

PASSIVITY AND PASSIVATION OF INTERCONNECTED TIME-DELAY MODELS OF REHEAT POWER SYSTEMS

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This paper investigates the problems of delay-dependent passivity and passivation of a class of linear interconnected time-delay systems with particular emphasis on multiarea reheat power systems. This class contains state delay in the dynamics and observation at the subsystem (local) level. A new state transformation is developed to exhibit the delay dependence in the system dynamics and a less conservative passivity-bounding inequality is incorporated. Through the analytical development, it is established that the passivity condition can be cast in a linear matrix inequality (LMI) format at the subsystem level thereby facilitating decentralized passivity analysis. For state-feedback passivation, it is proven that it is indifferent to use instantaneous or delayed decentralized state feedback. The case of dynamic output-feedback passivation is also treated. The analytical developments are simulated to a typical multiarea power system and the ensuing results show satisfactory performance.

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1. Introduction

Dynamical systems are being connected to form larger systems to meet the pressing demands and therefore problems of decentralized control and stabilization of interconnected systems are receiving considerable interests [1, 3–5, 11, 16, 21, 24, 26] where most of the effort focused on dealing with the interaction patterns. When the interconnected system involves delays [19], only few studies are available. In [12, 13], the focus has been on delays in the interaction patterns with the subsystem dynamics being known completely. There are various sources for delays including finite capabilities of information processing among different parts of the system, inherent phenomena like mass transport flow and recycling and/or byproduct of computational delays [19]. In [15], a class of uncertain systems is considered where the delays occur within the subsystems. Among

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the relevant topics is the passivity analysis and synthesis for classes of time-delay systems [18, 22, 23, 25] using classical definitions of passivity and positive realness [2, 7, 27]. These results show that passivity-based methods are highly effective and producing robust controllers to classes of time-delay with parametric uncertainties. In particular, when the time-delay factor is known, it is emphasized that delay-dependent passivity yields less conservative performance results. An alternative approach to interconnected power systems has been considered in [28] by representing the power system as Hamiltonian control system with dissipation and then using decentralized saturated nonlinear control to enhance the transient stability.

This paper has two major goals. First, it develops delay-dependent passivity and passivation criteria for a class of linear interconnected time-delay systems and designs decentralized state- and output-feedback controllers. Second, it applies these controllers to a class of interconnected reheat power systems. To achieve the first goal, we consider a less restrictive definition of passivity in dynamical systems [14] and employ a new state transformation to exhibit the delay-dependent system dynamics. Then we proceed to establish that the resulting passivity condition can be cast in a linear matrix inequality (LMI) format which can be conveniently solved by semidefinite programming techniques [7]. Following this, the problem of feedback passivation is examined and it is proven that when the state variables are available for measurements, it is indifferent to use instantaneous or delayed state feedback. This represents a pleasing avenue which enhances the effectiveness of the developed state transformation. Finally, the case of dynamic output-feedback passivation is treated and it is shown that the passivity conditions are cast in the form of system of equations and LMIs.

To achieve the second goal, we consider a class of interconnected reheat power systems, the model of the individual system possesses intrinsic time-delay features due to the feedback channel of reheat power as well as the finite processing capabilities in the governor and power system stabilizer. As a typical application, we consider a three-area reheat power system and use Matlab simulation to explore the effectiveness of the developed control strategy vis-à-vis standard control design approaches. The ensuing simulation results show the superiority of the analytical technique developed in this paper for a wide range of power operating conditions.

Looked at in this light, the developed results in this paper for delay-dependent passivity (Theorem 2.3), state-feedback passivation (Theorem 3.1), and output-feedback passivation (Theorem 4.1) give less conservative results than those of [9, 15–19, 21–25] since they provide a new state transformation for delay-dependent passive analysis and synthesis. Additionally, our control approach is technically superior to the techniques of [24, 29] since it establishes delay-dependent dynamic controllers for interconnected time-delay systems.

Notations and facts. In the sequel, we use W^t , W^{-1} , and $\lambda(W)$ to denote, respectively, the transpose, the inverse, and the eigenvalues of the induced norm of any square matrix W . We use $W > 0$ (< 0) to denote a symmetric positive definite (negative definite). Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise. The symbol \bullet will be used in some matrix expressions to induce a symmetric

structure, that is, if given matrices $L = L^t$ and $R = R^t$ of appropriate dimensions, then

$$\begin{bmatrix} L + M + \bullet & \bullet \\ N & R \end{bmatrix} = \begin{bmatrix} L + M + M^t & N^t \\ N & R \end{bmatrix}. \quad (1.1)$$

Fact 1.1. For any real matrices Σ_1 and Σ_2 with appropriate dimensions, it follows for all $\alpha > 0$ that

$$\Sigma_1 \Sigma_2 + \Sigma_2^t \Sigma_1^t \leq \alpha^{-1} \Sigma_1 \Sigma_1^t + \alpha \Sigma_2^t \Sigma_2. \quad (1.2)$$

2. Passivity of interconnected time-delay systems

We consider a class of interconnected time-delay systems Σ_o ,

$$\Sigma_o : \dot{x}(t) = [A_o + E]x(t) + A_d x(t - \tau) + \Gamma w(t), \quad (2.1)$$

$$x(s) = \kappa(s), \quad s \in [-\tau, 0], \quad (2.2)$$

$$z(t) = G_o x_j(t) + G_d x_j(t - \tau) + \Phi w(t), \quad (2.3)$$

to be composed of n_s coupled subsystems Σ_{oj} described by

$$\Sigma_{oj} : \dot{x}_j(t) = A_{oj} x_j(t) + A_{dj} x_j(t - \tau_j) + \sum_{k=1}^{n_s} E_{jk} x_k(t) + \Gamma_j w_j(t), \quad (2.4)$$

$$x_j(s) = \kappa_j(s), \quad s \in [-\tau_j, 0], \quad (2.5)$$

$$z_j(t) = G_{oj} x_j(t) + G_{dj} x_j(t - \tau_j) + \Phi_j w_j(t), \quad (2.6)$$

where $x_j(t) \in \mathbb{R}^{n_j}$ is the state vector, $w_j(t) \in \mathbb{R}^{q_j}$ is the exogenous input (either a control input or reference signal), $z_j(t) \in \mathbb{R}^{q_j}$ is the controlled output, and $\tau_j \in [0, \tau_j^*]$ is a known constant delay factor. The matrices A_{oj} , A_{dj} , G_{oj} , G_{dj} , Γ_j , and Φ_j are known real constant matrices of appropriate dimensions. The links between the matrices of system Σ_o and subsystems Σ_{oj} , $j = 1, \dots, n_s$, are quite standard. From now onwards, we focus on analysis and design at the subsystem level. Let $X_j(t, \kappa_j)$ denote the state trajectory of system Σ_{oj} from the initial condition $\{x_{oj}, \kappa_j\}$ and let $X_j(t, 0)$ be the corresponding trajectory with zero initial conditions.

Definition 2.1. System Σ_o is called *passive* if there exist scalars $\beta_j \geq 0$, $j = 1, \dots, n_s$, such that for all $t_{pj} \geq 0$ and for all $X_j(t, 0)$ of subsystems Σ_{oj} , $j = 1, \dots, n_s$,

$$2 \int_0^{t_{pj}} w_j^t(s) z_j(s) ds \geq -\beta_j \int_0^{t_{pj}} w_j^t(s) w_j(s) ds. \quad (2.7)$$

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Remark 2.2. It should be observed that Definition 2.1 provides less conservative performance results for a wide class of dynamical systems [14]. There are other alternative approaches based on port-controlled Hamiltonian systems with dissipation [8, 28] by considering the Hamiltonian function as the total energy (potential and kinetic energy) in the mechanical systems. These, however, require an additional a priori modeling effort in contrast to the direct state-space model (2.2)–(2.6) which permits incorporating time-delay factors.

Our purpose in this paper is to develop delay-dependent criteria for the interconnected time-delay system Σ_o using local information from subsystems (2.5)–(2.6) that render the system internally stable and passive based on Definition 2.1 and subsequently construct methods for feedback passivation.

2.1. State transformation. Introduce the state transformation

$$\sigma_j(t) = x_j(t) + \int_{t-\tau_j}^t A_{dj}x_j(s)ds \quad (2.8)$$

into (2.5) to yield

$$\begin{aligned} \dot{\sigma}_j(t) &= [A_{oj} + A_{dj}]x_j(t) + \Gamma_j w_j(t) + \sum_{k=1}^{n_s} E_{jk}x_k(t) \\ &= A_{odj}x(t) + \Gamma_j w_j(t) + \sum_{k=1}^{n_s} E_{jk}x_k(t). \end{aligned} \quad (2.9)$$

Define the augmented state vector

$$\zeta_j(t) = \begin{bmatrix} \sigma_j(t) \\ \varepsilon_j x_j(t) \end{bmatrix} \in \mathbb{R}^{2n}. \quad (2.10)$$

By combining (2.5) and (2.8)–(2.10) and taking the limit $\varepsilon_j \rightarrow 0$, we obtain the transformed system

$$(\Sigma_{T_{oj}}) : \dot{\zeta}_j(t) = \Lambda_{oj}\zeta_j(t) + \int_{t-\tau_j}^t \Upsilon_j\zeta_j(s)ds + \bar{\Gamma}_j w_j(t) + \sum_{k=1}^{n_s} \bar{E}_{jk}\zeta_k(t), \quad (2.11)$$

$$\zeta_j(t) = \bar{\kappa}_j(t), \quad t \in [-2\tau_j, 0], \quad (2.12)$$

$$z_j(t) = \bar{G}_j\zeta_j(t) + \bar{G}_{dj}\zeta(t - \tau_j) + \Phi_j w_j(t), \quad (2.13)$$

along with

$$\begin{aligned} \bar{\Gamma}_j &= \begin{bmatrix} \Gamma_j \\ 0 \end{bmatrix}, & \bar{G}_{oj} &= \begin{bmatrix} 0 & G_{oj} \end{bmatrix}, & \bar{G}_{dj} &= \begin{bmatrix} 0 & G_{dj} \end{bmatrix}, & \bar{E}_{jk} &= \begin{bmatrix} 0 & E_{jk} \\ 0 & 0 \end{bmatrix}, \\ \Lambda_{oj} &= \begin{bmatrix} 0 & A_{odj} \\ -I & I \end{bmatrix}, & Y_j &= \begin{bmatrix} 0 & 0 \\ 0 & A_{dj} \end{bmatrix}, & R_j &= \begin{bmatrix} R_{\sigma j} & 0 \\ 0 & R_{xj} \end{bmatrix}, \\ E_1 &= \begin{bmatrix} I \\ 0 \end{bmatrix}, & E_2 &= \begin{bmatrix} 0 \\ I \end{bmatrix}, & \bar{P}_j &= U_j \mathbb{P}_j, & U_j &= \begin{bmatrix} I_j & 0 \\ 0 & 0 \end{bmatrix}, & \mathbb{P}_j &= \begin{bmatrix} P_{\sigma j} & 0 \\ P_{dj} & P_{xj} \end{bmatrix}, \end{aligned} \quad (2.14)$$

$$\Pi_{oj} = \begin{bmatrix} -P_{dj} - P_{dj}^t + R_{\sigma j} & -P_{xj} + P_{dj}^t \\ & + P_{\sigma j}^t A_{odj} + \\ & P_{\sigma j}^t \sum_{k=1}^{n_s} E_{kj} \\ \bullet & P_{xj} + P_{xj}^t + \\ & R_{xj} + \tau_j Q_j \end{bmatrix}, \quad \Pi_{rj} = \begin{bmatrix} \tau_j P_{dj}^t A_{dj} \\ \tau_j P_{xj}^t A_{dj} \end{bmatrix}, \quad \Pi_{tj} = \begin{bmatrix} P_{\sigma j}^t \Gamma_j \\ -G_{oj}^t \end{bmatrix}. \quad (2.15)$$

THEOREM 2.3. Consider system $\Sigma_{T_{oj}}$. Given matrices $0 < Q_j = Q_j^t \in \mathbb{R}^{n_j \times n_j}$, $0 < R_j = R_j^t \in \mathbb{R}^{2n_j \times 2n_j}$, let matrices $0 < P_{\sigma j}$, P_{dj} , P_{xj} and scalar $\beta_j \geq 0$ satisfy the linear matrix inequalities (LMIs)

$$\begin{bmatrix} \Pi_{oj} & \Pi_{rj} & \Pi_{tj} \\ \bullet & -\tau_j Q_j & 0 \\ & & -\beta_j I - \Phi_j^t \\ \bullet & \bullet & -\Phi_j + G_{dj} R_{xj}^{-1} G_{dj}^t \end{bmatrix} < 0, \quad \begin{bmatrix} -\Phi_j - \Phi_j^t & \beta_j I & G_{dj} \\ \bullet & \beta_j I & 0 \\ \bullet & \bullet & -R_{xj} \end{bmatrix} < 0, \quad (2.16)$$

then system $\Sigma_{T_{oj}}$ is passive in the sense of Definition 2.1.

Proof. For given $Q_j = Q_j^t > 0$ and $R_j = R_j^t > 0$, let the Lyapunov functional $V_j(\cdot)$ of the transformed system (2.11)–(2.13) be selected as

$$V_j(t, \zeta_j) = \zeta_j^t(t) \bar{P}_j^t \zeta_j(t) + \int_{t-\tau_j}^t \zeta_j^t(s) R_j \zeta_j(s) ds + \int_{t-\tau_j}^t \int_{t+\theta_j}^t \zeta_j^t(s) E_2 Q_j E_2^t \zeta_j(s) ds d\theta_j. \quad (2.17)$$

Evaluating of the derivative of $V(t, \zeta_j)$ with respect to t yields

$$\begin{aligned} \dot{V}_j(t, \zeta_j) &= 2\zeta_j^t(t) \bar{P}_j^t \dot{\zeta}_j(t) + \tau_j \zeta_j^t(t) E_2 Q_j E_2^t \zeta_j(t) - \int_{-\tau_j}^0 \zeta_j^t(\theta_j) E_2 Q_j E_2^t \zeta_j(\theta_j) d\theta_j \\ &\quad + \zeta_j^t(t) R_j \zeta_j(t) - \zeta_j^t(t - \tau_j) R_j \zeta_j(t - \tau_j). \end{aligned} \quad (2.18)$$

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Focusing on the first term of (2.18) and using (2.13)–(2.17), we obtain

$$\begin{aligned}
2\zeta_j^t(t)\bar{P}_j^t\dot{\zeta}_j(t) &= 2\sigma^t(t)P_\sigma^t\dot{\sigma}(t) \\
&= 2\zeta_j^t(t)\mathbb{P}_j^t\begin{bmatrix} \dot{\sigma}_j(t) \\ 0 \end{bmatrix} = 2\zeta_j^t(t)\mathbb{P}_j^t\left[A_{odj}x(t) + \Gamma_j w_j(t) + \sum_{k=1}^{n_s} E_{jk}x_k(t)\right. \\
&\quad \left. - \sigma_j(t) + x_j(t) + \int_{t-\tau_j}^t A_{dj}x(s)ds\right] \\
&= 2\zeta_j^t(t)\mathbb{P}_j^t\Lambda_{oj}\zeta_j(t) + 2\zeta_j^t(t)\mathbb{P}_j^t\bar{\Gamma}_j w_j(t) + 2\int_{t-\tau_j}^t \zeta_j^t(s)\mathbb{P}_j^t\Upsilon_j\zeta_j(s)ds \\
&\quad + 2\zeta_j^t(t)\mathbb{P}_j^t\sum_{k=1}^{n_s}\bar{E}_{jk}\zeta_k(t).
\end{aligned} \tag{2.19}$$

Hence, it follows from (2.18)–(2.19) that

$$\begin{aligned}
\dot{V}_j(t, \zeta_j) &= \zeta_j^t(t)[\Lambda_{oj}^t\mathbb{P}_j + \mathbb{P}_j^t\Lambda_{oj} + R_j]\zeta_j(t) + 2\zeta_j^t(t)\mathbb{P}_j^t\bar{\Gamma}_j w_j(t) \\
&\quad + 2\int_{t-\tau_j}^t \zeta_j^t(s)\mathbb{P}_j^t\Upsilon_j\zeta_j(s)ds + \tau_j\zeta_j^t(t)E_2Q_jE_2^t\zeta_j(t) - \zeta^t(t-\tau_j)R_j\zeta_j(t-\tau_j) \\
&\quad - \int_{t-\tau_j}^t \zeta_j^t(s)E_2Q_jE_2^t\zeta_j(s)ds + 2\zeta_j^t(t)\mathbb{P}_j^t\sum_{k=1}^{n_s}\bar{E}_{jk}\zeta_k(t).
\end{aligned} \tag{2.20}$$

By Fact 1.1, we have

$$\begin{aligned}
2\int_{t-\tau_j}^t \zeta_j^t(s)\mathbb{P}_j^t\Upsilon_j\zeta_j(s)ds &= 2\int_{t-\tau_j}^t \zeta_j^t(s)\mathbb{P}_j^tE_2A_{dj}x_j(s)ds \\
&\leq \tau_j\zeta_j^t(t)\mathbb{P}_j^tE_2A_{dj}Q_j^{-1}A_{dj}^tE_2^t\mathbb{P}_j^t\zeta_j(t) + \int_{t-\tau_j}^t x_j^t(s)Q_jx_j(s)ds \\
&= \tau_j\zeta_j^t(t)\mathbb{P}_j^tE_2A_{dj}Q_j^{-1}A_{dj}^tE_2^t\mathbb{P}_j^t\zeta_j(t) + \int_{t-\tau_j}^t \zeta_j^t(s)E_2Q_jE_2^t\zeta_j(s)ds.
\end{aligned} \tag{2.21}$$

From (2.20)–(2.21), we get

$$\begin{aligned}
\dot{V}_j(t, \zeta_j) &= \zeta_j^t(t)\left[\Lambda_{oj}^t\mathbb{P}_j + \mathbb{P}_j^t\Lambda_{oj} + R_j + \tau_j\mathbb{P}_j^tE_2A_{dj}Q_j^{-1}A_{dj}^tE_2^t\mathbb{P}_j + \tau_jE_2Q_jE_2^t\right. \\
&\quad \left. + \mathbb{P}_j^t\sum_{k=1}^{n_s}\bar{E}_{jk} + \sum_{k=1}^{n_s}\bar{E}_{jk}^t\mathbb{P}_j\right]\zeta_j(t) + 2\zeta_j^t(t)\mathbb{P}_j^t\bar{\Gamma}_j w_j(t) - \zeta_j^t(t-\tau_j)R_j\zeta_j(t-\tau_j).
\end{aligned} \tag{2.22}$$

By incorporating the identity $\sum_{j=1}^{n_s} \sum_{k=1}^{n_s} \bar{E}_{jk} \zeta_k(t) = \sum_{k=1}^{n_s} \sum_{j=1}^{n_s} \bar{E}_{kj} \zeta_j(t)$ and (2.13) into (2.22) and completing the squares, it yields

$$\begin{aligned}
& \dot{V}_j(t, \zeta_j) - 2z_j^t(t)w_j(t) - \beta_j w_j^t(t)w_j(t) \\
&= \zeta_j^t(t) \left[\Lambda_{oj}^t \mathbb{P}_j + \mathbb{P}_j^t \Lambda_{oj} + R_j + \tau_j \mathbb{P}^t E_2 A_{dj} Q_j^{-1} A_{dj}^t E_2^t \mathbb{P}_j \right. \\
&\quad \left. + \tau_j E_2 Q_j E_2^t + \mathbb{P}_j^t \sum_{k=1}^{n_s} \bar{E}_{jk} + \sum_{k=1}^{n_s} \bar{E}_{jk}^t \mathbb{P}_j \right] \zeta_j(t) + 2\zeta_j^t(t) \mathbb{P}_j^t \bar{\Gamma}_j w_j(t) \\
&\quad - \beta_j w_j^t(t)w_j(t) - \zeta_j^t(t) \bar{G}_j^t w_j(t) - w_j^t(t) \bar{G}_j \zeta_j(t) - w_j^t(t) (\Phi_j + \Phi_j^t) w_j(t) \\
&\quad - \zeta_j^t(t - \tau_j) R_j \zeta_j(t - \tau_j) - \zeta_j^t(t - \tau_j) \bar{G}_{dj}^t w_j(t) - w_j^t(t) \bar{G}_{dj} \zeta_j(t - \tau_j) \\
&< \zeta_j^t(t) \left[\Lambda_{oj}^t \mathbb{P}_j + \mathbb{P}_j^t \Lambda_{oj} + R_j + \tau_j \mathbb{P}^t E_2 A_{dj} Q_j^{-1} A_{dj}^t E_2^t \mathbb{P}_j + \mathbb{P}_j^t \sum_{k=1}^{n_s} \bar{E}_{kj} + \sum_{k=1}^{n_s} \bar{E}_{kj}^t \mathbb{P}_j \right. \\
&\quad \left. + \tau_j E_2 Q_j E_2^t + (\mathbb{P}_j^t \bar{\Gamma}_j - \bar{G}_j^t) (\Phi_j^t + \Phi_j + \beta_j I - \bar{G}_{dj} R_j^{-1} \bar{G}_{dj}^t)^{-1} (\bar{\Gamma}_j^t \mathbb{P}_j - \bar{G}_j) \right] \zeta_j(t). \tag{2.23}
\end{aligned}$$

Applying the Schur complements to (2.23) and in view of LMIs (2.15) with (2.16), it follows by integrating (2.23) with respect to t over the period $0 \mapsto t_p$ that (2.7) holds and hence system Σ_{T_o} is passive in the sense of Definition 2.1. \square

3. State-feedback passivation

We now build on the foregoing results by considering the feedback passivation problem, that is, designing a feedback controller to render the closed-loop time-delay system passive. Extending on system (Σ_{J_o}) , we consider a class of time-delay systems of the form

$$\Sigma_j : \dot{x}_j(t) = A_{oj} x_j(t) + A_{dj} x_j(t - \tau_j) + \sum_{k=1}^{n_s} E_{jk} x_k(t) + B_{oj} u_j(t) + \Gamma_j w_j(t), \tag{3.1}$$

$$x_j(s) = \kappa_j(s), \quad s \in [-\tau_j, 0], \tag{3.2}$$

$$z_j(t) = G_{oj} x_j(t) + G_{dj} x_j(t - \tau_j) + \Phi_j w_j(t), \tag{3.3}$$

where $u_j(t) \in \mathbb{R}^{r_j}$ is the part of the control input used for feedback. Considering initially the case of instantaneous state-feedback passivation, we use the decentralized control law $u_j(t) = K_j x_j(t)$ such that its application to (3.2) under the transformation (2.8) yields

$$\dot{\sigma}_j(t) = A_{okj} x_j(t) + \sum_{k=1}^{n_s} E_{jk} x_k(t) + \Gamma_j w_j(t), \quad A_{okj} = A_{oj} + B_{oj} K_j. \tag{3.4}$$

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Then the transformed system becomes

$$\begin{aligned}
 (\Sigma_{Tk}) : \dot{\zeta}_j(t) &= \Lambda_{okj} \zeta_j(t) + \int_{t-\tau_j}^t Y_j \zeta_j(s) ds + \sum_{k=1}^{n_s} \bar{E}_{jk} \zeta_k(t) + \bar{\Gamma}_j w_j(t), \\
 \zeta_j(s) &= \bar{\kappa}_j(s), \quad s \in [-2\tau_j, 0], \\
 z_j(t) &= \bar{G}_{oj} \zeta_j(t) + \bar{G}_{dj} \zeta_j(t - \tau_j) + \Phi_j w_j(t),
 \end{aligned} \tag{3.5}$$

where

$$\Lambda_{okj} = \begin{bmatrix} 0 & A_{odj} + B_{oj} K_j \\ -I & I \end{bmatrix}. \tag{3.6}$$

Define the following matrices:

$$\begin{aligned}
 \bar{A}_{odj}^t &= \begin{bmatrix} A_{odj}^t & I \end{bmatrix}, \quad Z_j = \begin{bmatrix} 0 \\ X_{\sigma j} \end{bmatrix}, \quad \bar{B}_{oj} = \begin{bmatrix} B_{oj} \\ 0 \end{bmatrix}, \\
 Y_j &= \begin{bmatrix} X_{dj} & X_{xj} \end{bmatrix}, \quad W_j = \begin{bmatrix} W_{dj} & W_{xj} \end{bmatrix},
 \end{aligned} \tag{3.7}$$

where

$$\mathbb{P}_j^{-1} = \begin{bmatrix} X_{\sigma j} & 0 \\ X_{dj} & X_{xj} \end{bmatrix}, \quad X_{\sigma j} = P_{\sigma j}^{-1}, \quad X_{xj} = P_{xj}^{-1}, \quad X_{dj} = -X_{xj} P_{dj} X_{\sigma j}. \tag{3.8}$$

The following theorem establishes the main result.

THEOREM 3.1. *Consider system Σ_{Tk} . Given matrix $0 < Q_j = Q_j^t \in \mathbb{R}^{n_j \times n_j}$, let matrices Y_j , Z_j , W_j and scalar $\beta_j \geq 0$ satisfy the system of LMIs*

$$\begin{bmatrix} \Pi_{cj} & \tau_j E_2 A_{dj} & \tau_j Y_j^t E_2 & \bar{\Gamma}_j - Y_j^t G_{oj}^t \\ \bullet & -\tau_j Q_j & 0 & 0 \\ \bullet & \bullet & -\tau_j Q_j & 0 \\ \bullet & \bullet & \bullet & -\beta_j I - \Phi_j^t \\ & & & -\Phi_j + \bar{G}_{dj} R_j^{-1} \bar{G}_{dj}^t \end{bmatrix} < 0, \quad \begin{bmatrix} -\Phi_j - \Phi_j^t & \beta_j I & \bar{G}_{dj} \\ \bullet & \beta_j I & 0 \\ \bullet & \bullet & -R_j \end{bmatrix} < 0, \tag{3.9}$$

then system Σ_{Tk} is passive under the control law $u(t) = Kx(t)$ in the sense of Definition 2.1, where

$$\Pi_{cj} = Y_j^t \bar{A}_{odj}^t + \bar{A}_{odj} Y_j - E_1 Z_j^t - Z_j E_1^t + \bar{B}_{oj} W_j + W_j^t \bar{B}_{oj}^t + \sum_{k=1}^{n_s} E_{kj} E_1 Y_j + Y_j^t E_1^t \sum_{k=1}^{n_s} E_{kj}^t \tag{3.10}$$

and the state-feedback gain is given by $K_j = [W_j E_2][Y_j E_2]^{-1}$.

Proof. By Theorem 2.3 and taking into consideration (2.14)-(2.15), it is easy to see that passivity of system Σ_{Tk} implies that the inequality

$$\begin{aligned} & \Lambda_{oj}^t \mathbb{P}_j + \mathbb{P}_j^t \Lambda_{oj} + R_j + \tau_j \mathbb{P}^t E_2 A_{dj} Q_j^{-1} A_{dj}^t E_2^t \mathbb{P}_j + \mathbb{P}_j^t \sum_{k=1}^{n_s} \bar{E}_{kj} + \sum_{k=1}^{n_s} \bar{E}_{kj}^t \mathbb{P}_j \\ & + \tau_j E_2 Q_j E_2^t + (\mathbb{P}_j^t \bar{\Gamma}_j - \bar{G}_j^t) (\Phi_j + \bar{\Phi}_j + \beta_j I - \bar{G}_{dj} R_j^{-1} \bar{G}_{dj}^t)^{-1} (\bar{\Gamma}_j^t \mathbb{P}_j - \bar{G}_j) < 0 \end{aligned} \quad (3.11)$$

holds. Premultiplying (3.11) by \mathbb{P}_j^{-t} , postmultiplying by \mathbb{P}_j^{-1} , using (3.7)-(3.8) and the Schur complements, we obtain the LMI (3.9) and the proof is completed. \square

Remark 3.2. Had we considered delayed state-feedback passivation using the decentralized control law $u_j(t) = K_{dj} x_j(t - \tau_j)$, we would have used the state transformation as

$$\sigma_j(t) = x_j(t) + \int_{t-\tau_j}^t [A_{dj} + B_{oj} K_{dj}] x(s) ds. \quad (3.12)$$

On substituting (3.12) into (3.2) with (2.14)-(2.15), it yields

$$\dot{\sigma}_j(t) = [A_{odj} + B_{oj} K_{dj}] x_j(t) + \sum_{k=1}^{n_s} E_{jk} x_k(t) + \Gamma_j w_j(t). \quad (3.13)$$

A little algebra gives the transformed system

$$\begin{aligned} (\Sigma_{Tdk}) : \dot{\zeta}_j(t) &= \Lambda_{odkj} \zeta_j(t) + \int_{t-\tau_j}^t \Upsilon_j \zeta_j(s) ds + \sum_{k=1}^{n_s} \bar{E}_{jk} \zeta_k(t) + \bar{\Gamma}_j w_j(t), \\ \zeta_j(s) &= \bar{\kappa}_j(s), \quad s \in [-2\tau_j, 0], \\ z_j(t) &= \bar{G}_{oj} \zeta_j + \bar{G}_{dj} \zeta_j(t - \tau_j) + \Phi_j w_j(t), \end{aligned} \quad (3.14)$$

where

$$\Lambda_{odkj} = \begin{bmatrix} 0 & A_{odj} + B_{oj} K_{dj} \\ -I & I \end{bmatrix}. \quad (3.15)$$

Taking into account the matrices of (3.7)-(3.8), we could have established a result parallel to that of Theorem 3.1 by using $K_j \rightarrow K_{dj}$. It is thus not surprising to find that results on both instantaneous and delayed decentralized state-feedback passivation are equivalent. This, in fact, strengthens the state transformations (2.8) or (3.12) as vehicles to derive pertinent delay-dependent dynamic models.

4. Output-feedback passivation

We make another generalization of the previous section by considering passivation using dynamic output feedback for the system

$$(\Sigma_{JO}) : \dot{x}_j(t) = A_{oj}x(t) + A_{dj}x_j(t - \tau_j) + B_{oj}u_j(t) + \sum_{k=1}^{n_s} E_{jk}x_k(t) + \Gamma_j w_j(t), \quad (4.1)$$

$$x_j(s) = \kappa_j(s) \quad s \in [-\tau_j, 0], \quad (4.2)$$

$$y_j(t) = C_{oj}x_j(t) + E_{oj}x_j(t - \tau_j) + D_{oj}u_j(t), \quad (4.3)$$

$$z_j(t) = G_{oj}x_j(t) + G_{dj}x_j(t - \tau_j) + \Phi_j w_j(t), \quad (4.4)$$

where $y_j(t)$ is the vector of output measurements. We consider the following dynamic feedback controller:

$$(\Sigma_{Cj}) : \dot{x}_{cj}(t) = A_{oj}x_{cj}(t) + B_{oj}u_j(t) + K_{cj}[y_j(t) - C_{oj}x_{cj}(t)], \quad (4.5)$$

$$u_j(t) = G_{cj}x_{cj}(t),$$

where K_{cj} , G_{cj} are the unknown gains. From (4.2)–(4.5), we get the combined system

$$(\Sigma_{JCOj}) : \dot{\chi}_j(t) = \hat{A}_{oj}\chi_j(t) + \hat{A}_{dj}\chi_j(t - \tau_j) + \sum_{k=1}^{n_s} \bar{E}_{jk}\chi_k(t) + \bar{\Gamma}_j w_j(t), \quad (4.6)$$

$$z_j(t) = \hat{G}_{oj}\chi_j(t) + \hat{G}_{dj}\chi_j(t - \tau_j) + \Phi_j w_j(t),$$

where

$$\chi_j(t) = \begin{bmatrix} x_j(t) \\ x_{cj}(t) \end{bmatrix}, \quad \hat{A}_{oj} = \begin{bmatrix} A_{oj} & B_{oj}G_{cj} \\ K_{cj}C_{oj} & A_c + B_oG_c + K_c(D_oG_c - C_o) \end{bmatrix}, \quad (4.7)$$

$$\hat{A}_{dj} = \begin{bmatrix} A_{dj} & 0 \\ K_{cj}E_{oj} & 0 \end{bmatrix}, \quad \hat{G}_{oj} = \begin{bmatrix} G_{oj} & 0 \end{bmatrix}, \quad \hat{G}_{dj} = \begin{bmatrix} G_{dj} & 0 \end{bmatrix}.$$

Introducing the state transformation

$$\hat{\sigma}_j(t) = \chi_j(t) + \int_{t-\tau_j}^t \hat{A}_{dj}x(s)ds \quad (4.8)$$

which converts (4.6) into

$$\begin{aligned} \dot{\hat{\sigma}}_j(t) &= [\hat{A}_{oj} + \hat{A}_{dj}]\chi_j(t) + \sum_{k=1}^{n_s} \bar{E}_{jk}\chi_k(t) + \hat{\Gamma}_j w_j(t) \\ &= \hat{A}_{odj}\chi_j(t) + \sum_{k=1}^{n_s} \bar{E}_{jk}\chi_k(t) + \hat{\Gamma}_j w_j(t), \end{aligned} \quad (4.9)$$

the transformed system becomes

$$\begin{aligned}
 (\Sigma_{T_{COj}}) : \dot{\xi}_j(t) &= \begin{bmatrix} \hat{\sigma}_j \\ \hat{\chi}_j \end{bmatrix} = \Lambda_{coj} \xi_j(t) + \int_{t-\tau_j}^t \Upsilon_{coj} \xi_j(s) ds + \sum_{k=1}^{n_s} \hat{E}_{jk} \xi_k(t) + \hat{\Gamma}_j w_j(t), \\
 z_j(t) &= G_{coj} \xi_j(t) + G_{doj} \xi_j(t - \tau_j) + \Phi_j w_j(t),
 \end{aligned} \tag{4.10}$$

where

$$\begin{aligned}
 \hat{\Gamma}_j &= \begin{bmatrix} \bar{\Gamma}_j \\ 0 \end{bmatrix}, \quad G_{coj} = \begin{bmatrix} 0 & \hat{G}_{oj} \end{bmatrix}, \quad G_{doj} = \begin{bmatrix} 0 & \hat{G}_{dj} \end{bmatrix}, \quad \Lambda_{coj} = \begin{bmatrix} 0 & \hat{A}_{odj} \\ -I & I \end{bmatrix}, \\
 \Upsilon_{coj} &= \begin{bmatrix} 0 & 0 \\ 0 & \hat{A}_{dj} \end{bmatrix}, \quad \hat{E}_{jk} = \begin{bmatrix} 0 & \bar{E}_{jk} \\ 0 & 0 \end{bmatrix}.
 \end{aligned} \tag{4.11}$$

By using the Schur complements, it is not difficult to deduce from Theorem 2.3 that given matrix $\mathcal{Q}_j = \mathcal{Q}_j^t \in \mathbb{R}^{2n_j \times 2n_j}$, $\mathcal{R}_j = \mathcal{R}_j^t \in \mathbb{R}^{4n_j \times 4n_j}$, it follows that system $(\Sigma_{T_{COj}})$ is passive if there exists matrix $\Omega_j = \Omega_j^t \in \mathbb{R}^{4n_j \times 4n_j}$ satisfying the inequality

$$\begin{aligned}
 &\Lambda_{coj}^t \Omega_j + \Omega_j^t \Lambda_{coj} + \tau_j E_2 \hat{A}_{dj} \mathcal{Q}_j \hat{A}_{dj}^t E_2^t + \tau_j \Omega_j^t E_2 \mathcal{Q}_j E_2^t \Omega_j + \mathcal{R}_j + \Omega_j^t \sum_{k=1}^{n_s} \hat{E}_{jk} + \sum_{k=1}^{n_s} \hat{E}_{jk}^t \Omega_j \\
 &+ (\Omega_j^t \hat{\Gamma}_j - G_{coj}^t) (\Phi_j^t + \Phi_j + \beta_j I - G_{dj} R_j^{-1} G_{dj}^t)^{-1} (\hat{\Gamma}_j^t \Omega_j - G_{coj}) < 0,
 \end{aligned} \tag{4.12}$$

where

$$\begin{aligned}
 \Omega_j &= \begin{bmatrix} \Omega_{\sigma j} & 0 \\ \Omega_{dj} & \Omega_{xj} \end{bmatrix}, \quad \mathcal{Q}_j = \begin{bmatrix} \mathcal{Q}_{\sigma j} & 0 \\ 0 & \mathcal{Q}_{xj} \end{bmatrix}, \quad \mathcal{R}_j = \begin{bmatrix} R_j & 0 \\ 0 & 0 \end{bmatrix}, \\
 H_j &= \Phi_j^t + \Phi_j + \beta_j I - G_{dj} R_j^{-1} G_{dj}^t.
 \end{aligned} \tag{4.13}$$

We are now in position to present the following result.

THEOREM 4.1. *System Σ_{JCOj} is passive by the dynamic output feedback controller Σ_{Cj} if, given matrices $0 < \mathcal{Q}_{\sigma j} = \mathcal{Q}_{\sigma j}^t$, $0 < \mathcal{Q}_{xj} = \mathcal{Q}_{xj}^t$, $0 < \mathcal{R}_{\sigma 1j} = \mathcal{R}_{\sigma 1j}^t$, $0 < \mathcal{R}_{x1j} = \mathcal{R}_{x1j}^t$, $0 < \mathcal{R}_{x2j} = \mathcal{R}_{x2j}^t$, there exist matrices $\Omega_{\sigma 1j}$, $\Omega_{\sigma 2j}$, Ω_{d1j} , Ω_{d2j} , Ω_{x1j} , Ω_{x2j} and scalar $\beta_j \geq 0$ satisfying the*

system of equations and LMIs

$$\begin{aligned}
 & \begin{bmatrix} \Pi_{d1j} & \tau_j \Omega_{d1j}^t \\ \bullet & -\tau_j \mathcal{Q}_{\sigma j} \end{bmatrix} < 0, & \begin{bmatrix} \Omega_{x2j} + \Omega_{x2j}^t + \mathcal{R}_{x2j} & \tau_j \Omega_{x2j}^t & \tau_j L_j \\ \bullet & -\tau_j \mathcal{Q}_{2j} & 0 \\ \bullet & \bullet & -\tau_j \mathcal{Q}_{xj} \end{bmatrix} < 0, \\
 & \begin{bmatrix} \Pi_{d2j} & \tau_j \Omega_{d2j}^t \\ \bullet & -\tau_j \mathcal{Q}_{xj} \end{bmatrix} < 0, & \begin{bmatrix} \Omega_{x1j} + \Omega_{x1j}^t + \mathcal{R}_{x1j} & \tau_j \Omega_{x1j}^t & \tau_j A_{dj} & \Omega_{x1j} G_{oj}^t \\ \bullet & -\tau_j \mathcal{Q}_{\sigma} & 0 & 0 \\ \bullet & \bullet & -\tau_j \mathcal{Q}_{xj} & 0 \\ \bullet & \bullet & \bullet & -H_j \end{bmatrix} < 0,
 \end{aligned} \tag{4.14}$$

$$\Omega_{\sigma j} = \Omega_{d1j} + \Omega_{x1j}^t A_{odj}^t + \tau_j \Omega_{x1j}^t \mathcal{Q}_{\sigma j} \Omega_{d1j} - \Gamma_j H_j G_{oj} \Omega_{x1j}^t, \tag{4.15}$$

$$\Omega_{\sigma j} = \Omega_{d2j} + \Omega_{x2j}^t (A_{oj} + B_{oj} D_{oj}^\dagger C_{oj})^t + \tau_j \Omega_{x2j}^t \mathcal{Q}_{xj} \Omega_{d2j},$$

where

$$\begin{aligned}
 L_j &= \Omega_{d2j}^t G_{cj}^t B_{oj}^t \Omega_{d1j}^{-1} (C_{oj} + E_{oj})^\dagger E_{oj}, \\
 \Pi_{d1j} &= A_{odj} \Omega_{d1j} + \Omega_{d1j}^t A_{odj}^t + \Gamma_j H_j^{-1} \Gamma_j^t + \mathcal{R}_{\sigma 1j}, \\
 \Pi_{d2j} &= (A_{oj} + B_{oj} D_{oj}^\dagger C_{oj}) \Omega_{d2j} + \Omega_{d2j}^t (A_{oj} + B_{oj} D_{oj}^\dagger C_{oj})^t + \mathcal{R}_{\sigma 2j},
 \end{aligned} \tag{4.16}$$

and the feedback gains are given by

$$G_{cj} = D_{oj}^\dagger C_{oj}, \quad K_{cj} = -\Omega_{d2j}^t G_{cj}^t B_{oj}^t \Omega_{d1j}^{-1} (C_{oj} + E_{oj})^\dagger. \tag{4.17}$$

Proof. Using the block-diagonal matrices

$$\Omega_{\sigma j} = \begin{bmatrix} \Omega_{\sigma 1j} & 0 \\ 0 & \Omega_{\sigma 2j} \end{bmatrix}, \quad \Omega_{xj} = \begin{bmatrix} \Omega_{x1j} & 0 \\ 0 & \Omega_{x2j} \end{bmatrix}, \quad \Omega_{dj} = \begin{bmatrix} \Omega_{d1j} & 0 \\ 0 & \Omega_{d2j} \end{bmatrix}, \tag{4.18}$$

then straightforward expansion of (4.12) with some matrix manipulations, we get equations and LMIs (4.14)-(4.15) such that inequality (4.12) is satisfied. \square

Remark 4.2. It is readily seen that the developed results for delay-dependent passivity (Theorem 2.3), state-feedback passivation (Theorem 3.1), and output-feedback passivation (Theorem 4.1) give less conservative results than those of [9, 15–19, 21–25] since they provide a new state transformation for delay-dependent passive analysis and synthesis. This represents our first major contribution in this paper. On the other hand, our control approach is technically superior to the techniques of [24, 29] since it establishes delay-dependent dynamic controllers for interconnected time-delay systems.

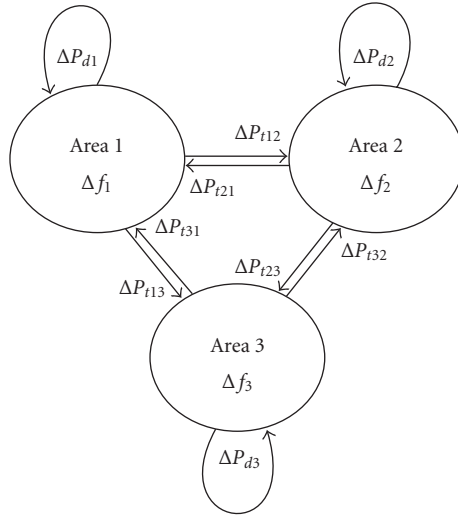


Figure 5.1. A schematic of three-area reheat power systems.

5. Application to power systems

In this section we demonstrate the application of the foregoing analytical developments to multiarea reheat power systems, a schematic layout of which in case of three-area is shown in Figure 5.1.

We will show that the resulting linearized dynamical model can be cast into the format (2.6). In modeling the power system, we note that the overall system model would essentially be an aggregate of the single-area reheat models in addition to the power interchange across the tie lines. By considering the dynamics of individual block of a single-area reheat power system as well as the integral area control [6], the corresponding governing relations are derived below. First, the dynamic model of the generator is described by [29]

$$\begin{aligned}\dot{\delta}(t) &= \omega_o \Delta\omega(t), \\ \Delta\dot{\omega}(t) &= \frac{1}{M} [P_M + r_g(t) + K_d \Delta\omega(t) - P_e], \\ \dot{E}'_q(t) &= \frac{1}{T'_{do}} [E_{fd}(t) - [x_d - x'_d]i_d - E'_q(t)],\end{aligned}\tag{5.1}$$

where

$$i_d = \frac{E_q - E \cos \delta}{x_e + x_q}, \quad E_q = E'_q + [x_d - x'_d]i_d, \quad P_e = \frac{E'_q E \sin \delta}{[x_d - x'_d]},\tag{5.2}$$

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which leads to

$$i_d = \frac{E_q - E \cos \delta}{x_e + x_q - x_d + x'_q}. \quad (5.3)$$

Then the automatic excitation voltage regulator is described by

$$\dot{E}_{fd}(t) = \frac{K_A}{T_A} [V_r - V_t + V_{\text{pss}}(t)] - \frac{K_{fd}(t)}{T_A}. \quad (5.4)$$

The governor and the power system stabilizer are represented by

$$\begin{aligned} \dot{g} &= \frac{1}{T_g} [b\Delta\omega(t - \tau) - g(t)], & r_g &= a\Delta\omega(t - \tau) + bg(t), \\ \dot{y}_1(t) &= y_2(t), & \dot{y}_2(t) &= \frac{-1}{T_2 T_Q} [y_1(t) + (T_2 + T_Q)y_2(t) - \Delta\omega(t - \tau)], \\ V_{\text{pss}} &= \frac{-K_J}{K_A} \left[\frac{-T_1}{T_2} y_1(t) + \frac{T_Q - T_1 T_2 - T_1 T_Q}{T_2} y_2(t) + \frac{T_1}{T_2} \Delta\omega(t - \tau) \right]. \end{aligned} \quad (5.5)$$

By linearizing the dynamical relationships (5.1)–(5.5) around an operating point $(\delta_o, \omega_o, E'_{qo}, E_{fdo}, g_o, y_{1o}, y_{2o})$ and defining the deviations

$$\begin{aligned} \Delta\delta &= \delta - \delta_o, & \Delta\omega &= \omega - \omega_o, & \Delta E'_q &= E'_q - E'_{qo}, & \Delta E_{fd} &= E_{fd} - E_{fdo}, \\ \Delta g &= g - g_o, & \Delta y_1 &= y_1 - y_{1o}, & \Delta y_2 &= y_2 - y_{2o}, \end{aligned} \quad (5.6)$$

we introduce the state and input vectors as $x^t = [\Delta\delta, \Delta\omega, \Delta E'_q, \Delta E_{fd}, \Delta g, \Delta y_1, \Delta y_2]$ and $u = [\Delta P_M, (V_{\text{ref}} - V_t)]$. Then the resulting linearized model can be cast into the form (2.6)-(2.7) with

$$A_o = \begin{bmatrix} 0 & \omega_o & 0 & 0 & 0 & 0 & 0 \\ G_1 & \frac{K_d}{M} & G_2 & 0 & \frac{b}{M} & 0 & 0 \\ G_3 & 0 & G_4 & \frac{1}{T_{do}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{T_A} & 0 & G_5 & G_6 \\ 0 & 0 & 0 & 0 & -\frac{1}{T_g} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & G_7 & G_8 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{a}{M} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{10} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_o = \begin{bmatrix} 0 & 0 \\ \frac{1}{M} & 0 \\ 0 & 0 \\ 0 & \frac{K_A}{T_A} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad C_o^t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad C_d^t = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$
(5.7)

where

$$G_1 = -\frac{E'_{q0} E \cos \delta_o}{M[x_e + x'_d]}, \quad G_2 = -\frac{E'_{q0} E \sin \delta_o}{M[x_e + x'_d]}, \quad G_3 = -E \sin \delta_o \frac{x_d - x'_d}{T'_{do}[x_e + x_q - x_d + x'_d]},$$

$$G_4 = -\frac{x_e + x_q}{T'_{do}[x_e + x_q - x_d + x'_d]}, \quad G_5 = \frac{K_J T_1}{T_A T_2}, \quad G_6 = -\frac{K_J [T_Q - T_1 T_2 - T_1 T_Q]}{T_A T_2},$$

$$G_7 = -\frac{1}{T_2 T_Q}, \quad G_8 = \frac{T_2 + T_Q}{T_2 T_Q}, \quad G_9 = -\frac{K_J T_1}{T_A T_2}, \quad G_{10} = \frac{b}{T_g}, \quad G_{11} = \frac{1}{T_2 T_Q}.$$
(5.8)

It should be observed that the delay factor τ occurs within the dynamics of the governor and the power system stabilizer and the turbine reheat power. This is typical in large power systems and an upper bound on the delay factor can be estimated using offline simulation. This confirms our observation that the linearized dynamical model of multiarea reheat power systems fits nicely the frame of interconnected time-delay systems.

5.1. Numerical data. Assuming identical power areas, the numerical values of the different parameters of any of the areas are given by

(i) synchronous machine parameters

$$\omega_o = 314.16 \text{ rad/s}, \quad M = 6.92, \quad K_d = -0.027, \quad x_d = 1.24 \text{ pu},$$

$$x_q = 0.743 \text{ pu}, \quad x'_d = 0.022 \text{ pu}, \quad x = 0.8 \text{ pu}, \quad P_M = 0.437 \text{ pu}, \quad (5.9)$$

$$E = 1 \text{ pu}, \quad T'_{do} = 5 \text{ s}, \quad \delta_o = 20^\circ, \quad E'_{q0} = 1.05;$$

(ii) voltage regulator and excitation system

$$K_A = 250, \quad T_A = 0.001 \text{ s}, \quad V_{\text{ref}} = 1 \text{ pu}; \quad (5.10)$$

(iii) governor system parameters

$$a = -0.000124, \quad b = -0.17, \quad T_g = 0.25 \text{ s}; \quad (5.11)$$

(iv) power system stabilizer

$$K_J = 12, \quad T_Q = 2.5 \text{ s}, \quad T_1 = 0.1 \text{ s}, \quad T_2 = 0.03 \text{ s}. \quad (5.12)$$

The upper bound on the time-delay τ^* is 0.65. Application of Theorem 4.1 yields the gains of the dynamic output feedback controller:

$$K_{cj} = \begin{bmatrix} 0 & -0.03 \\ -0.02 & 1.25 \\ 0 & -0.02 \\ 0 & -0.001 \\ 0 & -0.006 \\ 0 & -0.07 \\ 0.004 & -0.52 \end{bmatrix}, \quad G_{cj} = \begin{bmatrix} -0.04 & -0.0006 & 0.4 & 0.02 & 0.19 & 0 & 0 \\ 0 & -0.0001 & 0 & 0.004 & 0 - 0.16 & 3.6 & \end{bmatrix}. \quad (5.13)$$

5.2. Simulation results. Extensive simulations were carried out to examine the capabilities of the developed passivity-based control design and to compare the ensuing results with the \mathcal{H}_∞ -controller developed in [29]. The three-area interconnected power system was subjected to a constant load disturbance of 10% of the rated capacity. For purpose of comparison with other related works and due to symmetry, we consider the case when the disturbance is affecting one area only (say, area 1). Here the tie-line power interchange error between the other two areas (area 2 and area 3 in this case) is zero. It should be noted that all tie-line power interchange errors between the different areas with and without integral control are zero. This means that the tie lines between the different areas cannot carry any extra interchanging power due to the load increase in the areas.

The simulation results of the areas are displayed in Figures 5.2–5.7. In Figures 5.2 and 5.3, the frequency error and the inter-area power exchange error in area 1 are plotted for the cases with and without integral control using the developed passivity method. The corresponding results for areas 2 and 3 are plotted in Figures 5.4 and 5.5. Finally, in Figures 5.6 and 5.7, the frequency error and the inter-area power exchange error are plotted using three methods: standard linear-quadratic, the \mathcal{H}_∞ -controller of [29], and the passivity method developed in this paper. From these results, we observe that the steady-state frequency errors in each area were eliminated in a relatively short time. The inter-area tie-line power exchange errors are zero due to the fact that each area is capable of absorbing its own load increase.

In comparison to other established control design methods, the developed passivity-based control design for the interconnected power system has been proven to attain a satisfactory performance.

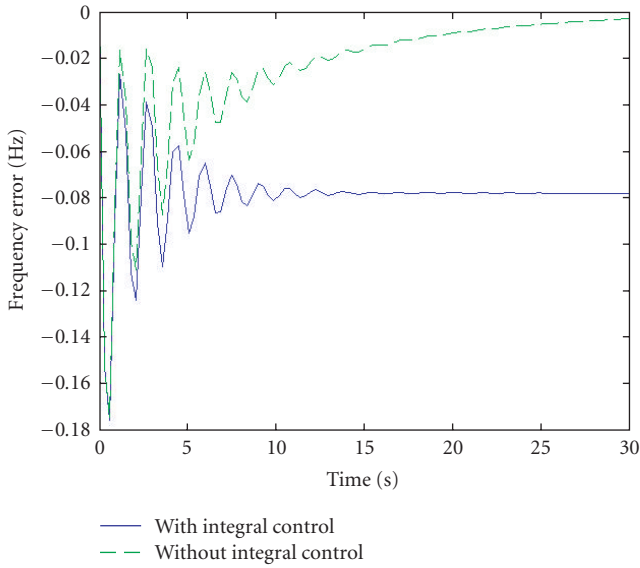


Figure 5.2. Frequency error in area 1: with integral control (solid line); without integral control (dashed line).

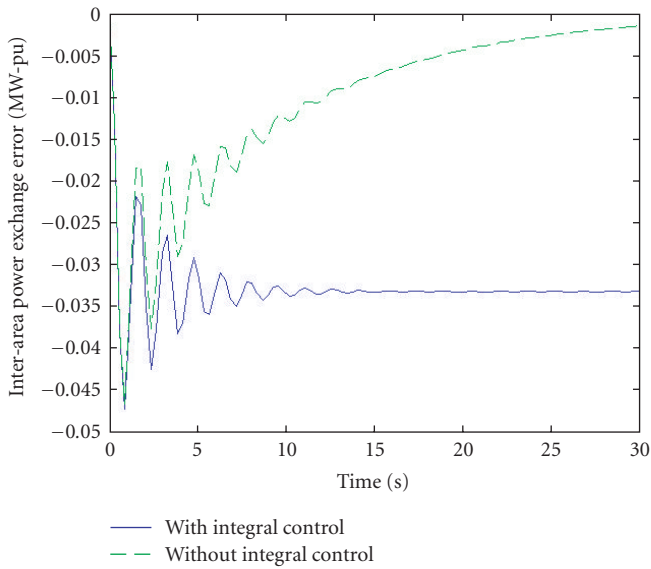


Figure 5.3. Inter-area power exchange error in area 1: with integral control (solid line); without integral control (dashed line).

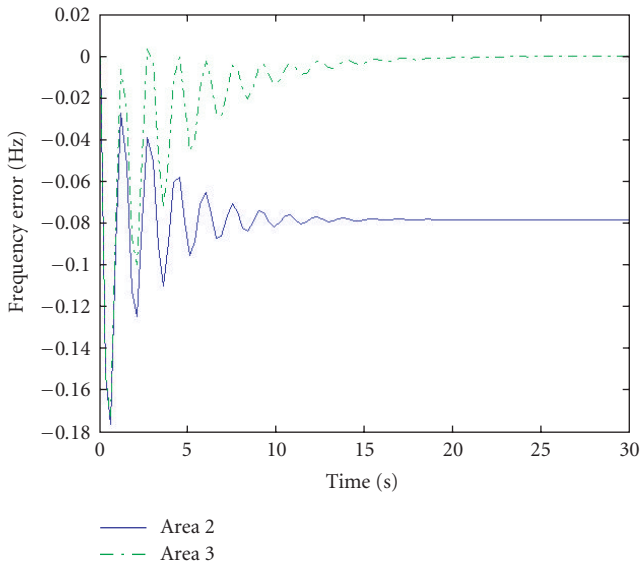


Figure 5.4. Frequency errors: in area 2 (solid line); in area 3 (dashed line).

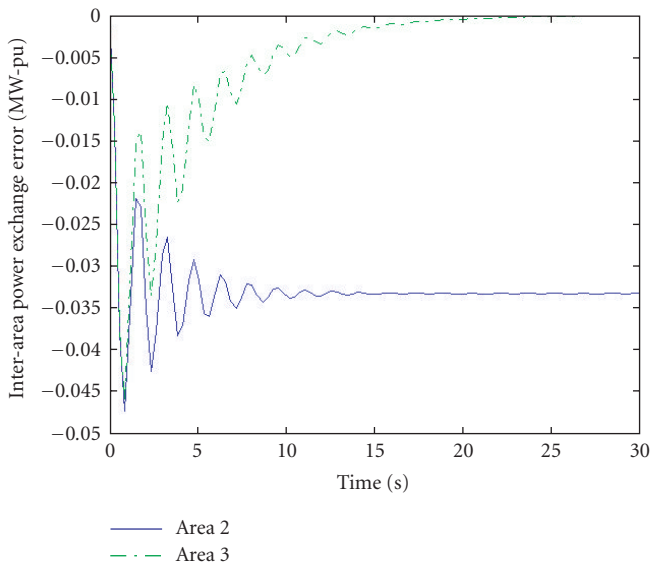


Figure 5.5. Inter-area power exchange errors: in area 2 (solid line); in area 3 (dashed line).

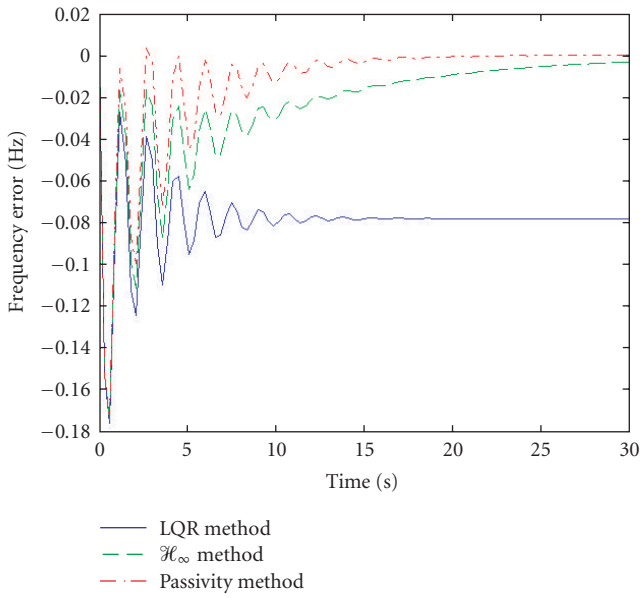


Figure 5.6. Frequency error in area 1: LQR method (solid line); \mathcal{H}_∞ method [29] (dashed line); passivity method (dashed-dotted line).

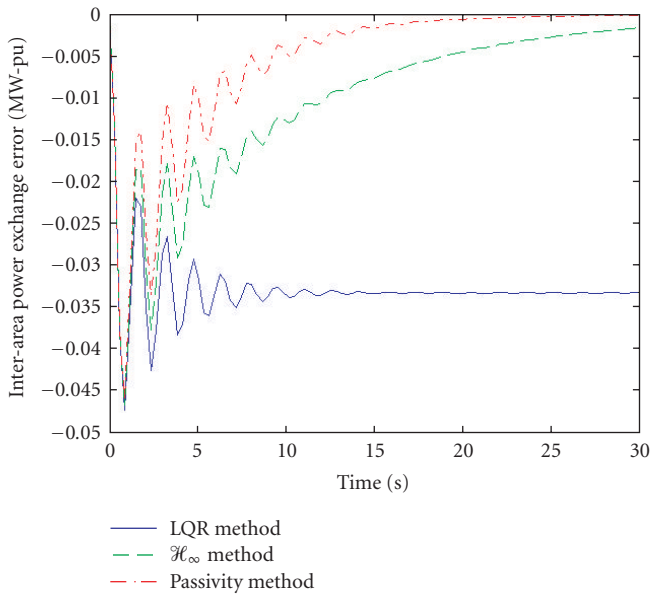


Figure 5.7. Inter-area power exchange error in area 1: LQR method (solid line); \mathcal{H}_∞ method [29] (dashed line); passivity method (dashed-dotted line).

6. Conclusions

A new state transformation has been developed to exhibit the delay-dependence dynamics of a class of linear interconnected time-delay systems for the purpose of investigating the problems of passivity and passivation. It has been established that the passivity condition can be cast in a linear matrix inequality (LMI) format at the subsystem level. For state-feedback passivation, it is proven that it is indifferent to use instantaneous or delayed decentralized state feedback. The case of output-feedback passivation is also treated using local dynamic controllers. Application to multiarea reheat power systems has been given to support the analytical results.

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