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Research Article

New Classes of Generalized Seminormed Difference Sequence Spaces

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The purpose of this paper is to introduce new classes of generalized seminormed difference sequence spaces defined by a Musielak-Orlicz function. We also study some topological properties and prove some inclusion relations between resulting sequence spaces.

1. Introduction and Preliminaries

Let ℓ^0 denote the space of all real sequences $x = \{x_k\}$. Let $\mathscr{C}$ denote the space whose elements are the sets of distinct positive integers. Given any element σ of $\mathscr{C}$, we denote by $c(\sigma)$ the sequence $\{c_n(\sigma)\}$ such that $c_n(\sigma) = 1$ if $n \in \sigma$, and $c_n(\sigma) = 0$ otherwise. Further

$$\mathscr{C}_s = \left\{\sigma \in \mathscr{C} : \sum_{n=1}^{\infty} c_n(\sigma) \leq s\right\}, \quad (1)$$

the set of those σ whose support has cardinality at most s, and

$$\Phi = \left\{\phi = \{\phi_k\} \in \ell^0 : \phi_1 > 0, \Delta\phi_k \geq 0, \Delta\left(\frac{\phi_k}{k}\right) \leq 0 \ (k = 1, 2, \ldots)\right\}, \quad (2)$$

where $\Delta\phi_k = \phi_k - \phi_{k-1}$.

For $\phi \in \Phi$, Sargent [1] defined the following sequence space:

$$m(\phi) = \left\{x = \{x_k\} \in \ell^0 : \sup_{s\geq1} \sup_{\sigma\in\mathscr{C}_s} \left(\frac{1}{\phi_s}\sum_{k\in\sigma}|x_k|\right) < \infty\right\}, \quad (3)$$

which was further studied in [2–4].

The space $m(\phi)$ was extended to $m(\phi, p)$ by Tripathy and Sen [5] as follows:

$$m(\phi, p) = \left\{x = \{x_k\} \in \ell^0 : \sup_{s\geq1} \sup_{\sigma\in\mathscr{C}_s} \left(\frac{1}{\phi_s}\sum_{k\in\sigma}|x_k|^p\right) < \infty\right\}. \quad (4)$$

The notion of the difference sequence space was introduced by Kızmaz [6] which was generalized by Mursaleen [7]. It was further generalized by Et and Çolak [8] as follows: $Z(\Delta^\mu) = \{x = (x_k) \in \omega : (\Delta^\mu x_k) \in z\}$ for $z = \ell_\infty$, c, and c_0, where μ is a nonnegative integer and

$$\Delta^\mu x_k = \Delta^{\mu-1}x_k - \Delta^{\mu-1}x_{k+1}, \qquad \Delta^0 x_k = x_k \quad \forall k \in \mathbb{N} \quad (5)$$

or equivalent to the following binomial representation:

$$\Delta^\mu x_k = \sum_{\nu=0}^{\mu}(-1)^\nu \binom{\mu}{\nu} x_{k+\nu}. \quad (6)$$

These sequence spaces were generalized by Et and Basarir [9] for $z = \ell_\infty(p)$, $c(p)$, and $c_0(p)$.

Dutta [10] introduced the following difference sequence spaces using a new difference operator:

$$Z\left(\Delta_{(\eta)}\right) = \left\{x = (x_k) \in \omega : \Delta_{(\eta)}x \in z\right\} \quad \text{for } z = \ell_\infty, c, \text{and } c_0, \quad (7)$$

where $\Delta_{(\eta)}x = (\Delta_{(\eta)}x_k) = (x_k - x_{k-\eta})$ for all $k, \eta \in \mathbb{N}$.

In [11], Dutta introduced the sequence spaces $\bar{c}(\|\cdot,\cdot\|, \Delta^{\mu}_{(\eta)}, p)$, $\bar{c}_0(\|\cdot,\cdot\|, \Delta^{\mu}_{(\eta)}, p)$, $\ell_\infty(\|\cdot,\cdot\|, \Delta^{\mu}_{(\eta)}, p)$, $m(\|\cdot,\cdot\|, \Delta^{\mu}_{(\eta)}, p)$, and $m_0(\|\cdot,\cdot\|, \Delta^{\mu}_{(\eta)}, p)$, where $\eta, \mu \in \mathbb{N}$ and $\Delta^{\mu}_{(\eta)}x_k = (\Delta^{\mu}_{(\eta)}x_k) = (\Delta^{\mu-1}_{(\eta)}x_k - \Delta^{\mu-1}_{(\eta)}x_{k-\eta})$ and $\Delta^{0}_{(\eta)}x_k = x_k$ for all $k, \eta \in \mathbb{N}$, which is equivalent to the following binomial representation:

$$\Delta^{\mu}_{(\eta)}x_k = \sum_{\nu=0}^{\mu}(-1)^{\nu}\binom{\mu}{\nu}x_{k-\eta\nu}. \tag{8}$$

The difference sequence spaces have been studied by several authors [12–19] and references therein. Başar and Altay [20] introduced the generalized difference matrix $B = (b_{mk})_{k,m\in\mathbb{N}}$ by

$$b_{mk} = \begin{cases} r, & k = m \\ s, & k = m-1 \\ 0, & (k > m) \text{ or } (0 \le k < m-1). \end{cases} \tag{9}$$

Başarir and Kayikçi [21] defined the matrix $B^{\mu}(b^{\mu}_{mk})$ which reduces to the difference matrix $\Delta^{\mu}_{(1)}$ if $r = 1, s = -1$. The generalized B^{μ}-difference operator is equivalent to the following binomial representation:

$$B^{\mu}x = B^{\mu}(x_k) = \sum_{\nu=0}^{\mu}\binom{\mu}{\nu}r^{\mu-\nu}s^{\nu}x_{k-\nu}. \tag{10}$$

Let $\wedge = (\wedge_k)$ be a sequence of nonzero scalars. Then, for a sequence space E, the multiplier sequence space $E_\wedge$, associated with the multiplier sequence $\wedge$, is defined as

$$E_\wedge = \{x = (x_k) \in \omega : (\wedge_k x_k) \in E\}. \tag{11}$$

Let $\omega(X)$ denote the space of all sequences with elements in (X, q), where (X, q) denotes a seminormed space, seminormed by q. The zero sequence is denoted by $\theta = (0, 0, 0, \ldots)$.

An Orlicz function M is a function, $M : [0, \infty) \to [0, \infty)$, which is continuous nondecreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$.

Lindenstrauss and Tzafriri [22] used the idea of Orlicz function to define the following sequence space:

$$\ell_M = \left\{x \in \omega : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty\right\}, \tag{12}$$

which is called an Orlicz sequence space. The space ℓ_M is a Banach space with the norm

$$\|x\| = \inf\left\{\rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \le 1\right\}. \tag{13}$$

It is shown in [22] that every Orlicz sequence space ℓ_M contains a subspace isomorphic to ℓ_p $(p \ge 1)$. The Δ_2-condition is equivalent to $M(Lx) \le KLM(x)$ for all values of $x \ge 0$ and for $L > 1$.

A sequence $\mathcal{M} = (M_k)$ of Orlicz functions is called a Musielak-Orlicz function. A sequence $\mathcal{N} = (N_k)$ defined by

$$N_k(v) = \sup\{|v|u - M_k(u) : u \ge 0\}, \quad k = 1, 2, \ldots, \tag{14}$$

is called the complimentary function of a Musielak-Orlicz function (see [23, 24]). For a given Musiclak-Orlicz function $\mathcal{M}$, the Musielak-Orlicz sequence space $t_{\mathcal{M}}$ and its subspace $h_{\mathcal{M}}$ are defined as follows:

$$\begin{aligned} t_{\mathcal{M}} &= \{x \in \omega : I_M(cx) < \infty \text{ for some } c > 0\}, \\ h_{\mathcal{M}} &= \{x \in \omega : I_M(cx) < \infty \ \forall\ c > 0\}, \end{aligned} \tag{15}$$

where $I_{\mathcal{M}}$ is a convex modular defined by

$$I_{\mathcal{M}}(x) = \sum_{k=1}^{\infty} M_k(x_k), \quad x = (x_k) \in t_M. \tag{16}$$

We consider $t_{\mathcal{M}}$ equipped with the Luxemburg norm,

$$\|x\| = \inf\left\{k > 0 : I_M\left(\frac{x}{k}\right) \le 1\right\}, \tag{17}$$

or equipped with the Orlicz norm,

$$\|x\|^0 = \inf\left\{\frac{1}{k}(1 + I_M(kx)) : k > 0\right\}. \tag{18}$$

A sequence space E is said to be solid if $(\alpha_k x_k) \in E$, whenever $(x_k) \in E$ for all sequences (α_k) of scalars such that $|\alpha_k| \le 1$ for all $k \in \mathbb{N}$.

A sequence space E is said to be monotone if E contains the canonical preimages of all its step spaces.

Remark 1. It is well known that a sequence space is solid implies that it is monotone (see Kamthan and Gupta [25]).

The sequence space $m(\phi)$ was introduced by Sargent [1]. He studied some of its properties and obtained its relationship with the space ℓ_p. Later on, it was investigated from sequence space point of view and related with summability theory by Bilgin [26], Esi [27], Tripathy and Mahanta [28], and many others.

The main goal of the present paper is to introduce new classes of generalized seminormed difference sequence spaces defined by Musielak-Orlicz function.

For a given infinite matrix $A = (a_{ik})_{i,k\ge1}$. The A-transform of a sequence $x = (x_k)_{k\ge1}$ is the sequence $Ax = (A_i)$ $(i \ge 1)$, where

$$A_i(x) = \sum_{k=1}^{\infty} a_{ik}x_k, \tag{19}$$

provided that the series on the right converges for each $i \ge 1$.

Let (X, q) be a seminormed space, $\mathcal{M} = (M_i)$ a Musielak-Orlicz function, and $p = (p_i)$ a bounded sequence of positive real numbers. Then we define the following classes of sequences:

$$\ell_\infty(\mathcal{M}, A, B_\Lambda^\mu, q, p) = \left\{ x = (x_k) \in w(X) : \sup_{i \geq 1} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu x_k)}{\rho} \right) \right)^{p_i} < \infty, \text{ for some } \rho > 0 \right\},$$

$$\ell_1(\mathcal{M}, A, B_\Lambda^\mu, q, p) = \left\{ x = (x_k) \in w(X) : \sum_{i=1}^{\infty} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu x_k)}{\rho} \right) \right)^{p_i} < \infty, \text{ for some } \rho > 0 \right\},$$

$$m(\mathcal{M}, A, B_\Lambda^\mu, \phi, q, p) = \left\{ x = (x_k) \in w(X) : \sup_{s \geq 1, \sigma \in \mathcal{C}_s} \frac{1}{\phi_s} \sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu x_k)}{\rho} \right) \right)^{p_i} < \infty, \text{ for some } \rho > 0 \right\}. \tag{20}$$

The following inequality will be used throughout the paper. If $0 < h = \inf p_k \leq p_k \leq \sup p_k = H$ and $D = \max(1, 2^{H-1})$ then

$$|a_k + b_k|^{p_k} \leq D \left\{ |a_k|^{pk} + |b_k|^{p_k} \right\}, \tag{21}$$

for all k and $a_k, b_k \in \mathbb{C}$. Also $|a|^{p_k} \leq \max(1, |a|^H)$ for all $a \in \mathbb{C}$.

We study here some topological properties and establish inclusion relations between these sequence spaces.

2. Main Results

Theorem 2. *Let $\mathcal{M} = (M_i)$ be a Musielak-Orlicz function and $p = (p_i)$ a bounded sequence of positive real numbers. Then the spaces $\ell_\infty(\mathcal{M}, A, B_\Lambda^\mu, q, p)$, $\ell_1(\mathcal{M}, A, B_\Lambda^\mu, q, p)$, and $m(\mathcal{M}, A, B_\Lambda^\mu, \phi, q, p)$ are linear spaces over the field of complex number $\mathbb{C}$.*

Proof. Let $x = (x_k)$, $y = (y_k) \in m(\mathcal{M}, A, B_\Lambda^\mu, \phi, q, p)$, and $\alpha, \beta \in \mathbb{C}$. Then there exist positive real numbers $\rho_1, \rho_2 > 0$ such that

$$\sup_{s \geq 1, \sigma \in \mathcal{C}_s} \frac{1}{\phi_s} \sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu x_k)}{\rho_1} \right) \right)^{p_i} < \infty,$$
$$\sup_{s \geq 1, \sigma \in \mathcal{C}_s} \frac{1}{\phi_s} \sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu y_k)}{\rho_2} \right) \right)^{p_i} < \infty. \tag{22}$$

Define $\rho_3 = \max(2|\alpha|\rho_1, 2|\beta|\rho_2)$. Since (M_i) is a nondecreasing, convex function and so by using inequality (21), we have

$$\begin{aligned} &\sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu(\alpha x + \beta y))}{\rho_3} \right) \right)^{p_i} \\ &\leq \sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu \alpha x)}{\rho_3} \right) + q \left(\frac{A_i(B_\Lambda^\mu \beta y)}{\rho_3} \right) \right)^{p_i} \\ &\leq D \sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu \alpha x)}{\rho_1} \right) \right)^{p_i} \\ &\quad + D \sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu \beta y)}{\rho_2} \right) \right)^{p_i}. \end{aligned} \tag{23}$$

Thus

$$\begin{aligned} &\sup_{s \geq 1, \sigma \in \mathcal{C}_s} \frac{1}{\phi_s} \sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu(\alpha x + \beta y))}{\rho_3} \right) \right)^{p_i} \\ &\leq \sup_{s \geq 1, \sigma \in \mathcal{C}_s} \frac{1}{\phi_s} D \sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu \alpha x)}{\rho_1} \right) \right)^{p_i} \\ &\quad + \sup_{s \geq 1, \sigma \in \mathcal{C}_s} \frac{1}{\phi_s} D \sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu \beta y)}{\rho_2} \right) \right)^{p_i} < \infty. \end{aligned} \tag{24}$$

Thus $(\alpha x + \beta y) \in m(\mathcal{M}, A, B_\Lambda^\mu, \phi, q, p)$. Hence $m(\mathcal{M}, A, B_\Lambda^\mu, \phi, q, p)$ is a linear space. Similarly, we can prove that the spaces $\ell_\infty(\mathcal{M}, A, B_\Lambda^\mu, q, p)$ and $\ell_1(\mathcal{M}, A, B_\Lambda^\mu, q, p)$ are linear spaces. This completes the proof of the theorem. □

Theorem 3. *Let $\mathcal{M} = (M_i)$ be a Musielak-Orlicz function and $p = (p_i)$ a bounded sequence of positive real numbers. Then $\ell_1(\mathcal{M}, A, B_\Lambda^\mu, q, p) \subset m(\mathcal{M}, A, B_\Lambda^\mu, \phi, q, p) \subset \ell_\infty(\mathcal{M}, A, B_\Lambda^\mu, q, p)$.*

Proof. Let $x = (x_k) \in \ell_1(\mathcal{M}, A, B_\Lambda^\mu, q, p)$. Then, for some $\rho > 0$, we have

$$\sum_{i=1}^{\infty} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu x_k)}{\rho} \right) \right)^{p_i} < \infty. \tag{25}$$

Since (ϕ_n) is a monotonic increasing, we have

$$\begin{aligned} &\frac{1}{\phi_s} \sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu x_k)}{\rho} \right) \right)^{p_i} \\ &\leq \frac{1}{\phi_1} \sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu x_k)}{\rho} \right) \right)^{p_i} \\ &\leq \frac{1}{\phi_1} \sum_{i=1}^{\infty} M \left(q \left(\frac{A_i(B_\Lambda^\mu x_k)}{\rho} \right) \right)^{p_i} < \infty. \end{aligned} \tag{26}$$

Hence,

$$\sup_{s \geq 1, \sigma \in \mathcal{C}_s} \frac{1}{\phi_s} \sum_{i \in \sigma} M_i \left(q \left(\frac{A_i(B_\Lambda^\mu x_k)}{\rho} \right) \right)^{p_i} < \infty. \tag{27}$$

Thus, $x = (x_k) \in m(\mathscr{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$. Therefore, $\ell_1(\mathscr{M}, A, B^{\mu}_{\Lambda}, q, p) \subset m(\mathscr{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$.

Next, let $x = (x_k) \in m(\mathscr{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$. Then, for some $\rho > 0$, we have

$$\sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x_k)}{\rho} \right) \right)^{p_i} < \infty. \tag{28}$$

Hence,

$$\sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x_k)}{\rho} \right) \right)^{p_i} < \infty \quad \text{(on taking cardinality of } \sigma \text{ to be 1)}. \tag{29}$$

Thus, $x = (x_k) \in \ell_\infty(\mathscr{M}, A, B^{\mu}_{\Lambda}, q, p)$. Therefore, $m(\mathscr{M}, A, B^{\mu}_{\Lambda}, \phi, q, p) \subset \ell_\infty(\mathscr{M}, A, B^{\mu}_{\Lambda}, q, p)$. This completes the proof of the theorem. □

Theorem 4. *Let $\mathscr{M} = (M_i)$ be a Musielak-Orlicz function and $p = (p_i)$ a bounded sequence of positive real numbers. Then the space $m(\mathscr{M}, A, B^{\mu}_{\Lambda}, \varphi, q, p)$ is a seminormed space, seminormed by*

$$g(x) = \inf \left\{ \rho > 0 : \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x)}{\rho} \right) \right)^{p_i} \leq 1 \right\}. \tag{30}$$

Proof. Clearly, $g(x) \geq 0$ for all $x = (x_k) \in m(\mathscr{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$ and $g(\theta) = 0$. Let $x = (x_k), y = (y_k) \in m(\mathscr{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$. Then there exist $\rho_1 > 0$ and $\rho_2 > 0$ such that

$$\begin{aligned} &\sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x)}{\rho_1} \right) \right)^{p_i} \leq 1, \\ &\sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} y)}{\rho_2} \right) \right)^{p_i} \leq 1. \end{aligned} \tag{31}$$

Let $\rho = \rho_1 + \rho_2$. Thus, we have

$$\begin{aligned} &\sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} (x+y))}{\rho} \right) \right)^{p_i} \\ &= \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} (x+y))}{\rho_1 + \rho_2} \right) \right)^{p_i} \\ &\leq \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} \left\{ \frac{\rho_1}{\rho_1+\rho_2} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} (x))}{\rho_1} \right) \right) \right. \\ &\qquad \left. + \frac{\rho_2}{\rho_1+\rho_2} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} (y))}{\rho_2} \right) \right) \right\}^{p_i} \\ &\leq \left(\frac{\rho_1}{\rho_1+\rho_2} \right) \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x)}{\rho_1} \right) \right)^{p_i} \\ &\quad + \left(\frac{\rho_2}{\rho_1+\rho_2} \right) \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} y)}{\rho_2} \right) \right)^{p_i} \leq 1. \end{aligned} \tag{32}$$

Since the ρ's are nonnegative, so we have

$$\begin{aligned} &g(x+y) \\ &= \inf \left\{ \rho > 0 : \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} (x+y))}{\rho} \right) \right)^{p_i} \leq 1 \right\} \\ &\leq \inf \left\{ \rho_1 > 0 : \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x)}{\rho_1} \right) \right)^{p_i} \leq 1 \right\} \\ &\quad + \inf \left\{ \rho_2 > 0 : \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} y)}{\rho_2} \right) \right)^{p_i} \leq 1 \right\}. \end{aligned} \tag{33}$$

Thus, $g(x + y) \leq g(x) + g(y)$. Next, for $\lambda \in \mathbb{C}$, without loss of generality, $\lambda \neq 0$, then

$$\begin{aligned} &g(\nu x) \\ &= \inf \left\{ \rho > 0 : \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} (\nu x))}{\rho} \right) \right)^{p_i} \leq 1 \right\} \\ &= \inf \left\{ \rho > 0 : \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x)}{r} \right) \right)^{p_i} \leq 1 \right\}, \\ &\qquad \text{where } r = \frac{\rho}{|\nu|}. \end{aligned} \tag{34}$$

This completes the proof of the theorem. □

Theorem 5. *Let $\mathscr{M} = (M_i)$ be a Musielak-Orlicz function and $p = (p_i)$ a bounded sequence of positive real numbers. Then*

(i) *the space $\ell_\infty(\mathscr{M}, A, B^{\mu}_{\Lambda}, q, p)$ is a seminormed space, seminormed by*

$$f(x) = \inf \left\{ \rho > 0 : \sup_{i\geq 1} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x)}{\rho} \right) \right)^{p_i} \leq 1 \right\}, \tag{35}$$

(ii) *the space $\ell_1(\mathscr{M}, A, B^{\mu}_{\Lambda}, q, p)$ is a seminormed space, seminormed by*

$$h(x) = \inf \left\{ \rho > 0 : \sum_{i=1}^{\infty} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x)}{\rho} \right) \right)^{p_i} \leq 1 \right\}. \tag{36}$$

Proof. It is easy to prove in view of Theorem 4, so we omit the details. □

Theorem 6. *Let $\mathscr{M} = (M_i)$ be a Musielak-Orlicz function and $p = (p_i)$ a bounded sequence of positive real numbers. Then $m(\mathscr{M}, A, B^{\mu}_{\Lambda}, \phi, q, p) \subset m(\mathscr{M}, A, B^{\mu}_{\Lambda}, \psi, q, p)$ if and only if $\sup_{s\geq 1}(\varphi_s/\psi_s) < \infty$.*

Proof. Suppose $\sup_{s\geq 1}(\varphi_s/\psi_s) < \infty$ and $x = (x_k) \in m(\mathscr{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$. Then, we have for some $\rho > 0$

$$\sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x)}{\rho} \right) \right)^{p_i} < \infty. \tag{37}$$

Thus,

$$\sup_{s\ge1,\sigma\in\mathscr{C}_s}\frac{1}{\psi_s}\sum_{i\in\sigma}M_i\left(q\left(\frac{A_i(B_\Lambda^\mu x)}{\rho}\right)\right)^{p_i}$$
$$\le\left(\sup_{s\ge1}\frac{\phi_s}{\psi_s}\right)\left(\sup_{s\ge1,\sigma\in\mathscr{C}_s}\frac{1}{\phi_s}\sum_{i\in\sigma}M_i\left(q\left(\frac{A_i(B_\Lambda^\mu x)}{\rho}\right)\right)^{p_i}\right)<\infty. \tag{38}$$

Therefore, $x=(x_k)\in m(\mathscr{M},A,B_\Lambda^\mu,\psi,q,p)$. Hence, $m(\mathscr{M},A,B_\Lambda^\mu,\phi,q,p)\subset m(\mathscr{M},A,B_\Lambda^\mu,\psi,q,p)$.

Conversely, let $m(\mathscr{M},A,B_\Lambda^\mu,\phi,q,p)\subset m(\mathscr{M},A,B_\Lambda^\mu,\psi,q,p)$. Suppose that $\sup_{s\ge1}(\phi_s/\psi_s)=\infty$. Then there exists a sequence of naturals $\{s_i\}$ such that $\lim_{i\to\infty}(\phi_{s_i}/\psi_{s_i})=\infty$. Let $x=(x_k)\in m(\mathscr{M},A,B_\Lambda^\mu,\phi,q,p)$. Then there exists $\rho>0$ such that

$$\sup_{s\ge1,\sigma\in\mathscr{C}_s}\frac{1}{\phi_s}\sum_{i\in\sigma}M_i\left(q\left(\frac{A_i(B_\Lambda^\mu x)}{\rho}\right)\right)^{p_i}<\infty. \tag{39}$$

Now, we have

$$\sup_{s\ge1,\sigma\in\mathscr{C}_s}\frac{1}{\psi_s}\sum_{i\in\sigma}M_i\left(q\left(\frac{A_i(B_\Lambda^\mu x)}{\rho}\right)\right)^{p_i}$$
$$\ge\left(\sup_{s\ge1}\frac{\phi_s}{\psi_s}\right)\left(\sup_{s\ge1,\sigma\in\mathscr{C}_s}\frac{1}{\phi_s}\sum_{i\in\sigma}M_i\left(q\left(\frac{A_i(B_\Lambda^\mu x)}{\rho}\right)\right)^{p_i}\right)=\infty. \tag{40}$$

Therefore, $x=(x_k)\notin m(\mathscr{M},A,B_\Lambda^\mu,\psi,q,p)$, which is a contradiction. Hence $\sup_{s\ge1}(\phi_s/\psi_s)<\infty$. □

We get the following corollary as a consequence of Theorem 6.

Corollary 7. *Let $\mathscr{M}=(M_i)$ be a Musielak-Orlicz function and $p=(p_i)$ a bounded sequence of positive real numbers. Then $m(\mathscr{M},A,B_\Lambda^\mu,\phi,q,p)=m(\mathscr{M},A,B_\Lambda^\mu,\psi,q,p)$ if and only if $\sup_{s\ge1}(\phi_s/\psi_s)<\infty$ and $\sup_{s\ge1}(\psi_s/\phi_s)<\infty$ for all $s=1,2,3,\ldots$.*

Theorem 8. *Let $\mathscr{M}'=(M_i)',\mathscr{M}''=(M_i)''$ be Musielak-Orlicz functions which satisfy Δ_2-conditions and $p=(p_i)$ a bounded sequence of positive real numbers. Then*

(i) $m(\mathscr{M}'^{\mu}_\Lambda,\phi,q,p)\subseteq m(\mathscr{M}\circ\mathscr{M}'^{\mu}_\Lambda,A,\phi,q,p)$;

(ii) $m(\mathscr{M}'^{\mu}_\Lambda,\phi,q,p)\cap m(\mathscr{M}''^{\mu}_\Lambda,\phi,q,p)\subseteq m(\mathscr{M}'+\mathscr{M}''^{\mu}_\Lambda,\phi,q,p)$.

Proof. (i) Let $x=(x_k)\in m(\mathscr{M}'^{\mu}_\Lambda,\phi,q,p)$. Then there exists $\rho>0$ such that

$$\sup_{s\ge1,\sigma\in\mathscr{C}_s}\frac{1}{\phi_s}\sum_{i\in\sigma}M_i'\left(q\left(\frac{A_i(B_\Lambda^\mu x)}{\rho}\right)\right)^{p_i}<\infty. \tag{41}$$

Let $0<\varepsilon<1$ and $0<\delta<1$ such that $M_i(t)<\varepsilon$ for $0\le t<\delta$. Let $y_k=M_i'(q(A_i(B_\Lambda^\mu x)/\rho))^{p_i}$ and, for any $\sigma\in\mathscr{C}_s$, let $\sum_{i\in\sigma}M_i(y_k)=\sum_1 M_i(y_k)+\sum_2 M_i(y_k)$, where the first summation is over $y_k\le\delta$ and the second summation is over $y_k>\delta$. Since (M_i) satisfies Δ_2-condition, we have

$$\sum_1 M_i(y_k)\le M_i(1)\sum_1 y_k\le M_i(2)\sum_1 y_k. \tag{42}$$

For $y_k>\delta$

$$y_k<\frac{y_k}{\delta}\le1+\frac{y_k}{\delta}. \tag{43}$$

Since (M_i) is nondecreasing and convex, so

$$M(y_k)<M\left(1+\frac{y_k}{\delta}\right)<\frac{1}{2}M(2)+\frac{1}{2}M_i\left(\frac{2y_k}{\delta}\right). \tag{44}$$

Since (M_i) also satisfies Δ_2-condition, so

$$M_i(y_k)<\frac{1}{2}K\frac{y_k}{\delta}M_i(2)+\frac{1}{2}K\frac{y_k}{\delta}M_i(2)=K\frac{y_k}{\delta}M_i(2). \tag{45}$$

Hence,

$$\sum_2 M_i(y_k)\le\max\left(1,K\delta^{-1}M_i(2)\right)\sum_2 y_k. \tag{46}$$

By (42) and (46), we have $x=(x_k)\in m(\mathscr{M}\circ\mathscr{M}'^{\mu}_\Lambda,\phi,q,p)$. Hence

$$m\left(\mathscr{M}'^{\mu}_\Lambda,\phi,q,p\right)\subseteq m\left(\mathscr{M}\circ\mathscr{M}'_\Lambda,\phi,q,p\right). \tag{47}$$

(ii) Let $x=(x_k)\in m(\mathscr{M}'^{\mu}_\Lambda,\phi,q,p)\cap m(\mathscr{M}''^{\mu}_\Lambda,\phi,q,p)$. Then there exists $\rho>0$ such that

$$\sup_{s\ge1,\sigma\in\mathscr{C}_s}\frac{1}{\phi_s}\sum_{i\in\sigma}M_i'\left(q\left(\frac{A_i(B_\Lambda^\mu x)}{\rho}\right)\right)^{p_i}<\infty,$$
$$\sup_{s\ge1,\sigma\in\mathscr{C}_s}\frac{1}{\phi_s}\sum_{i\in\sigma}M_i''\left(q\left(\frac{A_i(B_\Lambda^\mu x)}{\rho}\right)\right)^{p_i}<\infty. \tag{48}$$

The rest of the proof follows from the equality

$$\sum_{i\in\sigma}\left(\mathscr{M}_i'+\mathscr{M}_i''\right)\left(q\left(\frac{A_i(x)}{\rho}\right)\right)^{p_i}$$
$$=\sum_{i\in\sigma}M_i'\left(q\left(\frac{A_i(B_\Lambda^\mu x)}{\rho}\right)\right)^{p_i}+\sum_{i\in\sigma}M_i''\left(q\left(\frac{A_i(B_\Lambda^\mu x)}{\rho}\right)\right)^{p_i}. \tag{49}$$

This completes the proof of the theorem. □

Corollary 9. *Let $\mathscr{M}=(M_i)$ be a Musielak-Orlicz function and $p=(p_i)$ a bounded sequence of positive real numbers. Then, we have $m(A,B_\Lambda^\mu,\varphi,q,p)\subseteq m(\mathscr{M},A,B_\Lambda^\mu,\varphi,q,p)$.*

Proof. It follows from Theorem 8(i) on considering $\mathscr{M}'(x)=x$, for all $x\in[0,\infty)$. □

The following result is a consequence of Theorem 8 and Corollary 9.

Corollary 10. *Let $\mathcal{M} = (M_i)$ be a Musielak-Orlicz function and $p = (p_i)$ a bounded sequence of positive real numbers. Then $m(A, B^{\mu}_{\Lambda}, \varphi, q, p) \subseteq m(\mathcal{M}, A, B^{\mu}_{\Lambda}, \psi, q, p)$ if and only if $\sup_{s\geq 1}(\varphi_s/\psi_s) < \infty$.*

Theorem 11. *Let $\mathcal{M} = (M_i)$ be a Musielak-Orlicz function and $p = (p_i)$ a bounded sequence of positive real numbers. Then the space $m(\mathcal{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$ is solid.*

Proof. Let $x = (x_k) \in m(\mathcal{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$. Then

$$\sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x)}{\rho} \right) \right)^{p_i} < \infty. \tag{50}$$

Let (α_k) be a sequence of scalars with $|\alpha_k| \leq 1$ for all $k \in \mathbb{N}$. Then the result follows from (50) and the following inequality

$$\begin{aligned} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} \alpha x)}{\rho} \right) \right)^{p_i} &\leq \sum_{i\in\sigma} |\alpha|\, M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x)}{\rho} \right) \right)^{p_i} \\ &\leq \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i (B^{\mu}_{\Lambda} x)}{\rho} \right) \right)^{p_i}. \end{aligned} \tag{51}$$

This completes the proof of the theorem. □

In view of the above result, we get the following corollaries.

Corollary 12. *Let $\mathcal{M} = (M_i)$ be a Musielak-Orlicz function and $p = (p_i)$ a bounded sequence of positive real numbers. Then the space $m(\mathcal{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$ is monotone.*

We formulate the following result which can be established following the technique of Theorem 11 and Corollary 12.

Corollary 13. *Let $\mathcal{M} = (M_i)$ be a Musielak-Orlicz function and $p = (p_i)$ a bounded sequence of positive real numbers. Then the spaces $\ell_\infty(\mathcal{M}, A, B^{\mu}_{\Lambda}, q, p)$ and $\ell_1(\mathcal{M}, A, B^{\mu}_{\Lambda}, q, p)$ are solid and monotone.*

Theorem 14. *If (X, q) is complete, then $m(\mathcal{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$ is also complete.*

Proof. Let (x^j) be a Cauchy sequence in $m(\mathcal{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$, where $x^j = (x^j_k) = (x^j_1, x^j_2, x^j_3, \ldots) \in m(\mathcal{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$ for each $j \in \mathbb{N}$. Let $r > 0$ and $x_0 > 0$ be fixed. Then for each $\varepsilon/rx_0 > 0$, there exists a positive integer n_0 such that

$$g\left(x^j - x^l\right) < \frac{\varepsilon}{rx_0}, \quad \forall j, l \geq n_0. \tag{52}$$

This implies

$$\begin{aligned} &\inf \left\{ \rho : \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i \left(B^{\mu}_{\Lambda} \left(x^j - x^l\right)\right)}{\rho} \right) \right)^{p_i} \leq 1 \right\} \\ &\quad < \frac{\varepsilon}{rx_0}, \quad \forall j, l \geq n_0. \end{aligned} \tag{53}$$

We have for all j, $l \geq n_0$ and by (53)

$$\begin{aligned} &\sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i \left(B^{\mu}_{\Lambda} \left(x^j - x^l\right)\right)}{h\left(x^j - x^l\right)} \right) \right)^{p_i} \leq 1 \\ &\Longrightarrow \frac{1}{\phi_1} M_i \left(q \left(\frac{A_i \left(B^{\mu}_{\Lambda} \left(x^j - x^l\right)\right)}{h\left(x^j - x^l\right)} \right) \right)^{p_i} \leq 1 \\ &\Longrightarrow M_i \left(q \left(\frac{A_i \left(B^{\mu}_{\Lambda} \left(x^j - x^l\right)\right)}{h\left(x^j - x^l\right)} \right) \right)^{p_i} \leq \phi_1, \quad \forall j, l \geq n_0. \end{aligned} \tag{54}$$

We can find $r > 0$ such that $(rx_0/2)\eta(x_0/2) > \phi_1$, where η is the kernel associated with Musielak-Orlicz function $\mathcal{M}$, such that

$$\begin{aligned} &M_i \left(q \left(\frac{A_i \left(B^{\mu}_{\Lambda} \left(x^j - x^l\right)\right)}{h\left(x^j - x^l\right)} \right) \right)^{p_i} \leq \frac{rx_0}{2} \eta \left(\frac{x_0}{2} \right) \\ &\Longrightarrow q\left(A_i \left(B^{\mu}_{\Lambda} \left(x^j - x^l\right)\right)\right)^{p_i} < \frac{rx_0}{2} \cdot \frac{\varepsilon}{rx_0} = \frac{\varepsilon}{2}. \end{aligned} \tag{55}$$

Hence $A_i(B^{\mu}_{\Lambda} x^j)_{j\geq 1}$ is a Cauchy sequence in (X, q), which is complete. Therefore, for each $k \in \mathbb{N}$, there exist $x_k \in X$ and $x = (x_k)$ such that $q(A_i(B^{\mu}_{\Lambda}(x^j - x)))^{p_i} \to 0$ as $j \to \infty$. Using the continuity of $\mathcal{M}$, so for some $\rho > 0$, we have

$$\begin{aligned} &\sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{\lim_{i\to\infty} A_i \left(B^{\mu}_{\Lambda} \left(x^j - x^l\right)\right)}{\rho} \right) \right)^{p_i} \leq 1 \\ &\Longrightarrow \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i \left(B^{\mu}_{\Lambda} \left(x^j - x^l\right)\right)}{\rho} \right) \right)^{p_i} \leq 1. \end{aligned} \tag{56}$$

Now, taking the infimum of such ρ's by (53), we get

$$\begin{aligned} &\inf \Bigg\{ \rho > 0 : \\ &\quad \sup_{s\geq 1, \sigma\in\mathscr{C}_s} \frac{1}{\phi_s} \sum_{i\in\sigma} M_i \left(q \left(\frac{A_i \left(B^{\mu}_{\Lambda} \left(x^j - x^l\right)\right)}{\rho} \right) \right)^{p_i} \leq 1 \Bigg\} < \varepsilon, \\ &\quad \forall j \geq n_0. \end{aligned} \tag{57}$$

Since $m(\mathcal{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$ is a linear space and $(x - x^j)$ are in $m(\mathcal{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$, so it follows that $x = x^j + (x - x^j) \in m(\mathcal{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$. Hence $m(\mathcal{M}, A, B^{\mu}_{\Lambda}, \phi, q, p)$ is complete. This completes the proof of the theorem. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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