

which should be cited to refer to this work.

Note on Two-Phase Phenomena in Financial Markets *

JIANG Shi-Mei(蒋诗梅)^{1**}, CAI Shi-Min(蔡世民)¹, ZHOU Tao(周涛)^{1,2,3}, ZHOU Pei-Ling(周佩玲)¹

¹Department of Electronic Science and Technology, University of Science and Technology of China, Hefei 230026

²Department of Modern Physics, University of Science and Technology of China, Hefei 230026

³Department of Physics, University of Fribourg, Chemin du Muse 3, CH-1700 Fribourg, Switzerland

The two-phase behaviour in financial markets actually means the bifurcation phenomenon, which represents the change of the conditional probability from an unimodal to a bimodal distribution. We investigate the bifurcation phenomenon in Hang-Seng index. It is observed that the bifurcation phenomenon in financial index is not universal, but specific under certain conditions. For Hang-Seng index and randomly generated time series, the phenomenon just emerges when the power-law exponent of absolute increment distribution is between 1 and 2 with appropriate period. Simulations on a randomly generated time series suggest the bifurcation phenomenon itself is subject to the statistics of absolute increment, thus it may not be able to reflect essential financial behaviours. However, even under the same distribution of absolute increment, the range where bifurcation phenomenon occurs is far different from real market to artificial data, which may reflect certain market information.

PACS: 89.90.+n, 87.10.+e, 89.65.Gh, 89.75.Da

Financial markets are typical complex systems. To understand their dynamics requires interdisciplinary knowledge and exploration, including the application of concepts and tools of statistical physics. Since the early 1970s, a number of physicists have devoted their effort to the study of economic and financial phenomena.^[1–6] They have developed a wide range of concepts and models, including fractal and multifractal scaling, frustrated disordered systems, phenomena far from equilibrium, and so on.^[7–12]

Recently, using transactions and quotes data for 116 most-actively traded US stocks for the two-year period 1994–1995, Plerou–Gopikrishnan–Stanley empirically discovered a two-phase behaviour in financial markets.^[13] Introducing a parameter Σ describing the fluctuation during the time interval Δt , the conditional probability distribution $p(\Omega|\Sigma)$ of the volume imbalance Ω , is found to be with a single peak for $\Sigma < \Sigma_c$ and double peaks for $\Sigma > \Sigma_c$. At the critical value Σ_c , the transition from a single peak to double peaks occurs. The change of $p(\Omega|\Sigma)$ from an unimodal to a bimodal distribution (the bifurcation phenomenon) indicates that the market moves between an ‘equilibrium’ state and an ‘out-of-equilibrium’ state, and these two different states were interpreted as distinct phases. Following this idea, Zheng *et al.*^[14] investigated the bifurcation phenomenon in financial markets with the time series of the German DAX from 1994 to 1997. It was observed that the probability distribution of the return $Z(t)$ conditioned on the fluctuation of the financial index $r(t)$ displays a transition from a unimodal distribution for small r , to a bimodal distribution for large r . However, some recent works

on trading volume indicate that the bifurcation phenomenon is an artifact of the distribution of trade sizes q , which follows a power-law distribution with exponent $\zeta_q < 2$ in the Lévy stable domain.^[15–17]

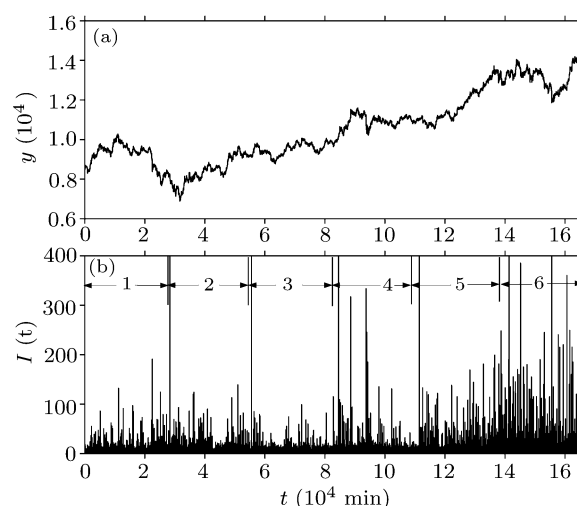


Fig. 1. (a) Hang-Seng index from 1 July 1994 to 28 May 1997 at the sampling intervals 1 minute. (b) The corresponding absolute increment $I(t)$.

In this Letter, the bifurcation phenomenon is investigated with the minute-by-minute records of Hang-Seng index from 1 July 1994 to 28 May 1997 (see Fig. 1 the index and its absolute increment). The trading time for a trading day in the data was not the same in the whole period. Although for all trading days, the Hong Kong stock market opened from 10:00 A.M. to 12:30 P.M. for the morning session, occupying a time interval of 150min and opened from 2:30

*Supported by the National Natural Science Foundation of China under Grant Nos 70571075 and 10635040, and the National Basic Research Programme of China under Grant No 2006CB705500.

**Email: msjiang@mail.ustc.edu.cn

P.M. in the afternoon, the closing times were not the same. From 1 July 1994 to 30 August 1995, the market closed at 3:45 P.M. with the total trading time 225 min per day. From 1 September 1995 to 30 December 1996, the market closed at 3:55 P.M. with the total trading time 235min per day. From 1 January 1997 to 28 May 1997, the market closed at 4:00 P.M. with the total trading time 240 min per day. The total number of data points is 165727. In order to carefully investigate the character of the Hang-Seng index, the total data is divided into six segments every half a year, as shown in Fig. 1(b). The largest number of data points in one segment is 29889, while the smallest is 24442. The number of data points is sufficiently large for a detailed statistical analysis.

Denote by $y(t)$ the time series of the Hang-Seng index, the corresponding absolute increment reads

$$I(t) = |y(t+1) - y(t)|. \quad (1)$$

The fluctuation $r_{\Delta t}(t)$ is simply the relative variation from t to $t + \Delta t$

$$r_{\Delta t}(t) = \langle |y(t+1) - y(t) - \langle y(t+1) - y(t) \rangle_{\Delta t}| \rangle_{\Delta t}, \quad (2)$$

where $\langle \rangle_{\Delta t}$ denotes the average from t to $t + \Delta t$, and $y(t+1)$ means $y(t+1 \text{ min})$. For a fixed Δt , we calculate the conditional probability distribution $p_{\Delta t}(Z, r) = p_{\Delta t}(Z|r)$ of the return $Z(t) = y(t + \Delta t) - y(t)$ with a specified r .

Figures 2(a)–2(f) show the empirical results of six segments, respectively. It can be found that for all the six segments, when the fluctuation r is very small, the distribution of return is single-peaked at about zero. However, for segment one to five, different from the expected two phase phenomena,^[13] when the fluctuation becomes bigger, the distribution of return is not a bimodal distribution. Instead, each of those five has more than two maxima. Actually, clear transition from unimodal to bimodal distribution can not be observed with the scale ranging from 1 min up to about a day. In contrast, as shown in Fig. 2(f), when the fluctuation $r < 4.8$ the distribution of return $p_{\Delta t}(Z, r)$ is single-peaked; when the fluctuation $r > 4.8$, the bigger the fluctuation is, the clearer the bimodal distribution becomes. The transition from unimodal to bimodal distribution holds for the scale ranging from 75 min to 125 min. Beyond the range this phenomenon fades away.

Table 1. Exponents and the existence of bifurcation phenomenon.

segment ID	exp (KS)	exp (LS)	exp (average)	unimodal to bimodal
1	2.16 ± 0.03	2.08 ± 0.03	2.12 ± 0.03	N
2	2.33 ± 0.04	2.31 ± 0.04	2.32 ± 0.04	N
3	2.43 ± 0.02	2.44 ± 0.02	2.44 ± 0.02	N
4	2.09 ± 0.03	2.05 ± 0.03	2.07 ± 0.03	N
5	2.09 ± 0.03	2.09 ± 0.03	2.09 ± 0.03	N
6	1.95 ± 0.03	1.91 ± 0.03	1.93 ± 0.03	Y

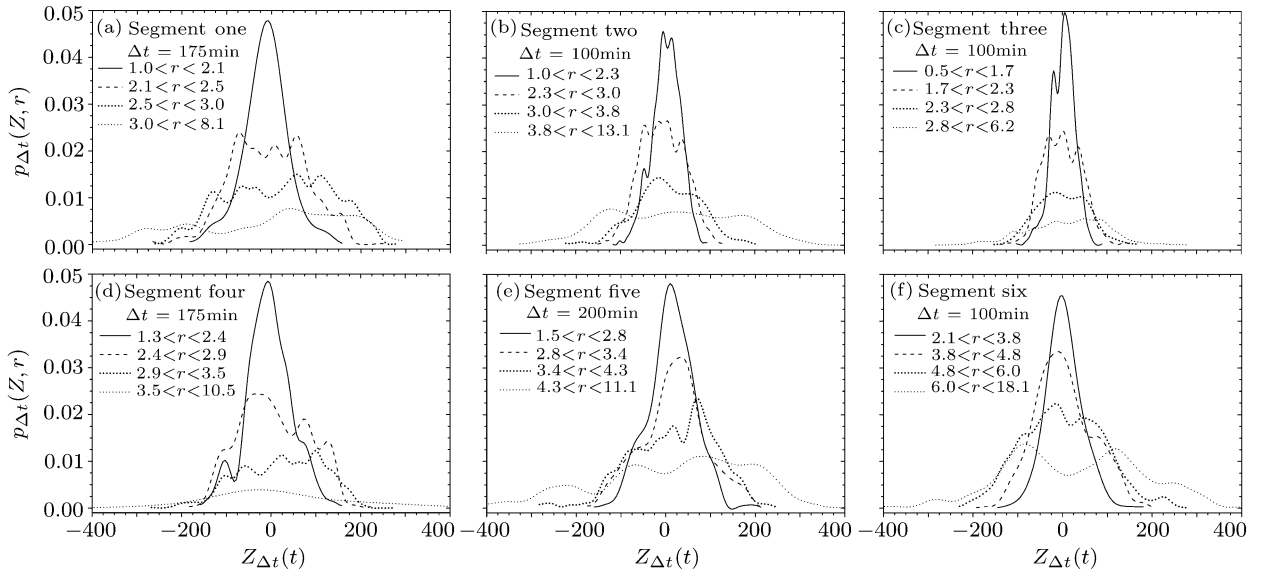


Fig. 2. Distribution $p_{\Delta t}(Z, r)$ of six segments: (a) segment one from 1 July to 30 December 1994 with the scale 150 min, (b) segment two from 3 January to 30 June 1995 with the scale 100 min, (c) segment three from 3 July to 29 December 1995 with the scale 100 min, (d) segment four from 2 January to 28 June 1996 with the scale 175 min, (e) segment five from 1 July to 31 December 1996 with the scale 200 min, (f) segment six from 2 January to 28 June 1997 with the scale 100 min.

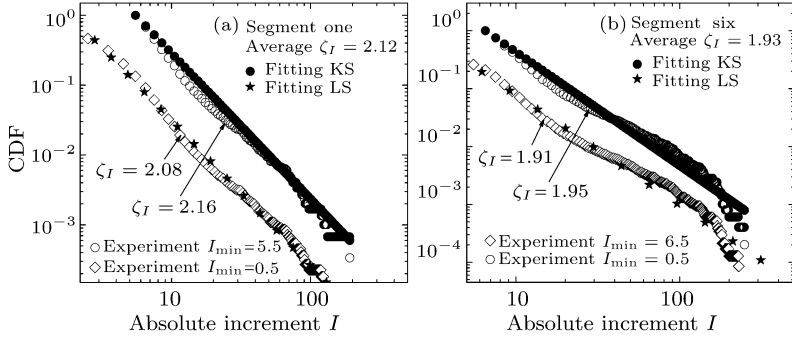


Fig. 3. Cumulative probability of absolute increment. (a) Segment one with average exponent $\zeta_I = 2.12 \pm 0.03$. (b) Segment six with average exponent $\zeta_I = 1.93 \pm 0.03$. CDF stands for the cumulative distribution function.

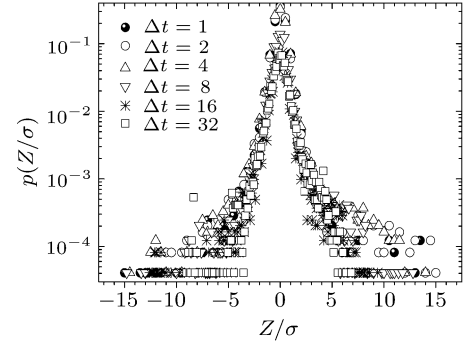


Fig. 4. Re-scaled plot of the probability distributions. The abscissa is the re-scaled returns, and the ordinate is the logarithm of re-scaled probability.

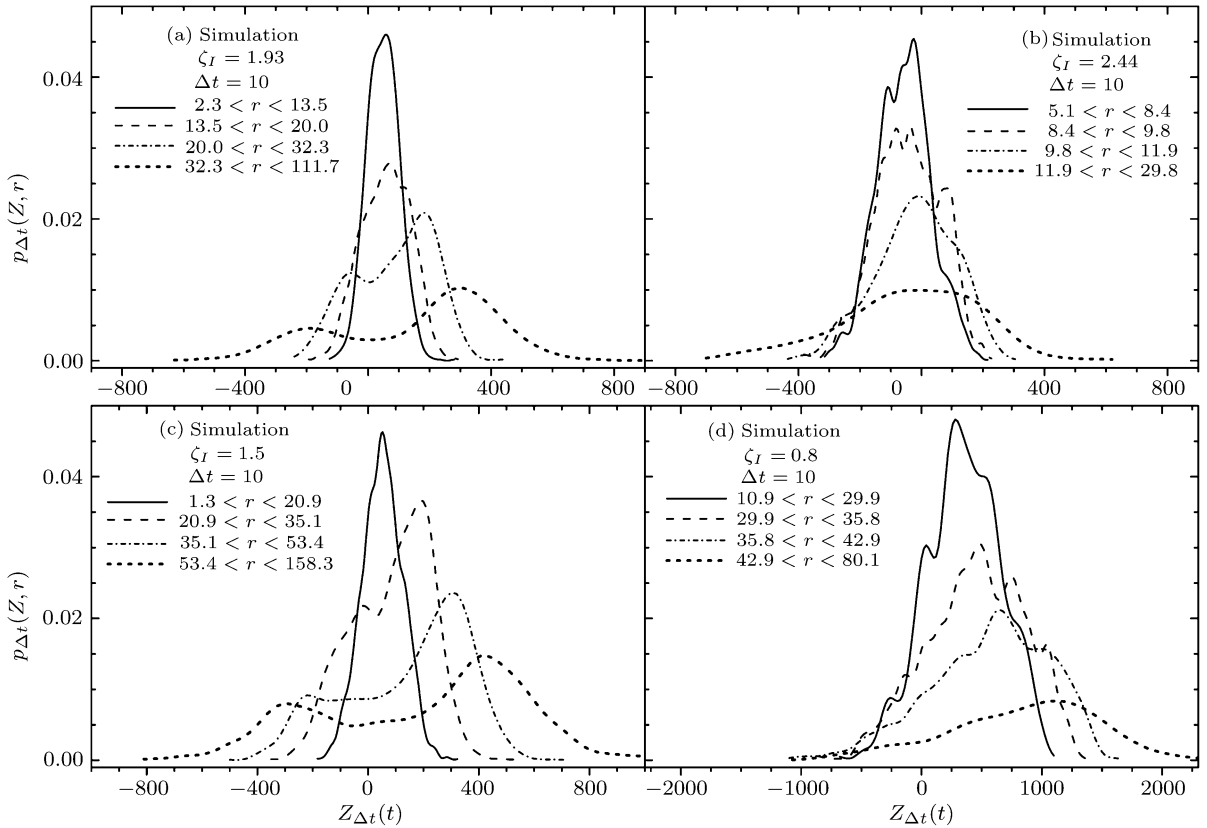


Fig. 5. Numerical simulation of the bifurcation phenomenon on artificial data. Power-law distributed I with the parameters: (a) $I_{\min} = 7$, $\zeta_I = 1.93$, 24442 data points as the same as segment six; (b) $I_{\min} = 5$, $\zeta_I = 2.44$, 28955 data points as same as segment three; (c) $I_{\min} = 7$, $\zeta_I = 1.50$, 25000 data points; (d) $I_{\min} = 7$, $\zeta_I = 0.80$, 25000 data points.

The study on trading volume shows that the transition of $p(\Omega|\Sigma)$ from an unimodal to a bimodal distribution is an artifact of the distribution of trade sizes q , which obeys a power-law distribution with exponent $\zeta_q < 2$ in the Lévy stable domain.^[15–17] Similar to the consideration in Refs. [16,17], we guess the existence of bifurcation phenomenon of stock index is dependent on the statistics of absolute increment I . Two typical cumulative distributions of I are reported in Fig. 3,

which both follow a power-law form above a lower bound I_{\min} . To demonstrate the stability of the distributions, PDFs of return for different time scales are analysed. As an example, Fig. 4 shows the re-scaled distributions for segment six. From Fig. 4 one can observe that the distributions for different time scales well collapse onto one master curve, which implies the stability of the distribution. We use the method of the best-fit power-law model and Kolmogorov–Smirnov

(KS) statistic^[18–20] to estimate parameters in the distribution, including both the lower bound I_{\min} and the power-law exponent ζ_I . In order to estimate carefully and accurately, we also apply the usual method of least-squares (LS) on the logarithm of the histogram. The exponents obtained by those two methods are nearly the same (see Fig. 3), and we report the two values and the average value (see Table 1). In Table 1, N means no bifurcation phenomenon appears and Y means the phenomenon can be observed. Compared with other five segments, the exponent of segment six is the smallest. Apparently, the bifurcation phenomenon of Hang–Seng index emerges when $\zeta_I < 2$.

Furthermore, given a power-law distribution of I , we generate artificial absolute increment time series $I(t)$ using the method introduced in Ref. [20]. The sign of increment (could be + or –) is randomly assigned, that is to say, the increment $i(t)$ is equal to $I(t)$ or $-I(t)$. Accordingly, $z(t)$ and $r(t)$ are

$$z(t) = \sum_{\tau=t}^{t+\Delta t} i(\tau), \quad (3)$$

and

$$r_{\Delta t}(t) = \langle |i(t) - \langle i(t) \rangle_{\Delta t}| \rangle_{\Delta t}. \quad (4)$$

The number of data points and the lower bound are set to be same as the real ones. The bifurcation phenomenon is clearly observed in Figs. 5(a) and 5(c), which holds for the scale ranging narrowly from 3 to 15 and from 3 to 18, respectively. Artificial data obeying power-law distribution with exponents ranging from 0 to 3 are carefully investigated, it is found that the obvious bifurcation phenomenon only holds when the power-law exponent ζ_I satisfies $1 < \zeta_I < 2$. There is no bifurcation phenomenon with $\zeta_I > 2$ and $0 < \zeta_I < 1$ whatever the scale is (see, for example, Figs. 5(b) and 5(d)).

Our findings suggest that the bifurcation phenomenon in financial index is not universal, but specific under certain conditions. For Hang–Seng index and randomly generated time series, the phenomenon just happens, within an appropriate period of time scale, when the power-law exponent of absolute increment distribution is between 1 and 2. The simulations on randomly generated time series suggest the bifurcation phenomenon itself is subject to the statistics

of absolute increment, thus it may not be able to reflect the essential financial behaviours (see also the relative comments from Refs. [15,21]). However, one should note that, even under the same distribution of absolute increment, the range where bifurcation phenomenon occurs is far different from real market to artificial data: for actual index the appropriate period is wide, while for the artificial data, it is very narrow (compare Fig. 2(f) with Fig. 5(a)). We expect that this difference could reflect certain market information, however, the underlying reason is not clear to us thus far.

References

- [1] Anderson P W and Pines D 1988 *The economy as an Evolving Complex System* (Santa Fe Institute Studies in the Science of Complexity Proceedings)
- [2] Arthur W B, Durlauf S N and Lane D A 1997 *The economy As an Evolving Complex System II: Proceedings* (Santa Fe Institute Studies in the Science of Complexity Lecture Notes)
- [3] Kertész J and Kondor I 1997 *Econophysics: An Emergent Science* (<http://newton.phy.bme.hu/~kullmann/Egyetem/konyv.html>)
- [4] Mantegna R N and Stanley H E 1999 *Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge: Cambridge University Press)
- [5] Johnson N F, Jefferies P and Hui P M 2003 *Financial Market Complexity* (Oxford: Oxford University)
- [6] Challet D, Marsili M and Zhang Y C 2005 *Minority Games* (Oxford: Oxford University Press)
- [7] Bouchaud J P and Potters M 2000 *Theory of Financial Risk* (Cambridge: Cambridge University Press)
- [8] Mandelbrot B B 1997 *Fractals and Scaling in Finance: Discontinuity, Concentration, Risk* (Berlin: Springer)
- [9] Sornette D 2000 *Critical Phenomena in Natural Science* (Berlin: Springer)
- [10] Wang B H and Hui P M 2001 *Eur. Phys. J. B* **20** 573
- [11] Cai S M et al 2006 *Chin. Phys. Lett.* **23** 754
- [12] Cai S M et al 2006 *Physica A* **367** 337
- [13] Plerou V, Gopikrishnan P and Stanley H E 2003 *Nature* **421** 130
- [14] Zheng B, Qiu T and Ren F 2004 *Phys. Rev. E* **69** 046115
- [15] Potters M et al 2003 *arXiv:cond-mat/0304514*
- [16] Plerou V, Gopikrishnan P and Stanley H E 2005 *Quantitative Finance* **5** 519
- [17] Matia K and Yamasaki K 2005 *Quantitative Finance* **5** 513
- [18] Kolmogorov A N 1958 *Dokl. Akad. Nauk SSSR* **119** 861
- [19] Sinai Y G 1959 *Dokl. Akad. Nauk SSSR* **124** 768
- [20] Clauset A et al 2007 *arxiv: 0706.1062*
- [21] Krzesinski A E et al 2007 *Int. J. Theor. Appl. Finance* **10** 89