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Research Article

A Derivation of the Nonlocal Volume-Averaged Equations for Two-Phase Flow Transport

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In this paper a detailed derivation of the general transport equations for two-phase systems using a method based on *nonlocal* volume averaging is presented. The *local* volume averaging equations are commonly applied in nuclear reactor system for optimal design and safe operation. Unfortunately, these equations are limited to length-scale restriction and according with the theory of the averaging volume method, these fail in transition of the flow patterns and boundaries between two-phase flow and solid, which produce rapid changes in the physical properties and void fraction. The *non-local* volume averaging equations derived in this work contain new terms related with *non-local* transport effects due to accumulation, convection diffusion and transport properties for two-phase flow; for instance, they can be applied in the boundary between a two-phase flow and a solid phase, or in the boundary of the transition region of two-phase flows where the *local* volume averaging equations fail.

1. Introduction

The technique of *local* volume averaging of microscopic conservation equations of motion and transport has received numerous research and analysis [1–16], in order to obtain macroscopic balance equations applicable to multiphase systems. The approximation of *local* volume-averaged conservation equation of two-phase flow is valid when the following length-scale restriction is fulfilled [10]:

$$\frac{\ell}{\Im} \ll 1, \tag{1}$$

where ℓ is the characteristic length of the dispersed phases, and $\Im$ is the characteristic length of the global system:

$$\frac{1}{\Im} = \frac{\mathrm{MAX}_{x\in V_k} \left| \nabla \langle \psi_k \rangle (\underline{x}, t) \right|}{\mathrm{MAX}_{x\in V_k} \left| \langle \psi_k \rangle (\underline{x}, t) \right|}. \tag{2}$$

In this equation ψ_k is the intrinsic property and $\langle \psi_k \rangle$ represents the average and V_k is the volume of k-phase. Then, ℓ is associated where ψ_k varies significantly, and $\Im$ with the changes in $\langle \psi_k \rangle$.

The imposition of the guarantees *good behavior* of the averaged variables. However, the most well-known multiphase flow systems where the length-scale restriction given by (1) are not true: geological systems [17], fractionation of hydrocarbons [18], transport of contaminants [19–21], elimination of contamination in aqueous streams [22], cuttings transport [23–25], and concentration of pharmaceutics [26], among others that include extraction and separation processes [27]. Specifically, in nuclear systems of BWR type and other industrial applications that involve multiphase flow, the length-scale restriction is no longer satisfied following the transition to churn or slug flow regimes, where the number of bubbles is highly decreased and their size is increased to the length of the magnitude order of the averaging volume, including the whole system, that is, pipe diameter.

The *local* volume averaging of the conservation equations (mass, momentum, and energy) involves averaging the product of a volume-averaged variable $\langle \psi_k \rangle$ (in this paper it was sought to use the nomenclature defined by Lahey and Drew [10]), with the unaveraged variable (φ_k), that is,

$\langle \varphi_k \langle \psi_k \rangle \rangle$ (here ψ_k and φ_k are intensive properties associated with the k phase). The conditions necessary to bring a volume-averaged variable outside the volume integral are the imposing of the length-scale restriction given by (1), that is, $\langle \varphi_k \langle \psi_k \rangle \rangle = \langle \varphi_k \rangle \langle \psi_k \rangle$, with the idea of obtaining manipulated variables associated with the processes of two-phase flow.

Another common case of the *local* volume averaging of the conservation equations is the average product of two unaveraged variables, that is, $\langle \varphi_k \psi_k \rangle$. The traditional representation is $\langle \varphi_k \psi_k \rangle = \alpha_k \langle \varphi_k \rangle \langle \psi_k \rangle + \langle \tilde{\varphi}_k \tilde{\psi}_k \rangle$, where $\tilde{\varphi}_k$ and $\tilde{\psi}_k$ represent the spatial deviations around averaged values of the *local* variables and are defined by the decomposition $\varphi_k = \langle \varphi_k \rangle + \tilde{\varphi}_k$ and $\psi_k = \langle \psi_k \rangle + \tilde{\psi}_k$ [5]. The removal of averaged quantities from the volume integrals is consistent with the length-scale restriction given by (1). The mathematical consequence of this type of inequality can be expressed as $\langle \tilde{\varphi}_k \rangle = 0$ and $\langle \tilde{\psi}_k \rangle = 0$.

However, for more realistic problems, this length-scale restriction given by (1) is not true. In general, this length-scale restriction is not valid within the boundary region (e.g., transition region in two-phase flows) due to significant spatial variations of the two-phase flow structure. The classical length-scale restriction which is implicit in the average transport equations are not satisfied.

In this paper a detailed derivation of the general transport equations for two-phase systems using a method based on *nonlocal* volume averaging, that is, without length-scale restriction is presented. The *nonlocal* volume averaging equations derived in this work contain new terms related to *nonlocal* transport effects due to accumulation, convection diffusion, and transport properties for two-phase flow heat transfer. The *nonlocal* terms were evaluated considering that these are a function of the *local* terms, which yield new coefficients or closure relationships.

2. Preliminaries

The two-phase flow is a system formed by a fluid mixture of l (liquid) and g (gas) phases flowing through a region V as is illustrated in Figure 1. Phase k (= l, g) has a variable volume V_k with a total interfacial area of A_k in the averaging volume V, which has an enveloping surface area (A) with a unit normal vector ($\underline{n}$) pointing outward. A portion of A_k is made of a liquid-gas interphase A_{lg} and a fluid-solid interface A_{kw}. The unit normal vector $\underline{n}_k$ of A_k is always drawn outwardly from phase k, regardless whether if it is associated with A_{lg} or A_{kw}.

Local averaging volume

$$V = V_l(t) + V_g(t). \tag{3}$$

Volume fraction g phase in fluid mixture

$$\alpha_g = \frac{V_g(t)}{V}. \tag{4}$$

The method of volume averaging is a technique that can be used to rigorously derive continuum equations for multiphase systems. This means that the equations valid for a particular phase can be spatially smoothed to produce equations that are valid everywhere, except in the boundaries which contain the multiphase systems.

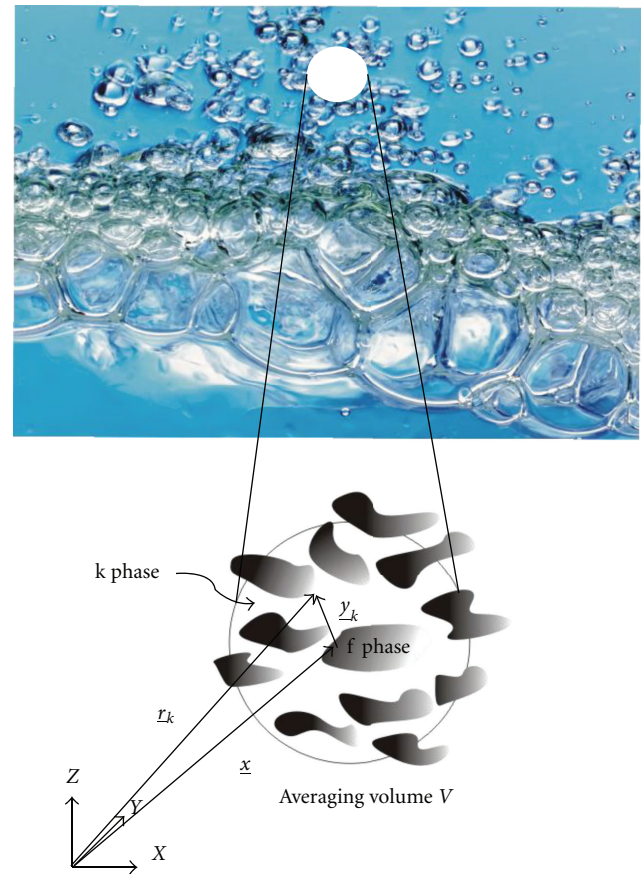

Figure 1: Schematic figure of a two-phase flow with a transition region, showing the position vectors and the averaging volume.

The volume average operator or superficial volume average $\langle \psi_k \rangle^s$ of some property ψ_k (scalar, vector, or tensor) associated with the k phase is given by

$$\langle \psi_k \rangle^s \big|_{\underline{x}} = \frac{1}{V} \int_{V_k(\underline{x},t)} \psi_k \left(\underline{x} + \underline{y}_k, t \right) dV, \tag{5}$$

where V is the averaging volume, V_k is the volume of the k phase (contained V), $\underline{x}$ is the position vector locating the centroid of the averaging volume, $\underline{y}_k$ is the position vector at any point in the k phase relative to the centroid, as is illustrated in Figure 1, and dV indicates that the integration is carried out with respect to the components of $\underline{y}_k$. Then, (5) indicates that the volume-averaged quantities are associated with the centroid. In order to simplify the notation, we will avoid the precise nomenclature used and represent the superficial average of ψ_k as

$$\langle \psi_k \rangle^s = \frac{1}{V} \int_{V_k} \psi_k \big|_{\underline{x}+\underline{y}_k} dV. \tag{6}$$

The *intrinsic average* is expressed in the form

$$\langle \psi_k \rangle = \frac{1}{V_k} \int_{V_k} \psi_k \big|_{\underline{x}+\underline{y}_k} dV. \tag{7}$$

These averages will be used in the theoretical development of the two-phase flow transport equations and are related by

$$\langle \psi_k \rangle^s = \alpha_k \langle \psi_k \rangle. \tag{8}$$

With $\psi_k = 1$, the result leads to

$$\langle 1 \rangle^s = \alpha_k. \tag{9}$$

As mentioned above V is a constant, which is invariant in both space and time as illustrated in Figure 1. In this case the volumes of each phase of the flow may change with the position and time, that is, $V_k(\underline{x}, t)$. It should be clear that the volume fraction α_k is a function of the position and time.

When the local instantaneous transport equations are averaged over the volume, terms arise which are averages of derivatives. In order to interchange differentiation and integration in the averaging transport equations, two special averaging theorems are needed. The first one is the spatial theorem [1, 28, 29]

$$\langle \nabla \psi_k \rangle^s = \nabla \langle \psi_k \rangle^s + \frac{1}{V} \int_{A_k} \psi_k \big|_{\underline{x}+\underline{y}_k} \underline{n}_k dA, \tag{10}$$

where ψ_k is a quantity associated with the k phase, $\underline{n}_k$ is the unit normal vector directed from the k phase towards the f phase, and A_k is the area of the k-f interface contained within V.

The second integral theorem is a special form of the Leibniz rule known as the transport theorem [10, 30]:

$$\left\langle \frac{\partial \psi_k}{\partial t} \right\rangle^s = \frac{\partial \langle \psi_k \rangle^s}{\partial t} - \frac{1}{V} \int_{A_k} \psi_k \big|_{\underline{x}+\underline{y}_k} \underline{W}_k \cdot \underline{n}_k dA, \tag{11}$$

where $\underline{W}_k$ is the velocity of the k-f interface in V,

$$A_k = A_{kw} + A_{lg}. \tag{12}$$

If $\psi_k = 1$, the previous theorems lead to

$$\nabla \alpha_k = -\frac{1}{V} \int_{A_k} \underline{n}_k dA, \tag{13}$$

$$\frac{\partial \alpha_k}{\partial t} = \frac{1}{V} \int_{A_k} \underline{W}_k \cdot \underline{n}_k dA. \tag{14}$$

In these theorems ψ_k should be continuous within the k phase.

It is important to note that these theorems are not restricted to the inequality given by (1).

In order to eliminate the point or *local* variable ψ_k in the spatial averaging theorem given by (10) we use the spatial decomposition ($\psi_k = \langle \psi_k \rangle + \widetilde{\psi}_k$) [5],

$$\begin{aligned} \langle \nabla \psi_k \rangle^s = \nabla \langle \psi_k \rangle^s &+ \frac{1}{V} \int_{A_k} \langle \psi_k \rangle \big|_{\underline{x}+\underline{y}_k} \underline{n}_k dA \\ &+ \frac{1}{V} \int_{A_k} \widetilde{\psi}_k \big|_{\underline{x}+\underline{y}} \underline{n}_k dA. \end{aligned} \tag{15}$$

In the homogeneous regions of the system, the following length-scale restriction given by (1) is usually satisfied, and the following simplification is considered:

$$\langle \psi_k \rangle \big|_{\underline{x}+\underline{y}_k} \cong \langle \psi_k \rangle, \quad \text{for } \frac{\ell}{\mathfrak{I}} \ll 1. \tag{16}$$

Then, the second term on the right side of (15) can be written as

$$\begin{aligned} &\frac{1}{V} \int_{A_k} \langle \psi_k \rangle \big|_{\underline{x}+\underline{y}_k} \underline{n}_k dA \\ &\quad = \left\{ \frac{1}{V} \int_{A_k} \underline{n}_k dA \right\} \langle \psi_k \rangle \\ &\quad = -\{\nabla \alpha_k\} \langle \psi_k \rangle, \quad \text{for } \frac{\ell}{\mathfrak{I}} \ll 1. \end{aligned} \tag{17}$$

In general, the averaged terms evaluated in the centroid can be removed from the integrals, where this result was obtained using the lemma given by (13). Then, (15) can be rewritten as follows:

$$\langle \nabla \psi_k \rangle^s = \alpha_k \nabla \langle \psi_k \rangle + \frac{1}{V} \int_{A_k} \widetilde{\psi}_k \big|_{\underline{x}+\underline{y}} \underline{n}_k dA, \quad \text{for } \frac{\ell}{\mathfrak{I}} \ll 1. \tag{18}$$

The similar form, the theorem given by (11) can be rewritten as

$$\begin{aligned} \left\langle \frac{\partial \psi_k}{\partial t} \right\rangle^s &= \alpha_k \frac{\partial \langle \psi_k \rangle^s}{\partial t} \\ &\quad - \frac{1}{V} \int_{A_k} \widetilde{\psi}_k \big|_{\underline{x}+\underline{y}_k} \underline{W}_k \cdot \underline{n}_k dA, \quad \text{for } \frac{\ell}{\mathfrak{I}} \ll 1, \end{aligned} \tag{19}$$

where this result was obtained using the lemma given by (14).

The *local* average volume in principle cannot describe significant variations or sudden changes where the characteristic length ℓ can be of the order of $\mathfrak{I}$. Then, it is necessary to extend the scope of the theorems given by (18) and (19), which is the goal of the next section.

3. *Nonlocal* Averaged Volume

The spatial decomposition given by $\psi_k = \langle \psi_k \rangle + \widetilde{\psi}_k$ represents a *decomposition of length scales*, that is, the average $\langle \psi_k \rangle$ undergoes significant change only over the large length scale, while that the spatial deviation $\widetilde{\psi}_k$ is dominated by the small length scale ℓ. However, this idea considered that the *nonlocal* effects are negligible.

Returning to (10) and (11), it clearly indicates that is a *nonlocal* spatial averaging theorem since the dependent variable $\langle \psi_k \rangle$ is evaluated at other points than the centroid (which is indicated by $\langle \psi_k \rangle|_{\underline{x}+\underline{y}_k}$). In this context, we use *nonlocal* in the sense that it does not involve the use of length-scale restriction in its derivation [31, 32].

3.1. Nonlocal Averaged Volume Approximation. The area integral of $\langle \psi_k \rangle|_{\underline{x}+\underline{y}_k} \underline{n}_k$ is evaluated in the k phase indicated by position vector $\underline{y}_k$ shown in Figure 1. Then,

$$\frac{1}{V} \int_{A_k} \langle \psi_k \rangle \big|_{\underline{x}+\underline{y}_k} \underline{n}_k dA, \tag{20}$$

which is essentially a *nonlocal* term since that the dependent variable $\langle \psi_k \rangle$ is not evaluated at the centroid, $\underline{x}$ (Figure 1).

The nature of the volume-averaged variable $\langle\psi_k\rangle|_{\underline{x}+\underline{y}_k}$ can be known applying a Taylor series expansion about the centroid of the averaging volume [33]:

$$\langle\psi_k\rangle\big|_{\underline{x}+\underline{y}_k} = \langle\psi_k\rangle + \underline{y}_k\cdot\nabla\,\langle\psi_k\rangle + \frac{1}{2}\underline{y}_k\underline{y}_k : \nabla\nabla\langle\psi_k\rangle + \cdots. \tag{21}$$

The second, third, and following terms on the left side correspond to *nonlocal* effects. Then, this equation can be approximate by

$$\langle\psi_k\rangle\big|_{\underline{x}+\underline{y}_k} = \langle\psi_k\rangle\big|_{\underline{x}} + \langle\psi_k\rangle_{NL}, \tag{22}$$

where $\langle\psi_k\rangle_{NL} = \underline{y}_k\cdot\nabla\langle\psi_k\rangle + (1/2)\underline{y}_k\underline{y}_k : \nabla\nabla\langle\psi_k\rangle + \cdots$. Inclusive can be treated as source term formed by $\langle\psi_k\rangle|_{\underline{x}+\underline{y}_k} - \langle\psi_k\rangle$. It is important to emphasize that the presence of the term $\langle\psi_k\rangle_{NL}$ involves that the *nonlocal* representation avoids imposing length-scale restrictions. Then, the general representation of *nonlocal* term is

$$\langle\psi_k\rangle_{NL} = \begin{cases} \langle\psi_k\rangle|_{\underline{x}+\underline{y}_k} - \langle\psi_k\rangle, & \text{without length-scale restrictions,} \\ 0, & \text{for } \dfrac{\ell}{\mathfrak{I}} \ll 1. \end{cases} \tag{23}$$

The physical interpretation of (23) indicates that the *nonlocal* contribution is negligible in the homogeneous region, that is, those portions of the two-phase flow that are not influenced by the rapid changes in the structure which occur in the boundary region. Therefore, *nonlocal* term can be important in the boundary region, where $\langle\psi_k\rangle|_{\underline{x}+\underline{y}_k} - \langle\psi_k\rangle$ is important and the length-scale constraints given by (1) are not valid.

Applying these ideas the theorems can be expressed in *nonlocal* terms:

$$\langle\nabla\psi_k\rangle^s = \alpha_k\nabla\langle\psi_k\rangle + \frac{1}{V}\int_{A_k}\langle\psi_k\rangle_{NL}\underline{n}_k dA + \frac{1}{V}\int_{A_k}\widetilde{\psi}_k\big|_{\underline{x}+\underline{y}}\underline{n}_k dA, \tag{24}$$

$$\left\langle\frac{\partial\psi_k}{\partial t}\right\rangle^s = \alpha_k\frac{\partial\langle\psi_k\rangle}{\partial t} - \frac{1}{V}\int_{A_k}\langle\psi_k\rangle_{NL}\underline{W}_k\cdot\underline{n}_k dA - \frac{1}{V}\int_{A_k}\widetilde{\psi}_k\big|_{\underline{x}+\underline{y}_k}\underline{W}_k\cdot\underline{n}_k dA. \tag{25}$$

The forms of these integral theorems are applied in this work to obtain *nonlocal* volume-averaged conservation equation for two-phase flow, that is, without restriction of the length scale.

3.2. Average Volume of the Product of Two Local Variables $\varphi_k\psi_k$. The explicit representation of the average volume of the product of two *local* variables is given by

$$\langle\varphi_k\psi_k\rangle^s = \frac{1}{V}\int_{V_k}(\varphi_k\psi_k)\big|_{\underline{x}+\underline{y}_k}dV. \tag{26}$$

Substituting the correspondent spatial deviations for the *local* variables φ_k and ψ_k leads to

$$\langle\varphi_k\psi_k\rangle^s = \alpha_k\langle\varphi_k\rangle\langle\psi_k\rangle + \langle\psi_k\rangle\langle\widetilde{\varphi}_k\rangle^s + \langle\varphi_k\rangle\langle\widetilde{\psi}_k\rangle^s + \langle\widetilde{\varphi}_k\widetilde{\psi}_k\rangle^s. \tag{27}$$

This is rewritten as

$$\langle\varphi_k\psi_k\rangle^s = \langle\varphi_k\psi_k\rangle_{NL} + \alpha_k\langle\varphi_k\rangle\langle\psi_k\rangle + \langle\widetilde{\varphi}_k\widetilde{\psi}_k\rangle^s, \tag{28}$$

where $\langle\varphi_k\psi_k\rangle_{NL}$ is a *nonlocal* term, since it involves, indirectly, values of $\langle\varphi_k\rangle$ and $\langle\psi_k\rangle$ that are not associated with the centroid of the averaging volume illustrated in Figure 1. The *nonlocal* contribution is given by

$$\langle\varphi_k\psi_k\rangle_{NL} = \langle\langle\varphi_k\rangle\langle\psi_k\rangle\rangle^s + \langle\widetilde{\varphi}_k\langle\psi_k\rangle\rangle^s + \langle\langle\varphi_k\rangle\widetilde{\psi}_k\rangle^s - \alpha_k\langle\varphi_k\rangle\langle\psi_k\rangle. \tag{29}$$

It can be demonstrated that

$$\langle\varphi_k\psi_k\rangle_{NL} = 0, \quad \text{for } \frac{\ell}{\mathfrak{I}} \ll 1. \tag{30}$$

3.3. Operators Applied to Two Local Variables $\varphi_k\psi_k$. The typical expressions in the transport phenomena in a two-phase flow involve an average differential operator with two *local* variables,

$$\langle\nabla\varphi_k\psi_k\rangle^s = \nabla\langle\varphi_k\psi_k\rangle^s + \frac{1}{V}\int_{A_k}\varphi_k\psi_k\big|_{\underline{x}+\underline{y}_k}\underline{n}_k dA,$$
$$\left\langle\frac{\partial\varphi_k\psi_k}{\partial t}\right\rangle^s = \frac{\partial\langle\varphi_k\psi_k\rangle^s}{\partial t} - \frac{1}{V}\int_{A_k}\varphi_k\psi_k\big|_{\underline{x}+\underline{y}_k}\underline{W}_k\cdot\underline{n}_k dA. \tag{31}$$

With the previous ideas we obtain expanded form of the theorems for the product of *two-local* variables

$$\begin{aligned}\langle \nabla \varphi_k \psi_k \rangle^s &= \nabla \langle \varphi_k \psi_k \rangle_{NL} + {}_k\nabla (\langle \varphi_k \rangle \langle \psi_k \rangle) + \nabla \langle \tilde{\varphi}_k \tilde{\psi}_k \rangle^s \\ &+ \underbrace{\left\{ \frac{1}{V} \int_{A_k} (\langle \varphi_k \rangle \langle \psi_k \rangle) \underline{n}_k dA \right\}_{NL}}_{nonlocal} \\ &+ \underbrace{\left\{ \frac{1}{V} \int_{A_k} (\tilde{\varphi}_k \tilde{\psi}_k) \big|_{\underline{x}+\underline{y}_k} \underline{n}_k dA \right\}_{D}}_{\text{dispersion}},\end{aligned} \tag{32}$$

$$\begin{aligned}\left\langle \frac{\partial \varphi_k \psi_k}{\partial t} \right\rangle^s &= \alpha_k \frac{\partial (\langle \varphi_k \rangle \langle \psi_k \rangle)}{\partial t} + \frac{\partial \langle \varphi_k \psi_k \rangle_{NL}}{\partial t} + \frac{\partial \langle \tilde{\varphi}_k \tilde{\psi}_k \rangle^s}{\partial t} \\ &+ \underbrace{\left\{ \frac{1}{V} \int_{A_k} (\langle \varphi_k \rangle \langle \psi_k \rangle) \underline{W}_k \cdot \underline{n}_k dA \right\}_{NL}}_{nonlocal} \\ &+ \underbrace{\left\{ \frac{1}{V} \int_{A_k} (\tilde{\varphi}_k \tilde{\psi}_k) \big|_{\underline{x}+\underline{y}_k} \underline{W}_k \cdot \underline{n}_k dA \right\}_{D}}_{\text{dispersion}}.\end{aligned} \tag{33}$$

4. *Nonlocal* Volume-Averaged General Balance Equations for Two-Phase Flow

The starting point for the development of the *nonlocal* volume-averaged conservation equations is the point conservation equations. In order to illustrate the application of the *nonlocal* theorems and related definitions, we considered the general balance equation for some ψ properties in the k phase:

$$\frac{\partial (\rho_k \psi_k)}{\partial t} + \nabla \cdot (\rho_k \underline{U}_k \psi_k) + \nabla \cdot \underline{D}_k = \rho_k f, \tag{34}$$

where ψ_k is the quantity conserved, $\underline{D}_k$ is the molecular flux, and f is a volumetric source. As summarized in Table 1, depending on the choice of the quantity to be conserved, either of these equations can be used to quantify the mass, momentum, and energy conservation of each phase.

The volume averaged of the general balance equation can be expressed as

$$\left\langle \frac{\partial (\rho_k \psi_k)}{\partial t} \right\rangle^s + \langle \nabla \cdot \rho_k \underline{U}_k \psi_k \rangle^s + \langle \nabla \cdot \underline{D}_k \rangle^s = \langle \rho_k f_k \rangle^s. \tag{35}$$

Table 1: Conservation terms.

Conservation principle	ψ_k	$\underline{D}_k$	f
Mass	1	0	0
Momentum	$\underline{U}_k$	$p_k \underline{\underline{I}} - \underline{\underline{\tau}}_k$	$\underline{g}_k$
Energy	$e_k - p_k/\rho_k$	$\underline{q}_k'' - \left(p_k \underline{\underline{I}} - \underline{\underline{\tau}}_k\right) \cdot \underline{U}_k$	$\underline{g}_k \cdot \underline{U}_k + q_k'''/\rho_k$

Note: $e_k = h_k + U_k^2/2$ is the total specific energy for the k phase, where h_k is the enthalpy.
$\underline{q}_k''$ denotes the heat flux vector for the k phase.

The *nonlocal* transport theorem of the product of two variables derived in this work given by (33) with $\varphi_k = \rho_k$ is used in order to express the first term of this equation:

$$\begin{aligned}\left\langle \frac{\partial \rho_k \psi_k}{\partial t} \right\rangle^s &= \alpha_k \frac{\partial (\langle \rho_k \rangle \langle \psi_k \rangle)}{\partial t} + \frac{\partial \langle \rho_k \psi_k \rangle_{NL}}{\partial t} + \frac{\partial \langle \tilde{\rho}_k \tilde{\psi}_k \rangle^s}{\partial t} \\ &- \left\{ \frac{1}{V} \int_{A_k} \langle \rho_k \rangle \langle \psi_k \rangle \underline{W}_k \cdot \underline{n}_k dA \right\}_{NL} \\ &- \left\{ \frac{1}{V} \int_{A_k} (\tilde{\rho}_k \tilde{\psi}_k) \big|_{\underline{x}+\underline{y}_k} \underline{W}_k \cdot \underline{n}_k dA \right\}_{D}.\end{aligned} \tag{36}$$

The *nonlocal* averaging theorem for the product of three variables can be developed following the same procedure given by (32). Then, the second term in (35) is given by

$$\begin{aligned}\langle \nabla \cdot \rho_k \underline{U}_k \psi_k \rangle^s &= \alpha_k \nabla \cdot (\langle \rho_k \rangle \langle \underline{U}_k \rangle \langle \psi_k \rangle) \\ &+ \nabla \cdot \langle \rho_k \underline{U}_k \psi_k \rangle_{NL} + \nabla \cdot \langle \rho_k \underline{U}_k \psi_k \rangle_{D} \\ &+ \left\{ \frac{1}{v} \int_{A_k} (\langle \rho_k \rangle \langle \underline{U}_k \rangle \langle \psi_k \rangle) \cdot \underline{n}_k dA \right\}_{NL} \\ &+ \left\{ \frac{1}{v} \int_{A_k} \left(\tilde{\rho}_k \tilde{\underline{U}}_k \tilde{\psi}_k\right) \Big|_{\underline{x}+\underline{y}_k} \cdot \underline{n}_k dA \right\}_{D},\end{aligned} \tag{37}$$

where the dispersion term is given by

$$\begin{aligned}\nabla \cdot \langle \rho_k \underline{U}_k \psi_k \rangle_D &= \nabla \cdot \langle \rho_k \rangle \left\langle \tilde{\psi}_k \tilde{\underline{U}}_k \right\rangle^s + \nabla \cdot \langle \underline{U}_k \rangle \langle \tilde{\rho}_k \tilde{\psi}_k \rangle^s \\ &+ \nabla \cdot \langle \psi_k \rangle \left\langle \tilde{\rho}_k \tilde{\underline{U}}_k \right\rangle^s + \nabla \cdot \left\langle \tilde{\rho}_k \tilde{\underline{U}}_k \tilde{\psi}_k \right\rangle^s.\end{aligned} \tag{38}$$

The *nonlocal* averaging theorem given by (24) with $\psi_k = \underline{D}_k$ is used in order to obtain the diffusive term

$$\begin{aligned}\langle \nabla \cdot \underline{D}_k \rangle^s &= \alpha_k \nabla \cdot \langle \underline{D}_k \rangle + \frac{1}{V} \int_{A_k} \langle \underline{D}_k \rangle_{NL} \cdot \underline{n}_k dA \\ &+ \frac{1}{V} \int_{A_k} \tilde{\underline{D}}_k \Big|_{\underline{x}+\underline{y}} \cdot \underline{n}_k dA.\end{aligned} \tag{39}$$

The terms $\langle \rho_k f_k \rangle^s$ are obtained with the application of (28)

$$\langle \rho_k f_k \rangle^s = \langle \rho_k f_k \rangle_{NL} + \alpha_k \langle \rho_k \rangle \langle f_k \rangle + \left\langle \widetilde{\rho}_k \widetilde{f}_k \right\rangle^s. \tag{40}$$

In order to simplify the previous equations, the following representations are proposed:

$$\frac{\partial \langle \rho_k \psi_k \rangle_{NL}}{\partial t} = \eta_a \alpha_k \frac{\partial (\langle \rho_k \rangle \langle \psi_k \rangle)}{\partial t}, \quad \text{nonlocal accumulation}, \tag{41}$$

$$\frac{\partial \langle \widetilde{\rho}_k \widetilde{\psi}_k \rangle^s |_{\underline{x}}}{\partial t} = \delta_a \alpha_k \frac{\partial (\langle \rho_k \rangle \langle \psi_k \rangle)}{\partial t}, \quad \text{dispersion for accumulation}, \tag{42}$$

$$\nabla \cdot \langle \rho_k \underline{U}_k \psi_k \rangle_{NL} = \eta_b \alpha_k \nabla \cdot (\langle \rho_k \rangle \langle \underline{U}_k \rangle \langle \psi_k \rangle), \quad \text{nonlocal convection}, \tag{43}$$

$$\nabla \cdot \langle \rho_k \underline{U}_k \psi_k \rangle_D = \delta_b \alpha_k \nabla \cdot (\langle \rho_k \rangle \langle \underline{U}_k \rangle \langle \psi_k \rangle), \quad \text{dispersion for convection}, \tag{44}$$

$$\langle \rho_k f_k \rangle_{NL} = \eta_c \alpha_k \langle \rho_k \rangle \langle f_k \rangle, \quad \text{nonlocal source}, \tag{45}$$

$$\left\langle \widetilde{\rho}_k \widetilde{f}_k \right\rangle^s = \delta_c \alpha_k \langle \rho_k \rangle \langle f_k \rangle, \quad \text{dispersion for source}, \tag{46}$$

where η and δ are dimensionless parameters. The parameter η is the *nonlocal* nature, while the δ parameter agglutinates the dispersion effects. Now, the diffusive flux of (39) can be expressed as

$$\langle \nabla \cdot \underline{D}_k \rangle^s = \alpha_k \nabla \cdot \langle \underline{D}_k \rangle + M_{kNL} + \widetilde{M}_k. \tag{47}$$

In this equation the following definitions were used:

$$M_{kNL} = \frac{1}{V} \int_{A_k} \langle \underline{D}_k \rangle_{NL} \cdot \underline{n}_k dA, \quad \text{interfacial Nonlocal diffusion}, \tag{48}$$

$$\widetilde{M}_k = \frac{1}{V} \int_{A_k} \widetilde{\underline{D}}_k \Big|_{\underline{x}+\underline{y}} \cdot \underline{n}_k dA, \quad \text{interfacial Diffusion due to dispersion}. \tag{49}$$

Finally, substituting (41)–(49), the *nonlocal* volume-averaged of the general balance equation (without length-scale restriction) finally is obtained:

$$\lambda_a \alpha_k \frac{\partial (\langle \rho_k \rangle \langle \psi_k \rangle)}{\partial t} + \lambda_b \alpha_k \nabla \cdot (\langle \rho_k \rangle \langle \underline{U}_k \rangle \langle \psi_k \rangle) + \alpha_k \nabla \cdot \langle \underline{D}_k \rangle = \lambda_c \alpha_k \langle \rho_k \rangle \langle f_k \rangle - M_{kNL} - \widetilde{M}_k - M^{\Gamma}_{kD} - M^{\Gamma}_{kNL}, \tag{50}$$

where

$$\lambda = \eta + \delta + 1, \tag{51}$$

$$M^{\Gamma}_{kD} = \left\{ \frac{1}{V} \int_{A_k} \left(\widetilde{\rho}_k \widetilde{\underline{U}}_k \widetilde{\psi}_k \right) \Big|_{\underline{x}+\underline{y}_k} \cdot \underline{n}_k dA \right\}_D - \left\{ \frac{1}{V} \int_{A_k} (\widetilde{\rho}_k \widetilde{\psi}_k) \Big|_{\underline{x}+\underline{y}_k} \underline{W}_k \cdot \underline{n}_k dA \right\}_D, \tag{52}$$

$$M^{\Gamma}_{kNL} = \left\{ \frac{1}{V} \int_{A_k} (\langle \rho_k \rangle \langle \underline{U}_k \rangle \langle \psi_k \rangle) \cdot \underline{n}_k dA \right\}_{NL} - \left\{ \frac{1}{V} \int_{A_k} \langle \rho_k \rangle \langle \psi_k \rangle \underline{W}_k \cdot \underline{n}_k dA \right\}_{NL}. \tag{53}$$

Recalling that a portion of A_k is made of a liquid-gas interphase and a fluid-solid interphase A_{kw}. Then, M^{Γ}_{kD} (and M^{Γ}_{kNL}) consider the transport phenomena related with interfacial mass transfer between fluid-fluid and fluid-solid interphase, that is, $M^{\Gamma}_k = M^{\Gamma}_{kfE} + M^{\Gamma}_{wkE}$ (with E = D, NL).

5. Discussion

The volume averaged of the balance equation with *length-scale* restriction can be obtained starting from the *nonlocal* averaging equation (50), which contains *local* and *nonlocal* terms of the averaged volume. When $\eta \to 0$, $\langle \rho_k \underline{U}_k \psi_k \rangle_{NL} \to 0$, $\langle \rho_k f_k \rangle_{NL} \to 0$, $M_{kNL} \to 0$, and $M^{\Gamma}_{kNL} \to 0$, the *local* averaging volume equation is recovered. Then, (46) simplifies to

$$(\delta_a + 1)\alpha_k \frac{\partial (\langle \rho_k \rangle \langle \psi_k \rangle)}{\partial t} + (\delta_b + 1)\alpha_k \nabla \cdot (\langle \rho_k \rangle \langle \underline{U}_k \rangle \langle \psi_k \rangle) + \alpha_k \nabla \cdot \langle \underline{D}_k \rangle = (\delta_c + 1)\alpha_k \langle \rho_k \rangle \langle f_k \rangle - \widetilde{M}_k - M^{\Gamma}_{kD}, \quad \frac{\ell}{\Im} \ll 1. \tag{54}$$

The fundamental difference between *local* and *nonlocal* equations is that (50) involves, indirectly, values of the variables that are not associated with the centroid of the averaged volume as illustrated in Figure 1, while in (54) all the values of the volume-averaged variables are associated with the centroid of the averaged volume. The physical interpretation indicates that (54) describes the *homogeneous* two-phase flow. In this work the homogeneous term is used to indicate that the two-phase flow system has a behavior close to that of a homogeneous system; then to ensure homogeneity the system under study is based in *length-scale* restriction used to perform the upscaling in the two-phase flow system. However, (50) has not *length-scale* restriction and in principle it can describe regions of a two-phase flow, where drastic changes occur in the void fraction and transport properties (e.g., diffusivity).

The *nonlocal* volume averaging equations derived in this work contain new terms related to *nonlocal* transport effects due to accumulation, convection diffusion, and transport properties for two-phase flow. In general, the *nonlocal* terms

were evaluated considering them as a function of the *local* terms, yielding new coefficients ($\eta's$) that can be called *nonlocal* coefficients due to its nature these coefficients were defined through (41), (43), and (45) along with (48) and (53). It is important to note that these last two equations can also be expressed in terms of the *local* terms.

The *nonlocal* coefficients ($\eta's$) are new closure relationships of the present novel formulation. For the application in a two-phase flow it is necessary as a first approximation to perform an analysis of order of magnitude, with the idea of identifying the predominant effects where the coefficients are not negligible (i.e., the temporal and diffusive effects are negligible). Then, the significant *nonlocal* coefficients can be evaluated with new or existing procedures in the experimental field, theoretical deduction, or numerical simulation, for instance.

The physical meaning of the *nonlocal* coefficients is related to the scaling process, that is, in the transition region as it can be observed in Figure 1. These coefficients act as coupling elements among the phenomena occurring in at least two different length scales. Outside of the interregion the length scales are smaller compared with those near the interregion (Figure 1).

Some examples where *nonlocal* general equation (50) can be applied are where α_k presents abrupt changes [33, 34], in particular transitions of flow patterns, interface with stratified or annular flow drops and bubbles, and others as in the boundary region of the two-phase flow and solid, where the length-scale restriction given by (1) is not valid.

6. Conclusions

In this paper a derivation of the general transport equations for two-phase systems using a method-based on *nonlocal* volume averaging was presented. The *nonlocal* volume averaging equations derived in this work (50) contain new terms related to *nonlocal* transport effects due to accumulation, convection diffusion, and transport properties for two-phase flow.

The *nonlocal* terms were evaluated as a first approximation considering that these are a function of the *local* terms (41), (43), and (45), given as result of the *nonlocal* volume averaging equations (50) for practical applications. The *nonlocal* coefficients ($\eta's$) are new closure relationships of the present novel formulation. The significant *nonlocal* coefficients can be evaluated with new or existent procedures: theoretical, numerical, and experimental. These coefficients act as coupling elements among the phenomena occurring in at least two different length scales, during the scaling process for pragmatic applications.

To illustrate the application of the representations of the *nonlocal* theorems and related definitions, the general balance equation for some ψ property in the k phase was considered, where it was demonstrated that a *nonlocal* volume averaging balance equation was obtained with meaningful averages. This general balance equation can be applied generally where α_k presents abrupt changes [34, 35], such as transitions of flow patterns, interfaces with stratified or annular flow drops and bubbles, and others such as in the boundary region of the multiphase system, where the length-scale restriction (1) are not valid.

The *nonlocal* averaging model derived in this work represents a novel proposal and its framework could be the beginning of extensive research, both theoretical and experimental, as well as numerical simulation.

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