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*Research Article*

# Computational Framework for Optimal Carbon Taxes Based on Electric Supply Chain Considering Transmission Constraints and Losses

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A modeling and computational framework is presented for the determination of optimal carbon taxes that apply to electric power plants in the context of electric power supply chain with consideration of transmission constraints and losses. In order to achieve this goal, a generalized electric power supply chain network equilibrium model is used. Under deregulation, there are several players in electrical market: generation companies, power suppliers, transmission service providers, and consumers. Each player in this model tries to maximize its own profit and competes with others in a noncooperative manner. The Nash equilibrium conditions of these players in this model form a finite-dimensional variational inequality problem (VIP). By solving this VIP via an extragradient method based on an interior point algorithm, the optimal carbon taxes of power plants can be determined. Numerical examples are provided to analyze the results of the presented modeling.

## 1. Introduction

Controlling carbon emissions by reducing fossil fuel use is the key to control the global warming problem. As a fundamental resource to supply modern economies and societies, electric power heavily relies on fossil fuel sources before conversion to electricity and thereby has an immense environmental impact. For example, more than one-third of the total US emissions of carbon dioxide ($CO_2$) and nitrous oxide ($NO_2$) comes from generating electricity [1]. In Taiwan, 74% of greenhouse gas arises from fossil fuel [2] and 50% of fossil fuel is consumed by the electric utility [3]. Any policy that aimed at mitigating the immense risks of unstable climate must directly consider the electricity industry [4, 5]. Carbon taxes and tradable carbon emission permits are now two popular instruments under evaluation internationally to control the increase of $CO_2$ emissions. Imposing taxes on carbon emissions will increase the price of delivered electricity, thereby making fossil fuel based power sources relatively expensive over renewable energy sources and decreasing the energy demand (due to negative price elasticity of the electricity demand). As a result, supply-side response takes place in the form of interfuel and technological substitution in power generation, for example, investment in energy production with less carbon intensive technologies to improve logistic planning and substitution of fossil fuels with renewable energy sources. Therefore, as noted in Nagurney et al. [1], a mathematic modeling framework capable to capture the interactions among decision-makers in an electric power supply chain network coupled with the incorporation of environmental policies such as carbon taxes is of great practical as well as policy-making importance.

There are several studies on carbon tax [6–15]. Kainuma et al. [6, 7] presented an end-use energy model for forecasting $CO_2$ emissions and assessing policy options to reduce greenhouse gas emissions. Their model evaluates the effects of imposing a carbon tax in combination with subsidies on various carbon-emitting technologies to reduce $CO_2$. Karki et al. [8] examined the substitution and price effects of carbon taxes using HOMER (Hybrid Optimization Model for Electric Renewables) model to mitigate $CO_2$ from distributed generators (DGs) in India. Shrestha and Marpaung [9] examined the implications of carbon tax for power sector development, demand-side management programs, and environmental emissions in the case of Indonesia from a long-term integrated resource planning perspective. Their study deals with the substitution possibilities among various centralized power plants such as hydro, thermal coal, thermal oil, thermal gas, and geothermal plants. Hua and Wu [10] evaluated the impact of carbon taxes on Taiwan's manufacturing sectors: chemical, paper, and printing, iron and steel, and cement sectors. Their study focuses on producers' interinput or interfuel substitution behavior as producers adjust to the carbon tax. Newcomer et al. [11] estimated the short-run carbon-reduction impacts of a policy where carbon emissions from electric power plants were taxed or otherwise priced and where all consumers saw and could respond to real-time market prices that reflected the cost of generation. Their analysis covers three regional transmission organizations in the U.S. Tamura and Kimura [12] formulated a dynamic model of a profit maximization problem for evaluating quantitatively how the policy of carbon tax and emissions trading would be effective to achieve the target reduction of the Kyoto Protocol. Wu et al. [13] proposed an electric power supply chain network equilibrium model with carbon taxes that were applied prior to distinct power generator/power plant combinations and demonstrated that the model could be reformulated and solved as a transportation network equilibrium problem with elastic demands. Nagurney and Liu [14] demonstrated that electric power supply chain network equilibrium problems can be reformulated and solved as transportation network equilibrium problems. They proved that the variational inequality formulations of the governing equilibrium conditions coincide with the corresponding variational inequalities of transportation network equilibrium problems over appropriately constructed supernetworks. Woolley et al. [15] developed a multipollutant permit trading model in the case of electric power supply chains in which there were different technologies associated with electric power production. Kockar et al. [16] investigated the effects that emissions constraints might have on market clearing prices in electricity markets. Their analysis is based on a two-step procedure in which the emissions generation scheduling problem is solved first, and then its solution is used in the dynamic optimal power flow problem that also accounts for emissions constraints despite the augmented cost function that includes possible purchases or sales of emissions allowances on the market. Their formulation allows for investigating how decisions of generators on how to use their $CO_2$ emission allocations over a period of time may affect market outcome and prices. Careri et al. [17] considered the impact of renewable energy sources (RES) incentives and $CO_2$ mitigation policies in the framework of the generation planning problem to be solved by a generation company (GENCO). Renewable energy quota and $CO_2$ emission limits were modeled as a set of new constraints in the traditional generation expansion planning. Expenditures for $CO_2$ emission right purchase, cash flows deriving from green certificate trading, and feed-in tariffs were formulated in the objective function to be maximized in the overall GENCO profit. This model allows the consideration of most of the present day incentives designed to support energy generation from renewable sources as well as most of the measures intended to discourage the use of fossil fuels. Chu et al. [18] applied economic Model Predictive Control (MPC) to a Regional dynamic Integrated model of Climate and the Economy (RICE model) as a test bed to design savings rates and global carbon tax for greenhouse gas emissions. The proposed general framework is based on feedback control theory to design sustainable policies to mitigate the climate change and global warming process and at the same time to minimize its effect on economic growth. He et al. [19] developed a simple optimization model from a microlevel to clarify impacts of administrative and market carbon emission abatement scheme on firm's production plan and expected net income when the firm is confronted with a random demand on its product. Chen et al. [20] tackled the optimal social welfare problem based on a mathematic modeling framework that is capable to capture the interactions among decision-makers in an electric power supply chain network with consideration of transmission power flows and constraints using a modified penalty function method. Reference [21] outlines several options and considerations in weighting a carbon tax, such as environmental integrity, cost-effectiveness, and distributional equity, as well as fundamental design issues. Imposing a carbon tax can help correct the market failure that exists when the value of environmental damages is not included in the market price of fossil fuels. Also, imposing carbon tax will raise significant revenue for government and impose costs on the economy, but the magnitude of those costs is directly related to how the revenue is used, which will ultimately be a political decision. Utilizing the revenue to reduce taxes on things we want to encourage, such as labor and capital investment, can maximize the economic benefits from the tax. However, a carbon tax could be subject to political compromises. Political pressure from powerful interest groups may make the decision-makers yield and therefore may dilute the effectiveness of the policy, compromising the environmental objective and reducing the availability of potentially lower cost emission reductions. Nagurney et al. [1] demonstrated how carbon taxes can be determined optimally and endogenously within a generalized electric power supply chain network equilibrium model. This model allows the government to impose bound(s) on the total amount of carbon emission and the optimal carbon taxes guarantee that the bound(s) are not exceeded. However, some physical laws existing in the power system network are not considered in their model, for example, transmission constraints and power flows. For a power system network, power flows are governed by Kirchhoff's Current and Voltage Laws and transmission line flow limits are constrained by operational security reasons.

In this paper, an extended model based on Nagurney et al.'s [1] is presented for the determination of optimal carbon taxes that applies to electric power plants in the context of electric power supply chain with consideration of

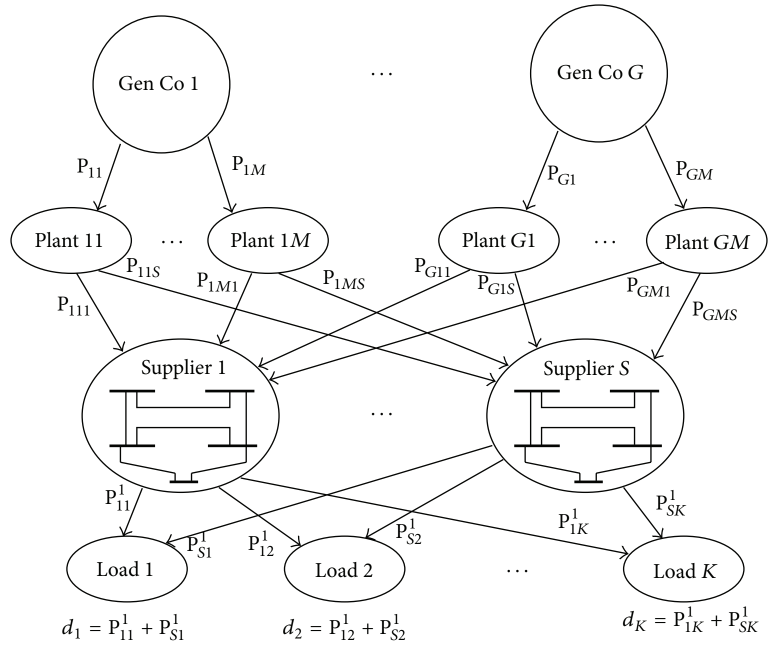


Figure 1: Electric power supply chain with transmission networks.

transmission constraints, transmission losses, and load flows. In order to achieve this goal, a generalized electric power supply chain network equilibrium model is used. Under deregulation, there are several players in the electrical market: generation companies, power suppliers, transmission service providers, and consumers. Each player in this model tries to maximize its own profit and competes with others in a noncooperative manner. The Nash equilibrium conditions of these players in this model form a finite-dimensional variational inequality problem (VIP). By solving this VIP via an extragradient method based on an interior point algorithm, the optimal carbon taxes of power plants can be determined.

This paper is organized as follows. In Section 2, an electric power supply chain with transmission constraints is presented. A computational solution methodology is addressed. In Section 3, several numerical examples are provided to analyze the results of the presented modeling. Conclusions are then drawn in the last section.

## 2. Electric Power Supply Chain with Transmission Constraints

Based on the electric power supply chain model presented in [1], a modified version [22] is presented in this paper, as shown in Figure 1. The modified electric power supply chain network consists of $G$ generator companies (Gen Cos), $M$ power plants for each power company, $S$ suppliers, and $K$ loads (demand markets) with one transmission provider. The power from supplier $s$ to load $k$, $p^1_{sk}$, is through only one transmission provider; that is, the superscript of $p^1_{sk}$ is 1. If there are two transmission providers in the electric power supply chain network, there will exist another link, $p^2_{sk}$, from supplier $s$ to load $k$. There are several players in this model: Gen Cos, suppliers, and demand markets. Each player is considered as a profit-maximizer. The transmission provider usually does not participate in the deregulated electricity market, since it needs to transmit power unbiased from the Gen Cos to loads. The role of transmission provider is like the Independent System Operation (ISO). It is not a player in the market. Therefore, no optimization problem is associated with it in this electric power supply chain network. The optimization problem of Gen Co $g$ can be expressed as follows:

$$\text{Max} \quad \sum_{m=1}^{M}\sum_{s=1}^{S}\rho^*_{1gms}p_{gms} - \sum_{m=1}^{M} f_{gm}\left(p_{gm}\right) - \sum_{m=1}^{M}\sum_{s=1}^{S}c_{gms}\left(p_{gms}\right) - \sum_{m=1}^{M}\tau^*_{gm}e_{gm}p_{gm}, \tag{1}$$

$$\text{s.t.} \quad \sum_{s=1}^{S}p_{gms} = p_{gm}, \quad m = 1,\dots,M, \tag{2}$$

$$p_{gm} \geq 0, \;\; p_{gms} \geq 0, \;\; m = 1,\dots M, \;\; s = 1,\dots,S, \tag{3}$$

where $p_{gm}$ is the quantity of electricity produced by Gen Co. $g$ using power plant $m$, $g = 1,\dots,G$, $m = 1,\dots,M$; $p_{gms}$ is the electric power flow between the power plant $m$ of the Gen Co. $g$ and the power supplier $s$, $g = 1,\dots,G$, $m = 1,\dots,M$, $s = 1,\dots,S$; $\rho^*_{1gms}$ is the price for selling $p_{gms}$ to supplier $s$; $f_{gm}(p_{gm})$ is the cost function of $p_{gm}$; $c_{gms}(p_{gms})$ is the transaction cost of $p_{gms}$; $e_{gm}$ is the amount of carbon emitted by Gen Co $g$ using power plant $m$ per unit of electric power produced; and $\tau^*_{gm}$ is the carbon tax imposed on $p_{gm}$.

The first term in (1) represents the revenue, and the next two terms are the power generation cost and transaction cost, respectively. The last term in (1) is the total payout in carbon taxes by the Gen Co $g$ based on the total pollution emitted by its power plants. Equation (2) is the conservation of flow equation. The Gen Cos compete in a noncooperative manner in the sense of Nash game [23]. The optimality conditions for all Gen Cos simultaneously coincide with the solution of the following variational inequalities:

$$\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}\left(p_{gm}^{*}\right)}{\partial p_{gm}}+\tau_{gm}^{*}e_{gm}\right]\left[p_{gm}-p_{gm}^{*}\right] + \sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}\left(p_{gms}^{*}\right)}{\partial p_{gms}}-\rho_{1gms}^{*}\right]\left[p_{gms}-p_{gms}^{*}\right] \geq 0, \quad \forall\left(p_{gm},p_{gms}\right)\in\kappa^{1}, \tag{4}$$

where $\kappa^1 \equiv \{(p_{gm}, p_{gms}) \mid (2) \text{ and } (3) \text{ hold}\}$.

The power suppliers are involved in transactions both with the Gen Cos and with the consumers at demand markets through the transmission service providers. The optimization problem associated with supplier $s$ can be expressed as follows:

$$\text{Max} \quad \sum_{k=1}^{K}\sum_{t=1}^{T}\rho_{2sk}^{t*}p_{sk}^{t}-c_{s}\left(h_{s}\right)-\sum_{g=1}^{G}\sum_{m=1}^{M}\rho_{1gms}^{*}p_{gms} - \sum_{k=1}^{K}\sum_{t=1}^{T}c_{sk}^{t}\left(p_{sk}^{t}\right)-\sum_{g=1}^{G}\sum_{m=1}^{M}\hat{c}_{gms}\left(p_{gms}\right) \tag{5}$$

$$\text{s.t.} \quad \sum_{g=1}^{G}\sum_{m=1}^{M}p_{gms}=h_{s} \tag{6}$$

$$\sum_{g=1}^{G}\sum_{m=1}^{M}p_{gms}=\sum_{k=1}^{K}\sum_{t=1}^{T}p_{sk}^{t}, \tag{7}$$

$$p_{gms}\geq 0, \; g=1,\ldots,G; \; m=1,\ldots M, \tag{8}$$

$$p_{sk}^{t}\geq 0, \; k=1,\ldots,K; \; t=1,\ldots,T, \tag{9}$$

where $p_{sk}^t$ is the power flow between supplier $s$ and demand market $k$ through transmission provider $t$, $s = 1,\ldots,S$, $k = 1,\ldots,K$, $t = 1,\ldots,T$; $\rho_{2sk}^{t*}$ is the price for selling $p_{sk}^t$ to demand market $k$ through transmission provider $t$; $h_s$ is the total power supply of supplier $s$; $c_s(h_s)$ is the operating cost function of supplier $s$; $c_{sk}^t(p_{sk}^t)$ is the transaction cost incurred by power supplier $s$ in transacting with demand market $k$ via transmission provider $t$; and $\hat{c}_{gms}(p_{gms})$ is the transaction cost incurred by power supplier $s$ in transacting with Gen Co. $g$ for power generated by power plant $m$.

The optimality conditions for all suppliers can be expressed as the following variational inequalities:

$$\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{sk}^{t}\left(p_{sk}^{t*}\right)}{\partial p_{sk}^{t}}-\rho_{2sk}^{t*}\right]\left[p_{sk}^{t}-p_{sk}^{t*}\right] + \sum_{s=1}^{S}\frac{\partial c_{s}\left(h_{s}^{*}\right)}{\partial h_{s}}\left[h_{s}-h_{s}^{*}\right] + \sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial \hat{c}_{gms}\left(p_{gms}^{*}\right)}{\partial p_{gms}}+\rho_{1gms}^{*}\right]\left[p_{gms}-p_{gms}^{*}\right] \geq 0, \quad \forall\left(p_{gms},p_{sk}^{t},h_{s}\right)\in\kappa^{2}, \tag{10}$$

where $\kappa^2 \equiv \{(p_{gms}, p_{sk}^t, h_s) \mid (6)\sim(9) \text{ hold}\}$.

For each demand market $k$, the following optimization is formulated:

$$\text{Max} \quad \rho_{3k}\left(d_{k}\right)d_{k}-\sum_{t=1}^{T}\sum_{S=1}^{S}\rho_{2sk}^{t*}p_{sk}^{t} - \sum_{k=1}^{K}\sum_{t=1}^{T}\hat{c}_{sk}^{t}\left(p_{sk}^{t}\right)p_{sk}^{t}, \tag{11}$$

$$\text{s.t.} \quad \sum_{s=1}^{S}\sum_{t=1}^{T}p_{sk}^{t}=d_{k}, \tag{12}$$

where $d_k$ is the demand at demand market $k$; $\rho_{3k}$ is the demand market price function at demand market $k$, $k = 1,\ldots,K$; and $\hat{c}_{sk}^t(p_{sk}^t)$ is the unit transaction cost incurred by consumers at demand market $k$ in transacting with power supplier $s$ via transmission provider $t$.

The optimality conditions for all demand markets can be expressed as the following variational inequalities:

$$\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\hat{c}_{sk}^{t}\left(p_{sk}^{t*}\right)+\rho_{2sk}^{t*}\right]\left[p_{sk}^{t}-p_{sk}^{t*}\right] - \sum_{k=1}^{K}\rho_{3k}\left(d_{k}^{*}\right)\left[d_{k}-d_{k}^{*}\right]\geq 0, \quad \forall\left(p_{sk}^{t},d_{k}\right)\in\kappa^{3}, \tag{13}$$

where $\kappa^3 \equiv \{(p_{sk}^t, d_k) \mid (12) \text{ holds}\}$.

As commonly done in a real-life environmental policy-making, bounds are applied in terms of the maximum carbon emissions that are allowed for each Gen Co or each power plant. If the carbon emission of a particular Gen Co or power plant is fewer than the imposed limit, then carbon tax is not assigned. If the emission equals the bound, then a tax is imposed. Mathematically, the equilibrium conditions for a carbon tax policy are as follows.

*Decentralized carbon tax* is as follows:

$$\overline{B}_{gm}-e_{gm}p_{gm}^{*}\begin{cases}=0, & \text{if } \tau_{gm}^{*}>0\\ \geq 0, & \text{if } \tau_{gm}^{*}=0.\end{cases} \tag{14}$$

*Centralized carbon tax with fixed upper bound* is as follows:

$$\overline{B}-\sum_{g=1}^{G}\sum_{m=1}^{M}e_{gm}p_{gm}^{*}\begin{cases}=0, & \text{if } \Gamma^{*}>0\\ \geq 0, & \text{if } \Gamma^{*}=0.\end{cases} \tag{15}$$

For the whole power supply chain system with decentralized carbon taxes, the optimal solution solves the following variational inequality problem:

$$
\begin{aligned}
&\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}\left(p_{gm}^{*}\right)}{\partial p_{gm}}+\tau_{gm}^{*}e_{gm}\right]\left[p_{gm}-p_{gm}^{*}\right]\\
&\quad+\sum_{s=1}^{S}\frac{\partial c_{s}\left(h_{s}^{*}\right)}{\partial h_{s}}\left[h_{s}-h_{s}^{*}\right]\\
&\quad+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}\left(p_{gms}^{*}\right)}{\partial p_{gms}}+\frac{\partial \hat{c}_{gms}\left(p_{gms}^{*}\right)}{\partial p_{gms}}\right]\\
&\quad\cdot\left[p_{gms}-p_{gms}^{*}\right]+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{sk}^{t}\left(p_{sk}^{t*}\right)}{\partial p_{sk}^{t}}\hat{c}_{sk}^{t}\left(p_{sk}^{t*}\right)\right]\\
&\quad\cdot\left[p_{sk}^{t}-p_{sk}^{t*}\right]-\sum_{k=1}^{K}\rho_{3k}\left(d_{k}^{*}\right)\left[d_{k}-d_{k}^{*}\right]\\
&\quad+\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\overline{B}_{gm}-e_{gm}p_{gm}^{*}\right]\left[\tau_{gm}-\tau_{gm}^{*}\right]\geq 0,\\
&\qquad\qquad\forall\left(p_{gm},p_{gms},p_{sk}^{t},h_{s},d_{k},\tau_{gm}\right)\in\kappa^{4},
\end{aligned}
\tag{16}
$$

where $\kappa^4 \equiv \{(p_{gm}, p_{gms}, p_{sk}^t, h_s, d_k, \tau_{gm}) \mid (2), (3), (6)\sim(9)$, and (12) hold$\}$.

For the whole power supply chain system with centralized carbon taxes and a fixed upper bound, the optimal solution solves the following variational inequality problem:

$$
\begin{aligned}
&\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}\left(p_{gm}^{*}\right)}{\partial p_{gm}}+\Gamma^{*}e_{gm}\right]\left[p_{gm}-p_{gm}^{*}\right]\\
&\quad+\sum_{s=1}^{S}\frac{\partial c_{s}\left(h_{s}^{*}\right)}{\partial h_{s}}\left[h_{s}-h_{s}^{*}\right]\\
&\quad+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}\left(p_{gms}^{*}\right)}{\partial p_{gms}}+\frac{\partial \hat{c}_{gms}\left(p_{gms}^{*}\right)}{\partial p_{gms}}\right]\\
&\quad\cdot\left[p_{gms}-p_{gms}^{*}\right]+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{sk}^{t}\left(p_{sk}^{t*}\right)}{\partial p_{sk}^{t}}+\hat{c}_{sk}^{t}\left(p_{sk}^{t*}\right)\right]\\
&\quad\cdot\left[p_{sk}^{t}-p_{sk}^{t*}\right]-\sum_{k=1}^{K}\rho_{3k}\left(d_{k}^{*}\right)\left[d_{k}-d_{k}^{*}\right]+\left[\overline{B}\right.\\
&\quad\left.-\sum_{g=1}^{G}\sum_{m=1}^{M}e_{gm}p_{gm}^{*}\right]\left[\Gamma-\Gamma^{*}\right]\geq 0,\\
&\qquad\qquad\forall\left(p_{gm},p_{gms},p_{sk}^{t},h_{s},d_{k},\Gamma\right)\in\kappa^{5},
\end{aligned}
\tag{17}
$$

where $\kappa^5 \equiv \{(p_{gm}, p_{gms}, p_{sk}^t, h_s, d_k, \Gamma) \mid (2), (3), (6)\sim(9)$, and (12) hold$\}$.

The above VIPs (16)~(17) can be formulated in a general form as follows:

$$\left\langle F\left(x^{*}\right),\left(x-x^{*}\right)\right\rangle \geq 0 \quad x^{*}\in C. \tag{18}$$

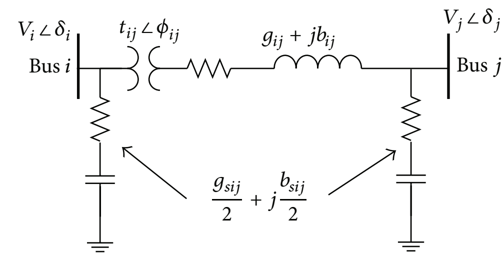

Figure 2: Two-bus power system.

Among many methods [13, 24–26] that are available to solve VIP, the projection method [27] is the simplest one, which, starting from any $x^0 \in C$, iteratively updates $x$ by the following equation:

$$x^{k+1}=Pc\left(x^{k}-\alpha F\left(x^{k}\right)\right), \tag{19}$$

where $Pc(\cdot)$ denotes the orthogonal projection map onto $C$ and $\alpha$ is a step length. $Pc(x^k - \alpha F(x^k))$ is the solution of the following quadratic programming problem, and $x^* \in C$ is a solution of (18) if only if $x^* = Pc(x^* - \alpha F(x^*))$ :

$$\min_{x\in C} x\frac{1}{2}x^{T}x-\left(x^{k}-\alpha F\left(x^{k}\right)\right)^{T}x. \tag{20}$$

In order to enhance the convergence of the projection method, a variant of projection methods, the extragradient method [27], was adopted in this paper. The general scheme of this method is as follows.

Given $x^0 \in C$, we generate a succession $\{x^k\}$ such that

$$
\begin{aligned}
\overline{x}^{k}&=Pc\left(x^{k}-\alpha_{k}F\left(x^{k}\right)\right),\\
x^{k+1}&=Pc\left(x^{k}-\eta_{k}F\left(\overline{x}^{k}\right)\right),
\end{aligned}
\tag{21}
$$

where $\alpha_k$ and $\eta_k$ are constant.

In the above formulations (16)~(17), the power flows governed by Kirchhoff's current and voltage laws and the transmission line flow limits constrained by operational security reasons are not considered. For a 2-bus power system shown in Figure 2, the real-power line flows $P_{ij}$ (from bus $i$ to bus $j$) and $P_{ji}$ (from bus $j$ to bus $i$) are approximately by (22a) and (22b)

$$
\begin{aligned}
P_{ij}&=V_{i}^{2}\left(\frac{g_{sij}}{2}+t_{ij}^{2}g_{ij}\right)\\
&\quad-t_{ij}V_{i}V_{j}\left[g_{ij}\cos\left(\left(\delta_{i}-\phi_{ij}\right)-\delta_{j}\right)\right.\\
&\quad\left.+b_{ij}\sin\left(\left(\delta_{i}-\phi_{ij}\right)-\delta_{j}\right)\right]\approx -b_{ij}\left[\left(\delta_{i}-\phi_{ij}\right)-\delta_{j}\right],
\end{aligned}
\tag{22a}
$$

$$
\begin{aligned}
P_{ji}&=V_{j}^{2}\left(\frac{g_{sij}}{2}+g_{ij}\right)\\
&\quad-t_{ij}V_{i}V_{j}\left[g_{ij}\cos\left(\delta_{j}-\left(\delta_{i}-\phi_{ij}\right)\right)\right.\\
&\quad\left.+b_{ij}\sin\left(\delta_{j}-\left(\delta_{i}-\phi_{ij}\right)\right)\right]\approx -b_{ij}\left[\delta_{j}-\left(\delta_{i}-\phi_{ij}\right)\right]\\
&=b_{ij}\left[\left(\delta_{i}-\phi_{ij}\right)-\delta_{j}\right],
\end{aligned}
\tag{22b}
$$

where $V_i$, $V_j$, and $t_{ij}$ are approximated as 1.0, $g_{ij} \ll b_{ij}$, $\sin\delta \approx \delta$ if $\delta$ is small enough, $\delta_i$ and $\delta_j$ are bus voltage angles, $g_{ij}$ and $b_{ij}$ are the active and reactive components of the complex branch series admittance, respectively, and $g_{sij}$ and $b_{sij}$ are the active and reactive components of the complex branch shunt admittance.

Based on (22a) and (22b), the network DC load flows thus can be expressed as follows:

$$\begin{aligned}
&\mathbf{P} = \mathbf{B}\delta + \mathbf{B}_\phi\phi = \mathbf{P}_g - \mathbf{P}_L \\
&\Longleftrightarrow \begin{bmatrix} P_2 \\ P_3 \\ \vdots \\ P_n \end{bmatrix} \\
&= \begin{bmatrix} B_{22} & B_{23} & \cdots & B_{2n} \\ B_{32} & B_{33} & \cdots & B_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ B_{n2} & B_{n3} & \cdots & B_{nn} \end{bmatrix} \begin{bmatrix} \delta_2 \\ \delta_3 \\ \vdots \\ \delta_n \end{bmatrix} \\
&+ \left[ (\mathbf{B}_\phi)_1 \ \cdots \ (\mathbf{B}_\phi)_K \right] \begin{bmatrix} \phi_{i1,j1} \\ \vdots \\ \phi_{iK,jK} \end{bmatrix} \\
&\Longleftrightarrow \delta = \mathbf{B}^{-1}\left(\mathbf{P}_g - \mathbf{P}_L - \mathbf{B}_\phi\phi\right),
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
B_{ii} &= -\sum_{j\in \text{node}i} b_{ij}, \\
B_{ij} &= B_{ji} = b_{ij}, \\
\left(\mathbf{B}_\phi\right)_k &= [0 \ \cdots \ 0 \ \underset{i\text{th}}{b_{ij}} \ 0 \ \cdots \ 0 \ \underset{j\text{th}}{-b_{ij}} \ 0 \ \cdots \ 0]^T,
\end{aligned} \tag{24}$$

and $\mathbf{P}_L$ is the bus load vector and $\mathbf{P}_g$ is the bus generation vector; $(\mathbf{B}_\phi)_k$ is the column in $\mathbf{B}_\phi$ associated with $k$th phase shifter variable $\phi_{ik,jk}$. The line flows of interest can be expressed as (25), and the constraints on line flows can be expressed as (26), where $\underline{\mathbf{P}}_\mathbf{f}$ and $\overline{\mathbf{P}}_\mathbf{f}$ are the lower and upper limits of the line flows, respectively. The transmission line loss can be approximated as (27).

$$\begin{aligned}
&\begin{bmatrix} \vdots \\ P_{ij} \\ \vdots \end{bmatrix} = \left[ 0 \ \cdots \ 0 \ \underset{\uparrow \atop i\text{th}}{-b_{ij}} \ 0 \ \begin{matrix}\vdots \\ \cdots \\ \vdots\end{matrix} \ 0 \ \underset{\uparrow \atop j\text{th}}{b_{ij}} \ 0 \ \cdots \ 0 \right] \begin{bmatrix} \delta_2 \\ \vdots \\ \delta_i \\ \vdots \\ \delta_j \\ \vdots \\ \delta_n \end{bmatrix} + \begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ 0 & b_{ij} & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ \phi_{ij} \\ \vdots \\ \vdots \end{bmatrix} \\
&\Longleftrightarrow \mathbf{P}_\mathbf{f} = \mathbf{M}\delta + \mathbf{M}'\phi
\end{aligned} \tag{25}$$

$$\underline{\mathbf{P}}_\mathbf{f} \le \mathbf{P}_\mathbf{f} = \mathbf{M}\delta + \mathbf{M}'\phi \le \overline{\mathbf{P}}_\mathbf{f} \tag{26}$$

$$P_{\text{Loss},ij} \cong g_{ij}\left(\delta_i - \delta_j\right)^2. \tag{27}$$

We denote $\psi = L\delta$ as the column vector of angular difference across lines, where $L$ is the line-bus incidence matrix. The total transmission line losses then can be expressed as follows:

$$\begin{aligned}
P_{\text{Loss}} &= \sum_{\text{alllines}} P_{\text{Loss},ij} = \psi^T G \psi = \delta^T L^T G L \delta \\
&= \left(B^{-1}\left(\mathbf{P}_g - \mathbf{P}_L - \mathbf{B}_\phi\phi\right)\right)^T \\
&\quad \cdot L^T G L \left(B^{-1}\left(\mathbf{P}_g - \mathbf{P}_L - \mathbf{B}_\phi\phi\right)\right),
\end{aligned} \tag{28}$$

where

$$G = \begin{bmatrix} g_{12} & & & 0 \\ & g_{13} & & \\ & & \ddots & \\ 0 & & & g_{(n-1)n} \end{bmatrix}. \tag{29}$$

Therefore, $P_{\text{Loss}}$ can be expressed as a quadratic form of the bus generation vector $\mathbf{P}_g$. Adding $P_{\text{Loss}}$ into the optimal carbon tax problem only affects the load balance equation

(7) for each supplier $s$, which can be expressed as (30) [22]. Adding the transmission line flow limits constrained by operational security reasons adds additional equation (26) into the VIP [22]. The other constraints (2), (3), (6), (8), (9), and (12) are intact

$$\sum_{g=1}^{G}\sum_{m=1}^{M} p_{gms} = \sum_{k=1}^{K}\sum_{t=1}^{T} p_{sk}^{t} + P_{\text{Loss}}. \tag{30}$$

During the solution process of the extragradient method, we need to solve quadratic programming problem twice every iteration, which contains nonlinear constraints. In this paper, an interior point algorithm [28] was utilized to enhance the overall computation efficiency. The interior point algorithm solves the nonlinear optimization problem as follows:

$$\begin{aligned} \text{Minimize} \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{x}) = \mathbf{0} \\ & \mathbf{h_l} \le \mathbf{h}(\mathbf{x}) \le \mathbf{h_u} \\ & \mathbf{x_l} \le \mathbf{x} \le \mathbf{x_u}. \end{aligned} \tag{31}$$

By adding slack variables $(s_h, s_{sh}, s_x)$ into (31), we obtain the optimization problem (32). Equation (33) is the modified Lagrangian function of the following equation:

$$\begin{aligned} \min \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & \text{(a)}\ \mathbf{g}(\mathbf{x}) = \mathbf{0} \\ & \text{(b)}\ \mathbf{h}(\mathbf{x}) + \mathbf{s_h} = \mathbf{h_u} \\ & \text{(c)}\ \mathbf{s_h} + \mathbf{s_{sh}} = \mathbf{h_u} - \mathbf{h_l} \\ & \text{(d)}\ \mathbf{x} + \mathbf{s_x} = \mathbf{x_u} \\ & \text{(e)}\ \mathbf{x} - \mathbf{x_l} \ge \mathbf{0},\ \mathbf{s_h}, \mathbf{s_{sh}}, \mathbf{s_x} \ge \mathbf{0} \end{aligned} \tag{32}$$

$$\begin{aligned} L_\mu = {} & f(\mathbf{x}) - \mathbf{y}^{\mathbf{T}}\mathbf{g}(\mathbf{x}) - \mathbf{y_h^T}\left[\mathbf{h_u} - \mathbf{s_h} - \mathbf{h}(\mathbf{x})\right] \\ & - \mathbf{y_{sh}^T}\left(\mathbf{h_u} - \mathbf{h_l} - \mathbf{s_h} - \mathbf{s_{sh}}\right) - \mathbf{y_x^T}\left(\mathbf{x_u} - \mathbf{x} - \mathbf{s_x}\right) \\ & - \mu \sum_{j=1}^{n} \ln\left(\mathbf{x} - \mathbf{x_l}\right)_j - \mu \sum_{j=1}^{n} \ln\left(\mathbf{s_x}\right)_j \\ & - \mu \sum_{i=1}^{m} \ln\left(\mathbf{s_h}\right)_i - \mu \sum_{i=1}^{m} \ln\left(\mathbf{s_{sh}}\right)_i . \end{aligned} \tag{33}$$

Then, Newton method or predictor-corrector method [27] is used to solve the KKT conditions ((34a), (34b), (34c), (34d), (34e), (34f), (34g), (34h), and (34i)) of (33). The value of $\mu$ decreases at every iteration

$$\begin{aligned} \nabla_x L_\mu & = \nabla f(\mathbf{x}) - \nabla \mathbf{g}(\mathbf{x})^{\mathbf{T}}\mathbf{y} + \nabla \mathbf{h}(\mathbf{x})^{\mathbf{T}}\mathbf{y_h} + \mathbf{y_x} \\ & \quad - \mu\left(\mathbf{x} - \mathbf{x_l}\right) = \mathbf{0} \end{aligned} \tag{34a}$$

$$\Longrightarrow \text{let } \mathbf{z} = \mu\left(\mathbf{x} - \mathbf{x_l}\right)$$

$$\text{then } \nabla f(\mathbf{x}) - \nabla \mathbf{g}(\mathbf{x})^{\mathbf{T}}\mathbf{y} + \nabla \mathbf{h}(\mathbf{x})^{\mathbf{T}}\mathbf{y_h} + \mathbf{y_x} - \mathbf{z} = \mathbf{0}$$

$$\begin{aligned} & \nabla_{s_h} L_\mu = \mathbf{y_h} + \mathbf{y_{sh}} - \mu\mathbf{S_h^{-1}}\mathbf{e} = \mathbf{0} \\ & \Longleftrightarrow \mathbf{S_h}\left(\mathbf{Y_h} + \mathbf{Y_{sh}}\right)\mathbf{e} = \mu\mathbf{e} \end{aligned} \tag{34b}$$

$$\begin{aligned} & \nabla_{s_{sh}} L_\mu = \mathbf{y_{sh}} - \mu\mathbf{S_h^{-1}}\mathbf{e} = \mathbf{0} \\ & \Longleftrightarrow \mathbf{S_{sh}}\mathbf{Y_{sh}}\mathbf{e} = \mu\mathbf{e} \end{aligned} \tag{34c}$$

$$\begin{aligned} & \nabla_{s_x} L_\mu = \mathbf{y_x} - \mu\mathbf{S_x^{-1}}\mathbf{e} = 0 \\ & \Longleftrightarrow \mathbf{S_x}\mathbf{Y_x}\mathbf{e} = \mu\mathbf{e} \end{aligned} \tag{34d}$$

$$\nabla_y L_\mu = -\mathbf{g}(\mathbf{x}) = \mathbf{0} \tag{34e}$$

$$\nabla_{y_h} L_\mu = \mathbf{h}(\mathbf{x}) + \mathbf{s_h} - \mathbf{h_u} = \mathbf{0} \tag{34f}$$

$$\nabla_{y_x} L_\mu = \mathbf{x} + \mathbf{s_x} - \mathbf{x_u} = \mathbf{0} \tag{34g}$$

$$\nabla_{y_{sh}} L_\mu = \mathbf{S_h} + \mathbf{s_{sh}} - \mathbf{h_u} + \mathbf{h_l} = \mathbf{0} \tag{34h}$$

$$\left(\mathbf{X} - \mathbf{X_l}\right)\mathbf{Z}\mathbf{e} = \mu\mathbf{e}. \tag{34i}$$

In Taiwan, deregulation has not been exercised yet and there is only one electric utility company (Taipower) that owns power plants and transmission/distribution systems. Although there are some independent power producers (IPPs) in Taiwan, they signed contracts with Taipower and are not allowed to sell electricity to consumers directly. Due to this regulated limitation, the electric power supply chain network in Figure 1 is modified to fit Taiwan power system. Figure 3 shows this modified electric power supply chain network with only one supplier (3-bus system), where the transmission network is represented.

With considering transmission constraints and losses, we replace the load balance equation (7) by (30) and add an additional constraint (26) in the original variational inequality problems (16) and (17). The resulting variational inequality problems are then solved by the extragradient method using an interior point algorithm as a solver. Numerical examples are presented in next section.

## 3. Numerical Results

In this paper, a 3-bus power system and a simplified Taipower Company network shown in Figure 4 were adopted for testing. The power plant data used in [1] with some modifications

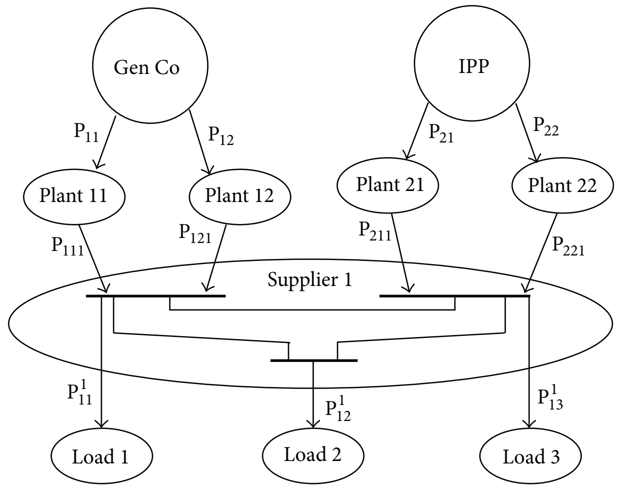


Figure 3: Electric power supply chain with one supplier consisting of a 3-bus network.

were utilized for the 3-bus system. These data are listed as follows:

$$
\begin{aligned}
f_{11}(p_{11}) &= 2.5p_{11}^2 + p_{11},\\
f_{12}(p_{12}) &= 2p_{12}^2 + 2p_{22},\\
f_{21}(p_{21}) &= 1.5p_{21}^2 + 3p_{21},\\
f_{22}(p_{22}) &= p_{22}^2 + 4p_{22},\\
e_{11} &= 1,\\
e_{12} &= 1,\\
e_{21} &= 1,\\
e_{22} &= 1,\\
c_{111}(p_{111}) &= 0.5p_{111}^2 + 3.5p_{111},\\
c_{121}(p_{121}) &= 0.5p_{121}^2 + 3.5p_{121},\\
c_{121}(p_{211}) &= 0.5p_{211}^2 + 2p_{211},\\
c_{221}(p_{221}) &= 0.5p_{221}^2 + 2p_{221},\\
c_1(h_1) &= 0.5h_1,\\
\rho_{31}(d_1) &= -1.33d_1 + 366.6,\\
\rho_{32}(d_2) &= -1.33d_2 + 366.6,\\
\rho_{33}(d_3) &= -1.33d_3 + 366.6,\\
\hat{c}_{11}^1(p_{11}^1) &= p_{11}^1 + 5,\\
\hat{c}_{12}^1(p_{12}^1) &= p_{12}^1 + 5,\\
\hat{c}_{13}^1(p_{13}^1) &= p_{13}^1 + 5,\\
\overline{B}_{11} &= 30,\\
\overline{B}_{12} &= 30,\\
\overline{B}_{21} &= 30,\\
\overline{B}_{22} &= 30,
\end{aligned}
\tag{35}
$$

for decentralized carbon taxes $\overline{B} = 100$, for centralized carbon taxes with fixed upper bound.

All other transaction costs were assumed to be equal to zero.

The 3-bus transmission data are listed as follows:

$$
\begin{aligned}
b_{12} &= -17.3611,\\
b_{13} &= -10.5107,\\
b_{23} &= -5.5882,\\
g_{12} &= 0.003,\\
g_{13} &= 1.9422,\\
g_{23} &= 1.282,\\
\overline{\mathbf{P}}_{\mathbf{f}12} &= 15,\\
\overline{\mathbf{P}}_{\mathbf{f}13} &= 10,\\
\overline{\mathbf{P}}_{\mathbf{f}23} &= 40\\
\underline{\mathbf{P}}_{\mathbf{f}12} &= -15,\\
\underline{\mathbf{P}}_{\mathbf{f}13} &= -10,\\
\underline{\mathbf{P}}_{\mathbf{f}23} &= -40,\\
p_{11\max} &= 10,\\
p_{12\max} &= 45,\\
p_{21\max} &= 45,\\
p_{22\max} &= 45,\\
p_{11\min} &= 0,\\
p_{12\min} &= 0,\\
p_{21\min} &= 0,\\
p_{22\min} &= 0.
\end{aligned}
\tag{36}
$$

The numerical solutions of two different carbon tax policies, decentralized carbon taxes, and centralized carbon tax for this 3-bus system with and without transmission constraints are listed in Table 1. For cases 1 and 3 under transmission constrained, transmission line (1–3) is at its upper bound. For case 3, transmission line (1-2) is at its upper bound. For case 1, the generations $p_{12}$, $p_{21}$, and $p_{22}$ are constrained by the $CO_2$ emission upper limits although they have not violated any physical generation limits ($p_{ij\max}$), and therefore their carbon taxes are nonzeros

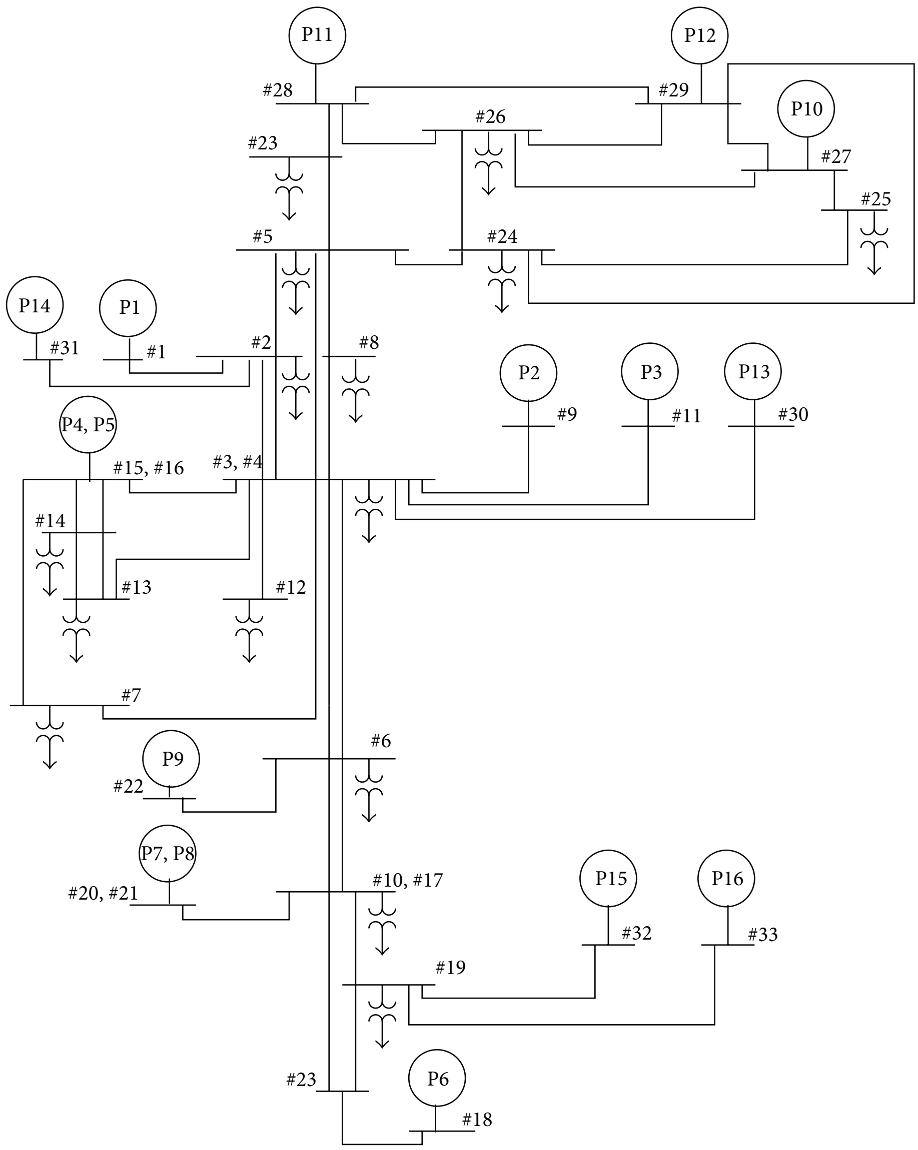


Figure 4: Simplified Taipower System.

(5.98, 49.3, and 108.28). For case 3 with the total emission limit 100, the centralized carbon tax is 62.56. In this case, generator $p_{22}$ has cheaper fuel cost and transaction cost than $p_{12}$ and $p_{21}$, and it would supply the demand as much as possible unless it hits its physical generation limit (=45). For cases 2 and 4 with unconstrained transmission, the loads are roughly uniform distributed. Because of this, the carbon taxes of these cases (2 and 4) are higher than those of their corresponding cases (1 and 3) with constrained transmission. The transmission constraints do affect the optimal carbon taxes.

Since Taiwan has only one power supplier (Taiwan Power Company), the power network shown in Figure 1 should be modified using one power supplier; that is, $S = 1$. In this study, a simplified power network system of Taiwan Power Company [22] is applied to verify the optimal carbon tax models. Figure 4 shows the simplified power system network of Taiwan Power Company, including 16 power plants, 17 demand markets, 33 buses, and 1 transmission provider. The user defined cost function of each power station is shown in Table 2. In such a model, since hydropower plants P1~P3 and P13 and nuclear power plants P6, P11, and P12 do not produce carbon and do not participate in carbon dispatch, the upper and lower limits of generation for those plants are set almost equivalent. The generation cost of fossil fuel power stations present positive correlations with their generation capacity. There are a total of nine fossil fuel power plants: five coal-fired plants, two petroleum-fired plants, and two natural gas-fired plants.

As Taiwan Power Company is a public utility, it owns both the power plants and the transmission lines, and thus $p_{gms} = p_{gm}$. We set the electricity sales per kilowatt hour of Taiwan Power Company as NT\$4.5 [29] and set the demand market price function as $\rho_k(d_k) = -1.33d_k + 450$. Here, we assume

Table 1: Numerical solutions for different types of carbon taxes for 3-bus system with and without transmission constraints.

| | Decentralized carbon taxes | | Centralized carbon tax with a fixed upper bound | |
|---|---|---|---|---|
| | Transmission constrained Case 1 | Transmission unconstrained Case 2 | Transmission constrained Case 3 | Transmission unconstrained Case 4 |
| $p_{11}$ | **10.00** | **10.00** | **10.00** | **10.00** |
| $p_{12}$ | 30.00 | 30.00 | 21.65 | 19.98 |
| $p_{21}$ | 30.00 | 30.00 | 23.35 | 25.02 |
| $p_{22}$ | 30.00 | 30.00 | **45.00** | **45.00** |
| $p_{f12}$ | −13.02 | −12.40 | **−15.00** | −20.66 |
| $p_{f13}$ | **10.00** | 18.68 | **10.00** | 16.94 |
| $p_{f23}$ | 9.51 | 13.92 | 10.15 | 15.66 |
| $d_1$ | 42.97 | 33.58 | 36.59 | 33.56 |
| $d_2$ | 37.47 | 33.68 | 43.20 | 33.70 |
| $d_3$ | 19.51 | 32.61 | 20.15 | 32.59 |
| $e_{11}p_{11}$ | $10.00 < \overline{B}_{11}$ | $10.00 < \overline{B}_{11}$ | 10.00 | 10.00 |
| $e_{12}p_{12}$ | **30.00** | **30.00** | 21.65 | 19.98 |
| $e_{21}p_{21}$ | **30.00** | **30.00** | 23.35 | 25.02 |
| $e_{22}p_{22}$ | **30.00** | **30.00** | 45.00 | 45.00 |
| $\overline{B}$ | | | 100 | 100 |
| $\tau_{11}$ | 96.98 | 118.87 | | |
| $\tau_{12}$ | 5.98 | 27.87 | | |
| $\tau_{21}$ | 49.30 | 58.13 | | |
| $\tau_{22}$ | 108.28 | 117.13 | | |
| $\Gamma$ | | | 62.56 | 78.00 |

Table 2: Generation cost function.

| Number | Fuel type | Constant (NT$) | 1st order coefficient (NT$/MW) | 2nd order coefficient (NT$/MW$^2$) | Generation lower bound (MW) | Generation upper bound (MW) |
|---|---|---|---|---|---|---|
| P1 | hydro | — | — | — | 936.7 | 937.7 |
| P2 | hydro | — | — | — | 870 | 871 |
| P3 | hydro | — | — | — | 200 | 201 |
| P4 | coal | 36295.83 | 442.586 | 0.01511 | 1020 | 3120 |
| P5 | coal | 36295.83 | 442.586 | 0.01511 | 1020 | 3120 |
| P6 | nuclear | — | — | — | 1820 | 1821 |
| P7 | coal | 41188.38 | 529.885 | 0.05477 | 728 | 2767.5 |
| P8 | coal | 41188.38 | 529.885 | 0.05477 | 728 | 2767.5 |
| P9 | coal | 36295.83 | 442.586 | 0.01511 | 1020 | 2880 |
| P10 | oil | 63425.35 | 928.957 | 0.25798 | 520 | 2910 |
| P11 | nuclear | — | — | — | 1150 | 1151 |
| P12 | nuclear | — | — | — | 1810 | 1811 |
| P13 | hydro | — | — | — | 600 | 601 |
| P14 | Gas | 226848.9 | 1112.22 | 0.09444 | 1145 | 1717 |
| P15 | oil | 45253.27 | 901.236 | 0.30542 | 700 | 3150 |
| P16 | gas | 145183.3 | 998.65 | 0.06044 | 867 | 1300.5 |

that all transmission costs $c_{gms}(p_{gms})$ are the same, which are equal to $0.25p_{gm}^2 + p_{gm}$ in a quadratic form. Other transaction costs are set as zero, and other relevant parameters in the model are set as follows:

$$\begin{aligned} e_{gm} &= 1\,(\text{coal-fired}), \\ e_{gm} &= 0.7\,(\text{petroleum-fired}), \\ e_{gm} &= 0.5\,(\text{natural gas-fired}), \\ c(h) &= 0.005h, \\ \hat{c}_{1i}^{1}\left(p_{1k}^{1}\right) &= 0.75p_{1k}^{1} + 5. \end{aligned} \quad (37)$$

*Decentralized carbon tax* is as follows:

$$\begin{aligned} \overline{B}_4 &= 2200, \\ \overline{B}_5 &= 1700, \\ \overline{B}_7 &= 1200, \\ \overline{B}_8 &= 1200, \\ \overline{B}_9 &= 1400, \\ \overline{B}_{10} &= 700, \\ \overline{B}_{14} &= 600, \\ \overline{B}_{15} &= 550, \\ \overline{B}_{16} &= 450. \end{aligned} \quad (38)$$

*Centralized carbon tax* with fixed upper bound: $\overline{B} = 10000$.

For the cases without transmission constraints imposed, the numerical solutions of decentralized carbon taxes, centralized carbon tax, and no carbon tax (no carbon emission limit) models are listed in Table 3. For the case without carbon tax, the total generation is about 20984 MW and the total carbon emission is 12182 tons. According to the regulations in Kyoto Protocol, the greenhouse gas emissions should be reduced down to 5.5% lower than the level in 1990 by 2012. Such a standard could be achieved by regulating carbon taxes to reduce power consumption and power generation. In this study, the reduction targets are set at 10~20% instead of 5.5% for comparing the situations after implementing carbon taxes. For the case with decentralized carbon taxes, the total carbon emission limit of thermal power plants is 10000, which is the same as the limit for the case with centralized carbon tax. From Table 3, the generations of power plants P14, P15, and P16 for the case without carbon tax are at their lower bounds due to their high generation costs. Nevertheless, the generations of these three plants for the cases with decentralized carbon taxes and centralized carbon tax reveal changes because of carbon emission limits and carbon taxes which make them no longer able to keep their generations at their lower bounds. The optimal carbon taxes of the thermal power plants (P4-P5, P7–P10, and P14–P16) for the case with decentralized carbon taxes range from 150 to 700. The optimal carbon tax is 615 for the case with centralized carbon tax. It is observed that the total transmission loss is below 1.5% of the total generation. The total generation of the system without carbon tax is 20984.59 MW, which is 2000 MW more than that with centralized carbon tax. Apparently, the total generation for the case with carbon taxes would be restrained by the elasticity of power demand and carbon taxes.

For the cases with transmission constraints imposed, the numerical solutions of decentralized carbon taxes, centralized carbon tax, and no carbon tax (no carbon emission limit) models are listed in Table 4. These results are compared to those without transmission constraints imposed. All the transmission line flow limits are set at 1600 MW, except lines 26–28, 28–23, 28-29, 18-17, and 18-19 that remain at the limit 2187 MW due to those fixed generation plants (P1–P3, P6, and P11–P13) that are connected to these lines.

Table 4 shows that, with centralized carbon taxes, the generations of those power plants with lower generation costs would be restricted because of line congestions; therefore, the total generation is evenly distributed to the power plants so as to reduce carbon taxes. For the case with decentralized carbon taxes, line congestion causes the power plant P5 to be unable to generate power up to the carbon emission limit, and therefore its optimum carbon tax is zero. Line congestion also causes the total generation for the case without carbon tax to drop down to 20041 MW as compared to the case without carbon tax and without transmission constraints (20984 MW, no carbon tax case in Table 3).

Comparing Table 3 with Table 4, the centralized carbon tax being collected for the case with line congestion is NT$5,061,300 (Table 4), which is less than the centralized carbon tax for the case without line congestion, NT$6,149,100 (Table 3). That is because, for the case without carbon tax, the total generation with line congestion (20041 MW) is less than that without line congestion (20984 MW); therefore, for the case with centralized carbon tax, the total generation with line congestion needs to be reduced by 1391 MW (= 20041 − 18649.73) due to transmission constraints, which is less than the one without line congestion, 2210 MW (= 20984 − 18774). The optimal centralized carbon tax being collected for the case with line congestion therefore is less than the one for the case without line congestion.

Regarding line losses, the transmission line flows for the cases with transmission constraints (when lines are congested) are less than the ones for the cases without transmission constraints (when lines are not congested). For this reason, the total line losses under different carbon tax models in Table 4 are lower than those in Table 3. However, the total line losses under various models in Table 4 are close due to line congestion.

Table 5 shows the congested lines for the cases with transmission constraints under various carbon tax models. Line 32 from bus 4 to bus 6 is found to be congested under all carbon tax models; therefore, such a line is a key line for improving the congestion.

To further analyze the effects of the carbon reduction amount on carbon taxes, the initial carbon emission limits are set for the cases with and without transmission constraints, shown in Table 6, based on the solutions of the cases without

Table 3: Numeric solutions under various carbon tax modes without transmission constraints.

| | Centralized tax | Decentralized taxes | No carbon tax |
|---|---|---|---|
| $p_1$ | 937.70 | 937.70 | 937.70 |
| $p_2$ | 871.00 | 871.00 | 871.00 |
| $p_3$ | 201.00 | 201.00 | 201.00 |
| $p_4$ | 1974.12 | 2200.00 | 2642.02 |
| $p_5$ | 1882.80 | 1700.00 | 2300.24 |
| $p_6$ | 1821.00 | 1821.00 | 1821.00 |
| $p_7$ | 1217.93 | 1200.00 | 1533.32 |
| $p_8$ | 1217.62 | 1200.00 | 1533.14 |
| $p_9$ | 1616.70 | 1400.00 | 1959.35 |
| $p_{10}$ | 720.43 | 933.33 | 910.91 |
| $p_{11}$ | 1151.00 | 1151.00 | 1151.00 |
| $p_{12}$ | 1811.00 | 1811.00 | 1811.00 |
| $p_{13}$ | 601.00 | 601.00 | 601.00 |
| $p_{14}$ | 1145.00 | 1200.00 | 1145.00 |
| $p_{15}$ | 700.00 | 733.33 | 700.00 |
| $p_{16}$ | 906.02 | 900.00 | 867.00 |
| $d_1$ | 1068.63 | 1076.99 | 1179.94 |
| $d_2$ | 1090.35 | 1090.71 | 1252.29 |
| $d_3$ | 1074.79 | 1083.13 | 1170.72 |
| $d_4$ | 1060.08 | 1069.30 | 1167.42 |
| $d_5$ | 1151.34 | 1152.55 | 1332.22 |
| $d_6$ | 1094.28 | 1093.52 | 1257.08 |
| $d_7$ | 1054.39 | 1063.13 | 1155.88 |
| $d_8$ | 1175.60 | 1176.93 | 1350.94 |
| $d_9$ | 1072.45 | 1081.05 | 1168.98 |
| $d_{10}$ | 1072.38 | 1081.99 | 1171.04 |
| $d_{11}$ | 1070.77 | 1080.53 | 1169.79 |
| $d_{12}$ | 1170.90 | 1172.08 | 1348.36 |
| $d_{13}$ | 1177.12 | 1178.45 | 1352.36 |
| $d_{14}$ | 1057.57 | 1067.29 | 1164.34 |
| $d_{15}$ | 1057.78 | 1068.16 | 1164.27 |
| $d_{16}$ | 1057.47 | 1068.13 | 1163.94 |
| $d_{17}$ | 1060.79 | 1071.33 | 1166.98 |
| Total generation | 18774 MW | 18860 MW | 20984 MW |
| $CO_2$ (t) | 10000 | 10000 | 12182 |
| Demand | 18566.37 MW | 18674.90 MW | 20736.51 MW |
| Carbon tax cost | NT$ 6149100 | NT$ 5194331 | NT$ 0 |
| Line losses | 207.63 MW | 185.10 MW | 248.16 MW |
| $\Gamma$ | 614.91 | | |
| $\tau_4$ | | 441.41 | |
| $\tau_5$ | | 680.78 | |
| $\tau_7$ | | 590.32 | |
| $\tau_8$ | | 590.10 | |
| $\tau_9$ | | 698.23 | |
| $\tau_{10}$ | | 262.68 | |
| $\tau_{14}$ | | 239.21 | |
| $\tau_{15}$ | | 149.39 | |
| $\tau_{16}$ | | 582.98 | |

Table 4: Numeric solutions under various carbon tax modes with transmission constraints.

| | Centralized tax | Decentralized taxes | No carbon tax |
|---|---|---|---|
| $p_1$ | 937.70 | 937.70 | 937.70 |
| $p_2$ | 871.00 | 871.00 | 871.00 |
| $p_3$ | 201.00 | 201.00 | 201.00 |
| $p_4$ | 2271.38 | 2200.00 | 2474.84 |
| $p_5$ | 1816.86 | 1700.00 | 1945.09 |
| $p_6$ | 1821.00 | 1821.00 | 1821.00 |
| $p_7$ | 1135.39 | 1200.00 | 1372.86 |
| $p_8$ | 1135.22 | 1200.00 | 1373.80 |
| $p_9$ | 1496.72 | 1400.00 | 1600.00 |
| $p_{10}$ | 817.91 | 933.33 | 1081.50 |
| $p_{11}$ | 1151.00 | 1151.00 | 1151.00 |
| $p_{12}$ | 1811.00 | 1811.00 | 1811.00 |
| $p_{13}$ | 601.00 | 601.00 | 601.00 |
| $p_{14}$ | 1145.00 | 1200.00 | 1231.78 |
| $p_{15}$ | 700.00 | 715.07 | 700.00 |
| $p_{16}$ | 867.00 | 900.00 | 867.00 |
| $d_1$ | 1063.02 | 1059.79 | 1118.34 |
| $d_2$ | 1144.68 | 1139.89 | 1253.62 |
| $d_3$ | 1032.95 | 1035.34 | 1061.45 |
| $d_4$ | 1038.57 | 1046.57 | 1085.15 |
| $d_5$ | 1219.65 | 1222.30 | 1348.06 |
| $d_6$ | 1147.87 | 1142.42 | 1214.26 |
| $d_7$ | 1022.39 | 1027.28 | 1059.52 |
| $d_8$ | 1235.83 | 1240.54 | 1393.77 |
| $d_9$ | 1031.16 | 1033.45 | 1059.77 |
| $d_{10}$ | 1032.91 | 1034.83 | 1061.71 |
| $d_{11}$ | 1031.63 | 1033.49 | 1147.86 |
| $d_{12}$ | 1233.32 | 1237.60 | 1345.65 |
| $d_{13}$ | 1237.55 | 1242.29 | 1386.83 |
| $d_{14}$ | 1036.63 | 1044.84 | 1083.42 |
| $d_{15}$ | 1037.37 | 1045.92 | 1084.61 |
| $d_{16}$ | 1037.27 | 1045.97 | 1084.75 |
| $d_{17}$ | 1040.52 | 1049.18 | 1087.77 |
| Total generation | 18649.73 MW | 18842 MW | 20041 MW |
| $CO_2$ (t) | 10000 | 9986.3 | 11152 |
| Demand | 18493.87 MW | 18681.61 MW | 19876.98 MW |
| Carbon tax cost | NT$ 5061300 | NT$ 4596627 | NT$ 0 |
| Line losses | 155.86 MW | 160.39 MW | 164.02 MW |
| $\Gamma$ | 506.13 | | |
| $\tau_4$ | | 539.53 | |
| $\tau_5$ | | 579.09 | |
| $\tau_7$ | | 456.15 | |
| $\tau_8$ | | 456.02 | |
| $\tau_9$ | | 552.94 | |
| $\tau_{10}$ | | 324.27 | |
| $\tau_{14}$ | | 310.93 | |
| $\tau_{15}$ | | 132.65 | |
| $\tau_{16}$ | | 317.62 | |

carbon tax model. The carbon emission limits are then gradually reduced by 1% ~21%. Figure 5 shows the carbon tax changes of the plants for the cases without transmission constraints under decentralized carbon taxes model. The carbon taxes of P4, P5, and P7~P10 are proportional to the reductions of carbon emission limits, while the carbon taxes

Table 5: Congested lines for cases with transmission constraints.

| | Centralized carbon taxes | Decentralized carbon tax | No carbon tax |
|---|---|---|---|
| Line number (from bus number to bus number) | 6 (5–2) | 6 (5–2) | 32 (4–6) |
| | 15 (20–17) | 32 (4–6) | |
| | 32 (4–6) | | |
| | 34 (6–22) | | |
| | 41 (16–7) | | |
| | 44 (15–13) | | |

Table 6: Carbon emission limits.

| | Decentralized carbon taxes (with transmission constraints) | Decentralized carbon taxes (without transmission constraints) | Centralized carbon tax |
|---|---|---|---|
| $\overline{B}_4$ | 2475 | 2642 | 11152 (with transmission constraints) |
| $\overline{B}_5$ | 1946 | 2300 | |
| $\overline{B}_7$ | 1373 | 1533 | |
| $\overline{B}_8$ | 1374 | 1533 | |
| $\overline{B}_9$ | 1600 | 1959 | 12182 (without transmission constraints) |
| $\overline{B}_{10}$ | 812 | 684 | |
| $\overline{B}_{14}$ | 750 | 750 | |
| $\overline{B}_{15}$ | 700 | 700 | |
| $\overline{B}_{16}$ | 600 | 600 | |

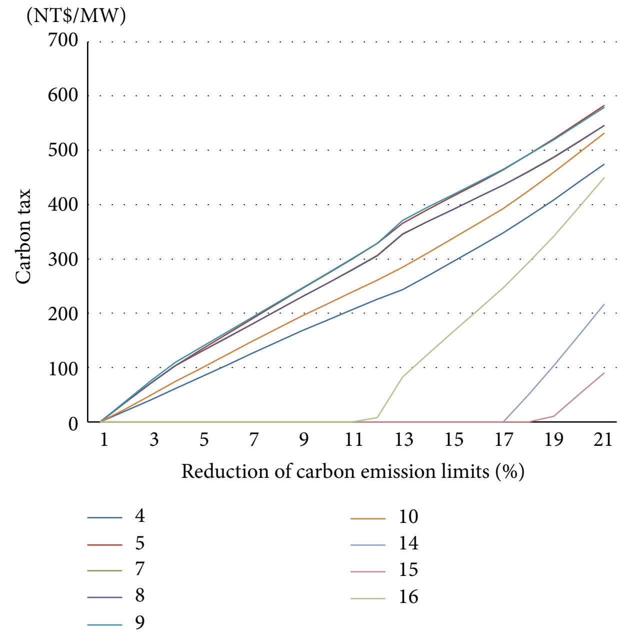


Figure 5: Carbon tax variations: decentralized carbon taxes model W/O transmission constraints.

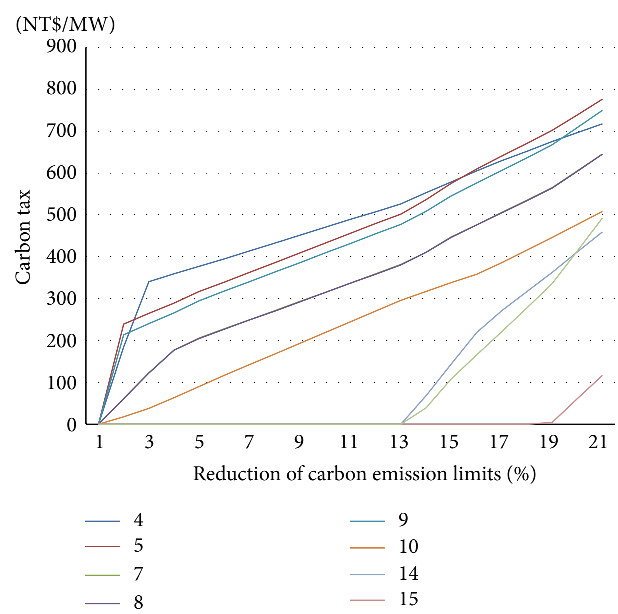


Figure 6: Carbon tax variations: decentralized carbon taxes model with transmission constraints.

of P14~P16, with high generation costs, change only when the percentage of the reduction of carbon emission limits is higher than 12%. Figure 6 shows the carbon tax changes of the plants for the cases with transmission constraints under decentralized carbon taxes model. Because of line congestion, the carbon tax changes of the plants reveal break points, and different linear slopes appear due to different levels of line congestions. The carbon tax changes under centralized carbon tax model are shown in Figure 7. As the carbon emission limit under centralized carbon tax model for the case with transmission constraints (with line congestion) is different from that without transmission constraints (no line congestion), the carbon emission limit with transmission constraints is lower than that without transmission constraints, and therefore the corresponding carbon tax for the case with transmission constraints is lower than that for the case without transmission constraints.

## 4. Conclusions

In this paper, a modeling and computational framework was presented for the determination of optimal carbon taxes that apply to electric power plants in the context of electric power supply chain with consideration of transmission constraints, line losses, and power flows. Two different types of carbon taxes within this generalized electric power supply chain network equilibrium model were discussed. The Nash equilibrium conditions of this model formed a finite-dimensional variational inequality problem (VIP). The resulting VIP was then solved by an extragradient method based on an interior point algorithm to determine the optimal carbon taxes of power plants.

The presented modeling and computational framework not only can calculate the economic dispatch solution and transmission line congestion for the model without carbon tax, but also can obtain the relationship between the optimal carbon taxes versus carbon emission limits for the models

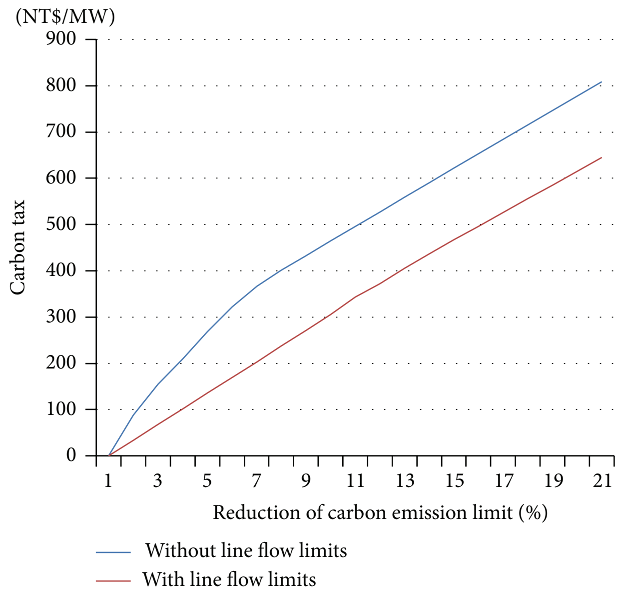


Figure 7: Carbon tax variations: centralized carbon tax model.

of decentralized and centralized carbon taxes. Numerical examples based on a 3-bus power system and a simplified Taipower Company network were provided to analyze the results of the presented modeling. The presented work is capable to capture the interactions in an electric power supply chain network coupled with the carbon taxes and therefore can be used as a decision-making tool for environmental policies.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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