3. The proton radius puzzle

This new value of the proton radius $r_{\rm p}$=0.84184(67) fm is 10 times more precise, but 5.0 σ smaller, than the 2006 CODATA value $r_{\rm p}$ = 0.8768(69) fm [10], which is dominated by spectroscopy in regular hydrogen (H). Our new $r_{\rm p}$ is 26 times more accurate, but 3.1 σ smaller, than the previously accepted, hydrogen-independent value extracted from electron proton scattering [15, 16] of $r_{\rm p}$ = 0.895(18) fm. Furthermore, new data from the Mainz MAMI electron accelerator [17] give $r_{\rm p}$ = 0.879(8) fm, in agreement with the hydrogen result as given by "adjustment 10" in Tab. XLV of Ref. [10].

Recent lattice QCD calculations [40], on the other hand, obtain $r_{\rm p}$ = 0.83(3) fm, favouring a lower radius than the one from H or electron scattering. Also, dispersion analysis of the nucleon form factors has recently [41] also produced smaller values of $r_{\rm p}$: Their "SC approach" gives $r_{\rm p}$= $0.844^{+0.008}_{-0.004}$ fm, in agreement with our accurate value, whereas their "explicit pQCD approach" gives an even smaller value of $r_{\rm p}$= $0.830^{+0.005}_{-0.008}$ fm. The situation is summarized in Fig. 5.

3.4. A new value of the Rydberg constant

Assuming for now the correctness of the QED calculations in hydrogen [2, 3] and μp [18–22], we can use our precise value of $r_{\rm p}$ and the most accurately measured transition frequency in hydrogen (1S-2S) [6, 7] to deduce a new value of the Rydberg constant.

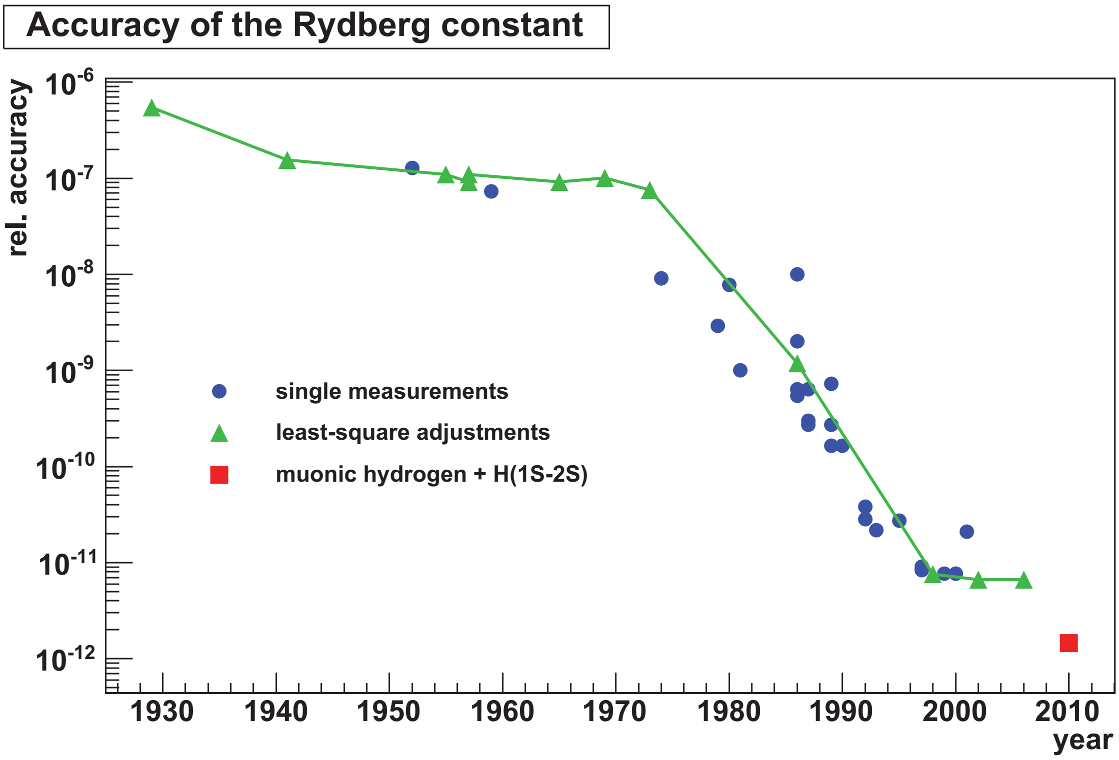

Figure 6. The Rydberg constant is a corner stone of the CODATA adjustment of fundamental constants [10]. Its accuracy is now 1.5 parts in 10^{12}, using our r_p and the super-accurate H(1S-2S) transition [6, 7].

This is $-110\,\mathrm{kHz/c}$ or $4.9\,\sigma$ away from the CODATA value [10], but 4.6 times more precise [1.5 parts in 10^{12}]. The new determination continues the astonishing improvement in the accuracy of the most accurately determined fundamental physical constant (Fig. 6).

3.5. The charge radius of the deuteron

The *difference* of the squared charge radii of the proton and the deuteron, $r_\mathrm{d}^2 - r_\mathrm{p}^2 = 3.82007(65)\,\mathrm{fm}^2$, is accurately determined from the precise measurement of the isotope shift of the 1S-2S transition in regular hydrogen and deuterium atoms [42]. This, together with our value of r_p , gives for the rms charge radius of the deuteron

$$r_\mathrm{d} = 2.12809(31)\,\mathrm{fm}. \tag{6}$$

Figure 7 compares this to recent results. The CODATA value of $r_\mathrm{d} = 2.1394(28)\,\mathrm{fm}$ is 4σ away. This can be understood because in the CODATA adjustment r_d is rigidly tied to r_p via the very precise isotope shift of the 1S-2S transition in regular H and D atoms. The proton charge radius deduced from H and μp disagree by $5\,\sigma$ for unknown reasons; hence the deuteron radius must also disagree by a large amount.

However, adjustment 11 of the 2006 CODATA adjustment (see Ref. [10] Tab. XLV), which uses only the deuterium data (scattering and spectroscopy), but ignores both electron scattering and spectroscopy on hydrogen, suggests a smaller value of r_d, in accord with our value. This is due to the fact that the value of r_d from electron-deuteron scattering [43] agrees with ours.

Neutron-proton scattering [44] gives a value which agrees both with the electron scattering

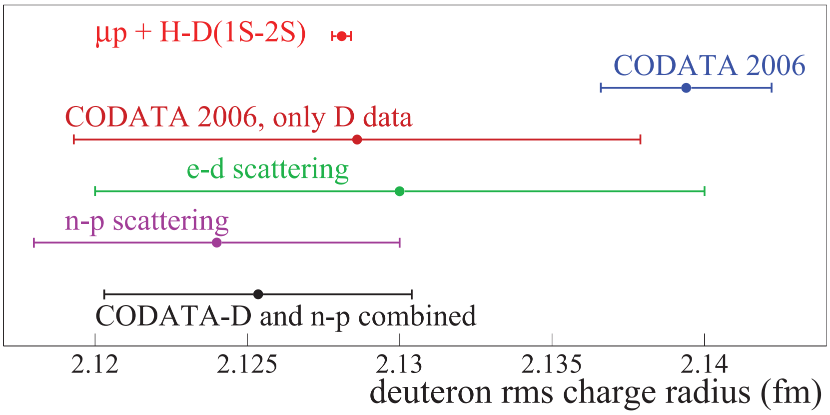

Figure 7. Our deuteron charge radius $r_{\mathrm{d}} = 2.12809(31)$ fm deduced from our r_{p} together with the H-D (1S-2S) isotope shift [42] disagrees with the CODATA value of r_{d}, but agrees with the CODATA 2006 adjustment 11 which uses only deuterium data (Ref. [10], see Tab. XLV, adj. 11), the value from electron scattering [43], and the value from neutron-proton scattering [44].

The average of the independent values "CODATA, D only" [10] and neutron scattering [44] is $r_{\mathrm{d}}= 2.1254(50)$ fm, 2.4σ away from the final CODATA value $r_{\mathrm{d}} = 2.1394(28)$ fm, but in good agreement with our result.

4. Conclusions and Outlook

The world's most precise value of the rms proton charge radius $r_{\mathrm{p}} = 0.84184(67)$ fm that we have obtained from laser spectroscopy of the Lamb shift in muonic hydrogen μp has created a puzzle. The disagreement with the previous values from hydrogen spectroscopy and electron scattering is stunning.

Using this new value of r_{p} and the accurately measured hydrogen-deuterium isotope shift [42] we obtain $r_{\mathrm{d}} = 2.12809(31)$ fm. This value agrees with several *hydrogen*-independent results.

Our new project, the measurement of the Lamb shift in muonic helium ions, will hopefully contribute to the solution of the "proton size puzzle".

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