The role of a matchmaker in buyer-vendor interactions

Linyuan Lü, M. Medo[a], and Y.-C. Zhang

[1] Physics Department, University of Fribourg, Pérolles, 1700 Fribourg, Switzerland
[2] Lab of Information Economy and Internet Research, University of Electronic Science and Technology of China, 610054 Chengdu, P.R. China

Abstract. We consider a simple market where a vendor offers multiple variants of a certain product and preferences of both the vendor and potential buyers are heterogeneous and possibly even antagonistic. Optimization of the joint benefit of the vendor and the buyers turns the toy market into a combinatorial matching problem. We compare the optimal solutions found with and without a matchmaker, examine the resulting inequality between the market participants, and study the impact of correlations on the system.

PACS. 89.65.Gh Economics; econophysics, financial markets, business and management – 89.75.-k Complex systems

1 Introduction

The study of complex economic systems has attracted attention of many physicists. They have contributed to the field with several highly simplified, yet influential, models such as the minority game [1], percolation [2], scaling models of financial markets [3], as well as with a set of useful tools and insights [4–7].

Adopting a simplifying point of view characteristic for the works mentioned above, in this paper we focus on the interactions between consumers and producers. These interactions represent a classical example of decision-making under uncertainty [9] where the limited information available to the contract participants results in a risk of making a wrong decision. In the standard economic literature, problems related to the interactions of consumers and producers are as diverse as the research of consumer behavior [8], the question of trust [10], the economics of information [11,12], and the behavior of entire firms and industries [13,14].

This work is particularly motivated by the classical stable-marriage problem [15,16] in which N men and N women all have their individual preferences and are to be matched one-to-one. Almost inevitably, it's impossible to satisfy everyone and hence stable matchings (where no one has the possibility to exchange the assigned partner for a better one) or the optimal matching (where the total satisfaction is maximal) are of interest. The stable marriage problem has implications in many economic and social systems. It can represent matching of job seekers and employers or that of lodgers and landlords; it is also a metaphor for problems in logistics [17] and in online marketing [18].

We study a situation where a certain product is available in multiple variants and the preferences of both the buyer and the vendor for each of the variants can be represented by numbers (the higher the number, the more appreciated the variant). The matching of the buyer and the vendor is then achieved by the selection of a single variant to be delivered. In the given framework, we first study outcomes achieved with the help of an external matchmaker – an idealized agent supervising the market and having perfect information about all the preferences. In particular, we investigate the inequality between profits enjoyed by the two involved parties and how correlations of the preferences influence the system's behavior.

For comparison, we study two simple matchmaker-free models of variant selection. In the first matchmaker-free model, the vendor makes consecutive offers and the buyer decides whether to accept an offer or not. Our results show that while this approach results in a small decrease of the total satisfaction, it considerably decreases the buyer-vendor inequality. In the second matchmaker-free model, the buyer is searching for the optimal variant by himself. We study the optimal number of examined variants and show that under some conditions, this number may be infinite: the buyer is tempted to search forever. Numerical simulations of the studied models are in most cases accompanied with approximate analytical results.

2 Trading under matchmaker's supervision

We assume that a given product is available in N different variants which can be prepared by a vendor and fulfill,

[a] e-mail: matus.medo@unifr.ch

to a greater or lesser degree, needs of a given buyer. The buyer's utility from purchasing variant α is denoted by x_α and the vendor's utility from providing this variant is denoted by y_α. The matchmaker optimizes the joint benefit by maximizing the total utility $u_\alpha(x_\alpha, y_\alpha)$. Obviously, the system's behavior depends on the choice of the utility function and on the nature of the utilities x_α, y_α – we shall study different settings in the following sections.

2.1 Linear utility function

The simplest form of the total utility is

$$u_\alpha(x_\alpha, y_\alpha) = x_\alpha + y_\alpha \tag{1}$$

where both utilities are merely summed with equal weights. In addition, we assume that both x_α and y_α are random variables drawn from the uniform distribution in the range $[-1, 1]$, $\mathcal{U}(-1, 1)$, and that they are uncorrelated. The distribution $f(u_\alpha)$ then has the tent-shaped form

$$f(u_\alpha) = \begin{cases} (2 + u_\alpha)/4 & u_\alpha \in [-2; 0), \\ (2 - u_\alpha)/4 & u_\alpha \in [0; 2]. \end{cases} \tag{2}$$

The probability that a randomly selected variant has the total utility greater than u_α is $P(u_\alpha) := \int_{u_\alpha}^{2} f(u')\,\mathrm{d}u'$. Since utilities of different variants are mutually independent, the largest utility $u_m := \max_{\alpha=1}^{N} u_\alpha$ has the distribution

$$g(u_m) = N f(u_m)[1 - P(u_m)]^{N-1}. \tag{3}$$

Here the factor N appears because any of N variants can have the largest utility, the factor $f(u_m)$ is the occurrence probability of u_m, and the factor $[1 - P(u_m)]^{N-1}$ is the probability that the remaining $N-1$ variants have utilities lower than u_m; equation (3) is also known as the extreme statistics of the random variable u_m. We study the model by computing $\langle u_m \rangle$ (by $\langle x \rangle$ we denote the average of x over all possible realizations). Since $P(u_\alpha < 0) = 1/2$ and $u_m < 0$ only when all N variants have $u_\alpha < 0$, it follows that $P(u_m < 0) = 2^{-N}$. Hence, assuming $N \gg 1$, we can confine our computation to $u_m > 0$ where $f(u_m) = (2 - u_m)/4$ and $P(u_m) = (2 - u_m)^2/8$. When N is large, $P(\langle u_m \rangle)$ is small and thus we can use the approximation $1 - P(u_m) \approx \exp[-P(u_m)]$. After replacing the lower integration bound in $\langle u_m \rangle$ with $-\infty$ (this is again justified by the negligible probability of $u_m < 0$) we obtain

$$\langle u_m \rangle \approx \frac{2N}{N-1} - \sqrt{\frac{2\pi N^2}{(N-1)^3}} \approx 2 - \sqrt{\frac{2\pi}{N}} \tag{4}$$

where we neglected terms of order $O(1/N)$ and higher. We see that as N increases, $\langle u_m \rangle$ rapidly approaches its upper bound – the difference scales with $N^{-1/2}$. Numerical computation of $\langle u_m \rangle$ shows that the relative error of equation (4) decreases fast with N: it is less than 1% already for $N = 17$.

Apart from the optimal total utility u_m, the inequality between the vendor and the buyer is also of interest. If variant β maximizes u_α, we say that the buyer-vendor inequality is $|x_\beta - y_\beta|$ and denote its expected value by Δ. To compute Δ, one needs to realize that for any given u_α, the term $|x_\alpha - y_\alpha|$ ranges from 0 to $2 - u_\alpha$ (this maximal difference is achieved when one of the two utilities is 1 and the other is $u_\alpha - 1$). Since x_α and y_α are uniformly distributed, all possible values of $|x_\alpha - y_\alpha|$ are equally probable and hence $\langle |x_\alpha - y_\alpha| \rangle = 1 - u_\alpha/2$. In turn, $\Delta = 1 - \langle u_m \rangle/2 = \sqrt{N/(2\pi)}$ which can be easily confirmed by numerical computation.

2.2 Affecting the inequality

Blind maximization of the total utility may not be the best policy because it can result in large inequalities between society members. In an effort to prevent that, the matchmaker may adopt the utility function

$$u'_\alpha(x_\alpha, y_\alpha) = \left(x_\alpha^k + y_\alpha^k\right)^{1/k} \qquad (k > 0). \tag{5}$$

where the variants that have one or both utilities negative are automatically excluded (they do not have the chance to become selected anyway). Choosing $k \gg 1$ in equation (5) favors those pairs (x_α, y_α) where at least one of the utilities is high, while $k < 1$ favors more equal splitting of the total utility. The expected value of $u'_m := \max_{\alpha=1}^{N} u'_\alpha$ can be computed in the same way as in the previous section, yielding

$$\langle u_m{}' \rangle \approx 2 - \left(\frac{\Gamma(\frac{1}{2} + \frac{1}{k})\sqrt{\pi}}{\Gamma(1 + \frac{1}{k}) 4^{1-1/k}} \right)^{1/2} N^{-1/2}. \tag{6}$$

Interestingly, $\langle u'_m \rangle$ scales with N in the same way as $\langle u_m \rangle$.

The expected buyer-vendor inequality is now a function of k, $\Delta(k)$. Numerical results shown in Figure 1 confirm our initial insight that $\Delta(k)$ grows with k. In particular, the value 0.5 achieved for $k \gtrsim 100$ corresponds to picking the variant which maximizes one of the utilities and paying no attention to the other utility. In that case, the selected variant has the larger of the two utilities close to 1 and the other one is 0.5 on average: together we have $\lim_{k\to\infty} \Delta(k) = 0.5$. Further, it can be seen in Figure 1 that $k < 1$ does not significantly decrease the inequality $\Delta(k)$ and thus to create a more social society, one has to use a different utility function. For example, for $N = 1\,000$, optimizing the outcome of the weaker (which corresponds to $u_\alpha(x_\alpha, y_\alpha) = \min[x_\alpha, y_\alpha]$) decreases the inequality by 29% while reducing the total utility by less than 0.3%.

2.3 Serving several buyers at once

When there are several buyers in the market, the matchmaker can either find the best variant for each buyer separately or she can compromise buyers' needs by finding one variant for all. While the former case is identical with our

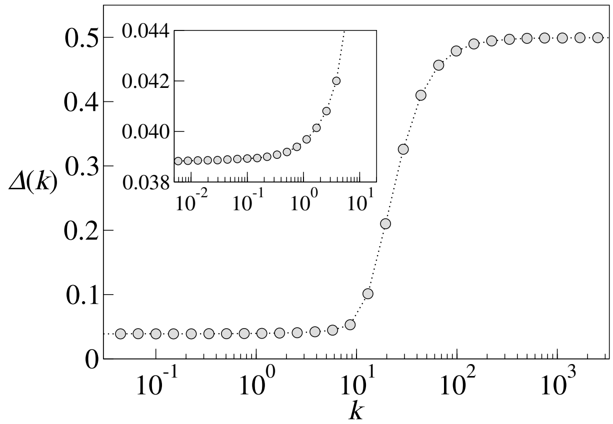

Fig. 1. Inequality $\Delta(k)$ as a function of k for $N = 1\,000$. The inset focuses on small values of k and the results are averaged over 10 000 independent realizations.

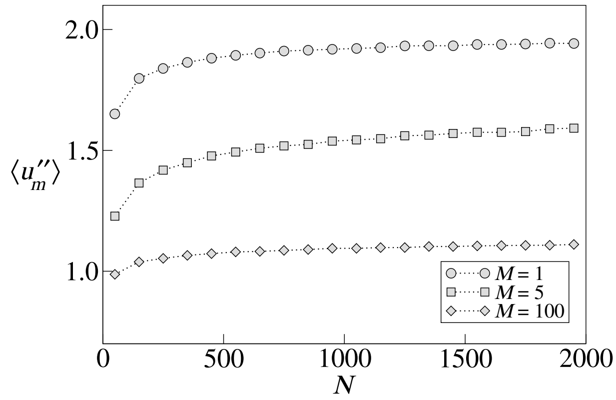

Fig. 2. Average utility with multiple buyers (results are averaged over 1 000 independent realizations).

analysis above, the latter case is different and requires the straightforward generalization of the total utility to the form $My_\alpha + \sum_{i=1}^{M} x_{i,\alpha}$ where M is the number of buyers and $x_{i,\alpha}$ is the utility of variant α for buyer i. For consistency with our previous formalism, we introduce the per-buyer utility

$$u''_\alpha(x_{1,\alpha}, \dots, x_{M,\alpha}, y_\alpha) = y_\alpha + \frac{1}{M}\sum_{i=1}^{M} x_{i,\alpha} := y_\alpha + a_\alpha \quad (7)$$

where a_α is the average buyers' utility of object α, the maximal utility we denote as u''_m. When utilities $x_{i,\alpha}$ are independent and the number of buyers is large, the central limit theorem states that the difference $a_\alpha - \langle x_{i,\alpha}\rangle$ is approximately a normally distributed quantity with variance proportional to $1/M$. It follows that due to the fast decay of the normal distribution, the matchmaker cannot find an object with the average utility a_α differing substantially from $\langle x_{i,\alpha}\rangle$. This is confirmed by Figure 2 where we show $\langle u''_m\rangle$ for various values of M. As M increases, fluctuations of a_α gets smaller and $\langle u''_m\rangle \approx 1$, corresponding to a negligible contribution of a_α to u''_α (it can be shown that to achieve non-negligible $\max_\alpha a_\alpha$, the necessary number of variants is proportional to e^M).

Now we see that it's impossible to satisfy several buyers with one variant. Since production of an individual variant for each buyer is often too expensive, it is then a natural question how to compromise between the buyers' satisfaction and the costs of personalized production. Within the given framework, one can introduce an additional cost which increases with the number of variants produced by the vendor – such a cost forces the vendor to narrow down the selection. This aspect of buyer-vendor interactions is extensively studied in [19,20] where they show that based on the compromise described above and a few simple additional assumptions, one can reproduce a rich variety of market phenomena.

2.4 Correlated utilities

So far we assumed that the vendor's and buyer's utilities are mutually uncorrelated. While convenient for analytical computation, this is not a realistic assumption because in general: what is good for the vendor is not good for the buyer and vice versa. In other words, one expects x_α and y_α to be negatively correlated.

To study the influence of correlations we first need to find a way how to introduce them into the system. In general it is easy to create correlated quantities by introducing a control parameter $t \in [0, 1]$ and assuming that both quantities have a common part which is proportional to t and independent parts which are proportional to $1 - t$. However, when each part itself is uniformly distributed, the resulting distribution depends on the value of t and this effect distorts further analysis of the system [19]. This motivates us to switch from uniform distributions to normal distributions which preserve their functional form under addition. We assume that utilities x_α and y_α are obtained as

$$x_\alpha = \sqrt{1-t}\,X_\alpha + \sqrt{t}\,C_\alpha, \quad y_\alpha = \sqrt{1-t}\,Y_\alpha + s\sqrt{t}\,C_\alpha \quad (8)$$

where $X_\alpha, Y_\alpha, C_\alpha$ are drawn from the standard normal distribution $\mathcal{N}(0, 1)$, $t \in [0, 1]$ is the parameter controlling the correlation strength and the parameter s switches between positive ($s = 1$) and negative ($s = -1$) correlations. Since when adding two normal distributions, individual means and individual variances sum up to give the resulting mean and variance respectively, both x_α and y_α have zero mean and unit variance. It is simple to compute the Pearson correlation coefficient of x_α and y_α which is $C_{xy} = st$.

To study the system of utilities produced by equation (8), we assume the linear total utility given by equation (1). Hence u_α is normally distributed with zero mean and its variance can be shown to be equal to $2(1+st) := v$. We are again interested in $u_m := \max_{\alpha=1}^{N} u_\alpha$ and $\langle u_m\rangle$. It can be shown (see Appendix 4 for details) that $\langle u_m\rangle$ approximately solves the equation

$$\langle u_m\rangle \exp\left[\frac{\langle u_m\rangle^2}{4(1+st)}\right] = N\sqrt{\frac{1+st}{\pi}}. \quad (9)$$

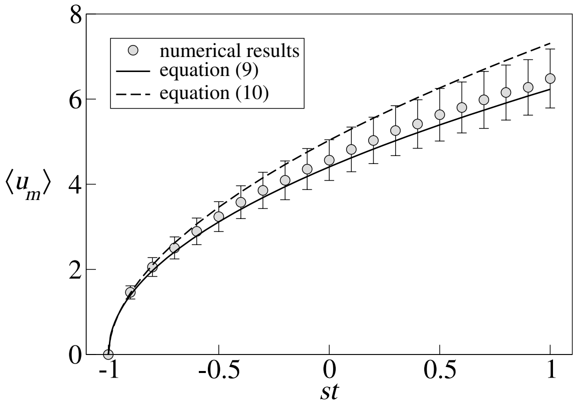

Fig. 3. The dependence of the average maximal utility $\langle u_m \rangle$ on st for $N = 1\,000$ (numerical results and their standard deviations are obtained from $1\,000$ realizations of the model).

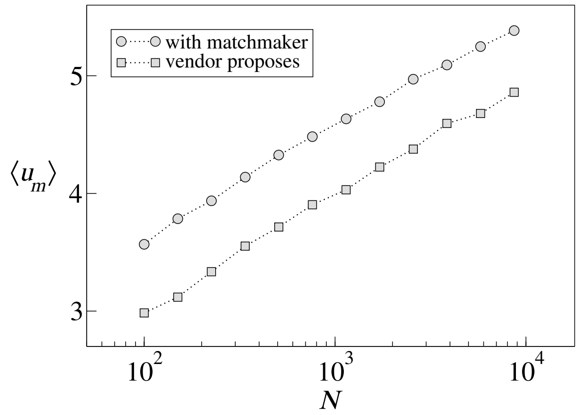

Fig. 4. The total utility of the selected variant with and without the matchmaker. Individual utilities are drawn from $\mathcal{N}(0,1)$, results are averaged over $1\,000$ realizations.

A comparison of this result with a numerical computation of $\langle u_m \rangle$ (where we randomly generate the utilities x_α and y_α, find the maximal total utility u_m and average over many realizations) is shown in Figure 3. As we can see, positive correlations amplify the variance of u_α and hence allow the matchmaker to reach a higher optimal utility. Notice that by setting $t = 0$ in equation (9), one automatically obtains the result for normally distributed uncorrelated utilities.

More insight can be gained if we attempt to find an approximate solution of equation (9). When N is large, the factor $\langle u_m \rangle$ on the left side of equation (9) is much smaller than the exponential term and hence it can be neglected. The simplified equation can be solved and gives us the approximate result

$$\langle u_m \rangle^2 \approx 4(1+st)\ln\left[N\sqrt{(1+st)/\pi}\,\right]. \qquad (10)$$

In contrast to equation (4), this time $\langle u_m \rangle$ grows with N without bounds. On the other hand, this growth is extremely slow: $\langle u_m \rangle$ is proportional to the square root of $\ln N$. For example, in the uncorrelated case increasing N from $1\,000$ to $1\,000\,000$ increases $\langle u_m \rangle$ only by 50%.

3 Trading without the matchmaker

Despite all the results obtained so far, one question remains open: what is the matchmaker's contribution to the studied vendor-buyer matchings? This question can be answered by investigating matchmaker-free methods of the variant selection.

3.1 Vendor proposes

In [19] they assumed that when the vendor offers a variant, the buyer accepts it only if his cost is smaller than the vendor's cost (instead of maximization of utilities, they studied minimization of costs). In our framework this means that the buyer accepts variant α only if $x_\alpha \geq y_\alpha$ (i.e., he wants to profit more than the vendor). The vendor's advantage is that he decides which variants to propose – obviously, it is optimal to begin with the variant that maximizes y_α. This concept, where proposing and accepting sides are well defined and distinguished, is similar to the classical Gale-Shapley algorithm known from the stable marriage problem [15].

Based on the matching described above and using equation (1), we can compute the average total utility of the selected variant and compare it to the case with the matchmaker. Assuming normally distributed utilities and zero correlations, we studied the system numerically. As can be seen in Figure 4, the total utility with the matchmaker is considerably higher. On the other hand, the inequality between the fellows decreases from approximately 1.1 (with the matchmaker) to approximately 0.4 (when the vendor proposes). When correlations are present, the difference between the two matching methods decreases and becomes zero for $C_{xy} = \pm 1$.

3.2 Buyer's search

As we have seen, the matchmaker optimizes the total utility at the cost of compromising the utilities of individuals. The buyer can avoid being "compromised" by searching for the best variant by himself. The drawback is that the search is costly (it consumes buyer's time and attention) and the corresponding cost has to be subtracted from the utility of the eventually selected variant. Since the time spent by searching grows linearly with the number of examined variants, it is natural to assume the linear cost term βN, where N is the number of examined variants and $\beta > 0$ is the cost per examined variant. The expected buyer's utility is

$$u_S(\beta, N) = \Big\langle \max_{1\leq\alpha\leq N} x_\alpha \Big\rangle - \beta N := \langle x_m \rangle - \beta N. \qquad (11)$$

This form of the buyer's utility was suggested in [11] where, however, the emphasis was on the discussion of the cost of information. In today's computerized and networked world, searching is easy. Hence to obtain approximate analytical results, we assume that β is much smaller than the typical utility value.

In equation (11), the term $\langle x_m \rangle$ grows with N but the cost βN eventually takes over and the total utility $u_S(\beta, N)$ decreases (see Fig. 5a for an illustration). This behavior is in agreement with the classical observation "Good things satiate, bad things escalate" by psychologists Coombs and Avrunin [21]. It is now natural to ask, what number of examined variants N_{opt} maximizes $u_S(\beta, N)$. We shall do that in three distinct cases. In addition to x_α drawn from the uniform distribution $\mathcal{U}(-1, 1)$ and from the normal distribution $\mathcal{N}(0, 1)$, which were studied before, we consider also the case when x_α is drawn from the power-law distribution $f(x) = (\gamma-1)x^{-\gamma}$, $x \in [1, \infty)$, $\gamma > 2$ (for accounts on the importance of power-law distributions in complexity management and organization see e.g. [6,22]).

In Appendix B we find expressions for $\langle x_m \rangle$ in all three cases. Therein, the optimal number of variants N_{opt} is found in the forms

$$\text{uniform:} \quad N_{\text{opt}} = (\beta/2)^{-1/2} - 1, \tag{12}$$

$$\text{normal:} \quad N_{\text{opt}} \approx \left(\beta\sqrt{-\ln(2\pi\beta^2)}\right)^{-1}, \tag{13}$$

$$\text{power-law:} \quad N_{\text{opt}} \approx \left(\beta(\gamma-1)/\Gamma(\delta)\right)^{-1/\delta}, \tag{14}$$

where $\delta = (\gamma-2)/(\gamma-1)$. Noticeably, in all three cases we observe a power-law dependency of N_{opt} on β. In the case of uniformly distributed utilities, $\langle x_m \rangle$ has an upper bound and with N it grows very slowly (that means: there is little to be gained by an extensive search). In consequence, N_{opt} is proportional to $\beta^{-1/2}$ and hence it is little sensitive to changes of β. When utilities are normally distributed, $\langle x_m \rangle$ grows with N without bounds – this allows N_{opt} to reach higher values than in the former case. In the power-law case, values of N_{opt} are largest and their dependency on β is strongest. Moreover, as the power-law exponent γ gets closer to 2, the growth of $\langle x_m \rangle$ with N becomes linear and hence when the growth rate is larger than β, it is optimal for the buyer to search "forever" (the exponent δ diverges).

To review the accuracy of the presented results, in Figure 5 we compare them with numerical simulations. As can be seen, good agreement is achieved in all three cases. Let us conclude with the remark that in the power-law case, the average, the mode, and the median of x_m may differ significantly. In consequence, it is an important question whether the buyer should rely on $\langle x_m \rangle$ which is strongly affected by rare extreme events.

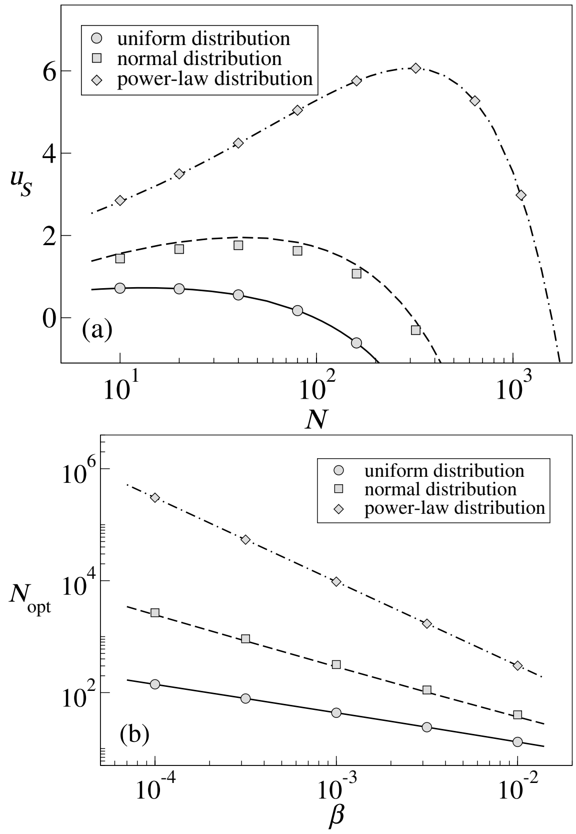

Fig. 5. The buyer's search for three distinct probabilistic distributions of utilities – comparison of numerical results (shown with symbols) and analytical results (shown with lines) for $u_S(0.01, N)$ (a) and N_{opt} (b); $\gamma = 4$ in both figures.

4 Conclusion

In this paper, we studied several simplified models of interactions between buyers and a vendor. Assuming that both the vendor and the buyers can attribute certain utility values to each available variant, the decision which variant is to be delivered can be formulated as a mathematical optimization problem.

Firstly we assumed that there is a matchmaker who can fairly select the optimal variant according to what is best for all participants of the contract. A plausible criterion for the matchmaker is to select the variant with the maximal total utility. Our results show that when the utilities are uniformly distributed, the matchmaker can achieve the total utility close to its upper bound even when the number of available variants is small. In other words, little choice is enough in this case. On the other hand, when the total utility is maximized, the difference between utilities of the vendor and the buyer may be too large to consider it to be the optimal choice. When the mere summation of individual utilities is replaced by a more refined expression, the inequality between the involved parties may be decreased. In particular, we found that when the matchmaker maximizes the smaller of the two individual utilities, the inequality decreases significantly while the total utility is almost unchanged.

When the matchmaker tries to find one variant for several buyers, the situation turns out to be far less favorable: the number of variants needed to approach the upper bound of the total utility grows exponentially with the number of buyers. As a result, production of multiple variants is advisable – this topic was extensively studied in our previous works [19,20]. Assuming normally distributed utilities, we studied the influence of correlations on the system behavior. While analytically more demanding, this generalization is important because it makes the system more realistic. Our results confirm that the optimal total utility depends strongly on the correlation of utilities.

Secondly we studied the variant selection without a matchmaker. In this case, the space of possible means of variant selection is vast and hence we focused on two particular situations. In the first one, the vendor offers and the buyer passively decides whether to accept the offered variant or not. While in comparison with the matchmaker-mediated outcome, the total utility is slightly lower (see Fig. 4), the inequality is decreased substantially. In the second selection method, the buyer chooses the variant by himself but he also has to pay the cost for examining the variants. Here we considered three different distributions of utilities and shown that when the distribution has a power-law tail, for the buyer it may be optimal to inspect a huge number of variants. In the extreme case of a power-law distribution with the exponent lower or equal than 2, the buyer does best by searching forever. Admittedly, these results are influenced by the linear growth of searching cost with the number of examined variants N assumed by equation (11). In contrast, in psychology it is well known that refusing the second best variant or having too many options may be frustrating for people [23]. These effects could be included by adding an additional cost term depending on the utility of the second best variant, or simply by a part of the searching cost proportional to a higher power of N.

Appendix A: Extreme statistics for normally distributed variables

In this appendix we show how $\langle u_m\rangle$ can be approximated when the number of variants, N, is large. Assuming $u_\alpha \in \mathcal{N}(0,v)$, $u_m := \max_{\alpha=1}^N u_\alpha$ has the distribution

$$f(u_m) = \frac{N\mathrm{e}^{-u_m^2/2v}}{\sqrt{2\pi v}}\left(1-\int_{u_m}^{\infty}\frac{\mathrm{e}^{-u^2/2v}}{\sqrt{2\pi v}}\mathrm{d}u\right)^{N-1}. \quad (A.1)$$

The error function $\mathrm{erf}[x] := \frac{2}{\sqrt{\pi}}\int_0^x \exp[-t^2]\,\mathrm{d}t$ allows us to write the integral in equation (A.1) as $\frac{1}{2}-\mathrm{erf}[u_m/\sqrt{2v}]/2$. When N is large, $u_m^2/(2v) \gg 1$ and hence we can use the asymptotic expansion $\mathrm{erf}[x] \approx 1-\exp[-x^2]/(x\sqrt{\pi})$ (taken from `mathworld.wolfram.com`, the next contributing term is proportional to $\exp[-x^2]/x^3$). Again, we use the approximation $1-y \approx \exp[-y]$ (which is valid for $y \ll 1$) to obtain

$$f(u_m) \approx \frac{N}{\sqrt{2\pi v}}\exp\left[-\frac{u_m^2}{2v}-\frac{(N-1)\exp[-u_m^2/2v]}{u_m\sqrt{2\pi/v}}\right].$$

Unfortunately, the integral corresponding to $\langle u_m\rangle$ cannot be solved. Since $f(u_m)$ does not have heavy tails, a reasonably precise result can be obtained by approximating $\langle u_m\rangle$ with $\tilde{u}_m$ maximizing $f(u_m)$, yielding

$$\langle u_m\rangle = (N-1)(1+v/\langle u_m\rangle^2)\sqrt{\frac{v}{2\pi}}\exp[-\langle u_m\rangle^2/2v].$$

When N is large, $v/\langle u_m\rangle^2 \ll 1$ and hence we neglect this term on the right side. In addition, $N-1\approx N$ and we get

$$\langle u_m\rangle \exp\left[\langle u_m\rangle^2/2v\right] = N\sqrt{v/2\pi} \quad (A.2)$$

which after substituting $v = 2(1+st)$ gives equation (9).

Appendix B: Analysis of the buyer's search

We begin with $x_\alpha \in \mathcal{U}(-1,1)$. Then x_m has the distribution $g(x_m) = \frac{N}{2}\left[1-(1-x_m)/2\right]^{N-1}$ and in consequence $\langle x_m\rangle = 1-2/(N+1)$. Maximizing $u_S(\beta,N)$ with respect to N we get $N_{\mathrm{opt}} = \sqrt{2/\beta}-1$.

Now let's consider $x_\alpha \in \mathcal{N}(0,1)$. After substituting $v=1$ in equation (A.2), the expected utility $u_S(\beta,N)$ can be maximized using implicit derivative, yielding the condition $\beta N\langle x_m\rangle = 1$. In this equation, "fast" and "slow" terms (N and $\langle x_m\rangle$ respectively) are mixed together and hence an approximate solution can be found by the following iterative procedure. Approximating $\langle x_m\rangle = 1$ gives the rough estimate $N_0 = 1/\beta$. Together with equation (A.2), this value leads to the improved estimate $\langle x_m\rangle \approx \sqrt{-\ln(2\pi\beta^2)}$ and in turn we get

$$N_{\mathrm{opt}} \approx \left(\beta\sqrt{-\ln(2\pi\beta^2)}\right)^{-1},$$

an improved estimate of N_{opt}.

Finally let's study the case where utilities x_α are constrained to the range $[1,\infty)$ and follow the power-law distribution $f(x_\alpha) = (\gamma-1)x^{-\gamma}$, $\gamma > 2$. Then the cumulative distribution is $P(x_\alpha) = x^{1-\gamma}$ and the distribution of x_m is $g(x_m) = N(\gamma-1)x^{-\gamma}(1-x^{1-\gamma})^{N-1}$. After approximating $1-x_m^{1-\gamma} \approx \exp[-x_m^{1-\gamma}]$ (the relevant values of x_m are large) and replacing the lower integration bound with 0 we obtain $\langle x_m\rangle \approx N^{1/(\gamma-1)}\Gamma[\delta]$ where $\delta = (\gamma-2)/(\gamma-1)$; this result is well defined for all $\gamma > 2$. When only a fraction r of all utilities follows the power law, the result generalizes to $\langle x_m\rangle \approx (Nr)^{1/(\gamma-1)}\Gamma[\delta]$. Maximization of the expected utility $u_S(\beta,N)$ is straightforward and yields $N_{\mathrm{opt}} \approx \left[\beta(\gamma-1)/\Gamma[\delta]\right]^{-1/\delta}$. For a different analysis of the largest value of a power-law distributed sample and a broader discussion of power laws see [24].

References

1. D. Challet, M. Marsili, Y.-C. Zhang, *Minority Games: interacting agents in financial markets* (Oxford University Press, 2005)
2. R. Cont, J.-P. Bouchaud, Macroeconomic Dynamics **4**, 170 (2000)
3. R.N. Mantegna, H.E. Stanley, Nature **376**, 46 (2002)

4. R.N. Mantegna, H.E. Stanley, *An Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge University Press, 1999)
5. D. Sornette, *Why Stock Markets Crash: Critical Events in Complex Financial Systems* (Princeton University Press, 2003)
6. M. Buchanan, Strategy+Business Enews **34**, 1 (2004)
7. D. Helbing, *Managing Complexity: Insights, Concepts, Applications*, edited by D. Helbing (Springer, 2008)
8. I. Simonson, Z. Carmon, R. Dhar, A. Drolet, S.M. Nowlis, Annual Review of Psychology **52**, 249 (2001)
9. D.E. Bell, H. Raiffa, A. Tversky, *Decision Making: Descriptive, Normative, and Prescriptive Interactions*, edited by D.E. Bell, H. Raiffa, A. Tversky (Cambridge University Press, 1988)
10. P.M. Doney, J.P. Cannon, Journal of Marketing **61**, 35 (1997)
11. G.J. Stigler, Journal of Political Economy **69**, 213 (1961)
12. J.E. Stiglitz, Quarterly Journal of Economics **115**, 1441 (2006)
13. J. Sutton, *Sunk Costs and Market Structure* (MIT Press, 1991)
14. D.W. Carlton, J.M. Perloff, *Modern Industrial Organization*, 4rd edn. (Addison-Wesley, 2005)
15. D. Gale, L.S. Shapley, Amer. Math. Monthly **69**, 9 (1962)
16. P. Laureti, Y.-C. Zhang, Physica A **324**, 49 (2003)
17. S.H. Ha, S.C. Park, IEEE Intelligent Systems **16**, 28 (2001)
18. Y. Bakos, Communications of the ACM **41**, 35 (1998)
19. M. Medo, Y.-C. Zhang, Physica A **387**, 2889 (2008)
20. L. Lü, M. Medo, Y.-C. Zhang, D. Challet, Eur. Phys. J. B **64**, 293 (2008)
21. C.H. Coombs, G.S. Avrunin, Psychological Review **84**, 216 (1977)
22. B. McKelvey, P. Andriani, Strategic Organization **3**, 219 (2005)
23. B. Schwartz, *The Paradox of Choice: Why More Is Less* (Ecco, 2003)
24. M.E.J. Newman, Contemporary Physics **46**, 323 (2005)