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Research Article

Fault Detection of Networked Control Systems Based on Sliding Mode Observer

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This paper is concerned with the network-based fault detection problem for a class of nonlinear discrete-time networked control systems with multiple communication delays and bounded disturbances. First, a sliding mode based nonlinear discrete observer is proposed. Then the sufficient conditions of sliding motion asymptotical stability are derived by means of the linear matrix inequality (LMI) approach on a designed surface. Then a discrete-time sliding-mode fault observer is designed that is capable of guaranteeing the discrete-time sliding-mode reaching condition of the specified sliding surface. Finally, an illustrative example is provided to show the usefulness and effectiveness of the proposed design method.

1. Introduction

Over the past few decades, the sliding mode control (SMC) (also known as variable structure control) problem originated in [1] has been extensively studied and widely applied, because of its advantage of strong robustness against model uncertainties, parameter variations, and external disturbances [2–11]. Since SMC possesses the ability to eliminate or compensate the system uncertainties and external disturbances, sliding mode observers techniques have been developed to deal with the state estimation problems for linear or nonlinear uncertain systems. For example, sliding mode observers techniques have been applied to deal with the fault detection problems [12–14].

In practical industrial process, various malfunction or imperfect behavior always occurs in normal operations resulted from the unexpected variations in external surroundings and sudden changes in signals and so forth. Such kind of phenomenons are categorized as sensor (actuator) faults. Faults in the measurement sensors, control actuators, or process equipment can result in serious degradation of the system performance and may even lead to a complete breakdown of the process operation. Due to this fact, the subject of fault detection (FD) has become a focus of increasing research investigation over the past few decades in both theoretical research and practical industrial areas [5, 15]. Fault diagnosis is aimed at detecting, isolating, and estimating the faults. Generally speaking, a fault detection process consists of constructing a residual signal and computing a residual evaluation function which can then be compared with a predefined threshold. When the residual exceeds the threshold, the fault is detected, and an alarm of fault is generated. Recently, the model-based approaches to fault detection problems have been widely adopted for dynamic systems. The main idea of these approaches is to introduce a performance index and then convert the fault detection problem into an associated optimization problem. Accordingly, a variety of important results have been reported in the literature. For example, the fault detection problems have been addressed in [16] for linear time-varying systems, in [17] for sampled-data systems, and in [18–21] for networked control systems (NCSs). However, it is less considered for the fault detection of networked control systems by slide mode approach.

On another active research front, rapid development of microelectronic, information, and communication technologies enhanced networking of intelligent sensors, actuators, controllers, and microprocessors and accelerated the application of NCSs in major industrial sectors. Therefore NCSs receive a great deal of attention [22, 23]. Integrating networks into automatic control systems can significantly increase the automation degree to meet the demands for high productivity and product qualit, and allow a flexible system configuration with less wiring and an easy maintenance. Remarkably different from classical control systems, the performance and behavior of the NCSs considerably depend on the technical characteristics of the network. In addition, accompanied with the growth of the integration and automation degree the overall failure rate will significantly increase. For example, in [24], the NCSs with stochastic mixed time delays and successive packet dropouts are represented by a T-S fuzzy model, based on such mode, and a fuzzy-observer-based approach for fault detection is developed. In [25], the fault detection problem for NCSs have been studied where the communication delays are described as a random Markov jump process. It is worth mentioning that most of the reported results have been concerned with the discrete time delays.

A thorough literature review on the fault detection problems for NCSs has revealed that, up to now, little attention has been paid to the study of fault detection for nonlinear NCSs with both communication delays and bounded disturbances by means of sliding mode observer. Summarizing the earlier discussion, in this paper, we are motivated to study the fault detection problem for a class of discrete-time nonlinear systems involving multiple communication delays and bounded disturbances input. In this paper, the fault detection problem is firstly converted into a sliding mode observer design problem. Sufficient conditions are established for the existence of the desired sliding-mode fault observer in terms of LMIs, and then, the sliding-mode reaching condition for the system is established. A numerical simulation example is provided to show the usefulness and effectiveness of the proposed design method.

The main contributions of this paper can be highlighted as follows: (1) the multiple communication delays and bounded disturbances are introduced for discrete-time networked control systems to reflect more realistic environment, (2) Slide mode observer approach is utilized to deal with the fault detection, and (3) to illustrate the applicability of the proposed results that the time of fault detect time is discussed.

Notation. The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. I denotes the identity matrix of compatible dimension. The notation $X \geq Y$ (resp., $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (resp., positive definite). A^T represents the transpose of A. $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. $\|x\|$ describes the Euclidean norm of a vector x. $\text{diag}\{F_1, F_2, \ldots\}$ stands for a block-diagonal matrix whose diagonal blocks are given by $F_1, F_2, \ldots$. The symbol $*$ in a matrix means that the corresponding term of the matrix can be obtained by symmetric property.

2. Problem Formulation

Consider a discrete-time nonlinear systems with multiple communication delays and bounded disturbances which can be represented by the following dynamic model:

$$\begin{aligned} x(k+1) &= Ax(k) + A_d\sum_{i=1}^{n} M_i x(k-\tau_i(k)) \\ &\quad + Bu(k) + g(x,k) + Dw(k) + Gf(k) \\ y(k) &= Cx(k) \\ x(k) &= \varphi(k), \quad k = -\overline{d}, -\overline{d}+1, \ldots, 0, \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ represents the state vector; $y(k) \in \mathbb{R}^p$ is the measured output; $u(k) \in \mathbb{R}^m$ is the control input; $f(k) \in \mathbb{R}^r$ is the fault of the system; $\omega(k) \in \mathbb{R}^q$ is a continuous vector function which represents the bounded exogenous disturbance input for the system; and $\varphi(k)$ is the given initial conditions. A, A_d, B, D, G, and C are known constant real-valued matrices with appropriate dimensions. $g(x,k)$ is a Lipschitz nonlinearity with a known Lipschitz constant γ [26]; that is,

$$\|g(x,k) - g(\hat{x},k)\| \leq \gamma \|x - \hat{x}\|, \tag{2}$$

where $\hat{x} \in \mathbb{R}^n$ is the observer state vector, and M_i ($i = 1, 2, \ldots, n$) has the form of

$$M_i = \text{diag}\left\{\underbrace{0, 0, \ldots, 0}_{i-1}, 1, 0, \ldots, 0\right\} \in \mathbb{R}^{n\times n} \quad i = 1, 2, \ldots, n. \tag{3}$$

Assumption 1. The pair (A, C) is observable, and the C is of full row rank; that is, all the states of system are available at every instant.

Assumption 2. The variables $\tau_i(k)$ ($i = 1, 2, \ldots, n$) represent the n different communication delays, which are assumed to be time-varying and satisfy $0 \leq \tau_i(k) \leq \overline{d}$ ($i = 1, 2, \ldots, n$), where $\overline{d}$ is constant positive scalars representing the upper bound on the communication delay. Since all the system state variables have time scale, we can obtain the $\tau_i(k)$ at every instant.

Assumption 3. The exogenous disturbance input is piecewise continuous bounded functions; that is, there exist known positive constants $\overline{w}$ such that

$$\|w(k)\| \leq \overline{w}. \tag{4}$$

Remark 4. In Assumption 2, although the n time-varying communication delays $\tau_i(k)$ are assumed to have the same upper bounds, they could actually take different values at the same sampling instant k. Such a description is suitable for networked control systems that have different communication delays when the signals are transferred via different channels at the same sample time k, which is very often the case in many practical applications.

In the next section, following the developments stated in [27], a sliding-mode observer is introduced to reconstruct the state vector and identify the system faults.

3. Sliding Mode Observer

In this section, a sliding mode observer will be designed. Considering the form of the model (1), the sliding mode observer can be designed:

$$\begin{aligned}\hat{x}(k+1) &= A\hat{x}(k) + A_d\sum_{i=1}^{n} M_i\hat{x}(k-\tau_i(k)) \\ &\quad + Bu(k) + g(\hat{x},k) + L(y(k)-\hat{y}(k)) + v(k) \\ \hat{y}(k) &= C\hat{x}(k),\end{aligned} \tag{5}$$

where $\hat{x}_k \in \mathbb{R}^n$ is the observer state vector, $\hat{y}_k \in \mathbb{R}^p$ is the output of the observer. $L \in \mathbb{R}^{n\times p}$ is the observer gain to be determined. The function $v(k)$ is the nonlinear input of observer to be determined.

Let $e_x(k) = x(k)-\hat{x}(k)$ and $e_y(k) = y(k)-\hat{y}(k)$. Then from (1) and (6), we have the fault detection dynamics governed by the following system:

$$\begin{aligned}e_x(k+1) &= (A-LC)e_x(k) + A_d\sum_{i=1}^{n} M_i e_x(k-\tau_i(k)) \\ &\quad + [g(x,k)-g(\hat{x},k)] + Dw(k) + Gf(k) - v(k) \\ e_y(k) &= Ce_x(k).\end{aligned} \tag{6}$$

Define the residual of the system $r(k)$ and the sliding mode manifold $s(k)$ as

$$r(k) = e_y(k), \qquad s(k) = e_y(k). \tag{7}$$

Our aim in this paper is to design a sliding mode observer that makes $r(k)$ zero when there is no fault. In this paper, we adopt a residual evaluation stage, including an evaluation function $J(k)$ and a threshold J_{th} of the following form:

$$J(k) = \left\{\sum_{k=s-\zeta}^{k=s} r^T(k)r(k)\right\}^{1/2} \qquad J_{\text{th}} = \sup_{f(k)=0} \mathbb{E}\{J(k)\}, \tag{8}$$

where ζ denotes the length of the finite evaluating time horizon. Based on (8), the occurrence of faults can be detected by comparing $J(k)$ with J_{th} according to the following rule:

$$\begin{aligned}J(k) > J_{\text{th}} &\Longrightarrow \text{with faults} \Longrightarrow \text{alarm} \\ J(k) \le J_{\text{th}} &\Longrightarrow \text{no faults}.\end{aligned} \tag{9}$$

Now, for that the system motion can get into the sliding surface in finite time and be stable, we aim to design a sliding mode observer such that the observer error system satisfies the following requirements (Q1) and (Q2), simultaneously:

(Q1) the error system (6) is asymptotically stable when $s(k+1) = s(k) = 0$;

(Q2) sliding mode manifold satisfies $\|s(k+1)\| < \|s(k)\|$.

The design problem stated above will be referred to as the sliding mode observer problem with finding the gain matrix L and the observer's nonlinear input $v(k)$.

4. Main Results

First of all, we introduce the following lemmas which will be used in this paper.

Lemma 5. *For any real vectors a, b and matrix $P > 0$ of compatible dimensions,*

$$a^T b + b^T a \le a^T P a + b^T P^{-1} b. \tag{10}$$

Lemma 6 (Schur complement). *Given constant matrices S_1, S_2, and S_3, where $S_1 = S_1^T$ and $0 < S_2 = S_2^T$, then $S_1 + S_3^T S_2^{-1} S_3 < 0$ if and only if*

$$\begin{bmatrix} S_1 & S_3^T \\ S_3 & -S_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -S_2 & S_3 \\ S_3^T & S_1 \end{bmatrix} < 0. \tag{11}$$

Theorem 7. *Consider that the system error model (6) associated with the surface (7) is asymptotically stable if there exist matrices $P > 0$, L satisfying*

$$\begin{bmatrix} (2\gamma^2-1)P & (A-LC)^T \\ * & -\frac{1}{2}P \end{bmatrix} < 0 \tag{12}$$

which holds and $v(k)$ is designed as

$$v(k) = v_1(k) + v_2(k) \tag{13}$$

with

$$\begin{aligned}v_1(k) &= l\frac{y(k)-\hat{y}(k)}{\|y(k)-\hat{y}(k)\|} \quad \text{if } y(k)-\hat{y}(k) \ne 0, \\ v_2(k) &= A_d\sum_{i=1}^{n} M_i e_x(k-\tau_i(k)),\end{aligned} \tag{14}$$

where $l = -\|D\|\overline{w}$ and γ is the Lipschitz constant for $g(x,k)$.

Proof. Choose a Lyapunov functional candidate as

$$V(k) = e_x^T(k)Pe_x(k) \tag{15}$$

with $P > 0$ being matrices to be determined. Then, along the trajectory of system (6), we have

$$\begin{aligned}\Delta V(k+1) &= e_x^T(k+1)Pe_x(k+1) - e_x^T(k)Pe_x(k)\\ &= e_x^T(k)\left[(A-LC)^T P(A-LC) - P\right]e_x(k)\\ &\quad + [g(x_k,k) - g(\hat{x}_k,k)]^T\\ &\quad \times P[g(x_k,k) - g(\hat{x}_k,k)]\\ &\quad + \sum_{i=1}^{n} e_x^T(k-\tau_i(k))M_i^T A_d^T P A_d\\ &\quad \times \sum_{i=1}^{n} M_i e_x(k-\tau_i(k))\\ &\quad + [Dw(k) - v(k)]^T P[Dw(k) - v(k)]\\ &\quad + 2e_x^T(k)(A-LC)^T P[g(x_k,k) - g(\hat{x}_k,k)]\\ &\quad + 2e_x^T(k)(A-LC)^T P A_d \sum_{i=1}^{n} M_i e_x(k-\tau_i(k))\\ &\quad + 2e_x^T(k)(A-LC)^T P[Dw(k) - v(k)]\\ &\quad + 2[g(x_k,k) - g(\hat{x}_k,k)]^T P A_d\\ &\quad \times \sum_{i=1}^{n} M_i e_x(k-\tau_i(k))\\ &\quad + 2[g(x_k,k) - g(\hat{x}_k,k)]^T P[Dw(k) - v(k)]\\ &\quad + 2\sum_{i=1}^{n} e_x^T(k-\tau_i(k))M_i^T A_d^T P[Dw(k) - v(k)].\end{aligned} \tag{16}$$

From (2), it is derived that

$$\begin{aligned}&[g(x_k,k) - g(\hat{x}_k,k)]^T P[g(x_k,k) - g(\hat{x}_k,k)]\\ &\quad \le \gamma^2 e_x^T(k)Pe_x(k).\end{aligned} \tag{17}$$

Letting $v(k) = v_1(k) + v_2(k)$, then

$$\begin{aligned}&[Dw(k) - v(k)]^T P[Dw(k) - v(k)]\\ &\quad = [Dw(k) - v_1(k) - v_2(k)]^T P[Dw(k) - v_1(k) - v_2(k)]\\ &\quad = [Dw(k) - v_1(k)]^T P[Dw(k) - v_1(k)]\\ &\qquad - 2[Dw(k) - v_1(k)]^T P v_2(k) + v_2(k)^T P v_2(k)\\ &\quad \le [\|Dw(k)\| + \|v_1(k)\|]^2 \|P\|\\ &\qquad + 2[\|Dw(k)\| + \|v_1(k)\|]\|P\|\|v_2(k)\|\\ &\qquad + v_2(k)^T P v_2(k)\\ &\quad \le [\|D\|\overline{w} + l]^2\|P\| + 2[\|D\|\overline{w} + l]\|P\|\|v_2(k)\|\\ &\qquad + v_2(k)^T P v_2(k)\\ &2e_x^T(k)(A-LC)^T P[Dw(k) - v(k)]\\ &\quad = 2e_x^T(k)(A-LC)^T P[Dw(k) - v_1(k)]\\ &\qquad - 2e_x^T(k)(A-LC)^T P v_2(k)\\ &\quad \le 2\|e_x^T(k)\|\|(A-LC)^T P\|[\|Dw(k)\| + \|v_1(k)\|]\\ &\qquad - 2e_x^T(k)(A-LC)^T P v_2(k)\\ &\quad \le 2\|e_x^T(k)\|\|(A-LC)^T P\|[\|D\|\overline{w} + l]\\ &\qquad - 2e_x^T(k)(A-LC)^T P v_2(k)\\ &2[g(x_k,k) - g(\hat{x}_k,k)]^T P[Dw(k) - v(k)]\\ &\quad = 2[g(x_k,k) - g(\hat{x}_k,k)]^T P[Dw(k) - v_1(k)]\\ &\qquad - 2[g(x_k,k) - g(\hat{x}_k,k)]^T P v_2(k)\\ &\quad \le 2\|g(x_k,k) - g(\hat{x}_k,k)\|\|P\|[\|D\|\overline{w} + l]\\ &\qquad - 2[g(x_k,k) - g(\hat{x}_k,k)]^T P v_2(k)\\ &2\sum_{i=1}^{n} e_x^T(k-\tau_i(k))M_i^T A_d^T P[Dw(k) - v(k)]\\ &\quad = 2\sum_{i=1}^{n} e_x^T(k-\tau_i(k))M_i^T A_d^T P[Dw(k) - v_1(k)]\\ &\qquad - 2\sum_{i=1}^{n} e_x^T(k-\tau_i(k))M_i^T A_d^T P v_2(k)\\ &\quad \le 2\left\|\sum_{i=1}^{n} e_x^T(k-\tau_i(k))M_i^T A_d\right\|\|P\|[\|D\|\overline{w} + l]\\ &\qquad - 2\sum_{i=1}^{n} e_x^T(k-\tau_i(k))M_i^T A_d^T P v_2(k).\end{aligned} \tag{18}$$

By using Lemma 5, we have

$$\begin{aligned}&2e_x^T(k)(A-LC)^T P[g(x_k,k) - g(\hat{x}_k,k)]\\ &\quad \le e_x^T(k)(A-LC)^T P(A-LC)e_x(k)\\ &\qquad + [g(x_k,k) - g(\hat{x}_k,k)]^T P[g(x_k,k) - g(\hat{x}_k,k)]\\ &\quad \le e_x^T(k)(A-LC)^T P(A-LC)e_x(k) + \gamma^2 e_x^T(k)Pe_x(k).\end{aligned} \tag{19}$$

The combination of (13)-(14) and (16)–(19) results in

$$\begin{aligned}\Delta V(k+1) &\le e_x^T(k)\left[2(A-LC)^T P(A-LC) + \left(2\gamma^2 - 1\right)P\right]\\ &\quad \times e_x(k) < 0.\end{aligned} \tag{20}$$

By the Schur complement, it is easy to obtain

$$\Delta V(k+1) \leq e_x^T(k) \begin{bmatrix} \left(2\gamma^2 - 1\right)P & (A-LC)^T \\ * & -\frac{1}{2}P \end{bmatrix} e_x(k) < 0 \quad (21)$$

which is equivalent to the inequality in (12), and therefore, the proof of Theorem 7 is complete. □

Having established the analysis results, we are now in a position to deal with the sliding motion reachability problem.

Theorem 8. *Considering the system* (6)*, assume $v(k)$ is chosen to satisfy* (13)*. If there exists a matrix L and $\varepsilon^2 < 1$ satisfying the condition*

$$\begin{bmatrix} \left(2\gamma^2 - \varepsilon^2\right)C^TC & (A-LC)^T \\ * & -\frac{1}{2}C^TC \end{bmatrix} < 0 \quad (22)$$

then the system motion will get into the sliding surface in finite time.

Proof. If sliding mode manifold satisfies $\|s(k+1)\| < \|s(k)\|$, that is,

$$\begin{aligned} & s(k+1)^T s(k+1) - \varepsilon^2 s(k)^T s(k) \\ &\quad = e_y^T(k+1)\, e_y(k+1) - \varepsilon^2 e_y^T(k)\, P e_y(k) \\ &\quad = e_x^T(k+1)\, C^T C e_x(k+1) - \varepsilon^2 e_x^T(k)\, C^T C e_x(k) \\ &\quad < 0, \end{aligned} \quad (23)$$

where $\varepsilon^2 < 1$, then substituting (6) into (23), we can obtain

$$\begin{aligned} & s(k+1)^T s(k+1) - \varepsilon^2 s(k)^T s(k) \\ &\quad = e_x^T(k)\left[(A-LC)^T C^T C(A-LC) - \varepsilon^2 C^T C\right] e_x(k) \\ &\qquad + \left[g(x_k,k) - g(\hat{x}_k,k)\right]^T C^T C\left[g(x_k,k) - g(\hat{x}_k,k)\right] \\ &\qquad + \sum_{i=1}^{n} e_x^T(k-\tau_i(k))\, M_i^T A_d^T C^T C A_d \sum_{i=1}^{n} M_i e_x(k-\tau_i(k)) \\ &\qquad + \left[Dw(k) - v(k)\right]^T C^T C\left[Dw(k) - v(k)\right] \\ &\qquad + 2e_x^T(k)(A-LC)^T C^T C\left[g(x_k,k) - g(\hat{x}_k,k)\right] \\ &\qquad + 2e_x^T(k)(A-LC)^T C^T C A_d \sum_{i=1}^{n} M_i e_x(k-\tau_i(k)) \\ &\qquad + 2e_x^T(k)(A-LC)^T C^T C\left[Dw(k) - v(k)\right] \\ &\qquad + 2\left[g(x_k,k) - g(\hat{x}_k,k)\right]^T C^T C A_d \sum_{i=1}^{n} M_i e_x(k-\tau_i(k)) \\ &\qquad + 2\left[g(x_k,k) - g(\hat{x}_k,k)\right]^T C^T C\left[Dw(k) - v(k)\right] \\ &\qquad + 2\sum_{i=1}^{n} e_x^T(k-\tau_i(k))\, M_i^T A_d^T C^T C\left[Dw(k) - v(k)\right]. \end{aligned} \quad (24)$$

From (2), it is derived that

$$\begin{aligned} & \left[g(x_k,k) - g(\hat{x}_k,k)\right]^T C^T C\left[g(x_k,k) - g(\hat{x}_k,k)\right] \\ &\quad \leq \gamma^2 e_x^T(k)\, C^T C e_x(k). \end{aligned} \quad (25)$$

Letting $v(k) = v_1(k) + v_2(k)$, then

$$\begin{aligned} & \left[Dw(k) - v(k)\right]^T C^T C\left[Dw(k) - v(k)\right] \\ &\quad = \left[Dw(k) - v_1(k) - v_2(k)\right]^T C^T \\ &\qquad \times C\left[Dw(k) - v_1(k) - v_2(k)\right] \\ &\quad = \left[Dw(k) - v_1(k)\right]^T C^T C\left[Dw(k) - v_1(k)\right] \\ &\qquad - 2\left[Dw(k) - v_1(k)\right]^T C^T C v_2(k) + v_2(k)^T C^T C v_2(k) \\ &\quad \leq \left[\|Dw(k)\| + \|v_1(k)\|\right]^2 \left\|C^T C\right\| \\ &\qquad + 2\left[\|Dw(k)\| + \|v_1(k)\|\right] \left\|C^T C\right\| \|v_2(k)\| \\ &\qquad + v_2(k)^T P v_2(k) \\ &\quad \leq \left[\|D\|\,\overline{w} + l\right]^2 \left\|C^T C\right\| + 2\left[\|D\|\,\overline{w} + l\right] \left\|C^T C\right\| \|v_2(k)\| \\ &\qquad + v_2(k)^T C^T C v_2(k). \end{aligned} \quad (26)$$

Substituting $l = -\|D\|\overline{w}$ into (26) we have

$$\left[Dw(k) - v(k)\right]^T C^T C\left[Dw(k) - v(k)\right] \leq v_2(k)^T C^T C v_2(k). \quad (27)$$

In a similar way,

$$\begin{aligned} & 2e_x^T(k)(A-LC)^T C^T C\left[Dw(k) - v(k)\right] \\ &\quad = 2e_x^T(k)(A-LC)^T C^T C\left[Dw(k) - v_1(k)\right] \\ &\qquad - 2e_x^T(k)(A-LC)^T C^T C v_2(k) \\ &\quad \leq 2\left\|e_x^T(k)\right\| \left\|(A-LC)^T C^T C\right\| \left[\|Dw(k)\| + \|v_1(k)\|\right] \\ &\qquad - 2e_x^T(k)(A-LC)^T C^T C v_2(k) \\ &\quad \leq 2\left\|e_x^T(k)\right\| \left\|(A-LC)^T C^T C\right\| \left[\|D\|\,\overline{w} + l\right] \\ &\qquad - 2e_x^T(k)(A-LC)^T C^T C v_2(k) \\ &\quad \leq -2e_x^T(k)(A-LC)^T C^T C v_2(k) \end{aligned}$$

$$\begin{aligned}
&2[g(x_k,k)-g(\hat{x}_k,k)]^T C^T C[Dw(k)-v(k)]\\
&\quad=2[g(x_k,k)-g(\hat{x}_k,k)]^T C^T C[Dw(k)-v_1(k)]\\
&\qquad-2[g(x_k,k)-g(\hat{x}_k,k)]^T C^T C v_2(k)\\
&\quad\le 2\|g(x_k,k)-g(\hat{x}_k,k)\|\left\|C^T C\right\|[\|D\|\overline{w}+l]\\
&\qquad-2[g(x_k,k)-g(\hat{x}_k,k)]^T C^T C v_2(k)\\
&\quad\le -2[g(x_k,k)-g(\hat{x}_k,k)]^T C^T C v_2(k)\\
&2\sum_{i=1}^{n} e_x^T(k-\tau_i(k)) M_i^T A_d^T C^T C[Dw(k)-v(k)]\\
&\quad=2\sum_{i=1}^{n} e_x^T(k-\tau_i(k)) M_i^T A_d^T C^T C[Dw(k)-v_1(k)]\\
&\qquad-2\sum_{i=1}^{n} e_x^T(k-\tau_i(k)) M_i^T A_d^T C^T C v_2(k)\\
&\quad\le 2\left\|\sum_{i=1}^{n} e_x^T(k-\tau_i(k)) M_i^T A_d\right\|\left\|C^T C\right\|[\|D\|\overline{w}+l]\\
&\qquad-2\sum_{i=1}^{n} e_x^T(k-\tau_i(k)) M_i^T A_d^T C^T C v_2(k)\\
&\quad\le -2\sum_{i=1}^{n} e_x^T(k-\tau_i(k)) M_i^T A_d^T C^T C v_2(k).
\end{aligned}\tag{28}$$

By using Lemma 5, we have

$$\begin{aligned}
&2e_x^T(k)(A-LC)^T C^T C[g(x_k,k)-g(\hat{x}_k,k)]\\
&\quad\le e_x^T(k)(A-LC)^T C^T C(A-LC)e_x(k)\\
&\qquad+[g(x_k,k)-g(\hat{x}_k,k)]^T\\
&\qquad\times C^T C[g(x_k,k)-g(\hat{x}_k,k)]\\
&\quad\le e_x^T(k)(A-LC)^T C^T C(A-LC)e_x(k)\\
&\qquad+\gamma^2 e_x^T(k) C^T C e_x(k).
\end{aligned}\tag{29}$$

Therefore

$$\begin{aligned}
&s(k+1)^T s(k+1)-\varepsilon^2 s(k)^T s(k)\\
&\quad\le e_x^T(k)\left[(A-LC)^T C^T C(A-LC)-\varepsilon^2 C^T C\right]e_x(k)\\
&\qquad+\gamma^2 e_x^T(k) C^T C e_x(k)\\
&\qquad+\sum_{i=1}^{n} e_x^T(k-\tau_i(k)) M_i^T A_d^T C^T C A_d\sum_{i=1}^{n} M_i e_x(k-\tau_i(k))\\
&\qquad+v_2(k)^T C^T C v_2(k)\\
&\qquad+e_x^T(k)(A-LC)^T C^T C(A-LC)e_x(k)\\
&\qquad+\gamma^2 e_x^T(k) C^T C e_x(k)\\
&\qquad+2e_x^T(k)(A-LC)^T C^T C A_d\sum_{i=1}^{n} M_i e_x(k-\tau_i(k))\\
&\qquad-2e_x^T(k)(A-LC)^T C^T C v_2(k)\\
&\qquad+2[g(x_k,k)-g(\hat{x}_k,k)]^T C^T C A_d\sum_{i=1}^{n} M_i e_x(k-\tau_i(k))\\
&\qquad-2[g(x_k,k)-g(\hat{x}_k,k)]^T C^T C v_2(k)\\
&\qquad-2\sum_{i=1}^{n} e_x^T(k-\tau_i(k)) M_i^T A_d^T C^T C v_2(k).
\end{aligned}\tag{30}$$

Since $v_2(k)=A_d\sum_{i=1}^{n} M_i e_x(k-\tau_i(k))$, substituting $v_2(k)$ into (30) we have

$$\begin{aligned}
&s(k+1)^T s(k+1)-\varepsilon^2 s(k)^T s(k)\\
&\quad\le e_x^T(k)\left[2(A-LC)^T C^T C(A-LC)+\left(2\gamma^2-\varepsilon^2\right)C^T C\right]\\
&\qquad\times e_x(k)<0.
\end{aligned}\tag{31}$$

By the Schur complement, it is easy to obtain

$$\begin{aligned}
&s(k+1)^T s(k+1)-\varepsilon^2 s(k)^T s(k)\\
&\quad\le e_x^T(k)\begin{bmatrix}\left(2\gamma^2-\varepsilon^2\right)C^T C & (A-LC)^T\\ * & -\frac{1}{2}C^T C\end{bmatrix}e_x(k)<0
\end{aligned}\tag{32}$$

which is equivalent to the inequality in (22), and therefore, the proof of Theorem 8 is complete. □

Remark 9. It is able to measure all the system states directly under normal circumstances, so we can use the above method to solve this problem.

Remark 10. According to Theorems 7 and 8, we can get the sliding mode observer unknown gain matrix L and nonlinear output $v(k)=-\|D\|\overline{w}((y(k)-\hat{y}(k))/\|y(k)-\hat{y}(k)\|)+A_d\sum_{i=1}^{n} M_i e_x(k-\tau_i(k))$.

5. Numerical Example

In this section, we present an illustrative example to demonstrate the effectiveness of the proposed algorithm. Consider the following nonlinear networked system with communication delays:

$$\begin{aligned}
x(k+1)&=Ax(k)+A_d\sum_{i=1}^{3} M_i x(k-\tau_i(k))+Bu(k)\\
&\quad+g(x,k)+Dw(k)+Gf(k)\\
y(k)&=Cx(k)\\
x(k)&=\varphi(k)=0,\quad k=-\overline{d},-\overline{d}+1,\ldots,0.
\end{aligned}\tag{33}$$

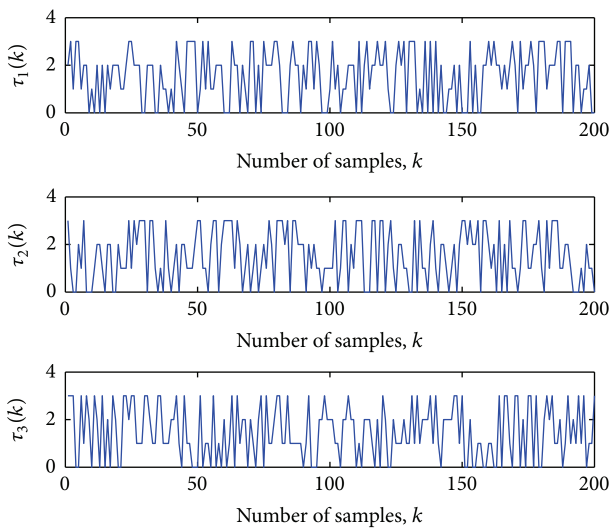

Figure 1: The time-varying communication delays $\tau(k)$.

The model parameters are given as follows:

$$A = \begin{bmatrix} 0.075 & -0.5 & 0 \\ 0 & 0.08 & 0 \\ 0.01 & 0 & -0.055 \end{bmatrix},$$
$$A_d = \begin{bmatrix} 0.03 & 0 & -0.01 \\ 0.02 & 0.03 & 0 \\ 0.04 & 0.05 & -0.01 \end{bmatrix},$$
$$B = \begin{bmatrix} -0.085 \\ 0.07 \\ 0.2 \end{bmatrix}, \qquad D = \begin{bmatrix} -0.012 \\ 0.05 \\ 0.02 \end{bmatrix}, \tag{34}$$
$$w(k) = 2\cos(0.1k), \qquad G = \begin{bmatrix} 0.01 \\ 0.03 \\ 0.05 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.3 & 0.1 & 0.2 \end{bmatrix},$$
$$g(x,k) = 0.0031 \begin{bmatrix} \sin(0.1x_1(k) - 0.2x_2(k)) \\ \sin(0.1x_1(k) + 0.5x_3(k)) \\ \sin(0.5x_2(k) + 0.3x_3(k)) \end{bmatrix},$$

and thus the Lipchitz condition (2) can be given as $\gamma = 0.0031$, and $\overline{w} = 2$.

Assume that time-varying communication delays $\tau_i(k)$ $(i = 1,2,3)$ are able to be obtained according to the time scale. Let us show the time delays of every system state in Figure 1. From Figure 1, we can know the $\overline{d} = 3$.

Applying Theorems 7 and 8, we can obtain the desired P, L, and ε as follows:

$$P = \begin{bmatrix} 0.0151 & -0.005 & 0.01 \\ -0.005 & 0.0017 & 0.0033 \\ 0.01 & 0.0033 & -0.0067 \end{bmatrix}, \qquad L = \begin{bmatrix} -0.0049 \\ 0.0012 \\ -0.0015 \end{bmatrix},$$
$$\varepsilon = 1.0773e^{-5}. \tag{35}$$

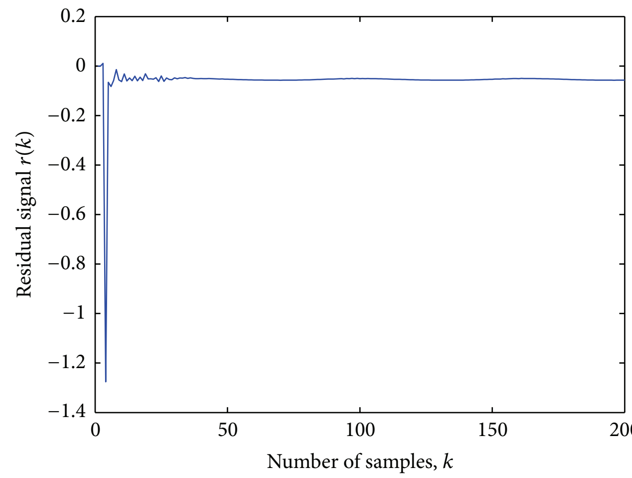

Figure 2: System residual signal with no faults.

When there is no fault in the system, the simulation result is shown in Figure 2. It is clear from Figure 2 that the state observer tracks the true state of the system well and the residual drifts in a region around 0.

To further illustrate the effectiveness of the designed sliding fault detection, for $k = 0, 1, \ldots, 200$, let the fault signal $f(k)$ be given as

$$f(k) = \begin{cases} 1, & 50 \le k \le 100 \\ 0, & \text{else.} \end{cases} \tag{36}$$

The residual signal $r(k)$ and evolution of residual evaluation function $J(k)$ are shown in Figures 3 and 4, respectively, which indicate that the designed observer can detect the fault effectively when it occurs.

For simulation purpose, the threshold is selected as $J_{\text{th}} = \sup_{f(k)=0} \mathbb{E}\{\sum_{k=0}^{200} r^T(k) r(k)\}^{1/2}$, and accordingly, it can be obtained that $J_{\text{th}} = 0.6770$ in Figure 4 represented the dotted curve after 200 Monte Carlo simulations with no faults. Solid curve represents the residual evaluation of the system. From Figure 4, it can be seen that $0.4864 = J(50) < J_{\text{th}} < J(51) = 0.8743$, which means that the fault can be detected in 1 time steps after its occurrence. From simulation results, it can be clearly observed that the smaller the threshold we obtain, the faster the fault detection will take.

6. Conclusions

In this paper, we have addressed the fault detection problem for a class of nonlinear discrete-time networked control systems comprising multiple communication delays and bounded disturbances. A sliding mode observer-based fault detection method has been presented. When the system states are completely measurable, the fault detection problem is converted into the sliding motion stable and reachable problem. A simulation example has been used to demonstrate the effectiveness of the fault detection techniques presented in this paper. The results in this paper could be further extended

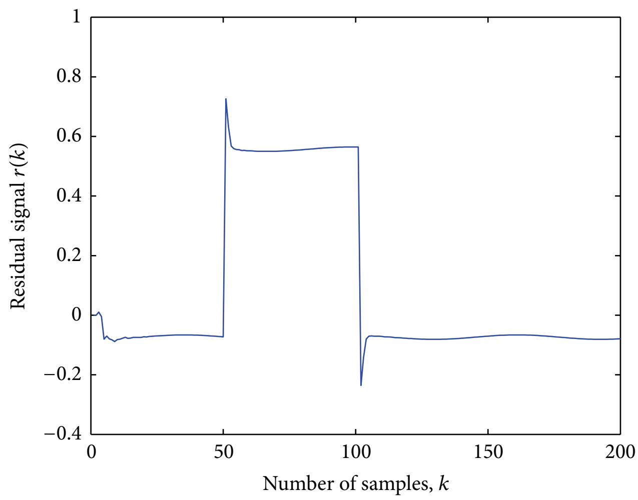

FIGURE 3: System residual signal with faults.

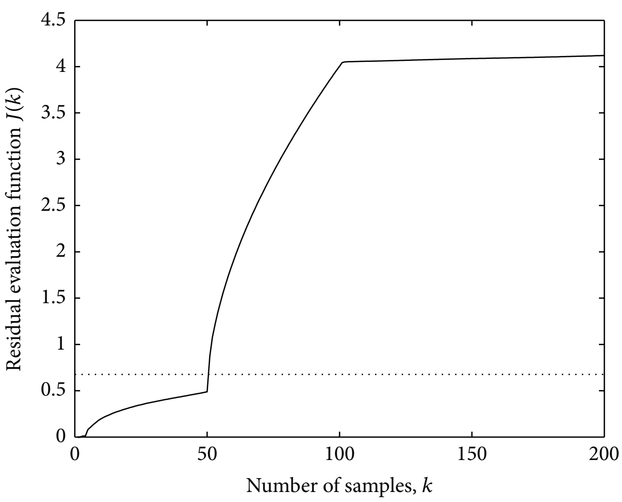

FIGURE 4: Evolution of residual evaluation function $J(k)$.

to state estimation for NCSs with multiple communication delays and bounded disturbances.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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