




Dynamic Software, Task Solving With or Without Guidelines, and Learning Outcomes

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Abstract The present study contributes to knowledge about how to design tasks that benefit from dynamic software in math education, comparing practice performance and learning outcomes among 129 students practicing on two different task designs using GeoGebra. The task designs differed with respect to the presence or absence of guidelines on how to solve the task. One student group practiced on the guided task while the other student group practiced on the unguided task, and 1 week later a posttest was conducted. Data were statistically analyzed and showed significant differences with regard to success during practice for students solving the guided task. Among the students who succeeded in solving the task (guided or unguided) during practice, however, the analysis showed significant differences in the posttest performance in favor of the unguided students.

Keywords GeoGebra · Guided and unguided task · Struggle · Reasoning · Learning outcome

1 Introduction

A frequently asked question in mathematics education research is how to design didactical situations that best support students in developing their mathematical knowledge. One suggestion for enhancing learning involves the use of technology, in this case dynamic software. Several studies have compared the learning outcomes of students using dynamic

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software with those of students using pen and paper, and they report that students using dynamic software outperform the latter, as the technology offers students an interactive environment in which to explore and engage in reasoning and creative problem solving (Chan and Leung 2014; Diković 2009). Another question concerns task design, and whether students learn better solving guided tasks, where students are given instructions on how to solve (parts of) the task, or if it is more beneficial to provide students with problems, that is, unguided non-routine tasks that they solve by constructing at least part of the methods themselves. There are studies comparing learning outcomes from these designs that are in favor of the unguided approach. Didactical situations designed to invite students to learn mathematics by engaging in mathematical reasoning and problem solving, constructing their own methods, are reported as beneficial for learning (Jonsson et al. 2014; Norqvist 2017). For teachers, these findings raise questions about task design when introducing dynamic software into classrooms with a view to improving students' learning. That is, do students' learning outcomes differ depending on whether students are required to construct their own methods for solving a task or whether, instead, they are given instructions for solving it?

The present study will investigate whether the reported learning benefits of using dynamic software will be realized, regardless of whether students are provided with guidance. This will be done by examining the practice performance and learning outcomes of students who, supported by GeoGebra, solve guided non-routine tasks compared to the performance and outcomes of students who solve unguided non-routine tasks. Hypotheses examined in this study will be presented later.

1.1 Dynamic Software, Problem Solving and Learning Outcomes

The number of studies looking into learning outcomes when dynamic software is used in problem solving has increased in the last decade. In a review, Chan and Leung (2014) examined nine quasi-experimental studies, which included a total of 587 participants, comparing groups of students using software such as Cabri Geometry and Geometer's Sketchpad to students working with pen and paper. They found a distinct effect size in favor of the use of dynamic software. Several similar studies, where students used GeoGebra for problem solving, show its use to have positive effects on students' learning outcomes. In posttests, students working with GeoGebra have outperformed students using pen and paper. These studies show significant positive effects on, for example, students' development of conceptual and procedural knowledge of functions (Diković 2009; Zulfaini and Zakaria 2012; Kepceoğlu 2016), as well as geometry (Bhagat and Chang 2015; Dogan and İcel 2011; Saha et al. 2010; Shadaan and Eu 2013; Zengin et al. 2012).

Numerous characteristics of dynamic software have been identified and suggested as explanations for these positive effects. Dynamic software such as GeoGebra allows students to interact with geometric and algebraic objects. For instance, functions can be defined algebraically and then changed dynamically (Hohenwarter and Jones 2007). That is, students may adjust the algebraic representation of the function and observe the change of the graph, or they can drag the graph and observe the way the formula changes in response. If anything is added or changed in any representation of a function, the others are automatically altered. This dynamic combination of the representation of functions, formula, graph, and table is seen as beneficial in helping students develop an understanding of functions (Coleman et al. 2015; Ferrara et al. 2006; Pierce et al. 2011). Furthermore, when students interact with the software they receive immediate visual responses to their actions. This kind of instant feedback of visualizing students' ideas and confirming or falsifying

their assumptions has been shown to make problem solving more efficient (Arcavi and Hadas 2000; Marrades and Gutiérrez 2000). Moreover, since the software takes care of time-consuming constructions, with tools such as drag and drop and sliders, students can construct multiple numerical variations of a mathematical object. These variations could be used to explore, contrast, and generalize concepts of, for example, functions (Leung 2008). Thus, dynamic software's ability to visualize relations between representations, offer feedback on students' actions, and provide multiple variations is outlined as beneficial for learning mathematics. That is, there are studies showing that dynamic software may support students' understanding of mathematics (Diković 2009; Leung 2008), enhance their reasoning (Natsheh and Karsenty 2014; Granberg and Olsson 2015), and encourage them to explore, that is, to try out multiple ideas during problem solving (Fahlberg-Stojanovska and Stojanovski 2009; Hohenwarter and Jones 2007; Hähkiöniemi and Lep-pääho 2012). The above-mentioned dynamic features provided by software such as Geo-Gerba are described as difficult to reproduce when working with pen and paper, and are therefore used to explain the positive effects of using such software.

These studies focus on differences in learning outcomes by comparing students using dynamic software with students using pen and paper. However, merely adding software to support students does not guarantee enhanced learning (Lou et al. 2001; Mullins et al. 2011). Other aspects, such as the way a given task is designed, may play an important role, and there is a fair amount of research looking into questions about task design and learning outcome.

1.2 Task Design and Learning Outcomes

A common question in educational research is whether students learn best from less guided approaches to mathematical content or if they need detailed instructions on how to approach specific tasks. Among the researchers advocating direct instructions, Kirschner et al. (2006) claim that approaches offering no or minimal guided instructions are likely to be ineffective. One of their arguments concerns the idea that this kind of problem-based learning makes heavy demands on working memory, which in turn prevents students from accumulating knowledge in their long-term memory. Mayer (2004) presents similar opinions in his review reporting on three decades of studies. Mayer argues that research, ever since the 1960s, has shown that guided methods of instruction are more effective for learning than pure discovery.

Hiebert and Grouws (2007), on the other hand, together with researchers such as Brousseau (1997) and Schoenfeld (1985), represent another approach. They claim that students develop a deeper understanding of mathematics when they are required to struggle, in a positive sense, with important mathematical concepts. According to Hiebert and Grouws, the struggle is initiated when a student's prior knowledge is insufficient to solve a given task and no solution method is provided. A productive struggle involves retrieval, reconstruction, and perhaps correction of prior knowledge, along with interpretation of the task at hand and construction of new knowledge in relation to what is already known (Hiebert and Grouws 2007).

In their review, Lee and Anderson (2013) compared didactical designs of direct instruction and designs of learning through discovery with no or limited guidance. Both designs were shown to have benefits; however, although direct instructions provide correct solutions, are time efficient, and reduce demands on working memory, this design was shown to lead to superficial and rote learning methods that are poorly remembered. Lithner (2008) describes how rote learning relates to students' line of thinking or reasoning, which

in turn relates to the design of the given task. When students are solving routine tasks they recall methods they learned earlier, or they make use of procedures provided in instructions for solving the task. These students engage in what Lithner defines as algorithmic reasoning (AR), that is, they follow procedures that are constructed to result in fast and accurate answers but that offer no broader context or meaning. Solving non-routine tasks, on the other hand, requires students to explore mathematical concepts, construct their own methods, and justify those methods using arguments anchored in intrinsic mathematics, a line of thinking Lithner describes as creative mathematical reasoning (CMR).

This kind of approach, engaging students in constructing (parts of) their methods, has implications for task design. To enhance learning and avoid rote learning methods, students need to struggle with mathematical problems; in other words, they must try to solve unguided non-routine tasks without the benefit of detailed instructions or memorized procedures. Jonsson et al. (2014) compared the learning outcomes between two groups of students: one that practiced on a series of guided tasks promoting algorithmic reasoning, and another that practiced on the same tasks with less guidance, so that they were encouraged to explore and engage in creative reasoning. In this study, the students in the former group were provided with formulas for solving the tasks, and the students in the latter (intervention) group were required to construct the formulas themselves. The students working with the guided tasks were more successful in correctly solving the tasks during practice. However, the students in the intervention group outperformed the others during the posttest. Norqvist (2017) replicated the study by Jonsson et al. (2014), but added explanations to the guided tasks as to why the provided methods would work. The study showed the same results; the guided students were more successful during practice, however, the unguided students outperformed the guided students during posttests. These findings support the idea that merely providing students with direct instructions could be seen as promoting “unproductive success,” which results from “conditions that maximize performance in the initial learning but may not maximize learning in the longer term” (Kapur 2016, p. 1).

Although it is not in focus in the present study, it is worth noting that there is a fair amount of research combining knowledge construction with instructions. For example, Kapur (2010, 2011) compared learning outcomes of students who were given instructions from the teacher either before or after solving a given task. The intervention group was given an unguided task involving a concept they had not yet learned and for which the students needed to construct a method. After the construction phase, the students were given a consolidation lesson in which they received instruction on the concept. The didactical design of the control group was reversed; first they were given direct instructions and then they worked on the same tasks as the intervention groups. The intervention groups scored higher on the posttests, indicating that the process of constructing has positive effects on students’ learning. Similar findings contrasting a didactical design of construction-before-instruction with a traditional design of instruction-before-practice are reported in other studies, for example, Hiebert and Stigler (2004), Schwartz and Martin (2004), and Schwartz et al. (2011). This way of combining construction and guidance is also advocated by Hmelo-Silver et al. (2007). They emphasize the importance of promoting competencies such as reasoning and problem solving, and argue that not all constructivist pedagogical approaches can be grouped together as unproductive unguided discovery learning. The cognitive load of working with unguided tasks may be reduced by providing students with supporting tools such as software, framing the activity with clear goals and rules, scaffolding students by, for instance, posing questions that challenge them to explain, and so on.

This question about task design, that is, about whether or not students should be provided with methods for solving the task, has been in focus for a long time and arises when it comes to designing didactical situations that include the use of dynamic software.

1.3 Task Design and Dynamic Software

The features of dynamic software such as GeoGebra that are reported as beneficial for learning, like dynamic visualization of multiple representations and instant feedback, are available to students regardless of task design. Students however, usually engage solely in the activities needed to solve a given problem (Joubert 2013). This is in line with studies that show that the very task design might influence how students use the software (Fahlberg-Stojanovska and Stojanovski 2009; Doorman et al. 2007; Hitt and Kieran 2009; Laborde 2001). Leung (2011), for example, points out that for students to benefit from the features of dynamic software, the given task should invite them to explore, reconstruct, and explain mathematical concepts and relations. This brings us to the question of whether and how differences in task design (guided or unguided) will affect learning outcomes when students are supported by dynamic software.

2 Aim and Hypothesis

The aim of the study is to investigate differences in practice performance and learning outcome depending on whether students have practiced and successfully solved guided or unguided non-routine tasks supported by dynamic software. The practice task involves constructing a mathematical rule, and the posttest examines the extent to which students are able to use the rule in solving tasks. The two groups, guided and unguided students, were matched with respect to their grades in mathematics. The students' grades were, furthermore, included in the analysis pursuing the stated hypothesis. That was done to control for the effects the students' grades might have on the results.

The hypotheses that will be examined are:

H1 Students practicing with the guided task will outperform the students practicing with the unguided task during practice (constructing the rule).

If students are provided with guidelines for solving a task and if they manage to follow these guidelines, they are likely to come to a correct solution (Brousseau 1997; Jonsson et al. 2014; Lithner 2008).

H2 Students who successfully solved the unguided task will perform better during the posttest (using the rule) compared to those who successfully solved the guided task.

There are studies pointing out that using dynamic software during problem solving enhances learning and conceptual understanding by providing an explorative milieu (Coleman et al. 2015; Pierce et al. 2011). If this is the case, that the very use of dynamic software can encourage students to become explorative and creative, the task design will consequently have less influence on the learning outcome. However, there is research showing that task design, with respect to the presence or absence of guidance, has a significant influence on students' learning outcomes (Jonsson et al. 2014; Norqvist 2017). In line with these results, there is research suggesting that students' use of the explorative and creative potential of dynamic software depends on how the given task is designed

(Leung 2011; Joubert 2013). Although these studies have not focused on learning outcomes, they suggest that students are more likely to be explorative and creative when working on unguided tasks, which in turn might positively influence the learning outcomes.

3 Method

The study was designed as an intervention study comparing practice performance and learning outcomes when students were presented with non-routine tasks with different designs. During the practice session, half of the students were given a guided non-routine task to solve and the other half were asked to solve an unguided non-routine task. Both tasks concerned the same learning goal with regard to linear functions, and both groups used GeoGebra for support. The students worked in pairs during practice, and their dialogue and screen activities were recorded. The recordings were examined after the intervention study to establish whether any of the students knew the answer in advance or whether any students who were given the guided task abandoned the guidelines and worked as if they had been presented with the unguided task. One week after the intervention study the students completed an individual posttest. The method is outlined in detail below.

3.1 Participants

The 141 students who participated in this study were between 15 and 16 years old and were enrolled in an upper secondary school in Sweden. The participants were studying the first year of the natural science program (three classes) or the technology program (two classes). Written informed consent was obtained from each student and all ethical requirements outlined by the Swedish Research Council (2001) were followed. Six participants were excluded since they did not participate during the posttest. Additionally, four participants were excluded since they worked on the posttest for less than 10 min, did not answer at least half of the questions, scored 0 points, and were assumed to not really have tried to solve the tasks. Finally, after examining the recordings from the practice, two students (one pair) were excluded since they, or at least one of them, were found to be familiar with the task before the practice session. None of the students who were given the tasks with guidelines ignored the instructions and solved the given problem as an unguided task. Altogether, 129 students were included in the analysis, 63 students working with the guided task and 66 with the unguided task.

3.2 Matched Groups

Earlier studies show that students' grades in mathematics and their cognitive abilities are strongly correlated (Furnham and Monsen 2009). Hence, the students' most recent grades in mathematics from the ninth grade in Swedish lower secondary school, with a maximum of 20, were collected and used to match the participants into two separate groups within each class. There were 129 students included in the analysis and the average grade was 15, 44 among the students who were given the unguided task and 15, 55 among those who were given the guided task. The groups were thus considered approximately equal in terms of grades. During the practice session, the students worked in pairs, and each pair was further matched according to their grades in mathematics. That was done to increase the

possibility that both students in each pair would be able to contribute equally to the task-solving process.

3.3 The Practice Session

The study was performed during regular mathematics lessons. To be able to study the task designs and compare practice as well as posttest performance, the authors, acting as teachers, introduced the tasks and provided pre-prepared support during practice. That is, the students could ask for help, but to maintain the design of the guided and unguided tasks, the authors had prepared answers to support the students working with the guided task in understanding the guidelines, whereas questions concerning the unguided task were replied to with answers such as “If you do not think your idea will work, try another one. The feedback aimed to encourage the students to continue with the task, and to come up with and try out ideas to solve it, but not to give them guidelines on how to reach a solution. None of the students had used GeoGebra before, but after a short introduction to the software, given by the authors, they all managed to submit formulas, adjust their functions, and measure angles. Students who asked technical questions about GeoGebra were given direct support on how to handle the software.

3.4 The Practice Tasks With or Without Guidelines

The aim was to give the students a non-routine task that they did not already know but could solve using their expected prior knowledge, identified as the anticipated learning outcome for students leaving the ninth grade of Swedish lower secondary school. Linear functions and the formula $y = mx + c$, where m represents the slope and c the intersection with the y -axis, is taught in the ninth grade. The target knowledge for both non-routine tasks was the same: to find a mathematical rule for how to choose the m -values to construct two linear functions with perpendicular graphs, that is $m_1 \times m_2 = -1$. The derivation of this rule is taught during the first year in upper secondary school but had not yet been presented to the students in this study. The non-routine task with no strict guidelines on how to derive the rule was constructed in such a way that the students were expected to create their own methods with no support besides GeoGebra (Fig. 1). The non-routine task with guidelines was designed to be similar to textbook tasks intended to invite students to explore mathematical concepts with technological support. Therefore, the guided tasks included guidelines on how to use GeoGebra to retrieve information that could be useful to the students in constructing the rule (Fig. 2). Hence, none of the students were presented

Construct a rule guiding you how to create perpendicular linear functions

1. Draw a linear function, $y=mx+c$ in GeoGebra.
2. Draw another linear function perpendicular to the first one.
3. Construct a rule that will guide you in choosing the m -values to draw two linear functions that are perpendicular to one another.
4. Test your rule by constructing three pairs of perpendicular linear functions and, if necessary, develop your rule. Make sure you agree with your friend and write the rule down.

Fig. 1 The non-routine task with no strict guidelines

Construct a rule guiding you how to create perpendicular linear functions

A linear function could be presented as $y=mx+c$, i.e. $y=2x+4$. Use GeoGebra to explore how to choose the m -values of two linear functions to make the functions perpendicular to one another.

- 1) Submit the linear functions, a to h, into GeoGebra and write each functions' m -value in column (1) **m_1 -value**, in the table.
- 2) Use GeoGebra to find the linear function that is perpendicular to function a) Write the m -value of that function in column (2) **m_2 -value**, in the table. Find the perpendicular function to b) write its m -value in the column and so on.
- 3) Multiply m_1 and m_2 in each row, write the answer in column (3) **$m_1 \times m_2$** .
- 4) Try to create a rule of how to choose two linear functions' m -values to make them perpendicular to one another. Make sure you agree with your friend and write the rule down.

	(1) m_1 -value	(2) m_2 -value	(3) $m_1 \times m_2$
a) $y = 2x+2$			
b) $y = 4x-2$			
c) $y = x-4$			
d) $y = -0,25x-2$			
e) $y = -x+3$			
f) $y = 0,5x-2$			
g) $y = -2x-3$			
h) $y = -0,5x-6$			

Fig. 2 The non-routine task with guidelines

with the target knowledge, as they all needed to construct the rule. However, half of the students solved the task with guidelines and the other half without guidelines. Examples of two successfully drawn perpendicular graphs are shown in Fig. 3.

3.5 The Posttest Tasks

One week after the intervention an individual posttest was conducted. All tasks were presented and answered by the students using laptops. GeoGebra was not used during the posttest. The software providing the posttest saved the students' answers and the time spent on each response. The students were allowed to use a simple virtual calculator, displayed with each task, for numerical computation. The posttest consisted of nine tasks. In the first task, students were asked to reproduce the previously constructed rule (testing whether they remembered the rule). The following eight tasks examined whether they were able to use the rule. These eight tasks were chosen from different upper secondary school

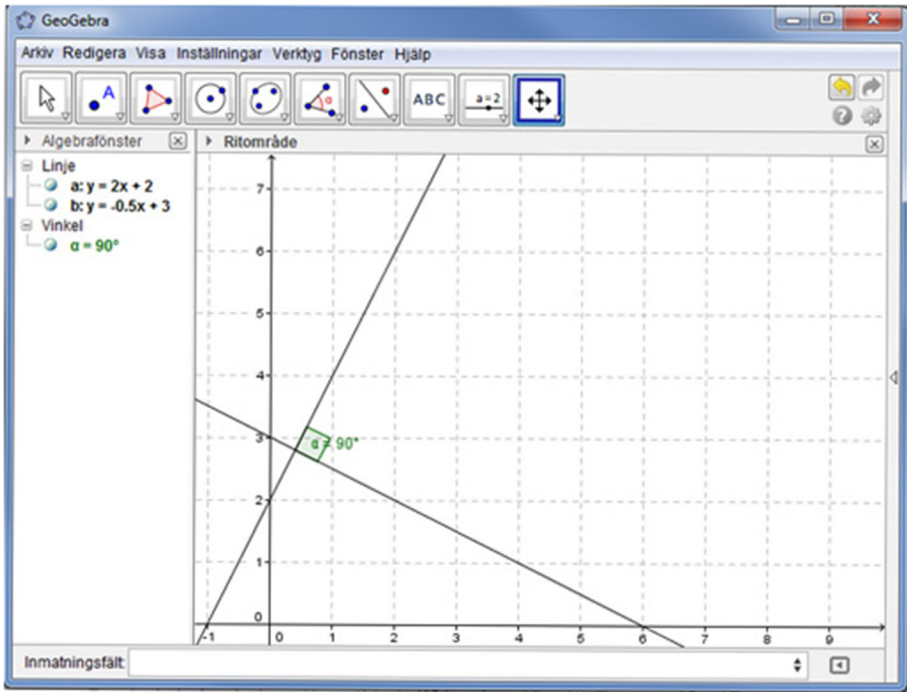


Fig. 3 GeoGebra showing the algebraic and graphical representations of two functions and the angle (90°) between the perpendicular graphs

mathematics textbooks. The tasks were selected to represent typical textbook tasks that would prompt students to use the mathematical rule in focus in this study. Some examples are presented in Fig. 4. The students were given 70 min to complete the posttest, and all students were able to do so within that timeframe.

3.6 Data

The hypotheses examined in this study concern students' performance during practice (constructing the rule) and during the posttest (using the rule). Furthermore, it is reasonable to examine whether students' ability to remember the rule will influence their ability to use it. Finally, it is likely that general mathematical ability will affect their achievements, and therefore the students' grades in mathematics will be included in the analysis. Data used for analysis will then consist of: (1) whether or not the students succeeded in constructing the rule during practice; (2) whether or not the students remembered the rule during the posttest; (3) the students' posttest scores (using the rule); and (4) the students' grades in mathematics.

1. The rule constructed during practice was regarded as correct if any of the following answers were given: $m_1 \times m_2 = -1$; if you multiply the m-values of two perpendicular functions it will equal -1 ; $m_2 = -1/m_1$; if the m-value of one function is "m" then the m of the perpendicular function is calculated by dividing 1 with the first "m" and then change the sign; or any other corresponding formula or procedure.

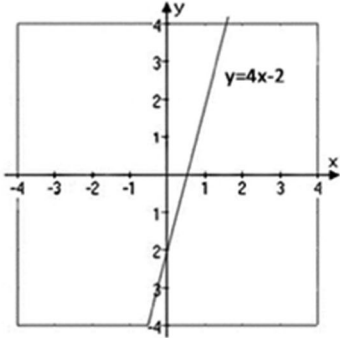
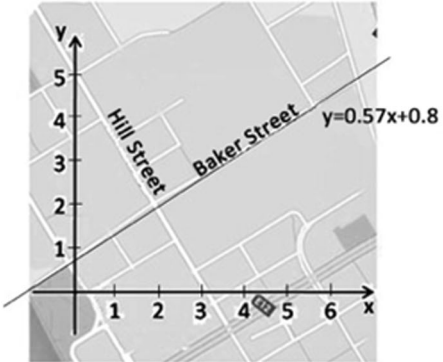
<p>1) Write down the mathematical rule that will guide you in choosing the m-values to create two linear functions having perpendicular graphs.</p>	
<p>2) Which one of these linear functions' graph is perpendicular to $y=2x-5$?</p> <p>a) $y=2x+0.5$ b) $y=-2x-0.5$ c) $y=-0.5x+2$ d) $y=0.5x-2$ e) none of the above</p>	
<p>3) Find a linear function whose graph is perpendicular to the following graph.</p>	
<p>4) Baker Street and Hill Street are perpendicular.</p> <p>The direction of Baker Street could be described by the linear function $y=0.57x+0.8$</p> <p>Find a linear function $y=mx+c$, that describes the direction of Hill Street</p>	

Fig. 4 Examples of posttest tasks asking students to present and use the rule

- The answer to the question examining to what extent they remembered the rule was regarded as correct if any of the above exemplified rules or any other corresponding formula or procedures were expressed.

3. The eight posttest tasks examining students' ability to make use of the rule were corrected and 1 point was given for each correct answer. Hence, a maximum of 8 points were given to students who correctly solved all eight tasks.
4. In Sweden, grades given to students range from A to F; A is the highest grade and F indicates that the student has failed. These grades correspond to the following numerical values: A = 20; B = 17.5; C = 15; D = 12.5; E = 10; and F = 0. So, the highest numeric grade is 20 and the lowest is 0. In this study the numeric grades are used for the analysis.

3.7 Statistical Analyses

The analysis will examine the proposed hypotheses one at a time.

H1 Students practicing with the guided task will outperform students practicing with the unguided task (constructing the rule).

To examine whether students practicing on the guided task outperformed the students practicing on the unguided task—constructing the rule—a Chi square test of independence was performed. Thereafter, an independent t-test was conducted to examine whether students' grades could be related to their success in solving the task during practice depending on task design.

H2 Students who successfully solved the unguided task will perform better during the posttest (using the rule) compared to those who successfully solved the guided task.

To examine the second hypothesis, an analysis in four steps was performed to explore whether there were any significant differences with regard to posttest performance between the student groups solving guided or unguided tasks. Firstly, an independent t-test was conducted to examine whether there were any differences in posttest performance between the guided and unguided students who successfully constructed the rule during practice. Secondly, the results were further examined, looking into factors underlying students' performance during the posttest. The hypothesis concerns the effects of task design (guided or unguided), and the authors also found it reasonable to look into whether successfully "remembering the rule" would influence students' ability to "use the rule" during the posttest. Furthermore, the authors wanted to take into account that even students who seemed to have failed during practice could have gained some understanding of the learning target that became useful during the posttest. Therefore, the authors included all students and entered "task design" (guided/unguided) and "remembering the rule" (yes/no) as factors in a 2×2 analysis of covariance (ANCOVA) with the posttest results of "use the rule" as dependent variable and "grades" as covariate. "Grades" was included as covariate since it could be expected that the grades within the two groups of students included in the analysis no longer matched. Thirdly, the interaction effect between "task design" and "remembering the rule" was analyzed, conducting an independent t-test comparing posttest performance among the students who constructed the rule with or without guidance. Finally, an independent t-test was conducted to examine whether students' grades could be related to their success in using the rule they remembered depending on whether they constructed the rule with or without guidelines. All statistical analyses were conducted using the Statistical Package for the Social Sciences (SPSS 23).

4 Results

The results related to each hypothesis are presented below.

4.1 Practice Performance (H1)

The students who were provided with guidance were more successful in constructing the rule during practice than the students working with the unguided task. Within the guided group, 43 out of 63 students (68%) constructed the rule compared to 22 out of 66 (33%) within the unguided group. A Chi square test showed a significant difference between the two groups in favor of the students solving the guided task $\chi^2(1, N = 129) = 15.723, p < .001$. The t-test showed that the students who successfully solved the unguided task had significantly higher grades ($m = 18.07, sd = 2.31$) than the students who successfully solved the guided task $m = 16.28, sd = 2.52, t(63) = 2.78, N = 65, p = .007$ (Table 1).

4.2 Posttest Performance (H2)

The students who successfully solved the unguided task during training scored significantly higher on the posttest compared to students who succeeded in solving the guided task. An initial t-test of independence showed a significant difference during the posttest in favor of the students who successfully solved the unguided task during practice, $m = 4.0$ compared to $m = 1.7, t(63) = 4.06, N = 65, p < .001$ (Table 2).

To pursue this hypothesis further, examining factors underlying students' performance during the posttest, three follow-up analyses were conducted. As described earlier, "task design" (guided/unguided) and "remembering the rule" (yes/no) were included as factors in a 2×2 analysis of covariance (ANCOVA) with "grades" as covariate and the posttest "using the rule" as dependent variable. The analyses revealed the main effects of both "remembering the rule" and "task design," $F(1,121) = 160, p < .001, \eta_p^2 = .57$; $F(1,121) = 11.46, p = .001, \eta_p^2 = .09$, respectively. These main effects were qualified by an interaction between "constructing the rule" and "task design," $F(1,121) = 13.4, p < .001, \eta_p^2 = .10$. Both the main effects and the interaction between "remember the rule" and "task design" can be seen in Fig. 5.

Thus, the analyses showed that the students who were provided with guidance were more successful in constructing the rule during practice than the students working with the unguided task. However, during the posttest, testing their ability to use the rule, students who constructed the rule without guidance significantly outperformed students who constructed the rule with guidance. Furthermore, only the students who actually remembered the rule during the posttest were able to use it. Still, the students who remembered the rule

Table 1 Practice performance and grades in mathematics, mean values

	Guided group	Unguided group
Number of students	N = 63	N = 66
Grades in mathematics, mean value	$m = 15.55$	$m = 15.44$
Number of students who succeeded during practice	N = 43 (68%)	N = 22 (33%)
Grades in mathematics, mean value	$m = 16.28$	$m = 18.07$

Table 2 Posttest performance among students who succeeded during practice

	Guided group	Unguided group
Number of students	N = 43	N = 22
Posttest result (max = 8)	<i>m</i> = 1.7	<i>m</i> = 4.0

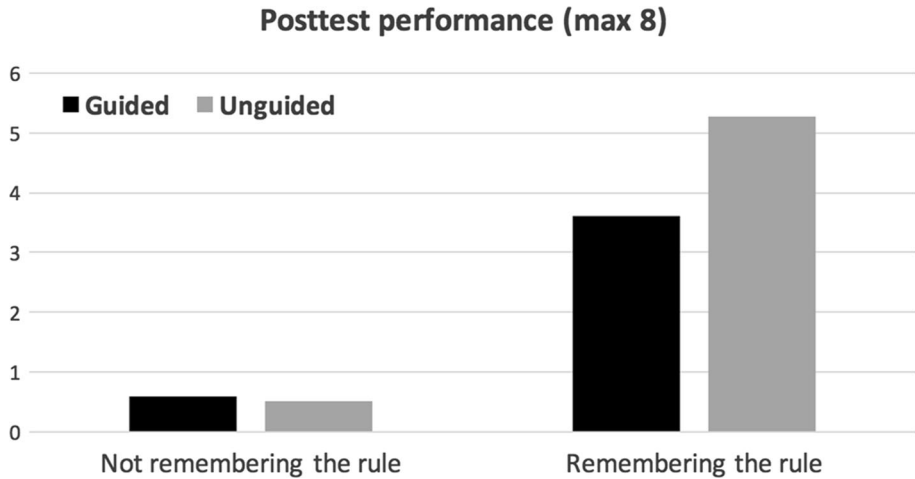


Fig. 5 ANCOVA, posttest performance among students practicing on guided and unguided task

they constructed without guidance significantly outperformed the students who remembered the rule they constructed with guidance. Finally, those students who failed during practice to construct the rule, or forgot the rule they constructed, failed during the posttest regardless of task design (guided or unguided).

To follow up on the interaction effect, an independent t-test comparing posttest performance in relation to guided and unguided practice for those who remembered the rule was conducted. The analysis revealed a significant effect, *m* = 5.27 compared to *m* = 3.60, *t*(28) = 2.24, *N* = 34, *p* = .03, in favor of the unguided students who successfully solved the practice task. A final t-test showed a non-significant difference *t*(28) = 0.45, *N* = 34, *p* = .7 with regard to the grades in mathematics among the students

Table 3 Posttest performance, using the rule, among the students who remembered the rule

	Guided group	Unguided group
Number of students who, during the posttest, remembered the rule they constructed during practice	N = 17 (40%)	N = 17 (77%)
Grades in mathematics, mean value	<i>m</i> = 18.50	<i>m</i> = 18.17
Posttest results (max = 8)	<i>m</i> = 3.60	<i>m</i> = 5.27

who remembered the rule they constructed with guidelines $m = 18.50$ ($sd = 1.85$) and without guidelines $m = 18.17$ ($sd = 2.21$) (Table 3).

5 Discussion

The analysis of students' performance during practice confirmed Hypothesis 1. The analysis showed a significant difference in favor of the students in the guided group, that is, the students who were provided with guidance were more successful in constructing the rule during practice.

The analysis of students' performance during posttest confirmed Hypothesis 2. The analysis showed a significant difference in favor of the students who succeeded in solving the unguided task during practice. An extended analysis showed that these results were driven by task design and successfully remembering the rule. Furthermore, the analysis showed that students who failed during practice, regardless of task design, failed during the posttest. It is worth noting that a significantly larger number of the unguided students did not solve the task compared to the students solving the guided task (this will be discussed later). However, a majority (60%) of the students who successfully solved the guided task did not remember the rule after 1 week, and those who did, compared to unguided students, performed significantly more poorly during the posttest using the rule. That is, the students who successfully solved the guided task were engaged in less effective learning than the guided students who succeeded during practice.

These results are discussed in terms of task design, the use of GeoGebra, and effects on learning outcome. The discussion will conclude with some implications for teaching.

5.1 Unguided and Guided Tasks, Practice Performance and Learning Outcome

The results in the present study are in line with earlier research, for example, Jonsson et al. (2014). During practice, students who are provided with guidelines, instructions, or a problem-solving method to follow will outperform students who are asked to construct their own methods. These kinds of results are hardly surprising since instructions in general are constructed to help students reach an answer and prevent mistakes (Brousseau 1997). Furthermore, it has been shown that following instructions reduces demands on students' working memory (Lee and Anderson 2013). Even students with somewhat lower grades in this study were able to solve the task using guidelines. However, the more important question is, under what circumstances does success during practice lead to long-term learning? As presented earlier, there is research arguing that direct instructions are more efficient for learning than problem solving with minimal instruction (Mayer 2004; Kirschner et al. 2006). The result of this study, however, supports claims from the other strand of research, that for effective learning to take place students need to engage in a fair amount of struggle constructing their own methods (Schoenfeld 1985; Brousseau 1997; Hiebert and Grouws 2007; Hmelo-Silver et al. 2007). Students who follow guidelines, instructions, or memorized procedures will not need to struggle. In other words, situations where students understand that prior knowledge is insufficient and that new information needs to be interpreted are unlikely to occur and therefore no struggle will take place. Hence, providing students with instructions will guide them to success during practice but might result in rote learning of superficial knowledge that is poorly memorized (Lee and

Anderson 2013; Lithner 2008). These circumstances might explain why students who successfully constructed the rule with guidance scored significantly lower on the posttest testing their ability to use the rule. Thus, successful performance during practice following instructions does not guarantee successful learning in the longer term, results that are consistent with, for example, Jonsson et al. (2014) and Kapur (2016). In order to enhance their learning during practice, students need to engage in a productive struggle, exploring and constructing (parts of) their methods.

5.2 GeoGebra, Unguided and Guided Tasks, and Learning Outcome

Learning activities such as exploring, creative reasoning, and constructing methods that are reported as beneficial for learning coincide with the activities that dynamic software has the potential to support (Coleman et al. 2015; Pierce et al. 2011; Dikovic 2009). Those features are available to students using the software regardless of task design. This study, however, shows that task design, unguided or guided, will still have significant effects on the learning outcome even if dynamic software such as GeoGebra is used. It is reasonable to assume that the potential of GeoGebra will be used differently when students are solving tasks with or without guidance. That could be explained by the idea that students will only do what they need to do to solve the task (Joubert 2013). If this is the case, the students who are provided with guidance will merely follow the instructions and, as discussed earlier, no struggle will be initiated. Therefore, these students will not engage in activities such as exploring, constructing, and developing methods, and thus will not need GeoGebra. For instance, in the present study, the guided students ended up with multiple representations of functions on the screen, but since no struggle was initiated they were not likely to use GeoGebra to dynamically explore the mathematical properties of these functions and relations between their representations. This is in contrast to students who had no instructions to follow. Students with no instructions concerning the solution method needed to come up with ideas, construct methods, interpret outcomes from activities, and so forth, and they needed GeoGebra to verify, falsify, explore, and develop their ideas. By doing so, they benefited from GeoGebra's potential to offer dynamic visualization of representations, instant feedback, and multiple displays of representations. Such features have been shown to be beneficial for learning functions (see, e.g., Dikovic 2009; Natsheh and Karsenty 2014). Thus, students are more likely to benefit from the features of GeoGebra, in terms of enhanced learning outcomes, when they are using the software to work with unguided tasks.

However, in the present study, the presence of GeoGebra was not sufficient to support all unguided students in successfully solving the task during practice. Two-thirds of the unguided students obviously struggled; however, the struggle was not productive enough to solve the task. Based on observations of the students during practice and from listening to the recordings, it is apparent that some of the unguided students' failure during practice could partially be understood as a lack of prior knowledge needed to solve the task. On the other hand, these students' shortcomings would more likely be noticed by the teacher during lessons than would those of students who managed to solve the guided task but without gaining any long-term knowledge. Another explanation for some of the unguided students' shortcomings could be that students in general might be more used to following provided methods than constructing methods. These circumstances bring us to the role of the teacher.

5.3 Implications for Teaching

In light of the present study we suggest that, in order to enhance learning benefits of dynamic software, students need to work with unguided tasks, that is, tasks in which they are not provided with solution methods. However, there are some challenges to be accepted; for example, the given task needs to be designed to (better) correspond with the students' prior knowledge. But overall, the teacher's challenge is to support students as they solve (parts of) the given task without turning the unguided task into a guided task. It is a question of helping students to turn fruitless struggles into productive struggles. That is, the teacher's role is to support students as they interpret the goal of the task and activate and reconstruct useful prior knowledge; to encourage them to come up with ideas for solving the task; and to challenge them to explain whether and how those ideas will bring them close to the solution. The presence of GeoGebra might facilitate teaching students how to engage in explorative task-solving strategies by constructing, testing, and evaluating ideas (e.g., Hohenwarter and Jones 2007). In other words, GeoGebra can be a means to concretize encouragement and explain approaches to and achievements of solving a task. For instance, instead of asking a student to explain the way they are thinking, a teacher may ask if there is a way to try an idea through an action in GeoGebra and encourage students to explain why they chose that particular action. Furthermore, once students have concluded their productive struggles and constructed (parts of) their methods or even failed, it can be beneficial to provide them with a consolidation lesson focusing on the experiences, successes, and failures they had while working with a task. As described earlier, there is a fair amount of research looking into learning effects comparing a didactical design of construction-before-instruction to instruction-before-practice pointing in favor of the former (e.g., Hiebert and Stigler 2004; Kapur 2010, 2011; Schwartz et al. 2011). In summary, students are more likely to benefit from a dynamic software's potential to enhance learning if they are working with and successfully solving tasks that do not include instructions on how to construct a solution method. Consequently, the challenge for teachers will be to design appropriate tasks and to provide feedback to support students to engage in productive struggles without turning unguided tasks into guided tasks.

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