

## Research Article

# Optimized Measurement Matrix Design Using Spatiotemporal Chaos for CS-MIMO Radar

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We investigate the possibility of utilizing the chaotic dynamic system for the measurement matrix design in the CS-MIMO radar system. The CS-MIMO radar achieves better detection performance than conventional MIMO radar with fewer measurements. For exactly recovering from compressed measurements, we should carefully design the measurement matrix to make the sensing matrix satisfy the restricted isometry property (RIP). A Gaussian random measurement matrix (GRMM), typically used in CS problems, is not satisfied for on-line optimization and the low coherence with the basis matrix corresponding to the MIMO radar scenario can not be well guaranteed. An optimized measurement matrix design method applying the two-dimensional spatiotemporal chaos is proposed in this paper. It incorporates the optimization criterion which restricts the coherence of the sensing matrix and singular value decomposition (SVD) for the optimization process. By varying the initial state of the spatiotemporal chaos and optimizing each spatiotemporal chaotic measurement matrix (SCMM), we can finally obtain the optimized measurement matrix. Its simulation results show that the optimized SCMM can highly reduce the coherence of the sensing matrix and improve the DOA estimation accuracy for the CS-MIMO radar.

## 1. Introduction

The application of compressive sensing (CS) to radar systems has received considerable attention in recent years [1, 2]. The CS theory asserts that a signal that exhibits sparsity in some domain can be recovered from far fewer samples than that required by the Nyquist theory [3]. According to the CS theory, employing CS in multiple-input multiple-output (MIMO) radar can recover the target scene information from significantly fewer samples than the traditional methods. For instance, in colocated CS-MIMO radar, each of the receive antennas compresses its received signal via a transformation matrix, referred to as the measurement matrix. The samples are subsequently forwarded to a fusion center, where an  $\ell_1$ -optimization problem will be solved for recovering the received signals. Actually, the measurement matrix plays an important role in this process. According to the RIP, an important property that measurement matrix  $\Phi$  should obey is the low coherence with the basis matrix  $\Psi$  [4]. With an orthonormal basis matrix  $\Psi$ , the use of a random

measurement matrix  $\Phi$  leads to a sensing matrix  $\Theta$  ( $\Theta \triangleq \Phi\Psi$ ) that meets the RIP requirement with overwhelming probability. Since the basis matrix is constructed specially based on the given signal model in a MIMO radar scenario, the measurement matrix is expected to facilitate an efficient and controllable implementation so as to match the known basis matrix.

Some works have addressed the measurement matrix design problem [5–8]. An iterative algorithm using shrinkage to decrease the average coherence of the sensing matrix was proposed in [5], and [6] proposed a gradient descent method to optimize the measurement matrix. In [7], an adaptive computational framework for optimizing the transmission waveform and Gaussian random measurement matrix separately and simultaneously was introduced, incorporating the target scene information for optimization. The framework leads to smaller cross-correlations between different target responses but has to bear great computation load when the scene is varying fast. The work in [8] proposed two approaches: the first one minimizes a performance penalty which is a linear

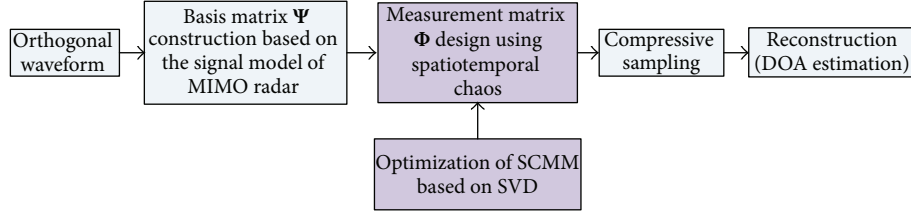


FIGURE 1: The scheme of using CS in MIMO radar.

combination of the coherence of the sensing matrix (CSM) and the inverse signal-to-interference ratio (SIR). It aims at improving the SIR and reducing the coherence of the sensing matrix at the same time. The second one, aiming at improving SIR only, imposes a structure on the measurement matrix and determines the parameters involved. It requires carefully chosen waveforms to guarantee the desired CS performance. Their simulation showed that the two measurement matrices with the proper waveform can improve detection accuracy as compared to the GRMM.

So far many of the CS schemes employ the Gaussian or Bernoulli matrix for their measurement matrix design or optimization. However, as proposed in [8], the Gaussian random matrix, typically used in the CS problems, is not necessarily the best choice for a given basis matrix in terms of the coherence of column pairs in the sensing matrix. Furthermore, in radar systems, it is difficult for designers to generate and control a perfect Gaussian random matrix in physical electric circuit with its randomness being well guaranteed.

Chaos is a nonlinear dynamical system, which can generate pseudorandom matrix in deterministic approach. It is easy to be implemented in physical electric circuit and only one initial state is necessary to be memorized. Moreover, since a chaotic dynamic system is quite sensitive to its initial state, slight changes will lead to quite different chaotic behaviors. This property can be applied for adjusting the chaotic measurement matrix to match the known basis matrix corresponding to a MIMO radar scenario, thus realizing on-line optimization in CS-MIMO radar systems. Several literatures have proposed the idea of using chaos in CS [9, 10], where different types of chaotic systems have been investigated, such as the logistic map, Chua's circuit, and Lorenz system. In [9], a mathematical statement is made to prove that the chaotic sequence is approximately independent and that the matrix constructed by the sampled chaotic sequence can satisfy RIP with overwhelming probability. The work in [10] capitalized on chaotic coded waveform to construct measurement operator in a monostatic MIMO noise radar system.

However, all of the proposed approaches have just applied one-dimensional chaotic systems for a simple CS recovery framework. The fact is, all of the methods have to tolerate the unexpected loss of the independence and randomness of the chaotic sequences during reshaping the sequence into a matrix for CS. In this paper, we propose a novel measurement matrix design method using spatiotemporal chaos for CS-MIMO radar. The measurement matrix for CS-MIMO radar is generated directly by the two-dimensional spatiotemporal

chaos. Compared to the low-dimensional chaotic system, spatiotemporal chaos possesses higher complexity and randomness [11, 12]. Therefore it is a more excellent candidate for measurement matrix. Considering that the basis matrix is known in a particular MIMO radar scenario, a joint optimization method with the goal of further reducing the coherence between the chaotic measurement matrix and the fixed basis matrix is proposed by varying the initial state of the spatiotemporal chaos and applying SVD on two specific structures to iteratively solve a minimization problem with respect to SCMM.

The scheme of CS-MIMO radar in this paper is demonstrated in Figure 1, in which the measurement matrix design and optimization are the key steps of our work.

The remainder of the paper is organized as follows. In Section 2, we provide the signal model of the monostatic CS-MIMO radar system. In Section 3 we propose our approach for designing the measurement matrix using the spatiotemporal chaos and provide the optimization method based on SVD. Experimental results are given in Section 4. Finally, some concluding remarks are presented in Section 5.

## 2. Signal Model of Monostatic MIMO Radar and Basis Matrix Construction

We consider a monostatic MIMO radar system consisting of  $N_t$  transmit antennas and  $N_r$  receiver antennas, both of which are closely distributed uniform linear arrays (ULA). Assume that  $K$  targets are in the far-field of the antenna plane. For simplicity, the Doppler shift is negligible. Let  $x_i$  denote the orthogonal waveform transmitted by the  $i$ th transmit antenna:

$$\mathbf{X} = [x_1(l), x_2(l), \dots, x_{N_t}(l)]^T, \quad l = 1, 2, \dots, L, \quad (1)$$

where  $L$  denotes the number of the snapshots during one pulse.

Let  $\lambda$  denote the wavelength of the transmitted signal and  $\theta$  denote the azimuth angle. The transmitting and receiving steering vectors can be, respectively, described by the following expressions:

$$\begin{aligned} \mathbf{A}(\theta) &= [a_1(\theta), a_2(\theta), \dots, a_{N_t}(\theta)]^T, \\ \mathbf{B}(\theta) &= [b_1(\theta), b_2(\theta), \dots, b_{N_r}(\theta)]^T, \end{aligned} \quad (2)$$

where

$$\begin{aligned} a_i(\theta) &= \exp\left(\frac{j2\pi(i-1)d\sin\theta}{\lambda}\right), \quad i = 1, 2, \dots, N_t, \\ b_j(\theta) &= \exp\left(\frac{j2\pi(j-1)d\sin\theta}{\lambda}\right), \quad j = 1, 2, \dots, N_r. \end{aligned} \quad (3)$$

Then the echo received by the  $j$ th receive antenna is given by

$$y_j(l) = \sum_{k=1}^K b_j(\theta_k) \mathbf{A}^T(\theta_k) \mathbf{X} \beta_k + e_j(l), \quad (4)$$

where  $(\cdot)^T$  denotes the transpose,  $\beta_k$  is the complex amplitude proportional to the radar cross-section (RCS) of the point target, and  $e_j$  denotes the interference-plus-noise term.

By discretizing the angle space as  $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_N]$ , where  $N$  is the number of the grid points, we can rewrite (4) as

$$y_j(l) = \sum_{n=1}^N b_j(\alpha_n) \mathbf{A}^T(\alpha_n) \mathbf{X} s_n + e_j(l), \quad (5)$$

where

$$s_n = \begin{cases} \beta_k, & \text{if there is a target at } \alpha_n \\ 0, & \text{else.} \end{cases} \quad (6)$$

In the matrix form we have  $y_j = \Psi_j \mathbf{s} + e_j$ , where  $\Psi_j = b_j \mathbf{A}^T \mathbf{X}$  is the basis matrix for the  $j$ th antenna and  $\mathbf{s} = [s_1, s_2, \dots, s_N]^T$ . According to the CS theory, if the number of the targets is smaller than  $N$ , the targets are sparse in the azimuth angle space and  $\mathbf{s}$  is a sparse vector. A nonzero element with index  $n$  in  $\mathbf{s}$  indicates that there is a target at the angle  $\alpha_n$ .

Using the measurement matrix  $\Phi$ , we can obtain the compressed measurements by placing the output of  $N_r$  receive antennas in vector  $\mathbf{Z}$ :

$$\mathbf{Z} = \Phi \Psi \mathbf{s} + \Phi \mathbf{e}. \quad (7)$$

The parameter  $\mathbf{s}$  is the unknown parameter to be estimated. As the basis matrix is fixed, the recovery accuracy for the targets scene mainly depends on the design of the measurement matrix.

### 3. Measurement Matrix Design Using Spatiotemporal Chaos

In this section, we present the idea of using spatiotemporal chaos for measurement matrix design. Subsequently, an SVD-based optimization is performed on the SCMM with the purpose of further reducing the coherence of column pairs in the sensing matrix.

*3.1. Measurement Matrix Design Using Spatiotemporal Chaos.* Spatiotemporal chaos typically shows disorder in both space

and time domain and is capable of exhibiting chaotic behavior for certain parameter values [13]. The spatiotemporal chaos model we applied here is the coupled map lattice (CML). The one-way coupled map lattice (OCOML) model is [14]

$$\begin{aligned} x_{n+1}(i) &= (1 - \varepsilon) f(x_n(i)) + \varepsilon f(x_n(i-1)), \\ i &= 1, 2, \dots, P, \end{aligned} \quad (8)$$

where  $x_n$  is the variable state,  $n$  is the time index,  $i$  is the lattice site index,  $P$  is the length of OCOML,  $\varepsilon$  is the coupling constant in the range  $[0, 1]$ , and  $f(x)$  is a nonlinear map. Given an initial sequence of length  $P$ , a spatiotemporal chaotic matrix can be obtained by (8). In this paper, we take the well-known logistic map as the nonlinear map to generate the initial sequence.

Compared with the low-dimensional chaotic systems, spatiotemporal chaos has more complex behavior and more abundant characteristics, which makes it an excellent candidate for pseudorandom matrix design.

According to the signal model proposed in Section 2, the length of the initial driving sequence should be set to  $N_r L$  ( $P = N_r \times L$ ), which is the number of columns in  $\Phi$ . We can obtain the following logistic sequence:

$$x_1(i+1) = \mu x_1(i) (1 - x_1(i)). \quad (9)$$

The boundary condition of the OCOML model is [14]

$$\begin{aligned} x_{n+1}(1) &= (1 - \varepsilon) \mu x_n(1) (1 - x_n(1)), \\ x_{n+1}(P) &= (1 - \varepsilon) \mu x_n(P) (1 - x_n(P)) \\ &\quad + \varepsilon \mu x_n(P-1) (1 - x_n(P-1)). \end{aligned} \quad (10)$$

By substituting (9) into (8), a measurement matrix  $\Phi$  of the size  $M \times N_r L$ , where  $M$  is the number of the compressive measurements, can be obtained and expressed as

$$\begin{aligned} \Phi_{n+1,i} &= x_{n+1}(i) \\ &= (1 - \varepsilon) \mu x_n(i) (1 - x_n(i)) \\ &\quad + \varepsilon \mu x_n(i-1) (1 - x_n(i-1)), \end{aligned} \quad (11)$$

where  $n = 1, 2, \dots, M$  and  $i = 1, 2, \dots, N_r \times L$ .

Despite the deterministic definition via ordinary difference expressions, spatiotemporal chaotic dynamical systems exhibit unpredictable behaviour. The detailed proof that chaotic matrix could satisfy RIP with overwhelming probability is presented in [9], which also works well for the SCMM  $\Phi$  in this paper.

*3.2. Optimization of the Spatiotemporal Chaotic Measurement Matrix.* The goal of the optimization is to further reduce the cross-correlations between the measurement matrices  $\Phi$  and  $\Psi$ . We firstly change the initial state of the spatiotemporal chaos to obtain different SCMMs. Then we optimize each SCMM by applying SVD to solve a minimization problem with a certain criterion. Finally, the optimized SCMM is selected depending on the minimum cross-correlations

with  $\Psi$ . The definition of the normalized cross-correlation between all columns in the sensing matrix proposed in [5] has the following expression:

$$cc_{\Theta} = \max_{i \neq j} \left| \frac{\langle \Theta_i, \Theta_j \rangle}{\|\Theta_i\|_2 \cdot \|\Theta_j\|_2} \right|. \quad (12)$$

However, the normalized cross-correlation is difficult for us to design  $\Phi$  as an optimization criterion. An alternative criterion is proposed in [7] as follows:

$$\arg \min_{\Theta} \|\mathbf{G} - \mathbf{G}^*\|_F^2, \quad (13)$$

where  $\mathbf{G} = \Theta^H \Theta$  is the Gram matrix,  $\mathbf{G}^*$  is a diagonal matrix,  $\mathbf{G}^* = \text{diag}(g_{1,1}, g_{2,2}, \dots, g_{N,N})$ , and  $g_{i,i} = \Theta_i^H \Theta_i$ . Actually,  $\mathbf{G}^*$  is obtained under the ideal assumption that minimum possible coherence occurs. Our goal is to make the Gram matrix as close to  $\mathbf{G}^*$  as possible.

With the successful case in [7], simpler criterion can be expressed to replace (13) and written as

$$\begin{aligned} \arg \min_{\Phi, \Gamma} \quad & \|\Phi \Psi(\mathbf{g}^*)^{-1} - \Gamma\|_F^2 \\ \text{s.t.} \quad & \Gamma^H \Gamma = \mathbf{I} \\ & \|\Phi\|_F^2 = c, \end{aligned} \quad (14)$$

where  $\mathbf{g}^* = \text{diag}(\sqrt{g_{1,1}}, \sqrt{g_{2,2}}, \dots, \sqrt{g_{N,N}})$ ,  $\Gamma$  is a semiunitary matrix (i.e.,  $\Gamma^H \Gamma = \mathbf{I}$ ), and  $c$  is a constant. To solve this minimization problem, we can calculate one variable with the other fixed and iterate this process until convergence is achieved.

SVD is applied twice during the whole process. The first SVD is performed on  $\Phi \Psi(\mathbf{g}^*)^{-1}$  to calculate  $\Gamma$ , and the second one is used to facilitate the LS estimator when part of  $\Psi(\mathbf{g}^*)^{-1}$  is filled with zero. The detailed steps are given below.

- (1) Calculate  $\Gamma$  with the known  $\Phi$  in (14) by the following solution:

$$\Gamma = \Gamma_1 \Gamma_2^H, \quad (15)$$

where  $\Gamma_1$  and  $\Gamma_2$  can be obtained by the SVD expression

$$\Phi \Psi(\mathbf{g}^*)^{-1} = \Gamma_1 \Sigma \Gamma_2^H, \quad (16)$$

where  $\Sigma$  is a diagonal matrix with the singular values as its diagonal elements.

- (2) Find  $\Phi$  with given  $\Gamma$  by the solution

$$\mathbf{Q}^H \Phi^H = \Gamma^H, \quad (17)$$

where  $\mathbf{Q} = \Psi(\mathbf{g}^*)^{-1}$ . Note that part of  $\mathbf{Q}$  is filled with zero and  $\mathbf{Q}\mathbf{Q}^H$  tends to be ill-posed. The LS estimator cannot be used directly. This problem can be solved by the SVD of  $\mathbf{Q}$  which can be expressed as

$$\mathbf{Q} = \mathbf{U}_1 \begin{pmatrix} \Delta & 0 \\ 0 & 0 \end{pmatrix} \mathbf{U}_2^H, \quad (18)$$

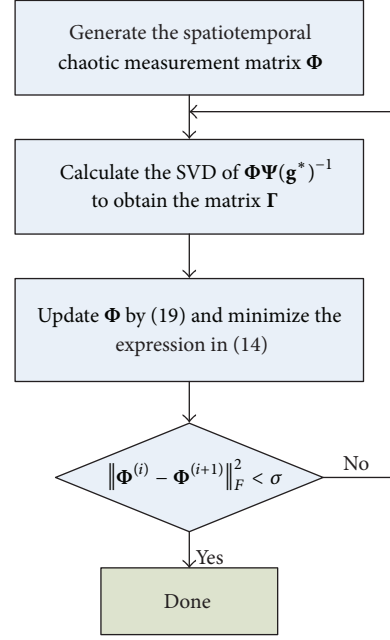


FIGURE 2: The optimization approach for each SCMM.

where  $\mathbf{U}_1$  and  $\mathbf{U}_2$  are unitary matrices,  $\Delta = \text{diag}(\Delta_1, \Delta_2, \dots, \Delta_q)$ , and  $q$  is the number of nonzero singular values of  $\mathbf{Q}$ .

- (3) Use the LS estimator to calculate the SCMM

$$\Phi = \left[ \mathbf{U}_1^H \begin{pmatrix} \Delta^{-1} & 0 \\ 0 & 0 \end{pmatrix} \mathbf{U}_2 \Gamma^H \right]^H. \quad (19)$$

In summary, the optimization approach is demonstrated in Figure 2.

By optimizing each SCMM, the optimized SCMM is finally chosen with the criterion that it has the minimum normalized cross-correlations with the fixed basis matrix.

## 4. Simulation Results

In this section, we will carry out computer simulations on three aspects. Firstly the coherence of the sensing matrix will be calculated to show the effectiveness of using spatiotemporal chaos for measurement matrix design. Secondly the examples of DOA estimation will be given to demonstrate the excellent performance of the CS-MIMO radar with the optimized SCMM. Thirdly the Monte Carlo simulation is employed to verify the recovery accuracy versus various system conditions.

**4.1. Coherence of the Sensing Matrix.** We consider the CS-MIMO radar with  $N_t = 32$  transmitting antennas and  $N_r = 15$  receiving antennas. The orthogonal Hadamard code of length  $L = 32$  is used as the transmit waveform. The CS-MIMO radar system observes the angle space in the range  $[-5^\circ, 5^\circ]$  with the grid step of  $0.2^\circ$ . We generate the SCMM by setting the initial value of the logistic map and the coupling

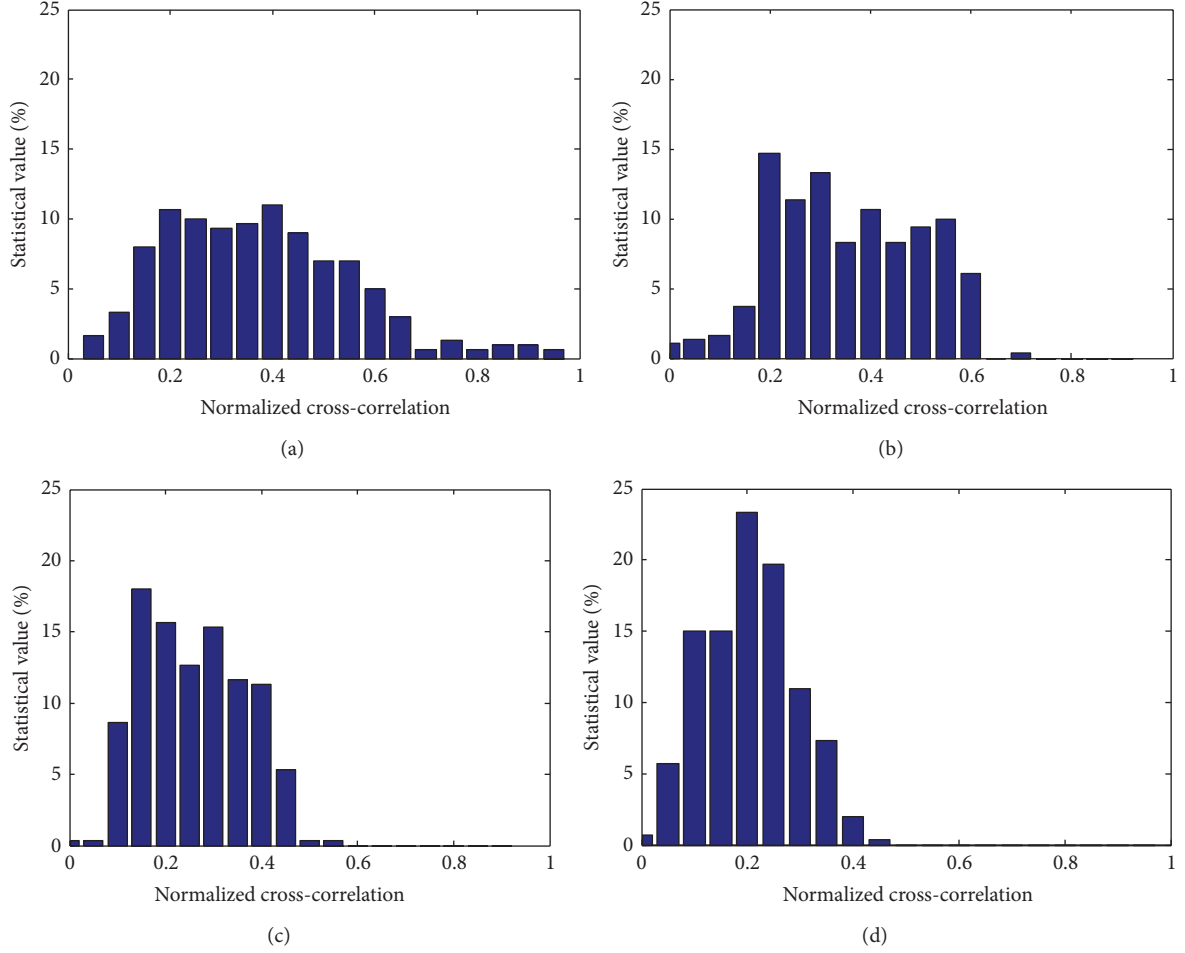


FIGURE 3: Histogram for the cross-correlations of the sensing matrix using (a) GRMM, (b) SCMM, (c) optimized GRMM, and (d) optimized SCMM.

TABLE 1: Average and maximum normalized cross-correlations.

Measurement matrix	Average	Maximum
GRMM	0.3624	0.9455
SCMM	0.3549	0.7075
Optimized GRMM	0.2844	0.5702
Optimized SCMM	0.2286	0.4383

constant of the spatiotemporal chaos  $\varepsilon$  to be 0.5015 and 0.95, respectively.

Figure 3 shows the histogram of the normalized cross-correlations of the sensing matrix  $\Theta$  with four different measurement matrices: the GRMM, the SCMM, the optimized GRMM, and the optimized SCMM. The detailed average and maximum normalized cross-correlations of the four different sensing matrices are given in Table 1. One can see that the GRMM and SCMM lead to almost similar coherence distributions but the maximum normalized cross-correlation of the latter is smaller. Both the average and maximum normalized cross-correlations of the optimized SCMM are smaller than the optimized GRMM. Moreover,

the coherence when using the optimized SCMM, shown in Figure 3(d), is obviously smaller than that of the GRMM. The average and maximum cross-correlations are reduced to about 0.1338 and 0.5072, respectively. Compared with the coherence distribution illustrated in [8], the maximum normalized cross-correlation is much smaller when using the proposed SCMM.

**4.2. DOA Estimation Using the Optimized SCMM.** We assume that several far-field point targets fall on the angle space and the received signal is mixed with zero mean Gaussian noise. At each receive antenna, the measurement matrix compresses the echo signal and acquires the measurements of length  $M = 50$ . Let  $N_t = 64$  and  $N_r = 20$ . The other parameters of the system are the same as in Section 4.1. Then the target scene will be reconstructed by the measurements from all the receiver antennas. The reconstruction algorithm we applied here is orthogonal matching pursuit (OMP) [15]:

$$\min \|s\|_1 \quad \text{s.t.} \quad \|Z - \Phi\Psi s\|_\infty \leq \delta, \quad (20)$$

where  $\delta > 0$  takes into account the possibility of noise in the linear measurements and of nonexact sparsity.



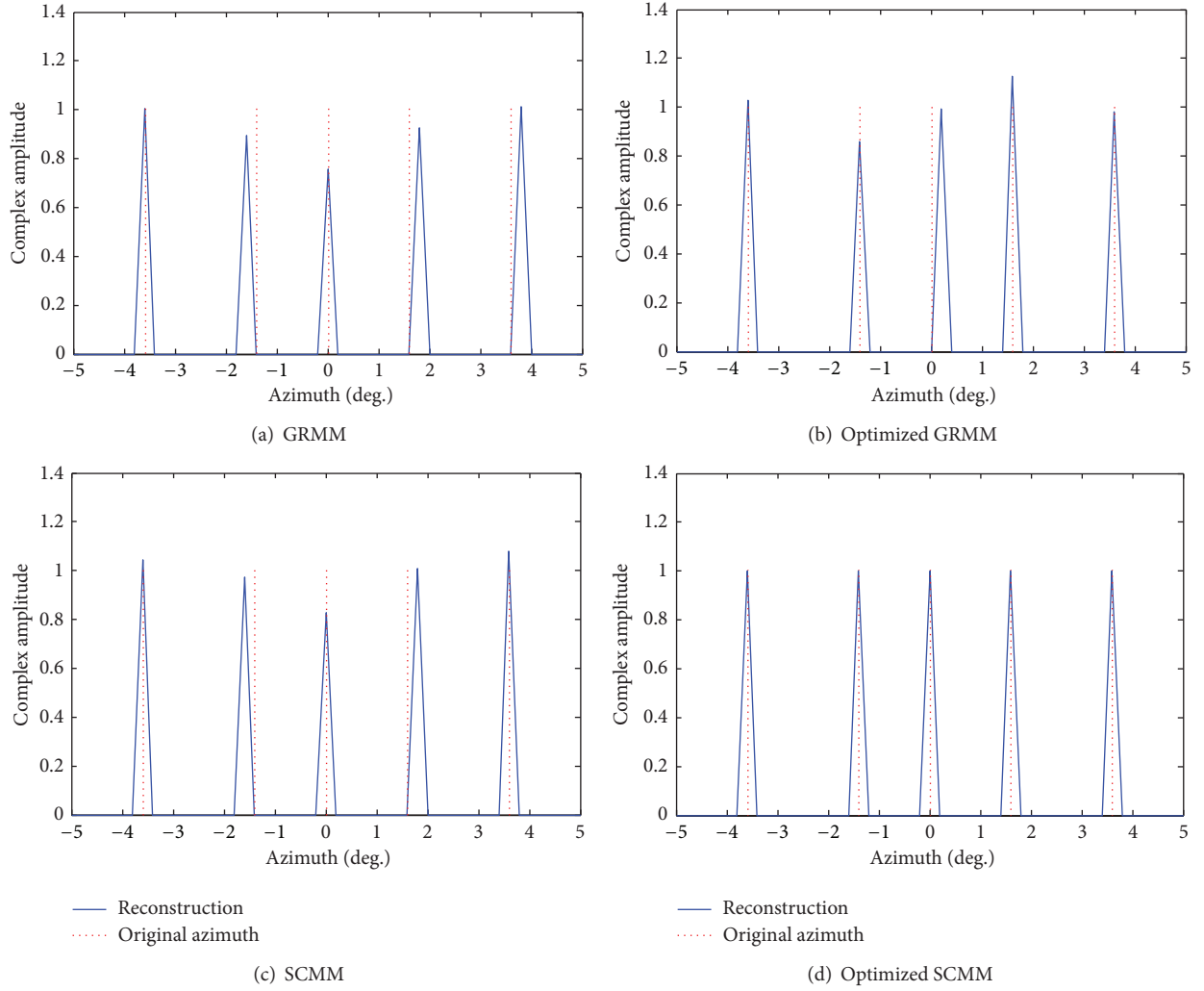


FIGURE 4: DOA estimation results when using (a) GRMM, (b) optimized GRMM, (c) SCMM, and (d) optimized SCMM.

We tested the CS-MIMO radar system with five targets to demonstrate the performance improvement induced by the optimized SCMM. The far-field targets are assumed to have the same complex amplitudes with  $\beta_k = 1$ . Figures 4(a) and 4(b) give the DOA estimation results when using GRMM and optimized GRMM. The estimation performance when using SCMM and optimized SCMM is shown in Figures 4(c) and 4(d). It is obvious that in Figure 4(d) the five targets are all successfully recovered, which is better than the results in Figure 4(b) when using the optimized GRMM. Although only three targets are correctly located, the reconstruction performance with SCMM is better than that of using GRMM.

**4.3. Monte Carlo Simulations for Proposed Method.** Monte Carlo simulations are applied here to verify the robustness of the proposed method. The parameters of the CS-MIMO radar system are similar to previous examples. Figure 5 shows the recovery error versus the target scene sparsity  $K$  with the other parameters fixed. It is obvious that the recovery errors of the optimized SCMM are stably smaller than that

of the optimized GRMM. Also, the SCMM achieves smaller recovery errors than the GRMM.

The recovery errors versus the initial value and the coupling constant of the spatiotemporal chaos in the OCOML model are given in Figures 6 and 7, respectively. One can see in Figure 6 that when the coupling constant is fixed, the recovery errors vary irregularly within an acceptable range according to different initial values. A similar result occurs in Figure 7. The fluctuating is caused by the sensitivity to the initial conditions of the spatiotemporal chaos and can be utilized to obtain the right measurement matrix for the given basis matrix, which is very practical for CS-MIMO radar systems to implement on-line experiments.

## 5. Conclusion

In this paper, a new notion of applying nonlinear dynamic chaotic system for measurement matrix design incorporating an SVD-based optimization method is proposed for CS-MIMO radar systems. Exploiting the statistical properties of

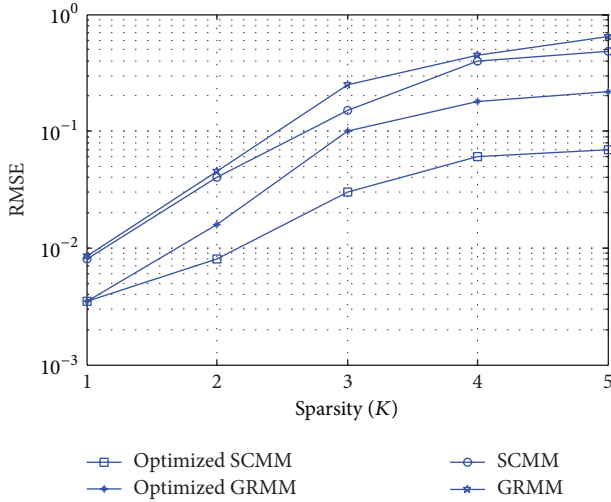
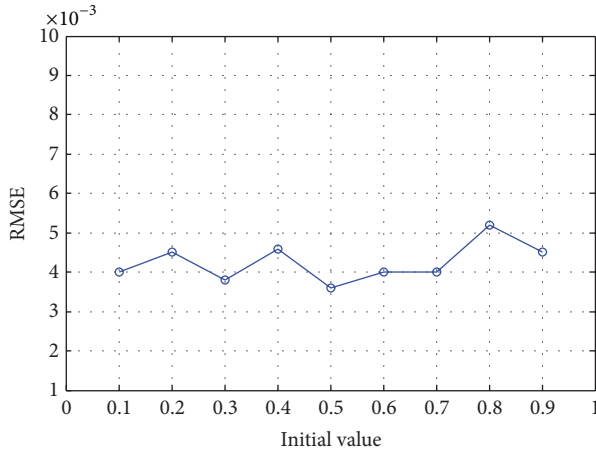
FIGURE 5: Recovery error versus the target sparsity  $K$ .

FIGURE 6: Recovery error versus the initial value of the spatiotemporal chaos.

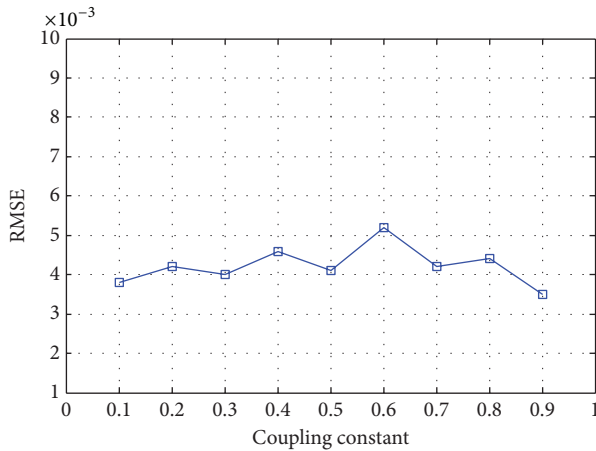


FIGURE 7: Recovery error versus coupling constant of the spatiotemporal chaos.

the spatiotemporal chaos, a pseudorandom but deterministic matrix is obtained to match the basis matrix constructed by the MIMO radar signal model. The iterative optimization is performed to update the generated chaotic matrix aiming at further reducing the coherence of the sensing matrix. Simulation results have proved that the proposed method outperforms that of the GRMM in exact recovery and DOA estimation. In summary, the proposed method has several advantages over the method using GRMM: it possesses low coherence of the sensing matrix which enables more accurate recovery and DOA estimation results for the MIMO radar system; it is easy to implement in electric circuit and only one initial state is necessary to be memorized; it can realize on-line optimization only by changing the initial states of the chaotic system, which is practical for the real radar system.

### Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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