

*Research Article*

# **Robust Stability of Switched Delay Systems with Average Dwell Time under Asynchronous Switching**

**Jun Cheng,<sup>1</sup> Hong Zhu,<sup>1</sup> Shouming Zhong,<sup>2,3</sup> and Yuping Zhang<sup>1</sup>**

<sup>1</sup> School of Automation Engineering, University of Electronic Science and Technology of China, Sichuan, Chengdu 611731, China

<sup>2</sup> School of Mathematical Sciences, University of Electronic Science and Technology of China, Sichuan, Chengdu 611731, China

<sup>3</sup> Key Laboratory for Neuroinformation of Ministry of Education, University of Electronic Science and Technology of China, Sichuan, Chengdu 611731, China

Correspondence should be addressed to Jun Cheng, jcheng6819@126.com

Received 10 August 2012; Accepted 7 September 2012

Academic Editor: Chong Lin

Copyright © 2012 Jun Cheng et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The problem of robust stability of switched delay systems with average dwell time under asynchronous switching is investigated. By taking advantage of the average dwell-time method and an integral inequality, two sufficient conditions are developed to guarantee the global exponential stability of the considered switched system. Finally, a numerical example is provided to demonstrate the effectiveness and feasibility of the proposed techniques.

## **1. Introduction**

In recent years, there has been increasing interest in the analysis and switched systems because of its applications in a variety of areas such as signal processing, signal estimation, pattern recognition, communications, control application, and many practical control systems. Switched linear systems comprise a collection of linear subsystems described by differential or difference equations and a switching law to specify the switching among these subsystems. A switched system is a combination of discrete and continuous dynamical systems. All of these systems arise as models for phenomena which cannot be described by exclusively continuous or exclusively discrete processes. Most recently, on the basis of Lyapunov functions and other analysis tools, the stability or stabilization for linear or nonlinear switched systems has been further investigated, and many valuable results have

been obtained, for a recent survey on this topic and related questions has attracted increasing attention [1–10]. The average dwell-time is an effective method for the switched systems which do not exist common Lyapunov function. Time delay commonly exists in engineering. Because of time delay, the system can become unstable or less capable, it is significant to study time delay. There are two kinds of stability for switched systems with time delay: time delay-independent stability and time delay-dependent stability. The time delay-independent stability is obviously conservative to the bounded time delay or small time delay, many results are obtained [11–13]. At present, there has been increasing interest in time-delay switched systems, and many valuable results have been obtained [14–18].

It is worth noting that the aforementioned results are all based on the basic assumption that the switching instants are simultaneous with those of the system. However, in actual operation, there inevitably exists asynchronous switching between the controllers and the practical subsystems, that is to say, the real switching instants of the controller exceed or lag behind those of the system, which will deteriorate performance of the systems. In fact, the necessity of taking into consideration the asynchronous switching is shown in efficient controller design in many mechanical and chemical systems. There are some results presented on control synthesis under asynchronous switching which have been proposed [19–28]. However, to the best of our knowledge, the issue of switched delay systems under asynchronous switching has not fully been investigated, which motivated this study for us.

In this paper, we deal with the problem of robust stability and  $L_2$ -gain of switched delay systems under asynchronous switching. In terms of the average dwell-time method and an integral inequality, two sufficient conditions are developed to guarantee the global exponential stability of the considered switched system. Finally, a numerical example is provided to illustrate the effectiveness and feasibility.

*Notations.* Throughout this paper,  $\mathcal{R}^n$  denotes the  $n$ -dimensional Euclidean space and  $\mathcal{R}^{n \times n}$  refers to the set of all  $n \times n$  real matrices. For real symmetric matrices  $X$  and  $Y$ , the notation  $X \geq Y$  (resp.,  $X > Y$ ) mean that the matrix  $X, Y$  are positive semidefinite, (resp., and positive definite).  $I$  is the identity matrix with appropriate dimensions.  $*$  represents the elements below the main diagonal of a symmetric matrix. The superscripts  $\top$  and  $-1$  stand for matrix transposition and matrix inverse, respectively;  $\|\cdot\|_c$  denotes the Euclidean norm.  $\lambda_{\max}(P)$  and  $\lambda_{\min}(P)$  denote the maximum and minimum eigenvalues of matrix  $P$ , respectively. The shorthand  $\text{diag}\{M_1, \dots, M_n\}$  denotes a block diagonal matrix with diagonal blocks being the matrices  $M_1, \dots, M_n$ . In this paper, if not explicit, matrices are assumed to have compatible dimensions.

## 2. Preliminaries

In this paper, we consider the following time-delay system described by

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + A_{\tau\sigma(t)}x(t - \tau(t)) + B_{\sigma(t)}\omega(t), \\ z(t) &= C_{\sigma(t)}x(t) + D_{\sigma(t)}\omega(t), \\ x(t) &= \phi(t), \quad t \in [-\tau_M, 0], \end{aligned} \tag{2.1}$$

where  $x(t) \in \mathcal{R}^n$  is the state of the system,  $\omega(t) \in \mathcal{R}^q$  is the noise signal. Switching signal  $\sigma(t)$  is a piecewise constant function of time  $t$ , and we take values in a finite set  $\mathbb{P} = \{1, 2, \dots, r\}$ ,  $r > 0$

denotes the number of subsystems.  $\sigma(t) = i \in \mathbb{P}$  means the  $i$ th subsystem is active, and the corresponding subsystem matrices are denoted by known constant matrices  $A_i$ ,  $A_{\tau i}$ ,  $B_i$ ,  $C_i$ , and  $D_i$  with appropriate dimensions and  $\tau(t)$  is the unknown time-varying delay satisfying

$$\tau_0 \leq \tau(t) \leq \tau_M, \quad \dot{\tau}(t) \leq \tau, \quad (2.2)$$

where  $\tau_0$ ,  $\tau_M$ , and  $\tau$  are constants;  $A$ ,  $A_\tau$ , and  $C_i$  are known real constant matrices;  $\phi(t)$  is a compatible vector-valued initial function on  $[-\tau_M, 0]$ .

The switching signal  $\sigma(t) : \mathbb{R}^+ \rightarrow \mathbb{P} = \{1, 2, \dots, r\}$  discussed in this paper is time dependent, that is,  $\sigma(t) : \{(t_0, \sigma(t_0)), (t_1, \sigma(t_1)), \dots, (t_{\mathfrak{S}}, \sigma(t_{\mathfrak{S}}))\}$ , where  $t_0$  is the initial instant. In this paper, we denote  $t_{\mathfrak{S}}$  represent the  $\mathfrak{S}$ th switching instant. for convenience,  $\underline{\sigma}(t)$  is used to denote the practical switching signal which can be written as

$$\underline{\sigma}(t) : \{(t_0 + \Delta_0, \sigma(t_0)), (t_1 + \Delta_1, \sigma(t_1)), \dots, (t_{\mathfrak{S}} + \Delta_{\mathfrak{S}}, \sigma(t_{\mathfrak{S}}))\}, \quad (2.3)$$

where  $\Delta_{\mathfrak{S}} < \inf_{\mathfrak{S} \geq 1} |t_{\mathfrak{S}+1} - t_{\mathfrak{S}}|$ ,  $k = 0, 1, \dots$ . Then there exist a matched period time interval  $[t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1})$  and mismatched period time interval  $[t_{\mathfrak{S}}, t_{\mathfrak{S}} + \Delta_{\mathfrak{S}})$  because of asynchronous switching. For simplicity, we assume that  $\Delta_{\mathfrak{S}} > 0$ ,  $\mathfrak{S} = 0, 1, \dots$

In this paper, we denote  $\Lambda_1$  and  $\Lambda_2$  as follows:

$$\begin{aligned} \Lambda_1 &= \{x(t) \in \mathbb{R}^n \mid \sigma(t_{\mathfrak{S}}) = i, \sigma(t_{\mathfrak{S}+1}) = j, t \in [t_{\mathfrak{S}} + \Delta_{\mathfrak{S}}, t_{\mathfrak{S}+1}), \mathfrak{S} = 0, 1, 2, \dots\}, \\ \Lambda_2 &= \{x(t) \in \mathbb{R}^n \mid \sigma(t_{\mathfrak{S}+1}) = j, t \in [t_{\mathfrak{S}+1}, t_{\mathfrak{S}+1} + \Delta_{\mathfrak{S}+1}), \mathfrak{S} = 0, 1, 2, \dots\}. \end{aligned} \quad (2.4)$$

Let

$$\varphi(t) = [x^\top(t), x^\top(t - \tau(t)), x^\top(t - \tau_M), x^\top(t - \tau_0), \omega^\top(t)]^\top. \quad (2.5)$$

This way, system (2.1) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= Y_{\sigma(t)} \cdot \varphi(t), \\ z(t) &= C_{\sigma(t)} x(t) + D_{\sigma(t)} \omega(t), \\ x(t) &= \phi(t), \quad t \in [-\tau_M, 0], \end{aligned} \quad (2.6)$$

where

$$Y_{\sigma(t)} = [A_{\sigma(t)}, A_{\tau\sigma(t)}, 0, 0, B_{\sigma(t)}]. \quad (2.7)$$

First of all, we will give some definitions and lemmas about system (2.6) which plays an important role in the derivation of our result.

*Definition 2.1* (see [29]). The unforced system is said to be exponential stable if there exist constants  $\nu > 0$  and  $\delta > 0$  such that

$$\|x(t)\| \leq \nu \sup_{-\tau_M \leq \zeta \leq 0} \|\varphi(\zeta)\|_c e^{-\delta t}. \quad (2.8)$$

*Definition 2.2* (see [30]). For  $\gamma > 0$ , the switched system (2.1) is said to have weighted  $L_2$ -gain, if under zero initial condition  $\phi(t)$ ,  $t \in [-\tau_M, 0]$ , it holds that

$$\int_0^\infty z^\top(s)z(s)ds \leq \gamma^2 \int_0^\infty \omega^\top(s)\omega(s)ds. \quad (2.9)$$

*Definition 2.3* (see [30]). For any  $T_2 > T_1 \geq 0$ , let  $N_\sigma(T_1, T_2)$  denote the switching number of discontinuities of  $\sigma(t)$  during on an interval  $(T_1, T_2)$ . If  $N_\sigma(T_1, T_2) \leq N_0 + (T_2 - T_1)/T_a$  holds for  $N_0 \geq 0$  and  $T_a > 0$ , then  $N_0$  and  $T_a$  are called chattering bound and average dwell time, respectively. Here we assume  $N_0 = 0$  for simplicity as commonly used in the literature.

**Lemma 2.4** (see [29]). For any given symmetric positive definite matrix  $X \in \mathfrak{R}^{n \times n}$ , and scalars  $\alpha > 0$ ,  $0 \leq d_1 < d_2$ , if there exists a vector function  $\dot{x}(t) : [-d_2, 0] \rightarrow \mathfrak{R}^n$  such that the following integration is well defined, then

$$\begin{aligned} & - \int_{-d_2}^{-d_1} \dot{x}(t+\theta)^\top e^{\alpha\theta} X \dot{x}(t+\theta) d\theta \\ & \leq \frac{\alpha}{e^{\alpha d_1} - e^{\alpha d_2}} \begin{bmatrix} x(t-d_1) \\ x(t-d_2) \end{bmatrix}^\top \begin{bmatrix} X & -X \\ -X & X \end{bmatrix} \begin{bmatrix} x(t-d_1) \\ x(t-d_2) \end{bmatrix}. \end{aligned} \quad (2.10)$$

**Lemma 2.5** (see [28]). Let  $\varrho \geq 0$  and  $\theta > \delta > 0$ . If there exists a real-value continuous function  $x(t) \geq 0$ ,  $t \geq t_0$ , such that the differential inequality

$$\frac{dx(t)}{dt} \leq -\theta x(t) + \delta \sup_{t-\varrho \leq s \leq t} x(s), \quad t \geq t_0 \quad (2.11)$$

holds, then

$$x(t) \leq \sup_{-\varrho \leq s \leq 0} x(t_0 + s) e^{-\mu(t-t_0)}, \quad t \geq t_0, \quad (2.12)$$

where  $\mu > 0$ , and satisfies  $\mu - \theta + \delta e^{\mu\varrho} = 0$ .

**Lemma 2.6** (see [9]). If a real scalar function  $x(t)$  satisfies the following differential inequality:

$$\dot{x}(t) \leq -\zeta x(t) + \eta v(t), \quad (2.13)$$

where  $\varsigma > 0$ ,  $\eta > 0$ , then

$$x(t) \leq e^{-\varsigma t} x(0) + \eta \int_0^t e^{-\varsigma s} v(t-s) ds. \quad (2.14)$$

### 3. Main Results

The following theorem presents a sufficient stability condition for system (2.1). We first present conditions  $\omega(t) = 0$ , the corresponding closed-loop system is given by

$$\begin{aligned} \dot{x}(t) &= \tilde{Y}_{\sigma(t)} \cdot \varphi(t), \\ x(t) &= \phi(t), \quad t \in [-\tau_M, 0], \end{aligned} \quad (3.1)$$

where

$$Y_{\sigma(t)} = [A_{\sigma(t)}, A_{\tau\sigma(t)}, 0, 0], \quad \varphi(t) = [x^\top(t), x^\top(t - \tau(t)), x^\top(t - \tau_M), x^\top(t - \tau_0)]^\top. \quad (3.2)$$

**Theorem 3.1.** For given scalars  $0 \leq \tau_0 \leq \tau_M$ ,  $\alpha > 0$ ,  $\beta > 0$ , then the system (2.1) is exponentially stable, if there exist positive-definite matrices  $P_i$ ,  $P_{ij}$ ,  $Q_{ki}$ ,  $Q_{kij}$  ( $k = 1, 2, 3$ ), and  $R_{li}$ ,  $R_{lij}$  ( $l = 1, 2$ ) such that the following LMIs hold:

$$\tilde{\Xi}_i = \begin{bmatrix} \tilde{\Xi}_{1,1} & \tilde{\Xi}_{1,2} & -\frac{\alpha}{1 - e^{\alpha\tau_M}} R_{1i} & 0 \\ * & \tilde{\Xi}_{2,2} & -\frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} R_{2i} & -\frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} R_{2i} \\ * & * & \tilde{\Xi}_{3,3} & 0 \\ * & * & * & -e^{-\alpha\tau_0} Q_{3i} + \frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} R_{2i} \end{bmatrix} < 0, \quad (3.3)$$

$$\Xi_i = \begin{bmatrix} \Xi_{1,1} & \Xi_{1,2} & \frac{\beta}{1 - e^{-\beta\tau_M}} R_{1ij} & 0 \\ * & \Xi_{2,2} & \frac{\beta}{e^{-\beta\tau_0} - e^{-\beta\tau_M}} R_{2ij} & \frac{\beta}{e^{-\beta\tau_0} - e^{-\beta\tau_M}} R_{2ij} \\ * & * & \Xi_{3,3} & 0 \\ * & * & * & -e^{\beta\tau_0} Q_{3ij} - \frac{\beta}{e^{-\beta\tau_0} - e^{-\beta\tau_M}} R_{2ij} \end{bmatrix} < 0 \quad (3.4)$$

with

$$\begin{aligned}
\tilde{\Xi}_{1,1} &= Q_{1i} + Q_{2i} + Q_{3i} + P_i A_i + A_i^\top P_i + \alpha P_i + \frac{\alpha}{1 - e^{\alpha\tau_M}} R_{1i} + \tau_M A_i^\top R_{1i} A_i \\
&\quad + (\tau_M - \tau_0) A_i^\top R_{2i} A_i, \\
\tilde{\Xi}_{1,2} &= P_i A_{\tau_i} + \tau_M A_i^\top R_{1i} A_{\tau_i} + (\tau_M - \tau_0) A_i^\top R_{2i} A_{\tau_i}, \\
\tilde{\Xi}_{2,2} &= -(1 - \tau) e^{-\alpha\tau_M} Q_{1i} + 2 \frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} R_{2i} + \tau_M A_{\tau_i}^\top R_{1i} A_{\tau_i} + (\tau_M - \tau_0) A_{\tau_i}^\top R_{2i} A_{\tau_i}, \\
\tilde{\Xi}_{3,3} &= -e^{-\alpha\tau_M} Q_{2i} + \frac{\alpha}{1 - e^{\alpha\tau_M}} R_{1i} + \frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} R_{2i}, \\
\Xi_{1,1} &= Q_{1ij} + Q_{2ij} + Q_{3ij} + P_{ij} A_i + A_i^\top P_{ij} - \beta P_{ij} + \frac{-\beta}{1 - e^{-\beta\tau_M}} R_{1ij} + \tau_M A_i^\top R_{1ij} A_i \\
&\quad + (\tau_M - \tau_0) A_i^\top R_{2ij} A_i, \\
\Xi_{1,2} &= P_{ij} A_{\tau_i} + \tau_M A_i^\top R_{1ij} A_{\tau_i} + (\tau_M - \tau_0) A_i^\top R_{2ij} A_{\tau_i}, \\
\Xi_{2,2} &= -(1 - \tau) e^{\beta\tau_M} Q_{1ij} - 2 \frac{\beta}{e^{-\beta\tau_0} - e^{-\beta\tau_M}} R_{2ij} + \tau_M A_{\tau_i}^\top R_{1ij} A_{\tau_i} + (\tau_M - \tau_0) A_{\tau_i}^\top R_{2ij} A_{\tau_i}, \\
\Xi_{3,3} &= -e^{\beta\tau_M} Q_{2ij} - \frac{\beta}{1 - e^{-\beta\tau_M}} R_{1ij} - \frac{\beta}{e^{-\beta\tau_0} - e^{-\beta\tau_M}} R_{2ij}.
\end{aligned} \tag{3.5}$$

In this case, for any switching signal with the average dwell-time satisfying

$$T_a > T_a^* = \frac{\ln \mu_2 \mu_1}{\kappa}, \tag{3.6}$$

$$\frac{T^+(t_0, t)}{T^-(t_0, t)} \geq \frac{\beta + \kappa}{\alpha - \kappa}, \quad 0 \leq \kappa < \alpha. \tag{3.7}$$

System (3.1) is exponentially stable with  $\mu_l \geq 1$  ( $l = 1, 2$ ) satisfying that

$$\begin{aligned}
P_j &\leq \mu_1 P_{ij}, \quad P_{ij} \leq \mu_2 P_i, \quad Q_{kj} \leq \mu_1 Q_{kij}, \quad Q_{kij} \leq \mu_2 Q_{ki} \quad (k = 1, 2, 3), \\
R_{lj} &\leq \mu_1 R_{lij}, \quad R_{lij} \leq \mu_2 R_{li} \quad (l = 1, 2), \quad \forall i \neq j, \quad i, j \in \mathbb{P}.
\end{aligned} \tag{3.8}$$

*Proof.* When  $t \in \Lambda_1$ , we consider the following Lyapunov-Krasovskii functional:

$$V_i(t, x(t)) = V_{1i}(t, x(t)) + V_{2i}(t, x(t)) + V_{3i}(t, x(t)), \tag{3.9}$$

where

$$\begin{aligned}
V_{1i}(t, x(t)) &= x^\top(t)P_i x(t), \\
V_{2i}(t, x(t)) &= \int_{t-\tau(t)}^t e^{-\alpha(t-s)} x^\top(s)Q_{1i}x(s)ds + \int_{t-\tau_M}^t e^{-\alpha(t-s)} x^\top(s)Q_{2i}x(s)ds \\
&\quad + \int_{t-\tau_0}^t e^{-\alpha(t-s)} x^\top(s)Q_{3i}x(s)ds, \\
V_{3i}(t, x(t)) &= \int_{-\tau_M}^0 \int_{t+\theta}^t e^{-\alpha(t-s)} \dot{x}^\top(s)R_{1i}\dot{x}(s)dsd\theta + \int_{-\tau_M}^{-\tau_0} \int_{t+\theta}^t e^{-\alpha(t-s)} \dot{x}^\top(s)R_{2i}\dot{x}(s)dsd\theta.
\end{aligned} \tag{3.10}$$

Taking the time derivative of  $V_i(t, x(t))$  for  $t \in [0, \infty)$  along the trajectory of the system (2.1) turns out to be

$$\begin{aligned}
\dot{V}_{1i}(t, x(t)) &= 2x^\top(t)P_i\tilde{Y}_i\psi(t), \\
\dot{V}_{2i}(t, x(t)) &\leq -\alpha V_{2i}(t, x(t)) + x^\top(t)(Q_{1i} + Q_{2i} + Q_{3i})x(t) \\
&\quad - (1 - \tau)e^{-\alpha\tau_M} x^\top(t - \tau(t))Q_{1i}x(t - \tau(t)) \\
&\quad - e^{-\alpha\tau_M} x^\top(t - \tau_M)Q_{2i}x(t - \tau_M) - e^{-\alpha\tau_0} x^\top(t - \tau_0)Q_{3i}x(t - \tau_0), \\
\dot{V}_{3i}(t, x(t)) &\leq -\alpha V_{3i}(t, x(t)) + \tau_M \psi^\top(t)\tilde{Y}_i^\top R_{1i}\tilde{Y}_i\psi(t) \\
&\quad + (\tau_M - \tau_0)\psi^\top(t)\tilde{Y}_i^\top R_{2i}\tilde{Y}_i\psi(t) \\
&\quad - \int_{t-\tau_M}^t \dot{x}^\top(s)e^{-\alpha(t-s)}R_{1i}\dot{x}(s)ds - \int_{t-\tau_M}^{t-\tau_0} \dot{x}^\top(s)e^{-\alpha(t-s)}R_{2i}\dot{x}(s)ds.
\end{aligned} \tag{3.11}$$

On the other hand, according to Lemma 2.4, we get that

$$\begin{aligned}
& - \int_{t-\tau_M}^t \dot{x}^\top(s)e^{-\alpha(t-s)}R_{1i}\dot{x}(s)ds \leq \frac{\alpha}{1 - e^{\alpha\tau_M}} \begin{bmatrix} x(t) \\ x(t - \tau_M) \end{bmatrix}^\top \begin{bmatrix} R_{1i} & -R_{1i} \\ -R_{1i} & R_{1i} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau_M) \end{bmatrix}, \\
& - \int_{t-\tau_M}^{t-\tau_0} \dot{x}^\top(s)e^{-\alpha(t-s)}R_{2i}\dot{x}(s)ds \\
& \leq - \int_{t-\tau(t)}^{t-\tau_0} \dot{x}^\top(s)e^{-\alpha(t-s)}R_{2i}\dot{x}(s)ds - \int_{t-\tau_M}^{t-\tau(t)} \dot{x}^\top(s)e^{-\alpha(t-s)}R_{2i}\dot{x}(s)ds \\
& \leq \frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} \begin{bmatrix} x(t - \tau_0) \\ x(t - \tau(t)) \end{bmatrix}^\top \begin{bmatrix} R_{2i} & -R_{2i} \\ -R_{2i} & R_{2i} \end{bmatrix} \begin{bmatrix} x(t - \tau_0) \\ x(t - \tau(t)) \end{bmatrix} \\
& \quad + \frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} \begin{bmatrix} x(t - \tau(t)) \\ x(t - \tau_M) \end{bmatrix}^\top \begin{bmatrix} R_{2i} & -R_{2i} \\ -R_{2i} & R_{2i} \end{bmatrix} \begin{bmatrix} x(t - \tau(t)) \\ x(t - \tau_M) \end{bmatrix}.
\end{aligned} \tag{3.12}$$

Then we can get

$$\dot{V}_i(t, x(t)) + \alpha V_i(t, x(t)) \leq \psi^\top(t) \tilde{\Xi}_i \psi(t), \quad (3.13)$$

where

$$\begin{aligned} \tilde{\Xi}_i = & \tau_M \tilde{Y}_i^\top R_{1i} \tilde{Y}_i + (\tau_M - \tau_0) \tilde{Y}_i^\top R_{2i} \tilde{Y}_i \\ & + \begin{bmatrix} \tilde{\Xi}_{1,1}^0 & P_i A_{\tau_i} & -\frac{\alpha}{1 - e^{\alpha\tau_M}} R_{1i} & 0 \\ * & \tilde{\Xi}_{2,2}^0 & -\frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} R_{2i} & -\frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} R_{2i} \\ * & * & \tilde{\Xi}_{3,3} & 0 \\ * & * & * & -e^{-\alpha\tau_0} Q_{3i} + \frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} R_{2i} \end{bmatrix} \end{aligned} \quad (3.14)$$

with

$$\begin{aligned} \tilde{\Xi}_{1,1}^0 &= Q_{1i} + Q_{2i} + Q_{3i} + P_i A_i + A_i^\top P_i + \frac{\alpha}{1 - e^{\alpha\tau_M}} R_{1i} + \alpha P_i, \\ \tilde{\Xi}_{2,2}^0 &= -(1 - \tau) e^{-\alpha\tau_M} Q_{1i} + 2 \frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} R_{2i}. \end{aligned} \quad (3.15)$$

In view of Schur complement, (3.3) implies that  $\tilde{\Xi}_i < 0$ . Then we have  $\dot{V}_i(t, x(t)) + \alpha V_i(t, x(t)) < 0$  for all  $\psi(t) \neq 0$ .

Then during the matched period, by Lemma 2.5,  $V_i(t, x(t))$  satisfy

$$\begin{aligned} V_i(t, x(t)) &\leq e^{-\alpha(t-t_0)} V_i(t_0, x_{t_0}), \quad t_0 \leq t < t_1, \\ V_i(t, x(t)) &\leq e^{-\alpha(t-t_{\mathfrak{S}-1} - \Delta_{\mathfrak{S}-1})} V_i(t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1}, x_{t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1}}), \quad t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1} \leq t < t_{\mathfrak{S}}, \quad \mathfrak{S} = 2, 3, \dots \end{aligned} \quad (3.16)$$

When  $t \in \Lambda_2$ , we consider the following Lyapunov-Krasovskii functional:

$$V_{ij}(t, x(t)) = V_{1ij}(t, x(t)) + V_{2ij}(t, x(t)) + V_{3ij}(t, x(t)), \quad (3.17)$$

where

$$\begin{aligned} V_{1ij}(t, x(t)) &= x^\top(t) P_{ij} x(t), \\ V_{2ij}(t, x(t)) &= \int_{t-\tau(t)}^t e^{\beta(t-s)} x^\top(s) Q_{1ij} x(s) ds + \int_{t-\tau_M}^t e^{\beta(t-s)} x^\top(s) Q_{2ij} x(s) ds \\ &\quad + \int_{t-\tau_0}^t e^{\beta(t-s)} x^\top(s) Q_{3ij} x(s) ds, \\ V_{3ij}(t, x(t)) &= \int_{-\tau_M}^0 \int_{t+\theta}^t e^{\beta(t-s)} \dot{x}^\top(s) R_{1ij} \dot{x}(s) ds d\theta + \int_{-\tau_M}^{-\tau_0} \int_{t+\theta}^t e^{\beta(t-s)} \dot{x}^\top(s) R_{2ij} \dot{x}(s) ds d\theta, \end{aligned} \quad (3.18)$$

where  $\beta > 0$ .



Similarly we have that

$$\dot{V}_{ij}(t, x(t)) - \beta V_{ij}(t, x(t)) \leq \psi^\top(t) \Xi_i \psi(t), \tag{3.19}$$

where

$$\begin{aligned} \Xi_{1i} &= \tau_M \tilde{Y}_i^\top R_{1ij} \tilde{Y}_i + (\tau_M - \tau_0) \tilde{Y}_i^\top R_{2ij} \tilde{Y}_i \\ &+ \begin{bmatrix} \Xi_{1,1}^0 & P_{ij} A_{\tau_i} & \frac{\beta}{1 - e^{-\beta\tau_M}} R_{1ij} & 0 \\ * & \Xi_{2,2}^0 & \frac{\beta}{e^{-\beta\tau_0} - e^{-\beta\tau_M}} R_{2ij} & \frac{\beta}{e^{-\beta\tau_0} - e^{-\beta\tau_M}} R_{2ij} \\ * & * & \Xi_{3,3} & 0 \\ * & * & * & -e^{\beta\tau_0} Q_{3ij} - \frac{\beta}{e^{-\beta\tau_0} - e^{-\beta\tau_M}} R_{2ij} \end{bmatrix} \end{aligned} \tag{3.20}$$

with

$$\begin{aligned} \Xi_{1,1}^0 &= Q_{1ij} + Q_{2ij} + Q_{3ij} + P_{ij} A_i + A_i^\top P_{ij} - \frac{\beta}{1 - e^{-\beta\tau_M}} R_{1ij} - \beta P_{ij}, \\ \Xi_{2,2}^0 &= -(1 - \tau) e^{\beta\tau_M} Q_{1ij} - 2 \frac{\beta}{e^{-\beta\tau_0} - e^{-\beta\tau_M}} R_{2ij}. \end{aligned} \tag{3.21}$$

Through Lemma 2.5, we also have

$$V_{ij}(t, x(t)) \leq e^{\beta(t-t_{\mathfrak{S}})} V_{ij}(t_{\mathfrak{S}}, x_{t_{\mathfrak{S}}}), \quad t_{\mathfrak{S}} \leq t \leq t_{\mathfrak{S}} + \Delta_{\mathfrak{S}}, \quad \mathfrak{S} = 1, 2, \dots \tag{3.22}$$

When  $t \in [t_{\mathfrak{S}}, t_{\mathfrak{S}} + \Delta_{\mathfrak{S}})$ ,  $\mathfrak{S} = 1, 2, \dots$ , we have the relationship  $N_{\underline{\sigma}(t)}(t_0, t) < N_{\sigma(t)}(t_0, t)$ , and it follows that

$$\begin{aligned} V(t, x(t)) &= V_{\underline{\sigma}(t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1})\sigma(t_{\mathfrak{S}})}(t) \leq V_{\underline{\sigma}(t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1})\sigma(t_{\mathfrak{S}})}(t_{\mathfrak{S}}) e^{\beta(t-t_{\mathfrak{S}})} \\ &\leq \mu_2 V_{\sigma(t_{\mathfrak{S}-1})}(t_{\mathfrak{S}}^-) e^{\beta(t-t_{\mathfrak{S}})} \leq \mu_2 V_{\sigma(t_{\mathfrak{S}-1})}(t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1}) e^{\beta(t-t_{\mathfrak{S}}) - \alpha(t_{\mathfrak{S}} - t_{\mathfrak{S}-1} - \Delta_{\mathfrak{S}-1})} \\ &\leq \mu_2 \mu_1 V_{\underline{\sigma}(t_{\mathfrak{S}-2} + \Delta_{\mathfrak{S}-2})}(t_{\mathfrak{S}}^-) e^{\beta(t-t_{\mathfrak{S}}) - \alpha(t_{\mathfrak{S}} - t_{\mathfrak{S}-1} - \Delta_{\mathfrak{S}-1})} \leq \dots \\ &\leq (\mu_2 \mu_1)^{N_{\sigma(t)}(t_0, t)} \mu_1^{-1} V_{\sigma(t_0)}(t_0) e^{\beta T^-(t_0, t) - \alpha T^+(t_0, t)}. \end{aligned} \tag{3.23}$$

From (3.7), we can obtain

$$T^-(t_0, t) \beta - T^+(t_0, t) \alpha \leq -\kappa(t - t_0). \tag{3.24}$$

Then through (3.23) and (3.24), we can easily get

$$V(t, x(t)) \leq \mu_1^{-1} (\mu_2 \mu_1)^{(t-t_0)/T_a} V_{\sigma(t_0)}(t_0) e^{-\kappa(t-t_0)} \leq \mu_1^{-1} V_{\sigma(t_0)}(t_0) e^{-(\kappa - (\ln \mu_2 \mu_1)/T_a)(t-t_0)}. \tag{3.25}$$

By (3.9) and (3.25), then we have

$$\begin{aligned} \lambda_{\min}(P_i)\|x(t)\|^2 \leq V(t, x(t)) \leq \mu_1^{-1} & \left[ \max_{\forall i \in \mathbb{P}} \lambda_{\max}(P_i) + \sum_{k=1}^2 \tau_M \max_{\forall i \in \mathbb{P}} \lambda_{\max}(Q_{ki}) \right. \\ & + \tau_0 \max_{\forall i \in \mathbb{P}} \lambda_{\max}(Q_{3i}) + \frac{\tau_M^2}{2} \max_{\forall i \in \mathbb{P}} \lambda_{\max}(R_{1i}) \\ & \left. + \frac{(\tau_M^2 - \tau_0^2)}{2} \max_{\forall i \in \mathbb{P}} \lambda_{\max}(R_{2i}) \right] \|x_{t_0}\|^2 e^{-(\kappa - (\ln \mu_2 \mu_1)/T_a)(t-t_0)}, \end{aligned} \quad (3.26)$$

which means

$$\|x(t)\| \leq \mu_1^{-1/2} \sqrt{\frac{a_1}{b_1}} \|x_{t_0}\| e^{-(1/2)(\kappa - \ln \mu_2 \mu_1/T_a)(t-t_0)}, \quad (3.27)$$

where

$$\begin{aligned} a_1 &= \max_{\forall i \in \mathbb{P}} \lambda_{\max}(P_i) + \sum_{k=1}^2 \tau_M \max_{\forall i \in \mathbb{P}} \lambda_{\max}(Q_{ki}) + \tau_0 \max_{\forall i \in \mathbb{P}} \lambda_{\max}(Q_{3i}) \\ &+ \frac{\tau_M^2}{2} \max_{\forall i \in \mathbb{P}} \lambda_{\max}(R_{1i}) + \frac{(\tau_M^2 - \tau_0^2)}{2} \max_{\forall i \in \mathbb{P}} \lambda_{\max}(R_{2i}), \end{aligned} \quad (3.28)$$

$$b_1 = \lambda_{\min}(P_i).$$

Similarly, when  $t \in \Lambda_1$ , we can also have

$$\|x(t)\| \leq \mu_1^{-1/2} \sqrt{\frac{a_2}{b_2}} \|x_{t_0}\| e^{-(1/2)(\kappa - \ln \mu_2 \mu_1/T_a)(t-t_0)}, \quad (3.29)$$

where

$$\begin{aligned} a_2 &= \max_{\forall i, j \in \mathbb{P}} \lambda_{\max}(P_{ij}) + \sum_{k=1}^2 \tau_M \max_{\forall i, j \in \mathbb{P}} \lambda_{\max}(Q_{kij}) + \tau_0 \max_{\forall i, j \in \mathbb{P}} \lambda_{\max}(Q_{3ij}) \\ &+ \frac{\tau_M^2}{2} \max_{\forall i, j \in \mathbb{P}} \lambda_{\max}(R_{1ij}) + \frac{(\tau_M^2 - \tau_0^2)}{2} \max_{\forall i, j \in \mathbb{P}} \lambda_{\max}(R_{2ij}), \end{aligned} \quad (3.30)$$

$$b_2 = \lambda_{\min}(P_{ij}).$$

For convenience, let  $a = \max\{a_1, a_2\}$ ,  $b = \min_{i \neq j, i, j \in \mathbb{P}} \lambda_{\min}(P_i, P_{ij})$ , through (3.27) and (3.29), we have

$$\|\eta_t\| \leq \mu_1^{-1/2} \sqrt{\frac{a}{b}} \|\eta_{t_0}\| e^{-(1/2)(\kappa - \ln \mu_2 \mu_1/T_a)(t-t_0)}. \quad (3.31)$$

□

**Theorem 3.2.** For given scalars  $0 \leq \tau_0 \leq \tau_M$ ,  $\alpha > 0$ ,  $\beta > 0$ , then the system (2.1) is exponentially stable with  $L_2$ -gain, if there exist positive-definite matrices  $P_i$ ,  $P_{ij}$ ,  $Q_{ki}$ ,  $Q_{kij}$  ( $k = 1, 2, 3$ ), and  $R_{li}$ ,  $R_{lij}$  ( $l = 1, 2$ ) such that the following LMIs hold:

$$\tilde{\Sigma}_{1i} = \begin{bmatrix} \tilde{\Sigma}_{1,1} & \tilde{\Xi}_{1,2} & -\frac{\alpha}{1 - e^{\alpha\tau_M}} R_{1i} & 0 & \tilde{\Sigma}_{1,5} \\ * & \tilde{\Xi}_{2,2} & -\frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} R_{2i} & -\frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} R_{2i} & \tilde{\Sigma}_{2,5} \\ * & * & \tilde{\Xi}_{3,3} & 0 & 0 \\ * & * & * & -e^{-\alpha\tau_0} Q_{3i} + \frac{\alpha}{e^{\alpha\tau_0} - e^{\alpha\tau_M}} R_{2i} & 0 \\ * & * & * & * & \tilde{\Sigma}_{5,5} \end{bmatrix}, \quad (3.32)$$

$$\underline{\Sigma}_{1i} = \begin{bmatrix} \underline{\Sigma}_{1,1} & \underline{\Xi}_{1,2} & \frac{\beta}{1 - e^{-\beta\tau_M}} R_{1ij} & 0 & \underline{\Sigma}_{1,5} \\ * & \underline{\Xi}_{2,2} & \frac{\beta}{e^{-\beta\tau_0} - e^{-\beta\tau_M}} R_{2ij} & \frac{\beta}{e^{-\beta\tau_0} - e^{-\beta\tau_M}} R_{2ij} & \underline{\Sigma}_{2,5} \\ * & * & \underline{\Xi}_{3,3} & 0 & 0 \\ * & * & * & -e^{\beta\tau_0} Q_{3ij} - \frac{\beta}{e^{-\beta\tau_0} - e^{-\beta\tau_M}} R_{2ij} & 0 \\ * & * & * & * & \underline{\Sigma}_{5,5} \end{bmatrix} < 0 \quad (3.33)$$

with

$$\begin{aligned} \tilde{\Sigma}_{1,1} &= Q_{1i} + Q_{2i} + Q_{3i} + P_i A_i + A_i^T P_i + \frac{\alpha}{1 - e^{\alpha\tau_M}} R_{1i} + \tau_M A_i^T R_{1i} A_i \\ &\quad + (\tau_M - \tau_0) A_i^T R_{2i} A_i + C_i^T C_i, \\ \tilde{\Sigma}_{1,5} &= P_i B_i + \tau_M A_i^T R_{1i} B_i + (\tau_M - \tau_0) A_i^T R_{2i} B_i + C_i^T D_i, \\ \tilde{\Sigma}_{2,5} &= \tau_M A_{\tau_i}^T R_{1i} B_i + (\tau_M - \tau_0) A_{\tau_i}^T R_{2i} B_i, \\ \tilde{\Sigma}_{5,5} &= \tau_M B_i^T R_{1i} B_i + (\tau_M - \tau_0) B_i^T R_{2i} B_i + D_i^T D_i - \gamma^2 I, \\ \underline{\Sigma}_{1,1} &= Q_{1ij} + Q_{2ij} + Q_{3ij} + P_{ij} A_i + A_i^T P_{ij} - \frac{\beta}{1 - e^{-\beta\tau_M}} R_{1ij} + \tau_M A_i^T R_{1ij} A_i \\ &\quad + (\tau_M - \tau_0) A_i^T R_{2ij} A_i + C_i^T C_i, \\ \underline{\Sigma}_{1,5} &= P_{ij} B_i + \tau_M A_i^T R_{1ij} B_i + (\tau_M - \tau_0) A_i^T R_{2ij} B_i + C_i^T D_i, \\ \underline{\Sigma}_{2,5} &= \tau_M A_{\tau_i}^T R_{1ij} B_i + (\tau_M - \tau_0) A_{\tau_i}^T R_{2ij} B_i, \\ \underline{\Sigma}_{5,5} &= \tau_M B_i^T R_{1ij} B_i + (\tau_M - \tau_0) B_i^T R_{2ij} B_i + D_i^T D_i - \gamma^2 I. \end{aligned} \quad (3.34)$$

*Proof.* For all nonzero  $\omega(t) \in L_2[0, \infty)$  and a scalar  $\gamma > 0$ , then we establish system (2.1) with  $L_2$ -gain performance  $\|\tilde{z}(t)\|_2 \leq \gamma \|\omega(t)\|$ . For convenience, denoting  $\Pi(t) = \tilde{z}^\top(t)\tilde{z}(t) - \gamma^2 \omega^\top(t)\omega(t)$ .

When  $t \in [t_0, t_1) \cup [t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1}, t_{\mathfrak{S}})$ ,  $k = 2, 3, \dots$ , by the system (2.1), we can obtain

$$\dot{V}(t, x(t)) + \alpha V_i(t, x(t)) + \Pi(t) = \varphi^\top(t) \tilde{\Sigma}_i \varphi(t). \quad (3.35)$$

From (3.32), we can easily get

$$\dot{V}_i(t, \eta_t) + \alpha V_i(t, \eta_t) + \Pi(t) < 0. \quad (3.36)$$

Integrate this inequality during  $[t_0, t]$ , it is known that

$$\begin{aligned} V_i(t, x(t)) &\leq e^{-\alpha(t-t_0)} V_i(t_0, x_{t_0}) - \int_{t-t_0}^t e^{-\alpha(t-s)} \Pi(s) ds, \quad t_0 \leq t < t_1, \\ V_i(t, x(t)) &\leq e^{-\alpha(t-t_{\mathfrak{S}-1}-\Delta_{\mathfrak{S}-1})} V_i(t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1}, x_{t_{\mathfrak{S}-1}+\Delta_{\mathfrak{S}-1}}) \\ &\quad - \int_{t_{\mathfrak{S}-1}+\Delta_{\mathfrak{S}-1}}^t e^{-\alpha(t-s)} \Pi(s) ds, \quad t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1} \leq t < t_{\mathfrak{S}}, \quad \mathfrak{S} = 2, 3, \dots \end{aligned} \quad (3.37)$$

When  $t \in [t_{\mathfrak{S}}, t_{\mathfrak{S}} + \Delta_{\mathfrak{S}})$ ,  $\mathfrak{S} = 2, 3, \dots$ , by the system (2.1), by the same way, we can obtain

$$\dot{V}(t, x(t)) - \beta V_i(t, x(t)) + \Pi(t) = \varphi^\top(t) \underline{\Sigma}_i \varphi(t) < 0. \quad (3.38)$$

Then we have

$$V_{ij}(t, x(t)) \leq V_{ij}(t_{\mathfrak{S}}, x_{t_{\mathfrak{S}}}) e^{\beta(t-t_{\mathfrak{S}})} - \int_{t_{\mathfrak{S}}}^t e^{\beta(t-s)} \Pi(s) ds, \quad t_{\mathfrak{S}} \leq t < t_{\mathfrak{S}} + \Delta_{\mathfrak{S}}, \quad \mathfrak{S} = 1, 2, \dots \quad (3.39)$$

When  $t \in [t_{\mathfrak{S}}, t_{\mathfrak{S}} + \Delta_{\mathfrak{S}})$ ,  $\mathfrak{S} = 1, 2, \dots$ , it follows that

$$\begin{aligned}
V(t, x(t)) &= V_{\underline{\sigma}(t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1})\sigma(t_{\mathfrak{S}})}(t_{\mathfrak{S}}) e^{\beta T^-(t_{\mathfrak{S}}, t) - \alpha T^+(t_{\mathfrak{S}}, t)} - \int_{t_{\mathfrak{S}}}^t e^{\beta T^-(t_{\mathfrak{S}}, t) - \alpha T^+(t_{\mathfrak{S}}, t)} \Pi(s) ds \\
&\leq \mu_2 V_{\sigma(t_{\mathfrak{S}-1})}(t_{\mathfrak{S}}^-) e^{\beta T^-(t_{\mathfrak{S}}, t) - \alpha T^+(t_{\mathfrak{S}}, t)} - \int_{t_{\mathfrak{S}}}^t e^{\beta T^-(t_{\mathfrak{S}}, t) - \alpha T^+(t_{\mathfrak{S}}, t)} \Pi(s) ds \\
&\leq \mu_2 \mu_1 V_{\underline{\sigma}(t_{\mathfrak{S}-2} + \Delta_{\mathfrak{S}-2})\sigma(t_{\mathfrak{S}-1})}(t_{\mathfrak{S}-1}) e^{\beta T^-(t_{\mathfrak{S}-1}, t) - \alpha T^+(t_{\mathfrak{S}-1}, t)} \\
&\quad - \mu_2 \mu_1 \int_{t_{\mathfrak{S}-1}}^{t_{\mathfrak{S}-1} - \Delta_{\mathfrak{S}-1}} e^{\beta T^-(t_{\mathfrak{S}}, t) - \alpha T^+(t_{\mathfrak{S}}, t)} \Pi(s) ds \\
&\quad - \mu_2 \int_{t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1}}^{t_{\mathfrak{S}}} e^{\beta T^-(t_{\mathfrak{S}}, t) - \alpha T^+(t_{\mathfrak{S}}, t)} \Pi(s) ds - \int_{t_{\mathfrak{S}}}^t e^{\beta T^-(t_{\mathfrak{S}}, t) - \alpha T^+(t_{\mathfrak{S}}, t)} \Pi(s) ds \leq \dots \\
&\leq - \int_{t_0}^t \mu_2^{N_{\sigma(t)}(s, t)} \mu_1^{N_{\underline{\sigma}(t)}(s, t)} e^{\beta T^-(s, t) - \alpha T^+(s, t)} \Pi(s) ds \\
&\quad + \mu_2^{N_{\sigma(t)}(t_0, t)} \mu_1^{N_{\underline{\sigma}(t)}(t_0, t)} V_{\sigma(t_0)}(t_0) e^{\beta T^-(t_{\mathfrak{S}}, t) - \alpha T^+(t_{\mathfrak{S}}, t)}.
\end{aligned} \tag{3.40}$$

Under the zero initial condition, Let  $t_0 = 0$ , (3.40) implies

$$\int_0^t \mu_2^{N_{\sigma(t)}(s, t)} \mu_1^{N_{\underline{\sigma}(t)}(s, t)} e^{\beta T^-(s, t) - \alpha T^+(s, t)} \Lambda(s) ds \leq 0. \tag{3.41}$$

Integrate (3.41) during  $[0, \infty)$ , then we can obtain

$$\begin{aligned}
&\int_0^\infty \int_0^t e^{[N_{\sigma(t)}(0, s) \ln \mu_2 + N_{\underline{\sigma}(t)}(0, s) \ln \mu_1]} e^{\beta T^-(s, t) - \alpha T^+(s, t)} \Pi(s) ds dt \\
&< \int_0^\infty e^{[N_{\sigma(t)}(0, s) \ln \mu_2 + N_{\underline{\sigma}(t)}(0, s) \ln \mu_1]} \Pi(s) \left( \int_s^\infty e^{-\kappa(t-s)} dt \right) ds \leq \int_0^\infty \Pi(s) ds \leq 0.
\end{aligned} \tag{3.42}$$

When  $t \in [t_0, t_1) \cup [t_{\mathfrak{S}-1} + \Delta_{\mathfrak{S}-1}, t_{\mathfrak{S}})$ ,  $\mathfrak{S} = 2, 3, \dots$ , by the same mathematical operations, we have  $\int_{t_0}^\infty \Pi(s) ds < 0$ .

From which we can get  $\|\tilde{z}(t)\|_2 \leq \|\omega(t)\|_2$ . This proof is completed.  $\square$

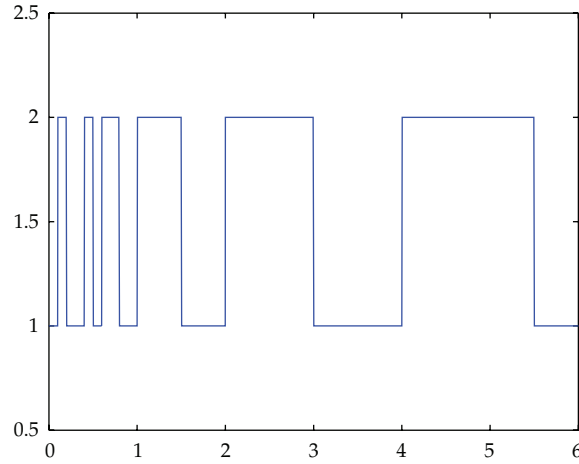
*Remark 3.3.* If  $\mu_1 = \mu_2 = 0$ , which implies that  $P_i = P_{ij} = P$ ,  $Q_{ki} = Q_{kij} = Q_k$ ,  $R_{li} = R_{lij} = R_l$ ,  $i, j \in \mathbb{P}$ , by (3.3)-(3.4) and (3.32)-(3.33), we have  $T_a = 0$ , then it requires a common Lyapunov functional for all subsystems, and the switching signals can be arbitrary. If  $\mu_k \rightarrow \infty$  ( $k = 1, 2$ ), we get from (3.3)-(3.4) and (3.32)-(3.33) that there is no switching, that is, switching signal will have a great dwell-time on the average.

#### 4. Illustrative Example

In this section, a numerical example is given to illustrate the effectiveness of the obtained results.

**Table 1:** Different  $T_a$  and  $\tau_M$  for  $\alpha = 0.25$ ,  $\beta = 0.2$ .

$\mu_1\mu_2$	1.0	1.5	2.0
$T_a$	0.0521	2.1022	5.2454
$\tau_M$	1.2036	1.5415	1.7154

**Figure 1:** Switching signal with ADT.

*Example 4.1.* Consider the system (2.1) with parameters as follows:

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -1.5 & 0.3 \\ 0 & -2.4 \end{bmatrix}, & A_2 &= \begin{bmatrix} -1.4 & 0.8 \\ 0.6 & -2.4 \end{bmatrix}, & A_{\tau 1} &= \begin{bmatrix} -2.2 & -0.3 \\ 0 & -1.6 \end{bmatrix}, \\
 A_{\tau 2} &= \begin{bmatrix} 1.2 & 0 \\ 0 & -1.8 \end{bmatrix}, & C_1 &= \begin{bmatrix} -0.4 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, & C_2 &= \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.35 \end{bmatrix}, & B_1 &= \begin{bmatrix} -0.35 \\ 0.28 \end{bmatrix}, \\
 B_2 &= \begin{bmatrix} -0.66 \\ 0.25 \end{bmatrix}, & D_1 &= \begin{bmatrix} -0.35 \\ -0.15 \end{bmatrix}, & D_2 &= \begin{bmatrix} 0.57 \\ -0.48 \end{bmatrix}.
 \end{aligned} \tag{4.1}$$

$\tau_0$  is fixed and assumed to be 0.2. The initial condition is assumed to be  $x(0) = [9, -9]^T$ ,  $\omega(t) = 0.5 \sin t$ . Then by solving the LMIs in Theorem 3.2, different  $T_a$  and  $\tau_M$  for different  $\mu_1\mu_2$ ,  $\alpha$ , and  $\beta$  can be obtained in Table 1. It can be seen that, for the given  $\tau_0$ , the upper bounds of the time delay  $\tau_M$  and the minimal average dwell-time  $T_a$  are dependent on  $\alpha$ ,  $\beta$ , and  $\mu_1\mu_2$ . Then the simulation result of the system is shown in Figures 1, 2, and 3. The switching signal  $\sigma(t)$  with average dwell-time  $T_a$  is shown in Figure 1. Figures 2–3 indicate that the state response of the switched system without asynchronous switching and with asynchronous switching, respectively.

## 5. Conclusions

In terms of the LMI approach, the problem of robust stabilization of switched delay systems with average dwell-time under asynchronous switching has been considered. Two sufficient

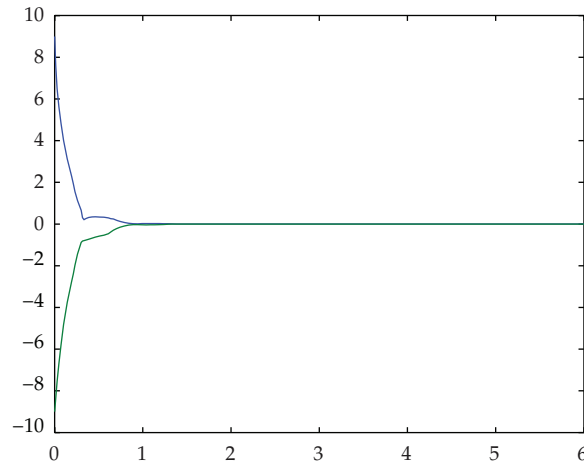


Figure 2: The state response without asynchronous switching.

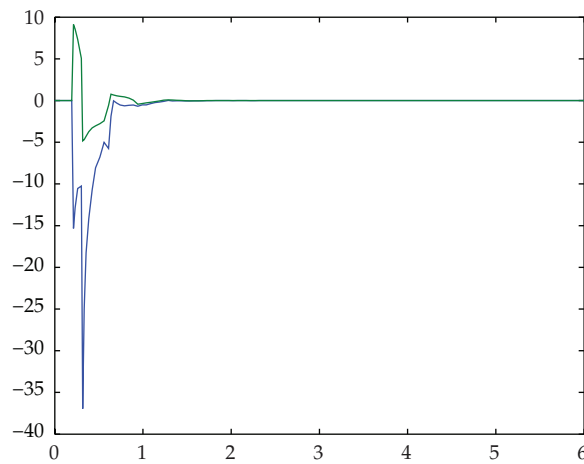


Figure 3: The state response with asynchronous switching.

conditions are developed to guarantee the global exponential stability of the considered switched system. At last, a numerical example is provided to demonstrate the effectiveness and feasibility of the proposed techniques.

### Acknowledgment

This work was supported by the Fundamental Research Funds for the Central Universities (103.1.2E022050205).

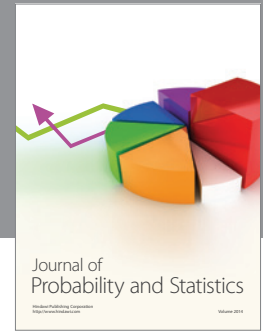
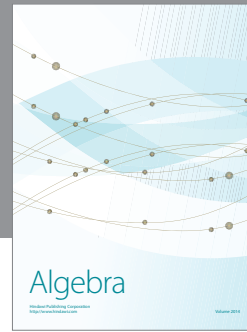
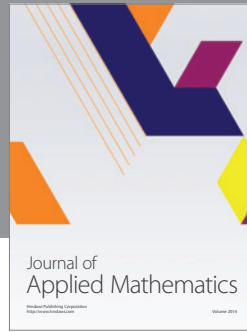
### References

- [1] V. Lakshmikantham, D. D. Bařnov, and P. S. Simeonov, *Theory of Impulsive Differential Equations*, World Scientific, Singapore, 1989.

- [2] S. H. Lee and J. T. Lim, "Stability analysis of switched systems with impulse effects," in *Proceedings of the IEEE International Symposium on Intelligent Control—Intelligent Systems and Semiotics*, pp. 79–83, September 1999.
- [3] P. Colaneri, J. C. Geromel, and A. Astolfi, "Stabilization of continuous-time switched nonlinear systems," *Systems & Control Letters*, vol. 57, no. 1, pp. 95–103, 2008.
- [4] G. Xie and L. Wang, "Periodic stabilizability of switched linear control systems," *Automatica*, vol. 45, no. 9, pp. 2141–2148, 2009.
- [5] C. Yang and W. Zhu, "Stability analysis of impulsive switched systems with time delays," *Mathematical and Computer Modelling*, vol. 50, no. 7-8, pp. 1188–1194, 2009.
- [6] V. N. Phat, T. Botmart, and P. Niamsup, "Switching design for exponential stability of a class of nonlinear hybrid time-delay systems," *Nonlinear Analysis. Hybrid Systems*, vol. 3, no. 1, pp. 1–10, 2009.
- [7] Z. Xiang, C. Liang, and Q. Chen, "Robust  $L_2$ - $L_\infty$  filtering for switched systems under asynchronous switching," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 8, pp. 3303–3318, 2011.
- [8] K. Hu and J. Yuan, "Improved robust  $H_\infty$  filtering for uncertain discrete-time switched systems," *IET Control Theory & Applications*, vol. 3, no. 3, pp. 315–324, 2009.
- [9] C. S. Tseng and B. S. Chen, " $L_\infty$ -gain fuzzy control for nonlinear dynamic systems with persistent bounded disturbances," in *Proceedings of the IEEE International Conference on Fuzzy Systems*, pp. 25–29, Budapest, Hungary, 2004.
- [10] D. Liberzon and A. S. Morse, "Basic problems in stability and design of switched systems," *IEEE Control Systems Magazine*, vol. 19, no. 5, pp. 59–70, 1999.
- [11] D. Liberzon, *Switching in Systems and Control*, Birkhäuser, Boston, Mass, USA, 2003.
- [12] X.-M. Sun, J. Zhao, and D. J. Hill, "Stability and  $L_2$ -gain analysis for switched delay systems: a delay-dependent method," *Automatica*, vol. 42, no. 10, pp. 1769–1774, 2006.
- [13] X. M. Sun, W. Wang, G. P. Liu, and J. Zhao, "Stability analysis for linear switched systems with time-varying delay," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 38, no. 2, pp. 528–533, 2008.
- [14] T.-F. Li, J. Zhao, and G. M. Dimirovski, "Stability and  $L_2$ -gain analysis for switched neutral systems with mixed time-varying delays," *Journal of the Franklin Institute*, vol. 348, no. 9, pp. 2237–2256, 2011.
- [15] D. Y. Liu, S. M. Zhong, and Y. Q. Huang, "Stability and  $L_2$ -gain analysis for switched neutral systems," in *Proceedings of the International Conference on Apperceiving Computing and Intelligence Analysis (ICACIA '08)*, pp. 247–250, 2008.
- [16] L. Xiong, S. Zhong, M. Ye, and S. Wu, "New stability and stabilization for switched neutral control systems," *Chaos, Solitons & Fractals*, vol. 42, no. 3, pp. 1800–1811, 2009.
- [17] C. Li, T. Huang, G. Feng, and G. Chen, "Exponential stability of time-controlled switching systems with time delay," *Journal of the Franklin Institute*, vol. 349, no. 1, pp. 216–233, 2012.
- [18] F. Gao, S. Zhong, and X. Gao, "Delay-dependent stability of a type of linear switching systems with discrete and distributed time delays," *Applied Mathematics and Computation*, vol. 196, no. 1, pp. 24–39, 2008.
- [19] Z. R. Xiang and R. H. Wang, "Robust stabilization of switched non-linear systems with time-varying delays under asynchronous switching," *Proceedings of the Institution of Mechanical Engineers I*, vol. 223, no. 8, pp. 1111–1128, 2009.
- [20] L. Zhang and P. Shi, "Stability,  $L_2$ -gain and asynchronous  $H_\infty$  control of discrete-time switched systems with average dwell time," *IEEE Transactions on Automatic Control*, vol. 54, no. 9, pp. 2192–2199, 2009.
- [21] W. Xiang, M. Che, C. Xiao, and Z. Xiang, "Stabilization of a class of switched systems with mismatched switching," in *Proceedings of the International Conference on Measuring Technology and Mechatronics Automation (ICMTMA '09)*, pp. 124–127, Zhangjiajie, China, April 2009.
- [22] L. Zhang and H. Gao, "Asynchronously switched control of switched linear systems with average dwell time," *Automatica*, vol. 46, no. 5, pp. 953–958, 2010.
- [23] W. Xiang, J. Xiao, and M. N. Iqbal, "Robust observer design for nonlinear uncertain switched systems under asynchronous switching," *Nonlinear Analysis. Hybrid Systems*, vol. 6, no. 1, pp. 754–773, 2012.
- [24] Z. Xiang, Y.-N. Sun, and Q. Chen, "Robust reliable stabilization of uncertain switched neutral systems with delayed switching," *Applied Mathematics and Computation*, vol. 217, no. 23, pp. 9835–9844, 2011.
- [25] Z. R. Xiang and R. H. Wang, "Robust control for uncertain switched non-linear systems with time delay under asynchronous switching," *IET Control Theory & Applications*, vol. 3, no. 8, pp. 1041–1050, 2009.
- [26] D. Xie and X. Chen, "Observer-based switched control design for switched linear systems with time delay in detection of switching signal," *IET Control Theory & Applications*, vol. 2, no. 5, pp. 437–445, 2008.



- [27] Z. Ji, X. Guo, S. Xu, and L. Wang, "Stabilization of switched linear systems with time-varying delay in switching occurrence detection," *Circuits, Systems, and Signal Processing*, vol. 26, no. 3, pp. 361–377, 2007.
- [28] Z. R. Xiang and R. H. Wang, "Robust control for uncertain switched non-linear systems with time delay under asynchronous switching," *IET Control Theory & Applications*, vol. 3, no. 8, pp. 1041–1050, 2009.
- [29] Z. G. Wu, P. Shi, H. Su, and J. Chu, "Delay-dependent stability analysis for switched neural networks with time-varying delay," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 41, no. 6, pp. 1522–1530, 2011.
- [30] C. Yang and W. Zhu, "Stability analysis of impulsive switched systems with time delays," *Mathematical and Computer Modelling*, vol. 50, no. 7-8, pp. 1188–1194, 2009.



# Hindawi

Submit your manuscripts at  
<http://www.hindawi.com>

