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Research Article

New Stability Criterion for Takagi-Sugeno Fuzzy Cohen-Grossberg Neural Networks with Probabilistic Time-Varying Delays

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A new global asymptotic stability criterion of Takagi-Sugeno fuzzy Cohen-Grossberg neural networks with probabilistic time-varying delays was derived, in which the diffusion item can play its role. Owing to deleting the boundedness conditions on amplification functions, the main result is a novelty to some extent. Besides, there is another novelty in methods, for Lyapunov-Krasovskii functional is the positive definite form of p powers, which is different from those of existing literature. Moreover, a numerical example illustrates the effectiveness of the proposed methods.

1. Introduction

Cohen-Grossberg neural networks (CGNNs) have many practical applications, like artificial intelligence, parallel computing, image processing and recovery, and so on ([1–6]). But the success of these applications largely depends on whether the system has some stability, and so people began to be interested in the stability analysis of the system. In recent decades, reaction-diffusion neural networks have received much attention ([7–13]), including various Laplacian diffusion ([6, 14–20]). Besides, people are paying more and more attention to fuzzy neural network system ([21–34]), due to encountering always some inconveniences such as the complicity, the uncertainty, and vagueness ([27, 35–37]). For example, in [27], Zhu and Li investigated the following fuzzy CGNNs model:

$$\begin{aligned} dx_i(t) = &\Bigg\{-a_i(x_i(t))\Bigg[b_i(x_i(t)) - \bigwedge_{j=1}^{n} a_{ij} f_j(x_j(t)) \\ &- \bigvee_{j=1}^{n} b_{ij} g_j(x_j(t)) - \bigwedge_{j=1}^{n} c_{ij} f_j(x_j(t-\tau)) \\ &- \bigvee_{j=1}^{n} d_{ij} g_j(x_j(t-\tau))\Bigg]\Bigg\} dt \\ &+ \sum_{j=1}^{n} \sigma_{ij}(x_j(t), x_j(t-\tau))\, dw_j(t), \\ &x_i(t) = \phi_i(t), \quad -\tau \leqslant t \leqslant 0. \end{aligned} \tag{1}$$

In [36], Muralisankar and Gopalakrishnan studied the following T-S fuzzy neutral type CGNNs with distributed delays:

$$\begin{aligned} dx_i(t) = &\sum_{j=1}^{r} h_j(\omega(t)) \Bigg\{-A_j(x(t))\Bigg[B_j(x(t)) \\ &- C_j f(x(t-\tau(t))) - M_j \int_{t-\rho(t)}^{t} f(x(s))\, ds \\ &- D_j \dot{x}(t - r(t))\Bigg]\Bigg\}. \end{aligned} \tag{2}$$

Besides, Balasubramaniam and Syed Ali discussed Takagi-Sugeno fuzzy Cohen-Grossberg BAM neural networks with discrete and distributed time-varying delays in [37].

Note that there is the following bounded condition on amplification functions in many literatures (see, e.g., [38, Theorem 4]) related to CGNNs:

$$0 < \underline{a}_i \leqslant a_i(r) \leqslant \overline{a}_i, \quad r \in R, \ i = 1, 2, \ldots, n. \tag{3}$$

So, in this paper, we try to delete this bounded condition on amplification functions. This is the main purpose of this paper.

2. Preliminaries

Consider the following fuzzy Takagi-Sugeno p-Laplace partial differential equations with distributed delay.

Fuzzy Rule j. IF $\omega_1(t)$ is μ_{j1} and $\cdots$ $\omega_s(t)$ is μ_{js} *THEN*

$$\begin{gathered}\frac{\partial u}{\partial t} = \nabla \cdot \left(\mathscr{D}(t,x,u) \circ \nabla_p u\right) - A(u)\Bigg[B(u) \\ - C_j f(u(t-\tau(t),x)) - M_j \int_{t-\rho(t)}^{t} f(u(s,x))\,ds\Bigg], \qquad (4) \\ u(\theta,x) = \phi(\theta,x), \quad (\theta,x) \in (-\infty,0] \times \Omega, \\ u(t,x) = 0 \in R^n, \quad (t,x) \in R \times \partial\Omega,\end{gathered}$$

where Ω is an arbitrary open bounded subset in R^m. $\omega_k(t)$ $(k = 1,2,\ldots,s)$ is the premise variable and μ_{jk} $(j = 1,2,\ldots,r;\ k = 1,2,\ldots,s)$ is the fuzzy set that is characterized by membership function. And r is the number of the *IF-THEN* rules; s is the number of the premise variables. $u(t,x) = (u_1(t,x), u_2(t,x), \ldots, u_n(t,x))^T \in R^n$, where $u_i(t,x)$ is the state variable of the ith neuron and the jth neuron at time t and in space variable x. Matrix $\mathscr{D}(t,x,u) = (\mathscr{D}_{ij}(t,x,u))_{n\times m}$ with each $\mathscr{D}_{ij}(t,x,u) \geqslant 0$, and $\mathscr{D}_{ij}(t,x,u)$ is diffusion operator. $\mathscr{D}(t,x,u) \circ \nabla_p u = (\mathscr{D}_{jk}(t,x,u))|\nabla u_i|^{p-2}(\partial u_i/\partial x_k)_{n\times m}$ denotes the Hadamard product of matrix $\mathscr{D}(t,x,u)$ and $\nabla_p u$ (see [39] for details). Matrices $A(u) = \operatorname{diag}(a_1(u_1), a_2(u_2), \ldots, a_n(u_n))$ and $B(u) = \operatorname{diag}(b_1(u_1), b_2(u_2), \ldots, b_n(u_n))$, where $a_i(u_i)$ and $b_i(u_i)$ represent an amplification function at time t and an appropriate behavior function at time t. $C_j = (c_{ik}^{(j)})_{n\times n}$ is the connection matrix. Time delays $\tau(t) \in [0,+\infty)$. $f(u(t-\tau(t),x)) = (f_1(u_1(t-\tau(t)t,x)), f_2(u_2(t-\tau(t),x)), \ldots, f_n(u_n(t-\tau(t),x)))^T$ is the activation function of the neurons. And the second and third equations of (4) imply the initial condition and the Dirichlet boundary condition, respectively.

By way of a standard fuzzy inference method, (4) can be inferred as follows.

$$\begin{gathered}\frac{\partial u}{\partial t} = \nabla \cdot \left(\mathscr{D}(t,x,u) \circ \nabla_p u\right) - A(u(t,x))\Bigg[B(u(t,x)) \\ - \sum_{j=1}^{r} h_j(\omega(t))\Bigg(C_j f(u(t-\tau(t),x)) \\ + M_j \int_{t-\rho(t)}^{t} f(u(s,x))\,ds\Bigg)\Bigg], \\ u(\theta,x) = \phi(\theta,x), \quad (\theta,x) \in (-\infty,0] \times \Omega, \\ u(t,x) = 0 \in R^n, \quad (t,x) \in R \times \partial\Omega,\end{gathered} \tag{5}$$

where $\omega(t) = [\omega_1(t), \omega_2(t), \ldots, \omega_s(t)]^T$ and $h_j(\omega(t)) = w_j(\omega(t))/\sum_{k=1}^{r} w_k(\omega(t))$, $w_j(\omega(t)) : R^s \to [0,1]$ $(j = 1,2,\ldots,r)$ is the membership function of the system with respect to the fuzzy rule j. h_j can be regarded as the normalized weight of each *IF-THEN* rule, satisfying $h_j(\omega(t)) \geqslant 0$ and $\sum_{j=1}^{r} h_j(\omega(t)) = 1$.

Next, we consider the following information for probability distribution of time delays $\tau(t)$:

$$\begin{gathered}\mathbb{P}\left(0 \leqslant \tau(t) \leqslant \tau_1\right) = c_0, \\ \mathbb{P}\left(\tau_1 < \tau(t) \leqslant \tau_2\right) = 1 - c_0.\end{gathered} \tag{6}$$

Here the nonnegative scalar $c_0 \leqslant 1$. Define a random variable as follows:

$$\mathfrak{C}(t) = \begin{cases} 1, & 0 \leqslant \tau(t) \leqslant \tau_1; \\ 0, & \tau_1 < \tau(t) \leqslant \tau_2. \end{cases} \tag{7}$$

So, in this paper, we consider the following Takagi-Sugeno (T-S) fuzzy system with probabilistic time-varying delays:

$$\begin{gathered}\frac{\partial u}{\partial t} = \nabla \cdot \left(\mathscr{D}(t,x,u) \circ \nabla_p u\right) - A(u(t,x))\Bigg\{B(u(t,x)) \\ - \sum_{j=1}^{r} h_j(\omega(t))\Big[c_0 C_j f\left(u\left(t-\tau_1(t),x\right)\right) + (1-c_0) \\ \cdot C_j f\left(u\left(t-\tau_2(t),x\right)\right) + \left(\mathfrak{C} - c_0\right) \\ \cdot \left(C_j f\left(u\left(t-\tau_1(t),x\right)\right) - C_j f\left(u\left(t-\tau_2(t),x\right)\right)\right) \\ + M_j \int_{t-\rho(t)}^{t} f(u(s,x))\,ds\Big]\Bigg\}, \\ u(\theta,x) = \phi(\theta,x), \quad (\theta,x) \in (-\infty,0] \times \Omega, \\ u(t,x) = 0 \in R^n, \quad (t,x) \in R \times \partial\Omega.\end{gathered} \tag{8}$$

System (8) includes the following integrodifferential equations:

$$\begin{aligned}
\frac{dx(t)}{dt} &= -A(x(t))\Bigg\{B(x(t)) - \sum_{j=1}^{r} h_j(\omega(t)) \\
&\cdot \Big[c_0 C_j f(x(t-\tau_1(t))) + (1-c_0) \\
&\cdot C_j f(x(t-\tau_2(t))) + (\mathfrak{C} - c_0) \\
&\cdot \big(C_j f(x(t-\tau_1(t))) - C_j f(x(t-\tau_2(t)))\big) \\
&+ M_j \int_{t-\rho(t)}^{t} f(x(s))\,ds\Big]\Bigg\}, \quad t \geqslant 0, \\
x(\theta) &= \phi(\theta), \quad \theta \in (-\infty, 0].
\end{aligned} \tag{9}$$

Particularly when $p = 2$, system (8) degenerates into the so-called reaction-diffusion CGNNs:

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \nabla \cdot (\mathscr{D}(t,x,u) \circ \nabla u) - A(u(t,x))\Bigg\{B(u(t,x)) \\
&- \sum_{j=1}^{r} h_j(\omega(t))\Big[c_0 C_j f(u(t-\tau_1(t),x)) + (1-c_0) \\
&\cdot C_j f(u(t-\tau_2(t),x)) + (\mathfrak{C} - c_0) \\
&\cdot \big(C_j f(u(t-\tau_1(t),x)) - C_j f(u(t-\tau_2(t),x))\big) \\
&+ M_j \int_{t-\rho(t)}^{t} f(u(s,x))\,ds\Big]\Bigg\}, \\
u(\theta,x) &= \phi(\theta,x), \quad (\theta,x) \in (-\infty,0] \times \Omega, \\
u(t,x) &= 0 \in R^n, \quad (t,x) \in R \times \partial\Omega.
\end{aligned} \tag{10}$$

Throughout this paper, we assume $p = p_1/p_2 > 1$ with p_1 being even number and p_2 being odd number. Besides, suppose that the following conditions hold:

(H1) There exist positive definite matrices $\underline{A} = \operatorname{diag}(\underline{a}_1, \underline{a}_2, \dots, \underline{a}_n)$ and $\overline{A} = \operatorname{diag}(\overline{a}_1, \overline{a}_2, \dots, \overline{a}_n)$ such that

$$0 < \underline{a}_i \leqslant \frac{a_i(s)}{s^{p-2}} \leqslant \overline{a}_i, \quad 0 \neq s \in R, \ i = 1, 2, \dots, n, \tag{11}$$

where $A(u) = \operatorname{diag}(a_1(u_1), a_2(u_2), \dots, a_n(u_n))$ and $u = (u_1, u_2, \dots, u_n)^T \in R^n$.

(H2) There exists a positive definite matrix $\mathbb{B} = \operatorname{diag}(b_1, b_2, \dots, b_n)$ such that $b_i(0) = 0$ and

$$\frac{b_i(s)}{s} \geqslant b_i, \quad 0 \neq s \in R, \ i = 1, 2, \dots, n. \tag{12}$$

(H3) There is a positive definite matrix $F = \operatorname{diag}(F_1, F_2, \dots, F_n)$ such that

$$|f_i(s)| \leqslant F_i|s|, \quad s \in R, \ i = 1, 2, \dots, n. \tag{13}$$

From (H1)–(H3), we know that $b_i(0) = f_i(0) = 0$ and $u = 0$ is an equilibrium of fuzzy system (8).

Remark 1. There are numerous functions satisfying (H1). For example, if $p = 4/3$, we may set

$$a_i(s) = \frac{0.1\left(1 + e^{-s^2}\right)}{\sqrt[3]{s^2}}, \quad \forall 0 \neq s \in R, \ a_i(0) = 0.2. \tag{14}$$

It is obvious that

$$\lim_{s\to 0} a_i(s) = \lim_{s\to 0} \frac{0.1\left(1 + e^{-s^2}\right)}{\sqrt[3]{s^2}} = +\infty. \tag{15}$$

So the function $a_i(s)$ is unbounded for $s \in R$. Moreover,

$$0.1 \leqslant \frac{a_i(s)}{s^{p-2}} = 0.1\left(1 + e^{-s^2}\right) \leqslant 0.2. \tag{16}$$

One can know from (16) that $0.1 \leqslant a_i(s)/s^{p-2} \leqslant 0.2$ with $\underline{a}_i = 0.1$ and $\overline{a}_i = 0.2$.

Remark 2. The amplification function $a_i(s)$ defined as (7) is actually unbounded for $s \in R$. However, various bounded conditions always imposed restrictions on the amplification functions of existing literature ([3–6, 9, 10, 24, 27, 28]). Hence, our condition (H1) is weaker, which will make a corollary with regard to ordinary integrodifferential equations (9) become novel.

For convenience's sake, we need to introduce the following standard notations similarly as [38]:

$$\begin{gathered}
Q = (q_{ij})_{n\times n} > 0\,(< 0), \\
Q = (q_{ij})_{n\times n} \geqslant 0\,(\leqslant 0), \\
Q_1 \geqslant Q_2\,(Q_1 \leqslant Q_2), \\
Q_1 > Q_2\,(Q_1 < Q_2), \\
\lambda_{\max}(\Phi), \\
\lambda_{\min}(\Phi), \\
|C| = (|c_{ij}|)_{n\times n}, \\
|u(t,x)|,
\end{gathered} \tag{17}$$

and the identity matrix with compatible I.

(i) The Sobolev space $= \{u \in L^p : \mathscr{D}u \in L^p\}$ (see [40] for details).

(ii) Denote by λ_1 the lowest positive eigenvalue of the boundary value problem (see [40] for details).

$$-\Delta_p \varphi(t,x) = \lambda\varphi(t,x), \quad x \in \Omega$$
$$\varphi(t,x) = 0, \quad x \in \partial\Omega. \tag{18}$$

Lemma 3. *One has*

$$a^{q-1}b \leqslant \frac{q-1}{q}a^q + \frac{b^q}{q}, \quad \forall a,b \in (0,+\infty), \; q > 1. \tag{19}$$

Note that Lemma 3 is the particular case of the famous Young inequality.

3. Results and Discussion

Lemma 4. *Let $P = \mathrm{diag}(p_1, p_2, \ldots, p_n)$ be a positive definite matrix and u be a solution of the fuzzy system (8). Then one has*

$$\int_\Omega u^T P\left(\nabla\cdot\left(D(t,x,u)\circ\nabla_p u\right)\right)dx \leqslant -\lambda_1 \underline{p} D\|u\|_p^p, \tag{20}$$

where $D = \min_{ik}(\inf_{t,x,u} | D_{ij}(t,x,u))$, $\|u\|_p^p = \sum_{i=1}^n \int_\Omega |u_i|^p dx$, and $\underline{p}$ is a positive scalar, satisfying $P > \underline{p}I$.

Proof. Since u is a solution of system (8), it follows by Gauss formula and the Dirichlet zero-boundary condition that

$$\begin{aligned}
&\int_\Omega \sum_{j=1}^n p_j u_j \sum_{k=1}^m \frac{\partial}{\partial x_k}\left(D_{jk}\left|\nabla u_j\right|^{p-2}\frac{\partial u_j}{\partial x_k}\right)dx \\
&\quad = -\sum_{k=1}^m\sum_{j=1}^n \int_\Omega p_j D_{jk}\left|\nabla u_j\right|^{p-2}\left(\frac{\partial u_j}{\partial x_k}\right)^2 dx \\
&\quad \leqslant -\lambda_1 D\underline{p}\sum_{j=1}^n \int_\Omega \left|u_j\right|^p dx = -\lambda_1 \underline{p} D\|u\|_p^p.
\end{aligned} \tag{21}$$

□

Remark 5. Lemma 4 extends the conclusion of [2, Lemma 2.1] and [10, Lemma 2.4] from Hilbert space $H_0^1(\Omega)$ to Banach space $W_0^{1,p}(\Omega)$. Particularly, in the case of $\Omega = (0,T) \subset R^1$ or $W_0^{1,p}(0,T)$, the first eigenvalue $\lambda_1 = ((2/T)\int_0^{(p-1)^{1/p}}(dt/(1-t^p/(p-1))^{1/p}))^p$ (see, e.g., [40]).

Theorem 6. *If there exists a positive definite matrix $P = \mathrm{diag}(p_1, p_2, \ldots, p_n)$ and two positive scalars $\underline{p}$, $\overline{p}$ such that the following inequalities hold:*

$$\begin{aligned}
\lambda_1 D\underline{p} + \underline{p}\lambda_{min}(\underline{A}\mathbb{B}) > \frac{n\overline{p}}{p}\sum_{j=1}^r &\Bigg((p-1)\left|c_j\right| \\
&+ \rho(p-1)\left|m_j\right| + \frac{c_0\left|c_j\right|}{1-\tau_1} + \frac{(1-c_0)\left|c_j\right|}{1-\tau_2} + \rho\left|m_j\right|\Bigg) \\
&\cdot \lambda_{max}\overline{A}\lambda_{max}F,
\end{aligned} \tag{22}$$

$$\begin{aligned}
&P > \underline{p}I, \\
&P < \overline{p}I,
\end{aligned} \tag{23}$$

then the null solution of fuzzy system (8) is globally asymptotically stable, where matrices $C_j = (c_{ik}^{(j)})_{n\times n}$, $M_j = (m_{ik}^{(j)})_{n\times n}$, $|c_j| = \max_{ik}|c_{ik}^{(j)}|$, $|m_j| = \max_{ik}|m_{ik}^{(j)}|$, and $\tau_1'(t) \leqslant \tau_1 < 1$, $\tau_2'(t) \leqslant \tau_2 < 1$, $0 \leqslant \rho(t) \leqslant \rho$.

Proof. Firstly, we can conclude from (H1)–(H3) that $u = 0$ is an equilibrium point for system (8).

Next, consider the Lyapunov-Krasovskii functional:

$$V(t) = V_1(t) + \frac{1}{1-\tau_1}V_2(t) + \frac{1}{1-\tau_2}V_3(t) + V_4(t), \tag{24}$$

where

$$\begin{aligned}
V_1(t) &= \int_\Omega u^T(t,x)Pu(t,x)\,dx = \sum_{i=1}^n\int_\Omega p_i u_i^2 dx, \\
V_2(t) &= 2c_0 n\frac{\overline{p}}{p}\left(\sum_{j=1}^r\left|c_j\right|\right) \\
&\quad \cdot \lambda_{\max}\overline{A}\lambda_{\max}F\sum_{k=1}^n\int_{t-\tau_1(t)}^t\int_\Omega\left|u_k(s,x)\right|^p dx\,ds, \\
V_3(t) &= 2(1-c_0)n\frac{\overline{p}}{p}\left(\sum_{j=1}^r\left|c_j\right|\right) \\
&\quad \cdot \lambda_{\max}\overline{A}\lambda_{\max}F\sum_{k=1}^n\int_{t-\tau_2(t)}^t\int_\Omega\left|u_k(s,x)\right|^p dx\,ds, \\
V_4(t) &= 2\frac{\overline{p}}{p}\left(\sum_{j=1}^r\left|m_j\right|\right) \\
&\quad \cdot \lambda_{\max}\overline{A}\lambda_{\max}F\sum_{i=1}^n\sum_{k=1}^n\int_\Omega\left(\int_{-\rho}^0 d\zeta\int_{t+\zeta}^t\left|u_k(s,x)\right|^p ds\right)dx.
\end{aligned} \tag{25}$$

Here, $u(t,x) = (u_1(t,x), u_2(t,x), \ldots, u_n(t,x))^T$ is a solution for stochastic fuzzy system (8). Below, we may denote $u(t,x)$ by u and $u_i(t,x)$ by u_i for simplicity. □

Remark 7. It is obvious that our Lyapunov-Krasovskii functional is the positive definite form of p powers, which is different from those of existing literature ([41–43]). For example, in [41], the model is also neural networks with discrete time delay and distributed delays:

$$\begin{aligned}
dx(t) &= \Big[-C_i x(t) + A_i f(y(t-\tau(t),i)) \\
&\quad + B_i\int_{t-\rho(t)}^t f(y(s),r(s))\Big]dt + \sigma_t dw_1(t), \\
dy(t) &= \Big[-\widetilde{C}_i y(t) + \widetilde{A}_i g(x(t-\widetilde{\tau}(t),i)) \\
&\quad + \widetilde{B}_i\int_{t-\widetilde{\rho}(t)}^t g(x(s),r(s))\Big]dt + \widetilde{\sigma}_t dw_2(t).
\end{aligned} \tag{26}$$

In [42, Theorem 1], the corresponding Lyapunov-Krasovskii functional is as follows:

$$\widetilde{V}_2 = \int_{-\rho}^{0} d\theta \int_{t+\theta}^{t} f^T(y(s), r(s)) Lf(y(s), r(s)) ds + \int_{-\widetilde{\rho}}^{0} d\theta \int_{t+\theta}^{t} g^T(x(s), r(s)) Lg(x(s), r(s)) ds, \tag{27}$$

which is the positive definite form of 2 powers. And the conclusion of [42, Theorem 1] is the asymptotical stability in the mean square, which is also similar to that of our Theorem 6. However, by means of our Lyapunov-Krasovskii functional with the positive definite form of p powers, we shall derive the asymptotical stability in the mean square for nonlinear p-Laplacian diffusion system (8).

Evaluating the time derivation of $V_1(t)$ along the trajectory of the fuzzy system (8), we can get by [38, Lemma 6] and Lemma 4

$$\begin{aligned} V_1'(t) &= 2\int_\Omega \left[u^T P\left(\nabla\cdot\left(D(t,x,u)\circ\nabla_p u\right)\right) - u^T PA(u) B(u)\right] dx + 2\sum_{j=1}^{r} h_j(\omega(t)) \\ &\quad \cdot\left(\int_\Omega u^T PA(u) c_0 C_j f(u(t-\tau_1(t),x)) dx + \int_\Omega u^T PA(u)(1-c_0) C_j f(u(t-\tau_2(t),x)) dx\right. \\ &\quad \left. + \int_\Omega u^T PA(u) M_j \int_{t-\rho(t)}^{t} f(u(s,x)) ds\, dx\right) \\ &\leqslant -2\lambda_1 \underline{p} D \|u\|_p^p - 2\int_\Omega u^T PA(u) B(u) dx + 2c_0 \sum_{j=1}^{r}\int_\Omega \left|u^T\right| PA(u)\left|C_j\right| \\ &\quad \cdot\left|f(u(t-\tau_1(t),x))\right| dx + 2(1-c_0)\sum_{j=1}^{r}\int_\Omega\left|u^T\right| \\ &\quad \cdot PA(u)\left|C_j\right|\left|f(u(t-\tau_2(t),x))\right| dx \\ &\quad + 2\sum_{j=1}^{r}\int_\Omega\left|u^T\right| PA(u)\left|M_j\right|\int_{t-\rho(t)}^{t}\left|f(u(s,x))\right| ds\, dx. \end{aligned} \tag{28}$$

Besides, gathering (H1) and (H2) gives

$$\int_\Omega u^T PA(u) B(u) dx \geqslant \underline{p}\lambda_{\min}(\underline{A}\mathbb{B})\|u\|_p^p. \tag{29}$$

It follows by (H1), (H3), and Lemma 3 that

$$\begin{aligned} c_0\int_\Omega\left|u^T\right| PA(u)\left|C_j\right|\left|f(u(t-\tau_1(t),x))\right| dx &= c_0\sum_{k=1}^{n}\sum_{i=1}^{n}\int_\Omega p_i\left|u_i a_i(u_i)\right|\left|c_{ik}^{(j)}\right|\left|f_k(u_k(t-\tau_1(t),x))\right| dx \\ &\leqslant c_0\overline{p}\sum_{k=1}^{n}\sum_{i=1}^{n}\int_\Omega \overline{a}_i\left|c_j\right|\left|u_i\right|^{p-1} F_k\left|u_k(t-\tau_1(t),x)\right| dx \\ &\leqslant c_0\overline{p}\left|c_j\right|\lambda_{\max}\overline{A}\lambda_{\max}F\sum_{k=1}^{n}\sum_{i=1}^{n}\int_\Omega\left(\frac{p-1}{p}\left|u_i\right|^p + \frac{\left|u_k(t-\tau_1(t),x)\right|^p}{p}\right) dx \\ &= c_0\frac{(p-1)\overline{p}}{p} n\left|c_j\right|\lambda_{\max}\overline{A}\lambda_{\max}F\|u\|_p^p \\ &\quad + c_0\frac{\overline{p}}{p} n\left|c_j\right|\lambda_{\max}\overline{A}\lambda_{\max}F\sum_{k=1}^{n}\int_\Omega\left|u_k(t-\tau_1(t),x)\right|^p dx. \end{aligned} \tag{30}$$

Similarly,

$$\begin{aligned} (1-c_0)\int_\Omega\left|u^T\right| PA(u)\left|C_j\right|\left|f(u(t-\tau_2(t),x))\right| dx &\leqslant (1-c_0)\frac{(p-1)\overline{p}}{p} n\left|c_j\right|\lambda_{\max}\overline{A}\lambda_{\max}F\|u\|_p^p \\ &\quad + (1-c_0)\frac{\overline{p}}{p} n\left|c_j\right|\lambda_{\max}\overline{A}\lambda_{\max}F\sum_{k=1}^{n}\int_\Omega\left|u_k(t-\tau_2(t),x)\right|^p dx, \end{aligned}$$

$$\begin{aligned}
&\int_{\Omega} \left|u^T\right| PA(u) \left|M_j\right| \int_{t-\rho(t)}^{t} \left|f(u(s,x))\right| ds\,dx = \sum_{k=1}^{n}\sum_{i=1}^{n} \int_{\Omega} p_i \left|u_i a_i(u_i)\right| \left|m_{ik}^{(j)}\right| \int_{t-\rho(t)}^{t} \left|f_k(u_k(s,x))\right| ds\,dx \\
&\leqslant \overline{p} \sum_{k=1}^{n}\sum_{i=1}^{n} \int_{\Omega} \int_{t-\rho(t)}^{t} \overline{a}_i \left|m_j\right| \left|u_i(t,x)\right|^{p-1} F_k \left|u_k(s,x)\right| ds\,dx \\
&\leqslant \overline{p} \left|m_j\right| \lambda_{\max}\overline{A}\lambda_{\max}F \sum_{k=1}^{n}\sum_{i=1}^{n} \int_{\Omega} \int_{t-\rho(t)}^{t} \left(\frac{p-1}{p}\left|u_i(t,x)\right|^p + \frac{\left|u_k(s,x)\right|^p}{p}\right) ds\,dx \leqslant \rho \frac{(p-1)\overline{p}}{p} n \left|m_j\right| \lambda_{\max}\overline{A}\lambda_{\max}F \|u\|_p^p \\
&+ \frac{\overline{p}}{p}\left|m_j\right| \lambda_{\max}\overline{A}\lambda_{\max}F \sum_{i=1}^{n}\sum_{k=1}^{n} \int_{\Omega} \int_{t-\rho(t)}^{t} \left|u_k(s,x)\right|^p ds\,dx.
\end{aligned} \tag{31}$$

On the other hand,

$$\begin{aligned}
V_2'(t) &= 2c_0 n \frac{\overline{p}}{p}\left(\sum_{j=1}^{r}\left|c_j\right|\right) \\
&\cdot \lambda_{\max}\overline{A}\lambda_{\max}F \left(\sum_{k=1}^{n} \int_{\Omega} \left|u_k(t,x)\right|^p dx \right. \\
&\left. - \sum_{k=1}^{n}\left(1-\tau_1'(t)\right) \int_{\Omega} \left|u_k(t-\tau_1(t),x)\right|^p dx \right) \\
&\leqslant 2c_0 n \frac{\overline{p}}{p}\left(\sum_{j=1}^{r}\left|c_j\right|\right) \lambda_{\max}\overline{A}\lambda_{\max}F \|u\|_p^p - 2c_0 n \\
&\cdot \frac{\overline{p}}{p}\left(\sum_{j=1}^{r}\left|c_j\right|\right) \lambda_{\max}\overline{A}\lambda_{\max}F(1-\tau_1) \\
&\cdot \sum_{k=1}^{n} \int_{\Omega} \left|u_k(t-\tau_1(t),x)\right|^p dx.
\end{aligned} \tag{32}$$

Similarly,

$$\begin{aligned}
V_3'(t) &\leqslant 2(1-c_0) n \frac{\overline{p}}{p}\left(\sum_{j=1}^{r}\left|c_j\right|\right) \lambda_{\max}\overline{A}\lambda_{\max}F \|u\|_p^p \\
&- 2(1-c_0) n \frac{\overline{p}}{p}\left(\sum_{j=1}^{r}\left|c_j\right|\right) \\
&\cdot \lambda_{\max}\overline{A}\lambda_{\max}F(1-\tau_2) \\
&\cdot \sum_{k=1}^{n} \int_{\Omega} \left|u_k(t-\tau_2(t),x)\right|^p dx.
\end{aligned} \tag{33}$$

Next, we need to recall some facts derived by mathematical analysis. Assume that $\eta(t,s)$ is continuous on variables t and s, and $\partial\eta/\partial t$ exists, utilizing the integral middle value theorem reaches

$$\begin{aligned}
\frac{d}{dt}\int_{\xi(t)}^{\varpi(t)} \eta(t,s)\,ds &= \varpi'(t)\,\eta(t,\varpi(t)) \\
&- \xi'(t)\,\eta(t,\xi(t)) \\
&+ \int_{\xi(t)}^{\varpi(t)} \frac{\partial\eta(t,s)}{\partial t} ds,
\end{aligned} \tag{34}$$

where both $\xi(\cdot)$ and $\varpi(\cdot)$ are differentiable.

Moreover, we can derive by employing (32) time and again

$$\begin{aligned}
V_4'(t) &= 2\frac{\overline{p}}{p}\left(\sum_{j=1}^{r}\left|m_j\right|\right) \lambda_{\max}\overline{A}\lambda_{\max}F \sum_{i=1}^{n}\sum_{k=1}^{n} \int_{\Omega} \left(\int_{-\rho}^{0} \left|u_k(t,x)\right|^p ds - \int_{-\rho}^{0} \left|u_k(t+s,x)\right|^p ds\right) dx \\
&= 2\frac{\overline{p}}{p}\left(\sum_{j=1}^{r}\left|m_j\right|\right) \lambda_{\max}\overline{A}\lambda_{\max}F \sum_{i=1}^{n}\sum_{k=1}^{n} \int_{\Omega} \left(\rho\left|u_k(t,x)\right|^p - \int_{t-\rho(t)}^{t} \left|u_k(s,x)\right|^p ds\right) dx \\
&= 2\frac{\overline{p}}{p}\left(\sum_{j=1}^{r}\left|m_j\right|\right) \lambda_{\max}\overline{A}\lambda_{\max}F \left(n\rho\|u\|_p^p - \sum_{i=1}^{n}\sum_{k=1}^{n} \int_{\Omega} \int_{t-\rho(t)}^{t} \left|u_k(s,x)\right|^p ds\,dx\right).
\end{aligned} \tag{35}$$

Combining (28)–(35) results in

$$V'(t) \leqslant -2\left[\lambda_1 D\underline{p} + \underline{p}\lambda_{\min}(\underline{A}\mathbb{B}) - \frac{n}{p} \cdot \sum_{j=1}^{r}\left(\overline{p}(p-1)|c_j| + \rho(p-1)\overline{p}|m_j| + c_0\frac{\overline{p}}{1-\tau_1}|c_j| + (1-c_0)\frac{\overline{p}}{1-\tau_2}|c_j| + \rho\overline{p}|m_j|\right) \cdot \lambda_{\max}\overline{A}\lambda_{\max}F\right]\|u\|_p^p \leqslant 0. \quad (36)$$

Now the standard Lyapunov functional theory derives that the null solution of the fuzzy system (8) is globally asymptotically stable.

Remark 8. In the case of Takagi-Sugeno fuzzy model, our Theorem 6 is better than [38, Theorem 4] because the condition (H1) is weaker than the bounded assumption (2).

Remark 9. In Theorem 6, (22) illustrates the influence of nonlinear diffusion on the stability of system (8) while its role was always ignored in existing results (see, e.g., [5, 39, 44]).

Theorem 6 derives the following corollary.

Corollary 10. *If there exists a positive definite matrix $P = \operatorname{diag}(p_1, p_2, \dots, p_n)$ and two positive scalars $\underline{p}$, $\overline{p}$ such that the following inequalities hold:*

$$\begin{gathered}\underline{p}\lambda_{min}(\underline{A}\mathbb{B}) > \frac{n\overline{p}}{p}\sum_{j=1}^{r}\left((p-1)|c_j| + \rho(p-1)|m_j| + \frac{c_0|c_j|}{1-\tau_1} + \frac{(1-c_0)|c_j|}{1-\tau_2} + \rho|m_j|\right)\lambda_{max}\overline{A}\lambda_{max}F, \\ P > \underline{p}I, \\ P < \overline{p}I,\end{gathered} \quad (37)$$

then the null solution of the ordinary integrodifferential equations (9) is globally asymptotically stable.

Furthermore, if both diffusion behaviors and distributed delay are ignored, we derive from Corollary 10.

Corollary 11. *If there exists a positive definite matrix $P = \operatorname{diag}(p_1, p_2, \dots, p_n)$ and two positive scalars $\underline{p}$, $\overline{p}$ such that the following inequalities hold:*

$$\begin{gathered}\underline{p}\lambda_{min}(\underline{A}\mathbb{B}) > \frac{n\overline{p}}{p} \cdot \sum_{j=1}^{r}\left((p-1)|c_j| + \frac{c_0|c_j|}{1-\tau_1} + \frac{(1-c_0)|c_j|}{1-\tau_2}\right) \cdot \lambda_{max}\overline{A}\lambda_{max}F, \\ P > \underline{p}I, \\ P < \overline{p}I,\end{gathered} \quad (38)$$

then the null solution of the following fuzzy system

$$\begin{gathered}\frac{dx(t)}{dt} = -A(x(t))\left\{B(x(t)) - \sum_{j=1}^{r}h_j(\omega(t)) \cdot \left[c_0C_jf(x(t-\tau_1(t))) + (1-c_0) \cdot C_jf(x(t-\tau_2(t))) + (\mathfrak{C}-c_0) \cdot \left(C_jf(x(t-\tau_1(t))) - C_jf(x(t-\tau_2(t)))\right)\right]\right\}, \\ t \geqslant 0, \\ x(\theta) = \phi(\theta), \quad \theta \in (-\infty, 0]\end{gathered} \quad (39)$$

is globally asymptotically stable.

Remark 12. Condition (H1) is weaker than the bounded conditions on amplification functions of existing literature ([3–6, 9, 10, 24, 27, 28]).

Discussion 1. In recent related literature ([27, 45–51]), some new conditions and methods were presented, and their results were very good. However, some of the methods and conditions are not applicable for system (8) with nonlinear p-Laplacian diffusion. How to apply the new conditions and methods of [45–49] to our system (8) is an interesting problem.

4. Methods and Numerical Example

4.1. Methods. In this paper, Lyapunov functional method is employed to derive the stability criterion. In this process, the integral middle value theorem together with the derivation formula on integral upper limit functions plays the important roles.

Example 1. Consider the following Takagi-Sugeno p-Laplace fuzzy T-S dynamic equations.

Fuzzy Rule 1. IF $\omega_1(t)$ is μ_{11}, and $\omega_2(t)$ is μ_{12}, THEN

$$\begin{aligned}\frac{\partial u}{\partial t} = {} & \nabla \cdot \left(\mathscr{D}(t,x,u) \circ \nabla_p u\right) - A(u)\Big[B(u) \\ & - c_0C_1f(u(t-\tau_1(t),x)) - (1-c_0) \\ & \cdot C_1f(u(t-\tau_2(t),x)) - (\mathfrak{C}-c_0) \\ & \cdot (C_1f(u(t-\tau_1(t),x)) - C_1f(u(t-\tau_2(t),x))) \\ & - M_1\int_{t-\rho(t)}^{t}f(u(s,x))\,ds\Big],\end{aligned}$$

$$u(\theta,x)=\phi(\theta,x),\quad (\theta,x)\in(-\infty,0]\times\Omega,$$
$$u(t,x)=0\in R^2,\quad (t,x)\in R\times\partial\Omega. \tag{40}$$

Fuzzy Rule 2. IF $\omega_1(t)$ is μ_{21}, and $\omega_2(t)$ is μ_{22}, *THEN*

$$\begin{aligned}\frac{\partial u}{\partial t}&=\nabla\cdot\left(\mathscr{D}(t,x,u)\circ\nabla_p u\right)-A(u)\Big[B(u)\\&-c_0C_2f\left(u\left(t-\tau_1(t),x\right)\right)-(1-c_0)\\&\cdot C_2f\left(u\left(t-\tau_2(t),x\right)\right)-(\mathfrak{C}-c_0)\\&\cdot\left(C_2f\left(u\left(t-\tau_1(t),x\right)\right)-C_2f\left(u\left(t-\tau_2(t),x\right)\right)\right)\\&-M_2\int_{t-\rho(t)}^{t}f\left(u(s,x)\right)ds\Big],\\u(\theta,x)&=\phi(\theta,x),\quad(\theta,x)\in(-\infty,0]\times\Omega,\\u(t,x)&=0\in R^2,\quad(t,x)\in R\times\partial\Omega,\end{aligned}\tag{41}$$

where $u(t,x)=(u_1(t,x),u_2(t,x))^T$, $\Omega=(0,\pi)$, $p=4/3$, and then Remark 1 gives

$$\lambda_1=\left(\frac{2}{\pi}\int_0^{(p-1)^{1/p}}\frac{dt}{\left(1-t^p/(p-1)\right)^{1/p}}\right)^p=0.7915. \tag{42}$$

Let $\tau_1(t)=t/3$, $\tau_2(t)=t/2$, and then $\tau_1=1/3$, $\tau_2=1/2$. Let $a_i(u_i)=0.1u_i^{-2/3}(1+e^{-iu_i^2})$, $i=1,2$, $b_1(u_1)=2u_1(1+\sin^2u_1)$, $b_2(u_2)=1.95u_2$, $f_1(u_1(t-\tau(t)))=0.16u_1(t-\tau(t))\sin u_1(t-\tau(t))$, $f_2(u_2(t-\tau(t)))=0.166u_2(t-\tau(t))$, $\tau=0.5$, $D=0.003$, $c_0=0.75$, $c_1=0.2$, $c_2=0.3$, $m_1=0.02$, $m_2=0.03$, and

$$\begin{aligned}\underline{A}&=\begin{pmatrix}0.01&0\\0&0.02\end{pmatrix},\\\overline{A}&=\begin{pmatrix}0.1&0\\0&0.2\end{pmatrix},\\\mathbb{B}&=\begin{pmatrix}2&0\\0&1.95\end{pmatrix},\\D(t,x,u)&=\begin{pmatrix}0.003&0.005\\0.004&0.006\end{pmatrix},\\M_1&=\begin{pmatrix}0.02&0.01\\0&0.01\end{pmatrix},\\F&=\begin{pmatrix}0.16&0\\0&0.166\end{pmatrix},\\C_1&=\begin{pmatrix}0.2&0.1\\0&0.15\end{pmatrix},\\C_2&=\begin{pmatrix}0.2&0.1\\0&0.3\end{pmatrix},\\M_2&=\begin{pmatrix}0.01&0.01\\0&0.03\end{pmatrix}.\end{aligned}\tag{43}$$

Now we use MATLAB to solve LMIs (22)-(23), obtaining the feasibility data

$$\begin{aligned}P&=\begin{pmatrix}0.9381&0\\0&1.013\end{pmatrix},\\\underline{p}&=0.9103,\\\overline{p}&=1.023.\end{aligned}\tag{44}$$

Now Theorem 6 derives that the null solution of this Takagi-Sugeno fuzzy equations is globally asymptotically stable (see Figures 1 and 2).

5. Conclusions

By constructing a novel Lyapunov function, we employed Young inequality and LMI technique to derive the asymptotic stability criteria for CGNNs with distributed delays and nonlinear diffusion. Since the stability of nonlinear p-Laplacian diffusion neural networks was originally investigated in [2], various p-Laplacian diffusion neural networks have attracted a lot of interest ([6, 17, 34, 39, 44]). As pointed out in Discussion 1, some new conditions and methods may not be applicable to CGNNs model with nonlinear p-Laplacian diffusion. So our results are a novelty to some extent.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

Xiongrui Wang wrote the original manuscript, carrying out the main part of this paper. Shouming Zhong checked it, and Ruofeng Rao is in charge of correspondence. All authors read and approved the final manuscript.

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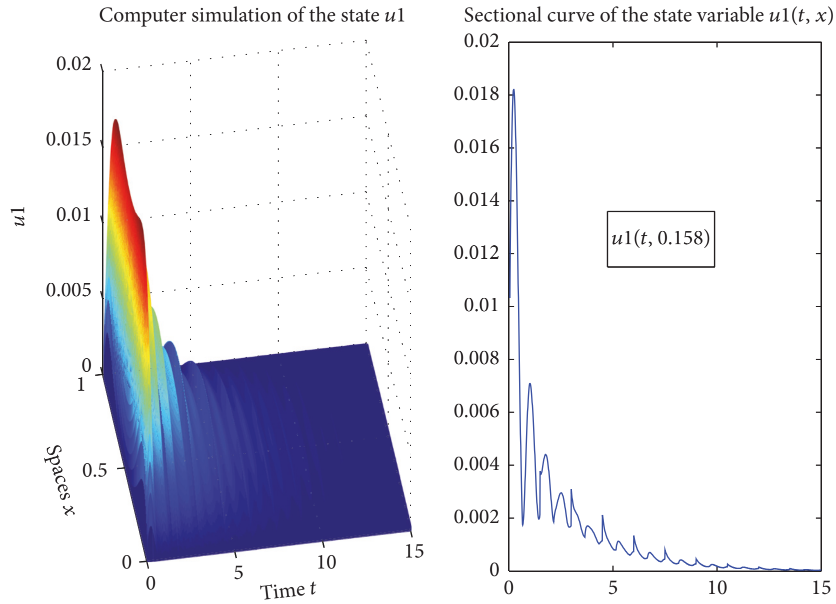

Figure 1: Computer simulations of the state $u_1(t, x)$.

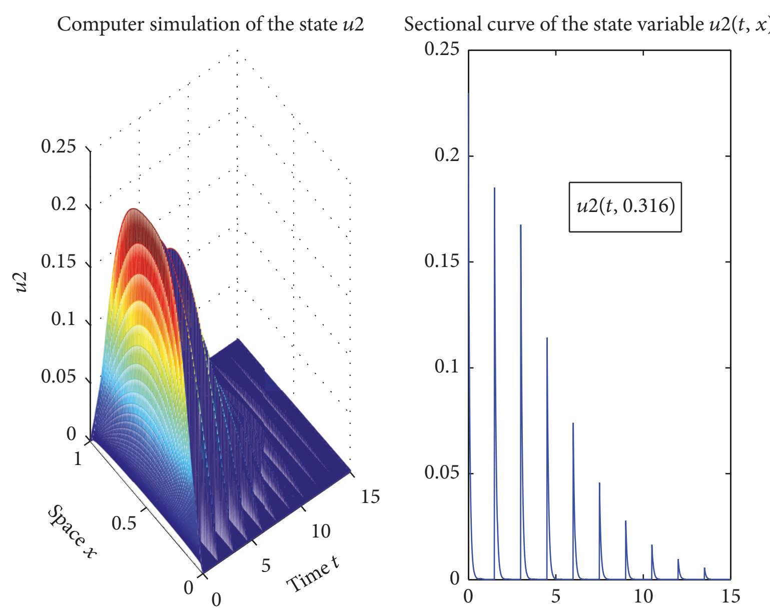

Figure 2: Computer simulations of the state $u_2(t, x)$.

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