

Reducing the heterogeneity of payoffs: An effective way to promote cooperation in the prisoner's dilemma game

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In this paper, the accumulated payoff of each agent is regulated so as to reduce the heterogeneity of the distribution of all such payoffs. It is found that there exists an optimal regulation strength at which cooperation in the prisoner's dilemma game is optimally promoted. If the heterogeneity is regulated to be either too weak or too strong, the promotive effect disappears and the evolution of cooperation may even be impaired if compared to the absence of the proposed regulatory mechanism. An explanation of the observed results is provided. In particular, it is found that cooperators on the spatial grid are not isolated but form compact clusters and that the distribution of these clusters is crucial for the promotion of cooperation. Our work provides insights into relations between the distribution of payoffs and the evolution of cooperative behavior in situations constituting a social dilemma.

I. INTRODUCTION

Cooperation is a widespread and important phenomenon and, indeed, it can be seen as the foundation for the sustainable development of many natural and social systems. Explaining the evolution of cooperation, from insect communities to human societies, has been a major goal of biologists since Darwin [1–4]. In a system, the individuals who are willing to contribute to the common good at a personal cost are referred to as cooperators, while those who choose to take advantage of such actions are the defectors. Cooperators are therefore not favored in terms of natural selection, as the latter is individually based and evaluated in terms of personal fitness. Accordingly, defection is often preferred and seems like the natural strategy to choose [4]. However, if all the individuals choose to defect, the social welfare suffers immensely and the system is set for a collapse. Therefore, the understanding of the conditions for the emergence and promotion of cooperation is fundamental importance biological, social, and economic science [5–13].

The problem of cooperation is frequently addressed in the framework of the evolutionary game theory. As one of the representative games, prisoner's dilemma game (PDG) seizes the characteristics of the conflict between the selfish individuals and the collective interests. In PDG, when most of the individuals take the cooperation strategy, the collective interests is optimized, but as to an individual, if it defects when its opponents cooperate, it will profit much greater, and meanwhile its opponents will profit little or none. Thus, due to the selection pressure routed in the fundamental Darwinian assumption, more and more individuals will choose to defect, and as a result the level of cooperation will decrease.

Ultimately, all the individuals will receive lower payoffs than if they chose to cooperate.

There are many mechanisms that can promote the cooperation of PDG such as repeated interaction [5], spatial extensions [14], reciprocity [15], and partly randomly contacts [16]. Very recently, the distribution of payoffs had also been found playing a crucial role in promoting cooperation in PDG [17–19]. Particularly, Perc found that Gaussian-distributed payoff variations is more successful in promoting cooperation than Levy distribution of payoffs [17], indicating that too strong heterogeneity of payoffs will harm the evolution of cooperation among selfish individuals. Then a natural question arises: what is the most effective way to regulate the total payoff of each agent to optimize the cooperation? Here in this paper, we aim to answer this question.

In the continuation, we will show that when the total payoffs are regulated to an intermediate extent, the evolution of cooperation is greatly promoted, and we will provide an explanation for this phenomenon. Furthermore, we find that if the regulation is too strong, the cooperation will be suppressed until the point of disappearance. We also present an explanation for this critical phenomenon and obtain the exact critical point. Moreover, we find the cooperators will gather together to form clusters and it is not the number but rather the size of the clusters that mainly determine the cooperation rate.

II. MODEL

In the spatial PDG, agents located on a square lattice follow two simple strategies: cooperation (C) or defection (D), described as the form of vector,

$$\phi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (1)$$

When a cooperator meets a cooperator, both of them get reward 1, and when two defectors meet, they each get 0, and

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when a cooperator meets a defector, it gets 0, but the defector receives temptation b , $1 < b < 2$. The above rule can be expressed by a matrix

$$\psi = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}, \quad (2)$$

which is called the payoff matrix, and the parameter b characterizes the temptation to defection against cooperation. Each agent plays PDG with its four neighbors. Therefore, the total payoff of the player i is the sum of payoffs after i interacts with its four neighbors, which is written as

$$P_i = \sum_{j \in \Lambda_i} \phi_i^T \psi \phi_j, \quad (3)$$

where Λ_i denotes four neighbors of individual i and the payoff in the following section is denoted as the total payoff in our paper. In classical PDG, an agent updates its strategy according to the following rule: the agent i plays PDG with its neighbors, then randomly selects a neighbor j , and adopts its strategy with probability

$$G_{i \rightarrow j} = \frac{1}{1 + \exp[(P_i - P_j)/T]}, \quad (4)$$

where T characterizes the stochastic noise. For $T=0$, the individual always adopt the best strategy determinately, while irrational changes are allowed for $T>0$. In numerical simulation, noise level is often set as $T=0.1$ because a few irrational behavior is common in real economic systems. With the probability defined above, if the selected neighbor j obtains more payoff than node i , node i will adopt the neighbor's strategy with larger probability, but if the neighbor j obtains less payoff than node i , the probability will be much smaller. It is worth noting that the parameter b has great effects on the cooperation process and with b 's increasing, more and more agents would be defectors.

In [17], Perc proposed several methods to regulate the distribution of payoffs and found that when the distribution becomes not so heterogeneous, the cooperation rate will be promoted greatly. In this paper, we present a scheme that regulates the total payoffs continuously and try to find the optimal regulation strength. In our regulation scheme, we define the regulated payoffs

$$W_i = P_i^\alpha, \quad (5)$$

where $\alpha > 0$ is the regulation parameter which determines the regulation strength and when $\alpha=1$, our model degenerates into the classical PDG. Clearly, when α decreases from 1, the distribution of regulated payoffs becomes narrower and narrower until $\alpha=0^+$, where the heterogeneity of the regulated payoffs distribution disappears and each agent obtains regulated payoff 1 or 0. In our work, we replaced the payoffs P_i and P_j in Eq. (4) by the regulated payoffs W_i and W_j , and we get generalized probability

$$G_{i \rightarrow j} = \frac{1}{1 + \exp[(W_i - W_j)/T]}. \quad (6)$$

We believe that when the distribution of regulated payoffs becomes a little more homogenous than the original one (α

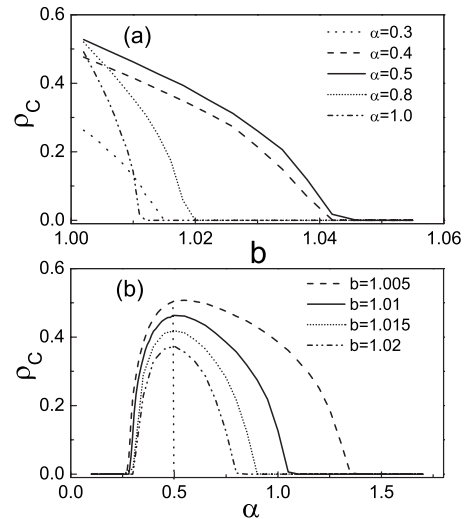


FIG. 1. Fraction of cooperation ρ_C as a function of b in (a) and α in (b). b is fixed at 1.01 in (a).

= 1 for regulated payoffs), the cooperation will be promoted greatly, however, when α is too small there will be less or even no cooperators in the system. We will give the explanation for this interesting phenomenon later in this paper.

III. SIMULATION AND ANALYSIS

In order to describe the evolution process of the game, we employ the fraction of cooperations as an order parameter

$$\rho_C = \frac{1}{L^2} \sum_{i=1}^{L^2} \phi_i^T \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (7)$$

Based on a periodic boundary lattice with size of 100×100 , an extensive Monte Carlo numerical simulation is performed with random initial states. After the system reaches dynamic equilibrium, ρ_C is calculated and the final results are obtained after the averaging of 10 000 times.

Figure 1(a) shows the cooperation fraction ρ_C as a function of b at different values of α . It displays that ρ_C decreases monotonically with the increasing of b , no matter what α is. Most interestingly, the cooperation is greatly affected by the parameter α for fixed b : in a large region of α , ρ_C will be increased, indicating the reduction in heterogeneity of payoffs will improve the cooperation. It is worth noting that there is at least one optimal value of α , where ρ_C takes its maximum, larger, or smaller α will cause the decreasing of ρ_C . Thus, to quantify the effects of α on the promotion of cooperation for different b , we present the dependence of ρ_C on α in Fig. 1(b). It is clearly seen that with α 's decreasing from 1, ρ_C will increase prominently and at the point $\alpha \approx 0.5$ ρ_C reaches its maximum, and after that the value ρ_C will decrease until no cooperators in the system. It is especially worth noting that our regulation scheme is more powerful for larger temptation b , for example, at $b=1.005$, for the classical PDG ($\alpha=1.0$), the fraction of cooperation $\rho_C=0.3855$ and for the best case of our regulation scheme ($\alpha \approx 0.5$), $\rho_C=0.5060$, the increment is $\Delta\rho_C=0.1205$; but for a larger

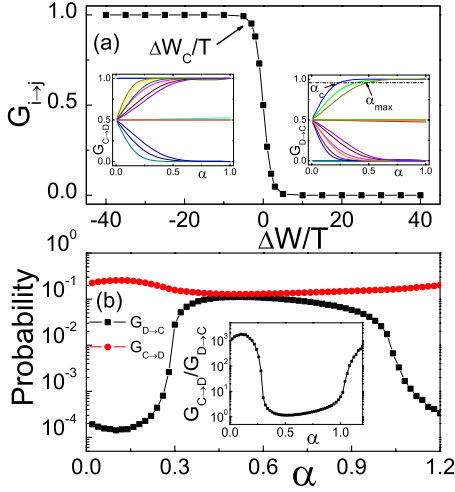


FIG. 2. (Color online) (a) The change in the generalized probability with the regulated payoffs difference. Insets in (a) shows $G_{C \rightarrow D}$ (left) and $G_{D \rightarrow C}$ (right) as a function of α , and the lines present 16 kinds of possible value of $\Delta W/T$. (b) Average transition probability as a function of α for $b=1.01$. Inset in (b) shows the value of $G_{C \rightarrow D}/G_{D \rightarrow C}$ varying with α .

temptation $b=1.020$, the fraction of cooperation ρ_C will increase from 0.0 to 0.3736, with the increment $\Delta\rho_C=0.3736$.

All the above simulation results are consistent with our estimation in Sec. II, and here we will give the explanation for this phenomenon and present the critical and maximal points of the curves in Fig. 1(b) in analysis. First of all, we go back to the generalized probability defined in Eq. (6), in Fig. 2(a) we present the change in $G_{i \rightarrow j}$ with the regulated payoffs difference between two neighboring agents $\Delta W = W_i - W_j$. From the figure, we can see when the difference is less than a critical point ΔW_c , the probability is near to 1, and we believe the changing of the strategy is determined; but when the difference is larger than this critical point, the probability will decrease sharply to 0, and it seems that the agent would never change its strategy. This curve shows that the difference of regulated payoffs plays an important role in the changing of strategy.

Now we investigate the possible values of ΔW 's and how they change with parameter α . According to the rule of changing strategy, if an agent could change its strategy, it must have at least one neighbor that takes the reverse strategy to it at the current moment. Thus, for a cooperator, the regulated payoff may be 0 , 1 , 2^α , and 3^α and, for a defector, the regulated payoff may take b^α , $(2b)^\alpha$, $(3b)^\alpha$, and $(4b)^\alpha$. When agent i is a cooperator and its selected neighbor j is a defector, the 16 possible $\Delta W_{C \rightarrow D}$'s will take values between $-(4b)^\alpha$ and $3^\alpha - b^\alpha$, and when agent i is a defector and its selected neighbor j is a cooperator, the 16 possible $\Delta W_{D \rightarrow C}$'s will take values between $b^\alpha - 3^\alpha$ and $(4b)^\alpha$. When $\alpha=1$, it is clear that the minimum of $\Delta W_{C \rightarrow D}$'s is much smaller than that of $\Delta W_{D \rightarrow C}$'s and there are ten values of 16 $\Delta W_{C \rightarrow D}$'s which are less than ΔW_c and, correspondingly, there are 10 $G_{C \rightarrow D}$'s which are about 1, but the number for $\Delta W_{D \rightarrow C}$'s is 3, as shown in the insets of Fig. 2(a). With α 's decreasing, all the possible nonzero regulated payoffs approach 1, which results in the decreasing of the absolute value of ΔW 's; thus

the ΔW 's that originally are less than ΔW_c leave this region by and by, which result in the decreasing of the probability that changing strategy [see right inset of Fig. 2(a)]. The first curve that leaves this region is a cooperator, which means the propensity for cooperators to change to defectors decrease; thus the fraction of cooperators ρ_C will increase. ρ_C reaches its maximum when the first curve belongs to defector leaves this region; the point α_{\max} can be obtained by the equation

$$(2b)^{\alpha_{\max}} - 3^{\alpha_{\max}} = \Delta W_c, \quad (8)$$

when $b=1.01$, $T=0.1$, and $\Delta W_c = -0.3476$, where $G=0.97$, $\alpha_{\max} \approx 0.53$ [right inset of Fig. 2(a)]. When α decreases to the point α_c , where all the curves belongs to defectors leave the region, the probability for the defectors changing to cooperators decreases sharply, yet there are still some curves of the cooperators that in the region—after a period of iteration—there will be no cooperators, the fraction of cooperations is 0. This critical point can be obtained by the equation

$$b^{\alpha_c} - 3^{\alpha_c} = \Delta W_c. \quad (9)$$

From this equation, it can be concluded that with the increasing of b , the critical point α_c will also increase, which is confirmed by the curves in Fig. 1(b). At the same parameters, we get $\alpha_c \approx 0.26$ [right inset of Fig. 2(a)]. In simulation, we get the two points $\alpha_{\max} \approx 0.50$ and $\alpha_c \approx 0.28$, which are consistent with the result obtained by the analysis very well. Because the fraction of cooperation transits from zero to nonzero at α_c , we can say a phase transition occurring from a noncooperation state to a cooperation existing state.

This kind of phenomenon can be conceptually identical to coherence resonance within the framework of noise-driven dynamical systems [20–24]. Similarly to results obtained by Perc [17] that cooperation is most promoted in an intermediate intensity of payoffs' noise; in our case there always exists an intermediate intensity of regulated payoffs for which cooperation is mostly maintained. To understand the resonancelike behavior induced by payoff regulation strength α , we also apply the mean-field method to investigate the motion of the fraction of cooperators ρ_C . $G_{C \rightarrow D}$ is denoted as the transition probability of cooperators changing into defectors and $G_{D \rightarrow C}$ is denoted as the transition probability of defectors changing into cooperators. Therefore, we get following equation [4,19]:

$$\frac{\partial \rho_C}{\partial t} = (1 - \rho_C)G_{D \rightarrow C} - \rho_C G_{C \rightarrow D}. \quad (10)$$

When the system has reached the steady state $\frac{\partial \rho_C}{\partial t} = 0$, and the fraction of cooperators in the steady state can be given as

$$\rho_C = \frac{G_{D \rightarrow C}}{G_{D \rightarrow C} + G_{C \rightarrow D}} = \frac{1}{1 + G_{C \rightarrow D}/G_{D \rightarrow C}}. \quad (11)$$

In our analysis, $G_{C \rightarrow D}$ and $G_{D \rightarrow C}$ are obtained from the simulation, as shown in Fig. 2(b), then we can calculate the value of cooperation fraction ρ_C according to Eq. (11). In Fig. 2(b),

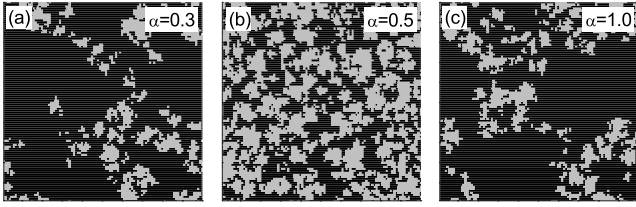


FIG. 3. For panels (a), (b), and (c), typical snapshots of the distribution of cooperators (light gray) and defectors (black) on a square 100×100 lattice obtained for a different value of α by $b = 1.01$. (a) $\alpha=0.3$, (b) $\alpha=0.5$, and (c) $\alpha=1.0$.

one can find that the transition probability $G_{C \rightarrow D}$ keeps in high value for all α , while $G_{D \rightarrow C}$ stays in low value for $\alpha < 0.3$ and $\alpha > 1.0$. As a result, the value of $G_{C \rightarrow D}/G_{D \rightarrow C}$ is large for $\alpha < 0.3$ and $\alpha > 1.0$ and is the smallest around $\alpha = 0.5$ for $b=1.01$ [see the inset of Fig. 2(b)]. Therefore, the cooperation level reaches its optimal value at around $\alpha = 0.5$. The facilitative effect of α on cooperation is directly related to $G_{D \rightarrow C}$. The described explanation is consistent with the general mechanism of cooperation in the spatial prisoner's dilemma game [17,19].

It has been confirmed that intermediate α promotes cooperation, while too small or too large value of α has the reverse effects. In Fig. 3, three typical snapshots are displayed to show how the system will be when α takes small, intermediate, and larger values. Clearly, most of the cooperators are not distributed in isolation but form some clusters. When the payoffs are not regulated ($\alpha=1.0$) or regulated too much ($\alpha=0.3$), there are only a few cooperator clusters in the system; but when $\alpha \approx 0.5$, there will emerge so many cooperator clusters that the cooperation is remarkably promoted. We also investigate the change in the number and the maximal and average size of cooperator clusters and find that the number of cooperator clusters has no notable changing for a large region of α although they will disappear at too small or too larger α ; but the maximal and average size of the cooperator clusters will be greatly increased in some region of α , which is shown in Fig. 4(a) and in the inset. Moreover, the distribution of cooperator clusters at the three values of α is also plotted in Fig. 4(b). It is clear, when $\alpha=0.5$, there are much more large clusters ($S_C > 86$) than when α is 0.3 or 1.0, and the small size clusters ($4 < S_C < 80$) are less than the two cases.

From the above simulation and analysis results we can assert, when α increases from 0.0, there is no cooperator in the system until α reaches some threshold, then the cooperators emerge, with α 's further increasing, more and more agents become cooperators and the cooperator clusters become larger and larger, when α reaches about 0.5, the number of cooperators and the maximal and average size of cooperator clusters all reach their maximums, and further increase α , where the payoffs are not regulated much, the cooperations are repressed again, more and more agents prefer to cheat, the cooperation is depressed, until it disappears. It seems that the emergence of larger cooperator clusters causes the promotion of cooperation since when the fraction of

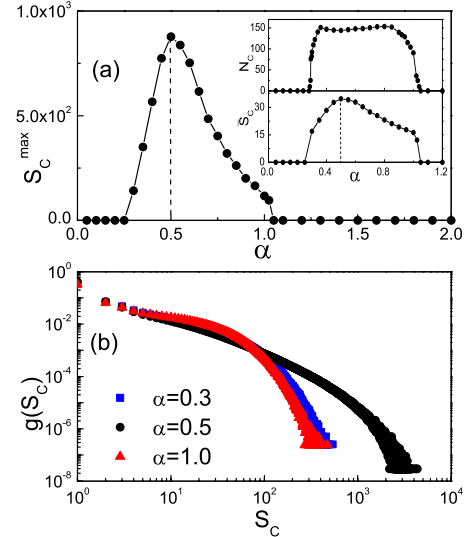


FIG. 4. (Color online) Panel (a) shows the size of the largest clusters formed by cooperators varying with α , and the inset shows the number of clusters N_C (upper) and average size of clusters formed by cooperators \bar{S}_C (lower). Clearly, N_C keeps about 150 from $\alpha=0.37$ to $\alpha=0.85$, while \bar{S}_C reaches its maximal value at $\alpha = 0.5$. Panel (b) displays the distribution of cooperators' cluster size for different values of α . $b=1.01$ in both panels (a) and (b).

cooperation has been promoted the number of cooperator clusters will not change but the size of them will be greatly enlarged, as shown in insets of Fig. 4(a). Our assertion is consistent with the previous researches that cooperators survive by forming compact clusters, and the cooperative agents along the boundary resisting against defectors can be enhanced by, for example, heterogeneous structure [25], attractiveness of the neighbors [26,27], and stochastic interactions [28]. It is found that the more heterogeneous distribution of the cluster size promotes cooperation, which is consistent with previous researches argued for heterogeneous states in general, including the distribution of degree of networks [29] and the teaching activity [30].

IV. CONCLUSION AND DISCUSSION

In conclusion, we regulate the total payoffs of each agent to narrow down the differences between agents in the spatial prisoner's dilemma game and find that there is an optimal regulation strength, where the cooperation is greatly promoted, especially for larger temptation. But too strong of the regulation will depress the cooperation, and even cause the disappearance of the cooperators. We reassure that it is the larger size and not the number of cooperator clusters that promote the cooperation. We also prove the existence of the optimal regulation strength and explain the formation of larger cooperator clusters. Similarly to other methods to regulate payoff, such as random variations in payoffs [31], our quantitative regulation of payoffs also reveal resonance-like cooperation. We hope that our study will prove useful for the understanding of the effects of heterogeneity on cooperation, ranging from payoffs [32] to social connections [33].

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- [1] L. A. Dugatkin, *Cooperation Among Animals* (Oxford University Press, New York, 1997).
- [2] R. Trivers, *Q. Rev. Biol.* **46**, 35 (1971).
- [3] M. A. Nowak, A. Sasaki, C. Taylor, and D. Fudenberg, *Nature* (London) **428**, 646 (2004).
- [4] G. Szabó and G. Fáth, *Phys. Rep.* **446**, 97 (2007).
- [5] R. Axelrod, *The Evolution of Cooperation* (Basic Books, New York, 1984).
- [6] J. Hofbauer and K. Sigmund, *Evolutionary Games and Population Dynamics* (Cambridge University Press, Cambridge, England, 1998).
- [7] M. A. Nowak, *Evolutionary Dynamics: Exploring the Equations of Life* (Harvard University Press, Harvard, 2006).
- [8] F. C. Santos, J. M. Pacheco, and T. Lenaerts, *Proc. Natl. Acad. Sci. U.S.A.* **103**, 3490 (2006).
- [9] Z.-X. Wu, X.-J. Xu, Z.-G. Huang, S.-J. Wang, and Y.-H. Wang, *Phys. Rev. E* **74**, 021107 (2006).
- [10] R. Boyd and S. Mathew, *Science* **316**, 1858 (2007).
- [11] W.-X. Wang, J.-H. Lü, G.-R. Chen, and P. M. Hui, *Phys. Rev. E* **77**, 046109 (2008).
- [12] M. Perc and A. Szolnoki, *Phys. Rev. E* **77**, 011904 (2008).
- [13] G. Szabó and A. Szolnoki, *Phys. Rev. E* **79**, 016106 (2009).
- [14] M. A. Nowak and R. M. May, *Nature* (London) **359**, 826 (1992).
- [15] M. A. Nowak and K. Sigmund, *Nature* (London) **437**, 1291 (2005).
- [16] J. Ren, W.-X. Wang, and F. Qi, *Phys. Rev. E* **75**, 045101(R) (2007).
- [17] M. Perc, *Phys. Rev. E* **75**, 022101 (2007).
- [18] J. Tanimoto, *Phys. Rev. E* **76**, 041130 (2007).
- [19] X.-J. Chen and L. Wang, *Phys. Rev. E* **77**, 017103 (2008).
- [20] M. Perc, *Phys. Rev. E* **72**, 016207 (2005).
- [21] M. Perc, *New J. Phys.* **8**, 22 (2006).
- [22] M. Perc and M. Marhl, *New J. Phys.* **8**, 142 (2006).
- [23] M. Perc and A. Szolnoki, *New J. Phys.* **9**, 267 (2007).
- [24] A. S. Pikovsky and J. Kurths, *Phys. Rev. Lett.* **78**, 775 (1997).
- [25] J. Gómez-Gardeñes, M. Campillo, L. M. Floría, and Y. Moreno, *Phys. Rev. Lett.* **98**, 108103 (2007).
- [26] J.-Y. Guan, Z.-X. Wu, Z.-G. Huang, X.-J. Xu, and Y.-H. Wang, *Europhys. Lett.* **76**, 1214 (2006).
- [27] P. Langer, M. A. Nowak, and C. Hauert, *J. Theor. Biol.* **250**, 634 (2008).
- [28] X.-J. Chen, F. Fu, and L. Wang, *Phys. Rev. E* **78**, 051120 (2008).
- [29] F. C. Santos and J. M. Pacheco, *Phys. Rev. Lett.* **95**, 098104 (2005).
- [30] A. Szolnoki and M. Perc, *New J. Phys.* **10**, 043036 (2008).
- [31] M. Perc, *New J. Phys.* **8**, 183 (2006).
- [32] H. M. Yang, Y. S. Ting, and K. Y. Michael Wong, *Phys. Rev. E* **77**, 031116 (2008).
- [33] F. C. Santos, M. D. Santos, and J. M. Pacheco, *Nature* (London) **454**, 213 (2008).