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Research Article

Pairwise Comparison and Distance Measure of Hesitant Fuzzy Linguistic Term Sets

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A hesitant fuzzy linguistic term set (HFLTS), allowing experts using several possible linguistic terms to assess a qualitative linguistic variable, is very useful to express people's hesitancy in practical decision-making problems. Up to now, a little research has been done on the comparison and distance measure of HFLTSs. In this paper, we present a comparison method for HFLTSs based on pairwise comparisons of each linguistic term in the two HFLTSs. Then, a distance measure method based on the pairwise comparison matrix of HFLTSs is proposed, and we prove that this distance is equal to the distance of the average values of HFLTSs, which makes the distance measure much more simple. Finally, the pairwise comparison and distance measure methods are utilized to develop two multicriteria decision-making approaches under hesitant fuzzy linguistic environments. The results analysis shows that our methods in this paper are more reasonable.

1. Introduction

Since Zadeh introduced fuzzy sets [1] in 1965, several extensions of this concept have been developed, such as type-2 fuzzy sets [2, 3] and interval type-2 fuzzy sets [4], type-n fuzzy sets [5], intuitionistic fuzzy sets [6, 7] and interval-valued intuitionistic fuzzy sets [8], vague sets [9] (vague sets are intuitionistic fuzzy sets [10]), fuzzy multisets [11, 12], nonstationary fuzzy sets [13], Cloud models [14–18] (Cloud models are similar to nonstationary fuzzy sets and type-2 fuzzy sets), and hesitant fuzzy sets [19, 20]. In the real world, there are many situations in which problems must deal with qualitative aspects represented by vague and imprecise information. So, in these situations, often the experts are more accustomed to express their assessments using linguistic terms rather than numerical values. In [21–23], Zadeh introduced the concept of linguistic variable as "a variable whose values are not numbers but words or sentences in a natural or artificial language." Linguistic variable provides a means of approximate characterization of phenomena which are too complex or too ill defined to be amenable to description in conventional quantitative ways. Since then, fuzzy sets and linguistic variables have been widely used in describing linguistic information as they can efficiently represent people's qualitative cognition of an object or a concept [24]. Thus, linguistic approaches have been so far used successfully in a wide range of applications, such as information retrieval [25–28], data mining [29], clinical diagnosis [30, 31], and subjective evaluation [32–37], especially in decision-making [38–49]. Usually, linguistic terms (words) are represented by fuzzy sets [50], type-2 fuzzy sets [51], interval type-2 fuzzy sets [52–54], 2-tuple linguistic model [40, 55], and so forth. In these linguistic models, an expert generally provides a single linguistic term as an expression of his/her knowledge. However, just as Rodriguez et al. [56] pointed out, the expert may think of several terms at the same time or look for a more complex linguistic term that is not defined in the linguistic term set to express his/her opinion. In order to cope with this situation, they recently introduced the concept of hesitant fuzzy linguistic term sets (HFLTSs) [56] under the idea of hesitant fuzzy sets introduced in [19, 20].

Similarly to a hesitant fuzzy set which permits the membership having a set of possible values, an HFLTS allows an expert hesitating among several values for a linguistic variable. For example, when people assess a qualitative criterion, they prefer to use a linguistic one such as "between medium and very high" which contains several linguistic terms {*medium*, *high*, *veryhigh*}, rather than a single linguistic term. In practical decision-making process, uncertainty and hesitancy are usually unavoidable problems. The HFLTSs can deal with such uncertainty and hesitancy more objectively, and thus it is very necessary to develop some theories about HFLTSs.

Comparisons and distance measures used for measuring the deviations of different arguments are fundamentally important in a variety of applications. In the existing literature, there are a number of studies on distance measures for intuitionistic fuzzy sets [57–60], interval-valued intuitionistic fuzzy sets [61], hesitant fuzzy sets [62, 63], linguistic values [64, 65], and so forth. Nevertheless, an HFLTS is a linguistic term subset, and the comparison among these elements is not simple. In [56], Rodriguez et al. introduced the concept of envelope for an HFLTS and then ranked HFLTSs using the preference degree method of interval values [66]. But, because an HFLTS is a set of discrete linguistic terms, it may seem problematical using the preference degree method for continuous interval to compare these discrete terms of HFLTSs. Up to now, just a few research has been done on the distance measure of HFLTSs [67]. Consequently, it is very necessary to develop some comparison methods and distance measure methods for HFLTSs. In [67], to calculate the distance of two HFLTSs, Liao et al. extend the shorter HFLTS by adding any value in it until it has the same length of the longer one according to the decision-maker's preferences and actual situations. In this paper, we present a new comparison method of HFLTSs based on pairwise comparisons of each linguistic term in the two HFLTSs. Then, a distance measure method based on the pairwise comparison matrix of HFLTSs is proposed without adding any value. Finally, we utilize the comparison method and distance measure method to develop some approaches to solve the multicriteria decision-making problems under hesitant fuzzy linguistic environments.

The rest of the paper is organized as follows. In Section 2, the concepts of hesitant fuzzy sets and HFLTSs are introduced; also the defects of the previous comparison method for HFLTSs are analyzed according to an example. Section 3 describes the comparison and distance measure of HFLTSs based on the proposed pairwise comparison method. In Section 4, a multicriteria decision-making problem is shown to illustrate the detailed processes and effectiveness of two ranking methods which are based on the comparisons and distance measures of HFLTSs, respectively. Finally, Section 5 draws our conclusions and presents suggestions for future research.

2. Preliminaries

2.1. Hesitant Fuzzy Sets. Hesitant fuzzy sets (HFSs) were first introduced by Torra [19] and Torra and Narukawa [20]. The motivation is that when determining the membership degree of an element into a set, the difficulty is not because we have a margin of error (such as an interval) but because we have several possible values.

Definition 1 (see [19]). Let X be a fixed set; a hesitant fuzzy set (HFS) on X is in terms of a function h that when applied to X returns a subset of $[0, 1]$.

To be easily understood, Zhu et al. [68] represented the HFS as the following mathematical symbol:

$$E = \{\langle x, h(x)\rangle \mid x \in X\}, \quad (1)$$

where $h(x)$ is a set of some values in $[0, 1]$, denoting the possible membership degrees of the element $x \in X$ to the set E. Liao et al. [67] called $h(x)$ a hesitant fuzzy element (HFE).

Example 2. Let $h = \{0.2, 0.3, 0.4\}$; then h is an HFE.

Definition 3 (see [69]). For an HFE h, the score function of h is defined as

$$s(h) = \frac{1}{\#h} \sum_{\gamma \in h} \gamma, \quad (2)$$

where $\#h$ is the number of the elements in h.

For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then h_1 is superior to h_2, denoted by $h_1 \succ h_2$; if $s(h_1) = s(h_2)$, then h_1 is indifferent with h_2, denoted by $h_1 \sim h_2$.

Example 4. Assume that we have three HFEs, $h_1 = \{0.2, 0.3, 0.4\}$, $h_2 = \{0.2, 0.35, 0.5\}$, and $h_3 = \{0.3, 0.4\}$; then according to the score function of HFE, (2), and Definition 3, we have $s(h_1) = (0.2 + 0.3 + 0.4)/3 = 0.3$, $s(h_2) = (0.2 + 0.35 + 0.5)/3 = 0.35$, and $s(h_3) = (0.3 + 0.4)/2 = 0.35$. Thus, $s(h_2) = s(h_3) > s(h_1)$; that is, the ranking is $h_2 \sim h_3 \succ h_1$.

The concept of HFS is very useful to express people's hesitancy in daily life. So, since it was introduced, more and more decision-making theories and methods under hesitant fuzzy environment have been developed [56, 62, 63, 67–73].

2.2. Hesitant Fuzzy Linguistic Term Sets. Similarly to the HFS, an expert may hesitate among several linguistic terms, such as "between medium and very high" or "lower than medium," to assess a qualitative linguistic variable. To deal with such situations, Rodriguez et al. [56] introduced the concept of hesitant fuzzy linguistic term sets (HFLTSs).

Definition 5 (see [56]). Suppose that $S = \{s_0, \ldots, s_g\}$ is a finite and totally ordered discrete linguistic term set, where s_i represents a possible value for a linguistic variable. An HFLTS, H_S, is defined as an ordered finite subset of the consecutive linguistic terms of S.

It is required that the linguistic term set S should satisfy the following characteristics:

(1) the set is ordered: $s_i > s_j$, if and only if $i > j$;

(2) there is a negation operator: $\mathrm{Neg}(s_i) = s_{g-i}$.

Example 6. Let S be a linguistic term set, $S = \{s_0 :$ n (nothing), s_1 : vl (very low), s_2 : l (low), s_3 : m (medium), s_4 : h (high), s_5 : vh (very high), and s_6 : p (perfect)$\}$; two different HFLTSs might be $H_S^1 = \{s_1 : \text{vl}, s_2 : \text{l}, s_3 : \text{m}\}$ and $H_S^2 = \{s_3 : \text{m}, s_4 : \text{h}\}$.

Definition 7. One defines the number of linguistic terms in the HFLTS H_S as the cardinality of H_S, denoted by $|H_S|$. In Example 6, $|H_S^1| = 3$ and $|H_S^2| = 2$.

Definition 8 (see [56]). The lower bound H_{S-} and upper bound H_{S+} of the HFLTS H_S are defined as $H_{S-} = \min\{s_i \mid s_i \in H_S\}$ and $H_{S+} = \max\{s_i \mid s_i \in H_S\}$.

Definition 9 (see [56]). The envelope of the HFLTS H_S, $\text{env}(H_S)$, is defined as the linguistic interval $[H_{S-}, H_{s+}] = [\text{Ind}(H_{S-}), \text{Ind}(H_{S+})]$, where Ind provides the index of the linguistic term; that is, $\text{Ind}(s_i) = i$. In Example 6, $\text{env}(H_S^1) = [s_1, s_3] = [1, 3]$ and $\text{env}(H_S^2) = [s_3, s_4] = [3, 4]$.

Based on the definition of envelope, Rodriguez et al. [56] compare two HFLTSs using the comparison method between two numerical intervals introduced by Wang et al. [66].

Definition 10 (see [66]). Letting $A = [a_1, a_2]$ and $B = [b_1, b_2]$ be two intervals, the preference degree of A over B (or $A > B$) is defined as

$$P(A > B) = \frac{\max(0, a_2 - b_1) - \max(0, a_1 - b_2)}{(a_2 - a_1) + (b_2 - b_1)} \quad (3)$$

and the preference degree of B over A (or $B > A$) is defined as

$$P(B > A) = \frac{\max(0, b_2 - a_1) - \max(0, b_1 - a_2)}{(a_2 - a_1) + (b_2 - b_1)}. \quad (4)$$

Example 11. Let $H_S^1 = \{s_1, s_2, s_3\}$, $H_S^2 = \{s_3, s_4\}$, and $H_S^3 = \{s_5, s_6\}$ be three different HFLTSs on S. According to Definition 9, we have $\text{env}(H_S^2) = [1, 3]$, $\text{env}(H_S^2) = [3, 4]$, and $\text{env}(H_S^3) = [5, 6]$. The preference degrees calculated by Definition 10, (3), and (4) are

$$\begin{aligned} &P(H_S^1 > H_S^2) = 0, \qquad P(H_S^2 > H_S^1) = 1; \\ &P(H_S^1 > H_S^3) = 0, \qquad P(H_S^3 > H_S^1) = 1; \\ &P(H_S^2 > H_S^3) = 0, \qquad P(H_S^3 > H_S^2) = 1. \end{aligned} \quad (5)$$

From Example 11 mentioned above, it can be observed that when we compare two HFLTSs using the preference degree method, there exist two defects as follows.

(1) The result $P(H_S^2 > H_S^1) = 1$ indicates that H_S^2 is absolutely superior to H_S^1. In fact, both H_S^1 and H_S^2 contain the linguistic term s_3. It means that the value of a linguistic variable may be equal in these two cases. Thus, it is unreasonable to say that H_S^2 is absolutely superior to H_S^1.

(2) The result $P(H_S^3 > H_S^1) = P(H_S^2 > H_S^1) = 1$, meaning that when compared with H_S^1, the two HFLTSs H_S^2 and H_S^3 are identical. In fact, H_S^3 is more superior to H_S^1 compared to H_S^2 to H_S^1. Thus, using the preference degree method to compare HFLTSs may result in losing some important information.

Based on the analysis mentioned above, we think that it is not suitable to compare discrete linguistic terms in HFLTSs using the comparison method for continuous numerical intervals. By the definition of an HFLTS, we know that every linguistic term in it is a possible value of the linguistic information. And noting that, the two HFLTSs for comparing may have different lengths. So, when comparing two HFLTSs, it needs pairwise comparisons of each linguistic term in them.

3. Comparison and Distance Measure of HFLTSs

3.1. Distance between Two Single Linguistic Terms. Let $s_i, s_j \in S$ be two linguistic terms. Xu [64] defined the deviation measure between s_i and s_j as follows:

$$d(s_i, s_j) = \frac{|i - j|}{T}, \quad (6)$$

where T is the cardinality of S; that is, $T = |S|$.

If only one preestablished linguistic term set S is used in a decision-making model, we can simply consider [49, 65]: $d(s_i, s_j) = |i - j| = s_{|i-j|}$.

Definition 12. Letting $s_i, s_j \in S$ be two single linguistic terms, then we call

$$d(s_i, s_j) = s_i - s_j = i - j \quad (7)$$

the distance between s_i and s_j.

The distance measure between s_i and s_j has a definite physical implication and reflects the relative position and distance between s_i and s_j. If $d(s_i, s_j) = 0$, then $s_i = s_j$. If $d(s_i, s_j) > 0$, then $s_i > s_j$. If $d(s_i, s_j) < 0$, then $s_i < s_j$.

Theorem 13. *Letting $s_i, s_j, s_k \in S$ be three linguistic terms, then*

(1) $d(s_i, s_j) = -d(s_j, s_i)$;

(2) $(|S| - 1) \leq d(s_i, s_j) \leq (|S| - 1)$;

(3) $d(s_i, s_k) = d(s_i, s_j) + d(s_j, s_k)$.

Proof. They are straightforward and thus omitted. □

3.2. Comparison of HFLTSs. The comparison of HFLTSs is necessary in many problems, such as ranking and selection. However, an HFLTS is a linguistic term subset which contains several linguistic terms, and the comparison among HFLTSs is not simple. Here, a new comparison method of HFLTSs, which is based on pairwise comparisons of each linguistic term in the two HFLTSs, is put forward.

Definition 14. Letting H_S^1 and H_S^2 be two HFLTSs on S, then one defines the pairwise comparison matrix between H_S^1 and H_S^2 as follows:

$$C(H_S^1, H_S^2) = \left[d(s_i, s_j)\right]_{|H_S^1| \times |H_S^2|}, \quad s_i \in H_S^1,\ s_j \in H_S^2. \quad (8)$$

Remark 15. The number of linguistic terms in the two HFLTSs, H_S^1 and H_S^2, may be unequal; that is, $|H_S^1| \neq |H_S^2|$. To deal with such situations, usually it is necessary to extend the shorter one by adding the stated value several times in it [62, 63], while our pairwise comparison method does not require this step.

Remark 16. From Definition 14, we have $[C(H_S^1, H_S^2)] = -[C(H_S^2, H_S^1)]^T$, where T is the transpose operator of matrix.

Example 17. Let $H_S^1 = \{s_1, s_2, s_3\}$ and $H_S^2 = \{s_2, s_3, s_4, s_5\}$ be two HFLTSs on S. According to Definition 14, the comparison matrix C between H_S^1 and H_S^2 is

$$\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \end{array} \begin{array}{c} \begin{array}{cccc} s_2 & s_3 & s_4 & s_5 \end{array} \\ \begin{bmatrix} -1 & -2 & -3 & -4 \\ 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \end{bmatrix} \end{array}. \tag{9}$$

Definition 18. Letting $C = C(H_S^1, H_S^2)$ be the pairwise comparison matrix between H_S^1 and H_S^2, the preference relations of H_S^1 and H_S^2 are defined as follows:

$$\begin{aligned} p\left(H_S^1 > H_S^2\right) &= \frac{\left|\sum_{C_{mn}>0} C_{mn}\right|}{\#\left\{C_{mn} = 0\right\} + \sum\left|C_{mn}\right|}, \\ p\left(H_S^1 = H_S^2\right) &= \frac{\#\left\{C_{mn} = 0\right\}}{\#\left\{C_{mn} = 0\right\} + \sum\left|C_{mn}\right|}, \\ p\left(H_S^1 < H_S^2\right) &= \frac{\left|\sum_{C_{mn}<0} C_{mn}\right|}{\#\left\{C_{mn} = 0\right\} + \sum\left|C_{mn}\right|}. \end{aligned} \tag{10}$$

It is obvious that $p(H_S^1 > H_S^2) + p(H_S^1 = H_S^2) + p(H_S^1 < H_S^2) = 1$. We say that H_S^1 is superior to H_S^2 with the degree of $p(H_S^1 > H_S^2)$, denoted by $H_S^1 \succ^{p(H_S^1 > H_S^2)} H_S^2$; H_S^1 is equal to H_S^2 with the degree of $p(H_S^1 = H_S^2)$, denoted by $H_S^1 \sim^{p(H_S^1 = H_S^2)} H_S^2$; and H_S^1 is inferior to H_S^2 with the degree of $p(H_S^1 < H_S^2)$, denoted by $H_S^1 \prec^{p(H_S^1 < H_S^2)} H_S^2$.

Considering Example 17, by Definition 18, (10), the preference relations of H_S^1 and H_S^2 were calculated as $p(H_S^1 > H_S^2) = 1/22$, $p(H_S^1 = H_S^2) = 2/22$, and $p(H_S^1 < H_S^2) = 19/22$. Thus, the comparison results are $H_S^1 \succ^{1/22} H_S^2$, $H_S^1 \sim^{2/22} H_S^2$, and $H_S^1 \prec^{19/22} H_S^2$.

3.3. Distance Measure of HFLTSs

Definition 19. Letting $C = C(H_S^1, H_S^2)$ be the pairwise comparison matrix between H_S^1 and H_S^2, the distance between H_S^1 and H_S^2 is defined as the average value of the pairwise comparison matrix:

$$d\left(H_S^1, H_S^2\right) = \frac{1}{\left|H_S^1\right| \times \left|H_S^2\right|} \sum_{m=1}^{|H_S^1|} \sum_{n=1}^{|H_S^2|} C_{mn}. \tag{11}$$

Considering Example 17, one has $d(H_S^1, H_S^2) = (-18)/(3 \times 4) = -1.5$.

To preserve all the given information, the discrete linguistic term set S is extended to a continuous term set $\overline{S} = \{s_\alpha \mid \alpha \in [-q, q]\}$, where q is a sufficiently large positive number. If $s_\alpha \in S$, then we call s_α an original linguistic term; otherwise, we call s_α a virtual linguistic term.

Remark 20. In general, the decision-maker uses the original linguistic terms to express his/her qualitative opinions, and the virtual linguistic terms can only appear in operations.

Definition 21. The average value of an HFLTS H_S is defined as

$$\operatorname{Aver}\left(H_S\right) = \frac{1}{\left|H_S\right|} \sum_{s_i \in H_S} s_i = \frac{1}{\left|H_S\right|} \sum_{s_i \in H_S} \operatorname{Ind}\left(s_i\right). \tag{12}$$

This definition is similar to the score function of an HFE, Definition 3.

Considering Example 17, we have $\operatorname{Aver}(H_S^1) = s_{(1+2+3)/3} = s_2 = 2$, and $\operatorname{Aver}(H_S^2) = s_{(2+3+4+5)/4} = s_{3.5} = 3.5$.

Theorem 22. *Letting H_S be an HFLTS on S, then*

$$0 \leq H_{s-} \leq Aver\left(H_S\right) \leq H_{s+} \leq \left(|S| - 1\right). \tag{13}$$

Proof. It is straightforward and thus omitted. □

Theorem 23. *Letting H_S^1 and H_S^2 be two HFLTSs on S, the distance between H_S^1 and H_S^2 defined by the average value of their pairwise comparison matrix is equal to the distance of the two average values of H_S^1 and H_S^2; that is, the distance between H_S^1 and H_S^2 can be easily obtained by*

$$d\left(H_S^1, H_S^2\right) = Aver\left(H_S^1\right) - Aver\left(H_S^2\right). \tag{14}$$

Proof. From Definitions 19 and 14, we have

$$\begin{aligned} d\left(H_S^1, H_S^2\right) &= \frac{1}{\left|H_S^1\right| \times \left|H_S^2\right|} \sum_{m=1}^{|H_S^1|} \sum_{n=1}^{|H_S^2|} C_{mn} \\ &= \frac{1}{\left|H_S^1\right| \times \left|H_S^2\right|} \sum_{s_i \in H_S^1} \sum_{s_j \in H_S^2} d\left(s_i, s_j\right) \\ &= \frac{1}{\left|H_S^1\right| \times \left|H_S^2\right|} \sum_{s_i \in H_S^1} \sum_{s_j \in H_S^2} \left(s_i - s_j\right) \\ &= \frac{1}{\left|H_S^1\right| \times \left|H_S^2\right|} \left(\sum_{s_i \in H_S^1} \sum_{s_j \in H_S^2} s_i - \sum_{s_i \in H_S^1} \sum_{s_j \in H_S^2} s_j \right) \\ &= \frac{1}{\left|H_S^1\right| \times \left|H_S^2\right|} \left(\left|H_S^2\right| \times \sum_{s_i \in H_S^1} s_i - \left|H_S^1\right| \times \sum_{s_j \in H_S^2} s_j \right) \\ &= \frac{1}{\left|H_S^1\right|} \sum_{s_i \in H_S^1} s_i - \frac{1}{\left|H_S^2\right|} \sum_{s_j \in H_S^2} s_j \\ &= \operatorname{Aver}\left(H_S^1\right) - \operatorname{Aver}\left(H_S^2\right), \end{aligned} \tag{15}$$

which completes the proof of Theorem 23. □

Considering Example 17, we have $d(H_S^1, H_S^2) = \text{Aver}(H_S^1) - \text{Aver}(H_S^2) = 2 - 3.5 = -1.5$.

By Theorem 23, we can easily obtain the following corollary.

Corollary 24. *Letting H_S^1, H_S^2 and H_S^3 be three HFLTSs on S, then*

(1) $(|S| - 1) \leq d(H_S^1, H_S^2) \leq (|S| - 1)$;

(2) $d(H_S^1, H_S^2) = -d(H_S^2, H_S^1)$;

(3) $d(H_S^1, H_S^3) = d(H_S^1, H_S^2) + d(H_S^2, H_S^3)$.

Proof. They are straightforward and thus omitted. □

If $d(H_S^1, H_S^2) > 0$ (or $\text{Aver}(H_S^1) > \text{Aver}(H_S^2)$), then we say that H_S^1 is superior to H_S^2 with the distance of $d(H_S^1, H_S^2)$, denoted by $H_S^1 \overset{d(H_S^1,H_S^2)}{\succ} H_S^2$; if $d(H_S^1, H_S^2) = 0$ (or $\text{Aver}(H_S^1) = \text{Aver}(H_S^2)$), then we say that H_S^1 is indifferent to H_S^2, denoted by $H_S^1 \sim H_S^2$; if $d(H_S^1, H_S^2) < 0$ (or $\text{Aver}(H_S^1) < \text{Aver}(H_S^2)$), then we say that H_S^1 is inferior to H_S^2 with the distance of $d(H_S^1, H_S^2)$, denoted by $H_S^1 \overset{d(H_S^1,H_S^2)}{\prec} H_S^2$.

4. Multicriteria Decision-Making Models Based on Comparisons and Distance Measures of HFLTSs

In this section, two new methods are presented for ranking and choice from a set of alternatives in the framework of multicriteria decision-making using linguistic information. One is based on the comparisons and preference relations of HFLTSs and the other is based on the distance measure of HFLTSs. We adopt Example 5 in [56] (Example 25 in our paper) to illustrate the detailed processes of the two methods.

Example 25 ([see [56]). Let $X = \{x_1, x_2, x_3\}$ be a set of alternatives, $C = \{c_1, c_2, c_3\}$ a set of criteria defined for each alternative, and $S = \{s_0$: n (nothing), s_1 : vl (very low), s_2 : l (low), s_3 : m (medium), s_4 : h (high), s_5 : vh (very high), s_6 : p (perfect)$\}$ the linguistic term set that is used to generate the linguistic expressions. The assessments that are provided in such a problem are shown in Table 1 and they are transformed into HFLTSs as shown in Table 2.

4.1. Multicriteria Decision-Making Based on the Comparisons of HFLTSs

Step 1. Considering each criterion c_i $(i = 1, 2, 3)$, calculate the preference degrees between all the alternatives x_j $(j = 1, 2, 3)$.

Considering criterion c_1, $H_S^{x_1} = \{s_1, s_2, s_3\}$, $H_S^{x_2} = \{s_2, s_3\}$ and $H_S^{x_3} = \{s_4, s_5, s_6\}$, so the preference degrees about criterion c_1 calculated using the comparison method of HFLTSs as described in Section 3.2 are $pc_1(x_1 > x_2) = 1/7$, $pc_1(x_1 = x_2) = 2/7$, $pc_1(x_1 < x_2) = 4/7$; $pc_1(x_1 > x_3) = 0/27$, $pc_1(x_1 = x_3) = 0/27$, $pc_1(x_1 < x_3) = 27/27$; $pc_1(x_2 > x_3) = 0/15$, $pc_1(x_2 = x_3) = 0/15$, $pc_1(x_2 < x_3) = 15/15$.

Table 1: Assessments that are provided for the decision problem.

	c_1	c_2	c_3
x_1	Between vl and m	Between h and vh	h
x_2	Between l and m	m	Lower than l
x_3	Greater than h	Between vl and l	Greater than h

Table 2: Assessments transformed into HFLTSs.

	c_1	c_2	c_3
x_1	$\{s_1, s_2, s_3\}$	$\{s_4, s_5\}$	$\{s_4\}$
x_2	$\{s_2, s_3\}$	$\{s_3\}$	$\{s_0, s_1, s_2\}$
x_3	$\{s_4, s_5, s_6\}$	$\{s_1, s_2\}$	$\{s_4, s_5, s_6\}$

Considering criterion c_2, $H_S^{x_1} = \{s_4, s_5\}$, $H_S^{x_2} = \{s_3\}$, and $H_S^{x_3} = \{s_1, s_2\}$, so the preference degrees about criterion c_2 calculated using the comparison method of HFLTSs as described in Section 3.2 are $pc_2(x_1 > x_2) = 3/3$, $pc_2(x_1 = x_2) = 0/3$, $pc_2(x_1 < x_2) = 0/3$; $pc_2(x_1 > x_3) = 12/12$, $pc_2(x_1 = x_3) = 0/12$, $pc_2(x_1 < x_3) = 0/12$; $pc_2(x_2 > x_3) = 3/3$, $pc_2(x_2 = x_3) = 0/3$, $pc_2(x_2 < x_3) = 0/3$.

Considering criterion c_3, $H_S^{x_1} = \{s_4\}$, $H_S^{x_2} = \{s_0, s_1, s_2\}$, and $H_S^{x_3} = \{s_4, s_5, s_6\}$, so the preference degrees about criterion c_3 calculated using the comparison method of HFLTSs as described in Section 3.2 are $pc_3(x_1 > x_2) = 9/9$, $pc_3(x_1 = x_2) = 0/9$, $pc_3(x_1 < x_2) = 0/9$; $pc_3(x_1 > x_3) = 0/4$, $pc_3(x_1 = x_3) = 1/4$, $pc_3(x_1 < x_3) = 3/4$; $pc_3(x_2 > x_3) = 0/36$, $pc_3(x_2 = x_3) = 0/36$, $pc_3(x_2 < x_3) = 36/36$.

Step 2. Aggregate the preference relations using the weighted average method: $p(x_j > x_k) = \text{sum}(w_i \times pc_i(x_j > x_k))$, $p(x_j = x_k) = \text{sum}(w_i \times pc_i(x_j = x_k))$, and $p(x_j < x_k) = \text{sum}(w_i \times pc_i(x_j < x_k))$, where w_i is the weight of criterion c_i, and $\text{sum}(w_i) = 1$. In this paper, $w_i = 1/3$, $i = 1, 2, 3$. Thus, the final preference relations are $(x_1 > x_2) = 15/21$, $p(x_1 = x_2) = 2/21$, $p(x_1 < x_2) = 4/21$; $p(x_1 > x_3) = 1/3$, $p(x_1 = x_3) = 1/12$, $p(x_1 < x_3) = 7/12$; $p(x_2 > x_3) = 1/3$, $p(x_2 = x_3) = 0$, $p(x_2 < x_3) = 2/3$.

Step 3. Rank the alternatives using the nondominance choice degree method as described in [56]. From the results of Step 2, it can be easily obtained that

$$P_D^S = \begin{pmatrix} - & \frac{11}{21} & 0 \\ 0 & - & 0 \\ \frac{1}{4} & \frac{1}{3} & - \end{pmatrix}. \tag{16}$$

Thus, $\text{NDD}_1 = \min\{(1 - 0), (1 - 1/4)\} = 3/4$, $\text{NDD}_2 = \min\{(1 - 11/21), (1 - 1/3)\} = 10/21$, and $\text{NDD}_3 = \min\{(1 - 0), (1 - 0)\} = 1$. Finally, the ranking of alternatives is $x_3 > x_1 > x_2$.

Table 3: Average values of the assessments.

	c_1	c_2	c_3
x_1	2	4.5	4
x_2	2.5	3	1
x_3	5	1.5	5

Table 4: Aggregation results of each alternative.

	x_1	x_2	x_3
Aggregation result	3.5	2.2	3.8

4.2. Multicriteria Decision-Making Based on the Distance Measures of HFLTSs

Step 1. Considering each criterion c_i $(i = 1, 2, 3)$, calculate the average values of HFLTSs for all the alternatives x_j $(j = 1, 2, 3)$. The results are shown in Table 3.

Step 2. Aggregate the average values using the weighted average method. The results are shown in Table 4.

Step 3. Rank the alternatives using the distance measure method. Thus, the ranking of alternatives is $x_3 \overset{0.3}{\succ} x_1 \overset{1.3}{\succ} x_2$.

4.3. Results Analysis. In [56], the ranking of alternatives is $x_1 \succ x_3 \succ x_2$, while both methods in this paper are $x_3 \succ x_1 \succ x_2$. Note that the practical decision-making problem is quite different from other applications where well-established measures can be used to quantify the performance for validation. In decision-making, usually there is no ground truth data or quantitative measures to assess the performance of a method [37]. This is why "plausibility" is used rather than "validation." Here, we analyze the original assessments about each criterion of alternatives x_1 and x_3. Considering criterion c_1, the original assessments of x_1 and x_3 are "between vl and m" and "greater than h," respectively, so it is obviously $x_3 \succ x_1$ about criterion c_1. Considering criterion c_2, the original assessments of x_1 and x_3 are "between h and vh" and "between vl and l," respectively, so this time $x_1 \succ x_3$. Considering criterion c_3, the original assessments of x_1 and x_3 are "h" and "greater than h," respectively, so $x_3 \succ x_1$ again. Summarily, $x_3 \succ x_1$ occurs twice, while $x_1 \succ x_3$ only once. Thus, we believe that our result is more plausible.

5. Conclusion

The comparison and distance measure of HFLTSs are fundamentally important in many decision-making problems under hesitant fuzzy linguistic environments. From an example, we found that there existed two defects when comparing HFLTSs using the previous preference degree method. By analyzing the definition of an HFLTS, a new comparison method based on pairwise comparisons of each linguistic term in the two HFLTSs has been put forward. This comparison method does not need the assumption that the values in all HFLTSs are arranged in an increasing order and two HFLTSs have the same length when comparing them. Then, we have defined a distance measure method between HFLTSs based on pairwise comparisons. Further, we have proved that this distance is equal to the distance of the average values of HFLTSs, which makes the distance measure much simpler. Finally, two new methods for multicriteria decision-making in which experts provide their assessments by HFLTSs have been proposed. The encouraging results demonstrate that our methods in this paper are more reasonable.

In the future, the application of HFLTSs to group decision-making problems will be explored. We will also investigate how to obtain the weights of criteria under hesitant fuzzy linguistic environments.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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