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Opinion dynamics on directed small-world networks

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Abstract. In this paper, we investigate the self-affirmation effect on formation of public opinion in a directed small-world social network. The system presents a non-equilibrium phase transition from a consensus state to a disordered state with coexistence of opinions. The dynamical behaviors are very sensitive to the density of long-range-directed interactions and the strength of self-affirmation. When the long-range-directed interactions are sparse and individual generally does not insist on his/her opinion, the system will display a continuous phase transition, in the opposite case with strong self-affirmation and dense long-range-directed interactions, the system does not display a phase transition. Between those two extreme cases, the system undergoes a discontinuous phase transition.

PACS. 89.75.-k Complex systems – 89.65.-s Social and economic systems – 05.70.Fh Phase transitions: general studies – 05.50.+q Lattice theory and statistics (Ising, Potts, etc.)

1 Introduction

Recently, much effort has been devoted to the studying opinion dynamics [1–3]. Statistical physics provides quantitative tools to reveal the underlying laws that govern the opinion dynamics [4]. Agent-based models have been proposed to study complex phenomena of opinion formation. With process of social influence [5,6], consensus in opinion formation achieves. For example, the opinion of an individual may be affected by its nearest neighbors, as described in the Sznajd model [7,8], the Galam's majority rule [9,10], and the Axelrod multicultural model [11]. On the other hand, the contrarian effect is introduced to account for the phenomenon of a transition from a polarized opinion state to a coexistent opinions state [12,13]. The similar results are also obtained in references [14–16] in which social temperature is considered. The real-life system often seems a black box to us: the outcome can be observed, but the hidden mechanism is not visible. If we see many individuals hold the same opinion, we say *The Spiral of Silence* phenomenon [17] occurs. It is common in real world that people adhere to their own opinion even opposite to most of their friends [18–20], which we call *self-affirmation* of individuals, similar to the contrarian effect [12,13]. In our early work, the influence of inflexible units has been investigated in a simple social model [16]. It is found that this kind of effect can lead to a nontrivial phase diagram. However, traditional opinion models fail to account for the self-affirmation effect of individuals as well as directed relations between agents.

In the real world, interactions between individuals are not only short ranged, but also long ranged [21,22]. The interaction usually displays a directed feature, which means an individual who receives influence from a provider may not affect the provider. We apply the directed small-world networks proposed by Sánchez et al. [14] to represent this kind of relations between individuals. In this paper, we present an opinion dynamics model including individual self-affirmation psychological feature and long-range-directed correlations between individuals. The main difference between this model and other physics' inspired models is that this model takes into account both effects of self-affirmation and social structure. The former lies in the microscopic level, while the latter concerns the macroscopic impacts. The parameter space can be roughly divided into three regions, in which, respectively, we observe continuous phase transition, discontinuous phase transition and no phase transition.

2 Model

In this section, we introduce a directed small-world network model and an opinion dynamics model. We start with a two-dimensional regular lattice, in which every node is connected with adjacent four nodes inwardly and outwardly respectively, then, with probability p, rewire each outward link to a randomly chosen nonadjacent node.

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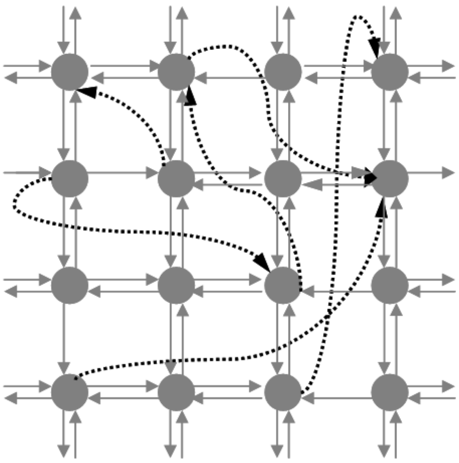

Fig. 1. Illustration of the structure of a directed small-world network for $p = 0.1$ [14].

In this way, as shown in Figure 1, a directed network with a density p of long-range-directed links is obtained. In this network, nodes represent individuals in the social system and the outward links represent the influences from others. Each node connects with four nodes outwardly which are called its mates.

In the network, an individual state represents its viewpoint, which evolves according to the social process, determined not only by other correlative surrounding effects but also by its own character. It is supposed that there are two kinds of possible opinions in the system, just as the agreement and disagreement in the election, and each individual takes only one of them. Therefore, the state of a node i can be described as σ_i, $\sigma_i \in \{+1, -1\}$. We describe the difference of σ_i from its mates by $W(\sigma_i) = 2\,\sigma_i \sum_{j=1}^{4} \sigma_j$, where σ_j $(j = 1, 2, 3, 4)$ are states of i's mates. In addition, q $(0 < q \leq 1)$ is used to describe the probability, with which individuals follow their mates' dominant opinion. Meanwhile $1 - q$ represents the self-affirmation probability of individuals, with which an individual insists on his/her own opinion though it is opposite to the majority of his/her mates.

According to the illumination above, we introduce the dynamical rule as follows: $W(\sigma_i) > 0$ indicates that σ_i is the same as the majority of σ_j $(j = 1, 2, 3, 4)$ and σ_i overturns with probability $\exp[-W(\sigma_i)/T]$ which depends on a temperature-like parameter T. $W(\sigma_i) < 0$ indicates that σ_i is opposite to the majority of σ_j $(j = 1, 2, 3, 4)$, and σ_i overturns with probability q. When $W(\sigma_i) = 0$, the state of node i overturns with probability q also. So that the overturning probability $P(\sigma_i)$ of σ_i is given by

$$P(\sigma_i) = \begin{cases} \exp[-W(\sigma_i)/T], & \text{for } W(\sigma_i) > 0 \\ q, & \text{for } W(\sigma_i) \leq 0. \end{cases} \quad (1)$$

From the dynamical rule (1), we can see that, when $q = 1$, the current model restores to the network-based Ising model [14]. However, our model is non-equilibrium because the overturning probability of a state does not satisfy the detailed equilibrium condition.

3 Simulations

In order to describe the evolution process of the model, we employ a magnetization-like order parameter

$$m = \left| \frac{1}{L^2} \sum_{i=1}^{L^2} \sigma_i \right|, \sigma_i \in \{+1, -1\}. \quad (2)$$

The network size is $L \times L$ and m is the absolute average value of the states of all nodes. An extensive Monte Carlo numerical simulation has been performed on our model with a random initial configuration and a periodic boundary. Results are calculated after the system reaches a non-equilibrium stationary state. In order to reduce the occasional errors, for network size $L = 16$, 32, 64, and 100, we have averaged the result over 40 000, 10 000, 2000, and 1000 runs, respectively, with different network structures under different random initial configurations. Obviously, when $\langle m \rangle$ tends to 1, the system enters into an ordered state, i.e., individuals in the system reach a consensus opinion. Meanwhile, if the system stays in a disordered state, the order parameter scales as $\langle m \rangle \sim \frac{1}{L}$. As shown in Figure 2, the system reaches an ordered state when T is less than a critical temperature T_c. The system displays a continuous phase transition for $p = 0.1$ and $q = 0.9$, while a discontinuous phase transition for $p = 0.9$ and $q = 0.9$. From the probability density functions (PDFs) of the order parameter near the phase transition point of the phase diagram (p, T), one can distinguish between the continuous phase transition and discontinuous phase transition clearly. According to PDFs inserted in the upper and lower panels of Figure 2, it is found that the most probable values of m, which correspond to the highest peaks of PDFs, jump little from nonzero to zero in the continuous phase transition from Figure 2a to Figure 2b, while sharply in the discontinuous one from Figure 2c to Figure 2d. It seems that the long-range correlations can change the nature of phase transition.

Evidently, given $q = 0.9$, the system varies from the continuous phase transition to discontinuous phase transition when the density of long-range-directed connections is high enough. It is natural to ask how these topology structures influence the opinion dynamics. To solve this problem, we define the domain size s as the number of neighborhood nodes in the same state. As shown in Figure 3a, it is found that the domain size s distributes in a power law, $g(s) \sim s^{-\tau}$ for $s \ll L^2$ at the critical point, where $g(s)$ is the probability function. One can find that there is a local maximum probability of large domain size for $p = 0.1$. Smaller p indicates more localized interactions between individuals, and a large domain emerges more easily. Besides, we calculate the number of time steps, t, during which an individual holds the same opinion. As shown in Figure 3b, one can find that individuals change their own opinion for $p = 0.9$ more frequently than for $p = 0.1$ at $T = 0.1$, and the probability of t obeys a power-law distribution $f(t) \sim t^{-\gamma}$ $(t < t_0)$ for $p = 0.1$. Clearly, p plays the key role in determining the communication strength between different opinion domains, and

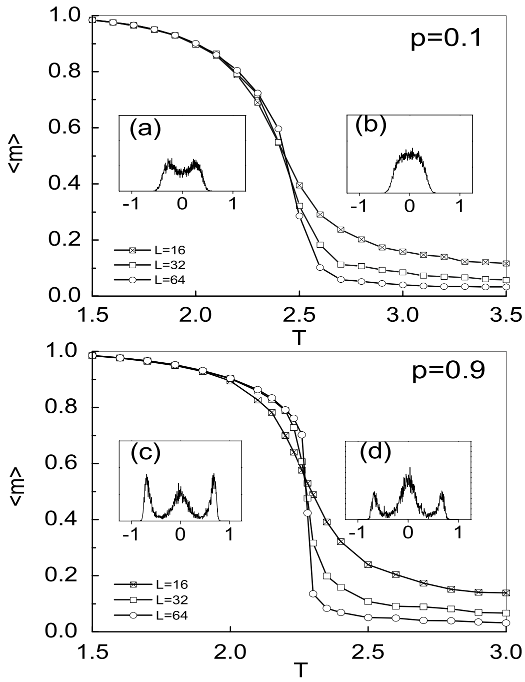

Fig. 2. $\langle m \rangle$ varies with T for different system sizes. The upper and lower plots are for $p = 0.1$ and $p = 0.9$, with $q = 0.9$ fixed. Insets are PDFs nearby the phase transition point: (a) $T \to T_c^-$, (b) $T \to T_c^+$, (c) $T \to T_c^-$, (d) $T \to T_c^+$.

individuals change their own opinions more frequently due to the long-range connections between different opinion domains.

Figures 4a–4c show the phase diagram of opinion dynamics determined by the network structure parameter p as well as the individual self-affirmation psychology characteristic parameter $1 - q$. In Figure 4a for $q = 0.9$, the system displays continuous phase transition for $p < p_c$, while discontinuous phase transition for $p \geq p_c$. The system displays discontinuous phase transition for $q = 0.5$ in Figure 4b. The system displays discontinuous phase transition for $p < p_0$ and $q = 0.3$ in Figure 4c, while the system does not have a phase transition for $p > p_0$ and $q = 0.3$ in Figure 4c. As shown in Figure 4d, the continuous phase transition takes place in the area I, the discontinuous phase transition appears in the area II and the system stays in disordered state without phase transition in the area III. When both the parameters p and $1 - q$ are large enough, indicating weak interactions between individuals in both local and global levels, the system keeps disordered at any temperature, i.e., the phase transition can not take place in the system, as in the area III of Figure 4d.

A finite-size scaling analysis is employed to study the critical behavior of continuous phase transition for $p = 0.1$ and $q = 0.9$. In the neighborhood of the critical point T_c, $\langle m \rangle \propto (T_c - T)^{\beta}$, $(T < T_c)$, where β is the order parameter exponent. Besides, when T is near to critical point T_c of the second order phase transition, a character length scale ξ denotes the correlation length in space. $\xi \propto (T_c - T)^{-\nu}$, $(T < T_c)$, where ν is a correlation length

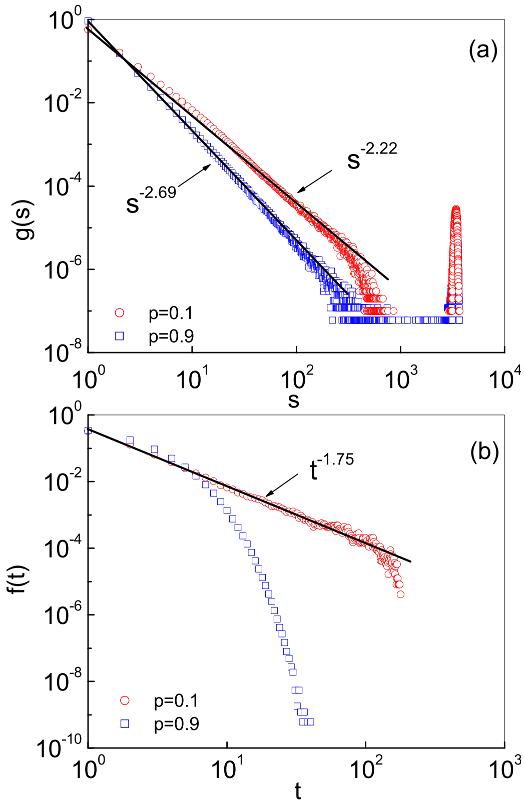

Fig. 3. (Color online) Distributions of domain size $g(s)$ (a) and opinion holding time $f(t)$ (b) in different networks with $q = 0.9$ fixed, (a) for $T = T_c$ and (b) for $T = 0.1$. The data points are obtained from 10^5 samples with fixed network size, $L = 64$.

exponent in the space direction. At critical point, various ensemble-averaged quantities depend on the ratio of system size and the correlation length L/ξ. Therefore, the order parameter $\langle m \rangle$ satisfies the scaling law in the neighborhood of the critical point: $\langle m \rangle \propto L^{-\beta/\nu} f[(T_c - T)L^{1/\nu}]$. At T_c, $\langle m \rangle \propto L^{-\beta/\nu}$, and we obtain $\beta/\nu = 0.530(5)$ for $p = 0.1$ and $q = 0.9$ in Figure 5a. Figure 5b reports $\langle m \rangle L^{\beta/\nu}$ versus $(1 - T/T_c)L^{1/\nu}$ on a double-logarithmic plot for $q = 0.1$ and $q = 0.9$. It is shown that with the choices $\beta/\nu = 0.530(5)$ and $\nu = 0.92(1)$ the data for different network sizes are well collapsed on a single master curve [23]. The slope of the line is $\beta = 0.488 \pm 0.005$, which gives the asymptotic behavior for $\langle m \rangle L^{\beta/\nu}$ as $L \to \infty$. So that, we have $\beta = 0.488(5)$, $\nu = 0.92(1)$ for $p = 0.1$ and $q = 0.9$. Comparing to critical exponents of 0.50 and 0.94 for $p = 0.1$ and $q = 1.0$ in reference [14], 0.30 and 0.80 for $p = 0.5$ and $q = 1.0$ also in reference [14], 0.118 and 0.8 for $p = 0.0$ and $q = 0.8$ in reference [16], 0.11 and 0.85 for $p = 0.0$ and $q = 0.6$ also in reference [16], we can conclude that critical exponents β and ν depend on both p and q.

4 Conclusion

In conclusion, the effect of long-range-directed links between individuals on the opinion formation is

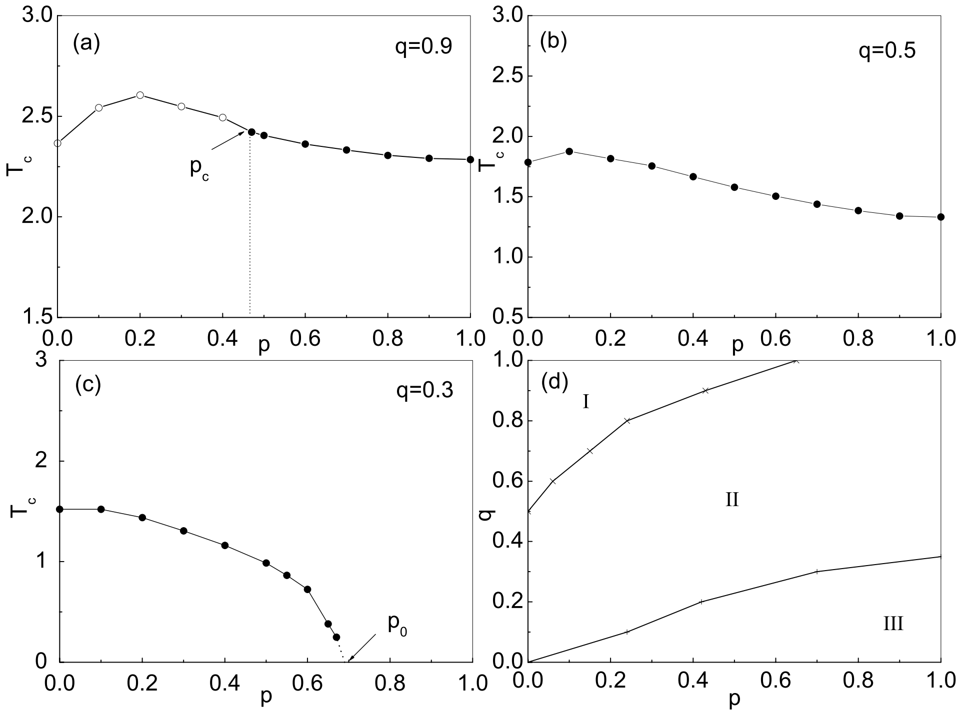

Fig. 4. The phase diagram of the model in the $p-q$ plane. Points are numerical determinations of the critical temperatures T_c for different p. The open circles correspond to continuous transition, while the solid ones correspond to discontinuous transition. Plot (d) reports the phase diagram: the system displays continuous phase transition in region I, discontinuous phase transition in region II, and no phase transition in region III.

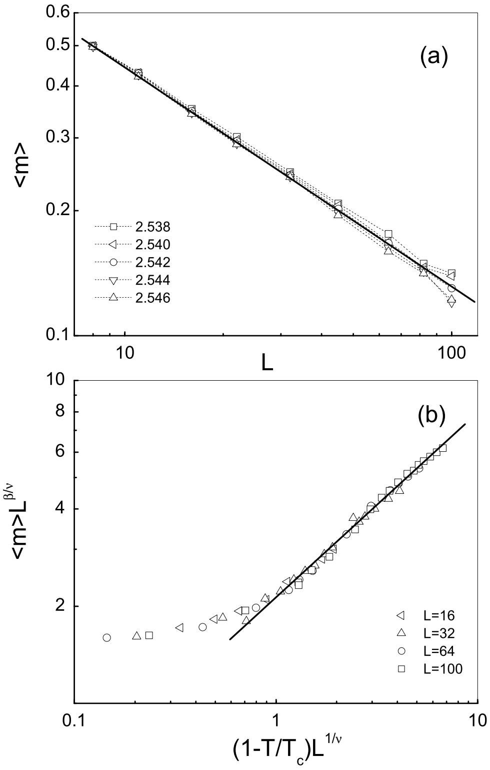

Fig. 5. Finite size scaling of continuous phase transition for $p = 0.1$ and $q = 0.9$. (a) A log-log plot of the order parameter $\langle m \rangle$ against L. (b) Double logarithmic plot of $\langle m \rangle\ L^{\beta/\nu}$ versus $(1 - T/T_c)L^{1/\nu}$ for $L = 16$, 32, 64, and 100.

systematically explored. The results show that the system takes on a non-equilibrium phase transition from a consensus state to a state of coexistence of different opinions. With increasing density of long-range-directed links, a continuous phase transition changes into a discontinuous one. The reason why the phase transition behavior varies is that the long-range links make individuals change their own opinions more frequently. It is worth mentioning that the system keeps in a disordered state when there are sufficient long-range links. Those long-range interactions break the possibly local order, thus hinder the global consensus. The similar phenomenon of order-disorder nonequilibrium phase transition emerging in the system is also observed in contrarians' models [12,13,15] or a non-conservative voters' model [24].

Phase transitions from a consensus state to a disordered state are common features of opinion dynamics seized by both contrarians' models and the present model. A contrarian is defined as an agent adopting the choice opposite to the prevailing choice of others whatever this choice is (see Ref. [12]), while self-affirmation effect is presented as a probability at which an agent insists on his/her opinion opposite to majority of his/her neighborhood. However, directionaly is not considered in classical contrarians' models, though the social interactions between agents are, in general, not symmetric. In fact, the present directed small-world topology plays a crucial rule in opinion dynamics. The present model uncovers that behaviors of phase transition are simultaneously determined by strength of self-affirmation and density of long-range-directed links. In addition, a new kind of fantastic phenomena is observed in the present model: resulting from the coupling effects of strong self-affirmation and dense long-range-directed links, the system stays in a disordered state at any temperature.

In opinion dynamics, the self-affirmation psychology character sometime may lead to polarized decision [25,26]. Moreover, interactions between individuals in social system depend on the topology of social networks [27–29]. In macroscopic level, the opinion dynamics is highly affected by social structure, while in the microscopic, it is sensitive to the dynamical mechanism of individual. Our work shows a systematic picture of opinion dynamics, and provides a deep insight into effects of these two factors.

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