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## Research Article

# An Explicit Method for the Split Feasibility Problem with Self-Adaptive Step Sizes

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An explicit iterative method with self-adaptive step-sizes for solving the split feasibility problem is presented. Strong convergence theorem is provided.

## 1. Introduction

Since its publication in 1994, the split feasibility problem has been studied by many authors. For some related works, please consult [1–18]. Among them, a more popular algorithm that solves the split feasibility problems is Byrne's CQ method [2]:

$$x_{n+1} = P_C(x_n - \tau A^*(I - P_Q)Ax_n), \quad (1.1)$$

where  $C$  and  $Q$  are two closed convex subsets of two real Hilbert spaces  $H_1$  and  $H_2$ , respectively, and  $A : H_1 \rightarrow H_2$  is a bounded linear operator. The CQ algorithm only involves the computations of the projections  $P_C$  and  $P_Q$  onto the sets  $C$  and  $Q$ , respectively, and is therefore implementable in the case where  $P_C$  and  $P_Q$  have closed-form expressions.

Note that CQ algorithm can be obtained from optimization. If we set

$$f(x) := \frac{1}{2} \|Ax - P_Q Ax\|^2, \quad (1.2)$$

then the convex objective  $f$  is differentiable and has a Lipschitz gradient given by

$$\nabla f(x) = A^*(I - P_Q)A. \quad (1.3)$$

Thus, the CQ algorithm can be obtained by minimizing the following convex minimization problem

$$\min_{x \in C} f(x). \quad (1.4)$$

We can use a gradient projection algorithm below to solve the split feasibility problem:

$$x_{n+1} = P_C(x_n - \tau_n \nabla f(x_n)), \quad (1.5)$$

where  $\tau_n$ , the step size at iteration  $n$ , is chosen in the interval  $(0, 2/L)$ , where  $L$  is the Lipschitz constant of  $\nabla f$ .

However, we observe that the determination of the step size  $\tau_n$  depends on the operator (matrix) norm  $\|A\|$  (or the largest eigenvalue of  $A^*A$ ). This means that in order to implement the CQ algorithm, one has first to compute (or, at least, estimate) the matrix norm of  $A$ , which is in general not an easy work in practice. To overcome the above difficulty, the so-called self-adaptive method which permits step size  $\tau_n$  being selected self-adaptively was developed. See, for example, [10, 14, 15, 19–23].

Inspired by the above results and the self-adaptive method, in this paper, we present an explicit iterative method with self-adaptive step sizes for solving the split feasibility problem. Convergence analysis result is given.

## 2. Preliminaries

Let  $H_1$  and  $H_2$  be two real Hilbert spaces and  $C$  and  $Q$  two closed convex subsets of  $H_1$  and  $H_2$ , respectively. Let  $A : H_1 \rightarrow H_2$  be a bounded linear operator. The split feasibility problem is to find a point  $x^*$  such that

$$x^* \in C, \quad Ax^* \in Q. \quad (2.1)$$

Next, we use  $\Gamma$  to denote the solution set of the split feasibility problem, that is,  $\Gamma = \{x \in C : Ax \in Q\}$ .

We know that a point  $x^* \in C$  is a stationary point of problem (1.4) if it satisfies

$$\langle \nabla f(x^*), x - x^* \rangle \geq 0, \quad \forall x \in C. \quad (2.2)$$

Given  $x^* \in H_1$ .  $x^*$  solves the split feasibility problem if and only if  $x^*$  solves the fixed point equation

$$x^* = P_C(x^* - \gamma A^*(I - P_Q)Ax^*). \quad (2.3)$$

Next we adopt the following notation:

- (i)  $x_n \rightarrow x$  means that  $x_n$  converges strongly to  $x$ ;
- (ii)  $x_n \rightharpoonup x$  means that  $x_n$  converges weakly to  $x$ ;
- (iii)  $\omega_\omega(x_n) := \{x : \exists x_{n_j} \rightharpoonup x\}$  is the weak  $\omega$ -limit set of the sequence  $\{x_n\}$ .

Recall that a function  $f : H \rightarrow \mathbb{R}$  is called convex if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y), \quad (2.4)$$

for all  $\lambda \in (0, 1)$  and  $\forall x, y \in H$ . It is known that a differentiable function  $f$  is convex if and only if there holds the relation:

$$f(z) \geq f(x) + \langle \nabla f(x), z - x \rangle, \quad (2.5)$$

for all  $z \in H$ . Recall that an element  $g \in H$  is said to be a subgradient of  $f : H \rightarrow \mathbb{R}$  at  $x$  if

$$f(z) \geq f(x) + \langle g, z - x \rangle, \quad (2.6)$$

for all  $z \in H$ . If the function  $f : H \rightarrow \mathbb{R}$  has at least one subgradient at  $x$  is said to be sub-differentiable at  $x$ . The set of subgradients of  $f$  at the point  $x$  is called the subdifferential of  $f$  at  $x$ , and is denoted by  $\partial f(x)$ . A function  $f$  is called sub-differentiable if it is subdifferentiable at all  $x \in H$ . If  $f$  is convex and differentiable, then its gradient and subgradient coincide. A function  $f : H \rightarrow \mathbb{R}$  is said to be weakly lower semi continuous (w-lsc) at  $x$  if  $x_n \rightharpoonup x$  implies

$$f(x) \leq \liminf_{n \rightarrow \infty} f(x_n). \quad (2.7)$$

$f$  is said to be w-lsc on  $H$  if it is w-lsc at every point  $x \in H$ .

A mapping  $T : C \rightarrow C$  is called *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|, \quad (2.8)$$

for all  $x, y \in C$ .

Recall that the (nearest point or metric) projection from  $H$  onto  $C$ , denoted  $P_C$ , assigns, to each  $x \in H$ , the unique point  $P_C(x) \in C$  with the property

$$\|x - P_C(x)\| = \inf\{\|x - y\| : y \in C\}. \quad (2.9)$$

It is well known that the metric projection  $P_C$  of  $H$  onto  $C$  has the following basic properties:

- (a)  $\|P_C(x) - P_C(y)\| \leq \|x - y\|$  for all  $x, y \in H$ ;
- (b)  $\langle x - y, P_C(x) - P_C(y) \rangle \geq \|P_C(x) - P_C(y)\|^2$  for every  $x, y \in H$ ;
- (c)  $\langle x - P_C(x), y - P_C(x) \rangle \leq 0$  for all  $x \in H, y \in C$ .

**Lemma 2.1** (see [24]). Assume that  $\{a_n\}$  is a sequence of nonnegative real numbers such that

$$a_{n+1} \leq (1 - \gamma_n)a_n + \delta_n, \quad (2.10)$$

where  $\{\gamma_n\}$  is a sequence in  $(0, 1)$  and  $\{\delta_n\}$  is a sequence such that

- (1)  $\sum_{n=1}^{\infty} \gamma_n = \infty$ ;
- (2)  $\limsup_{n \rightarrow \infty} (\delta_n / \gamma_n) \leq 0$  or  $\sum_{n=1}^{\infty} |\delta_n| < \infty$ .

Then  $\lim_{n \rightarrow \infty} a_n = 0$ .

**Lemma 2.2** (see [25]). Let  $(s_n)$  be a sequence of real numbers that does not decrease at infinity, in the sense that there exists a subsequence  $(s_{n_i})$  of  $(s_n)$  such that  $s_{n_i} \leq s_{n_i+1}$  for all  $i \geq 0$ . For every  $n \geq n_0$ , define an integer sequence  $(\tau(n))$  as

$$\tau(n) = \max\{k \leq n : s_{n_i} < s_{n_i+1}\}. \quad (2.11)$$

Then  $\tau(n) \rightarrow \infty$  as  $n \rightarrow \infty$  and for all  $n \geq n_0$

$$\max\{s_{\tau(n)}, s_n\} \leq s_{\tau(n)+1}. \quad (2.12)$$

### 3. Main Results

In this section, we will introduce our algorithm and prove our main results.

Let  $C$  and  $Q$  be nonempty closed convex subsets of real Hilbert spaces  $H_1$  and  $H_2$ , respectively. Let  $A : H_1 \rightarrow H_2$  be a bounded linear operator. In the sequel, we assume that the split feasibility problem is consistent, that is  $\Gamma \neq \emptyset$ .

*Algorithm 3.1.* For  $u \in C$  and given  $x_0 \in C$ , let the sequence  $\{x_{n+1}\}$  defined by

$$\begin{aligned} y_n &= \alpha_n u + (1 - \alpha_n)x_n, \\ x_{n+1} &= P_C \left( y_n - \tau_n \frac{f(y_n) \nabla f(y_n)}{\|\nabla f(y_n)\|^2} \right), \quad n \geq 0, \end{aligned} \quad (3.1)$$

where  $\{\alpha_n\} \subset (0, 1)$  and  $\{\tau_n\} \subset (0, 2)$ .

*Remark 3.2.* In the sequel, we may assume that  $\nabla f(y_n) \neq 0$  for all  $n$ . Note that this fact can be guaranteed if the sequence  $\{y_n\}$  is infinite; that is, Algorithm 3.1 does not terminate in a finite number of iterations.

**Theorem 3.3.** Assume that the following conditions are satisfied:

- (i)  $\lim_{n \rightarrow \infty} \alpha_n = 0$  and  $\sum_{n=1}^{\infty} \alpha_n = \infty$ ;
- (ii)  $\inf_n \tau_n (2 - \tau_n) > 0$ .

Then  $\{x_n\}$  defined by (3.1) converges strongly to  $P_{\Gamma}(u)$ .

*Proof.* Let  $v \in \Gamma$ . It follows that  $\nabla f(v) = 0$  for all  $v \in \Gamma$ . From (2.5), we deduce that

$$f(y_n) = f(y_n) - f(v) \leq \langle \nabla f(y_n), y_n - v \rangle. \quad (3.2)$$

Thus, by (3.1) and (3.2), we have

$$\begin{aligned} \|x_{n+1} - v\|^2 &= \left\| P_C \left( y_n - \tau_n \frac{f(y_n) \nabla f(y_n)}{\|\nabla f(y_n)\|^2} \right) - v \right\|^2 \\ &\leq \left\| y_n - \tau_n \frac{f(y_n) \nabla f(y_n)}{\|\nabla f(y_n)\|^2} - v \right\|^2 \\ &= \|y_n - v\|^2 - 2\tau_n \frac{f(y_n)}{\|\nabla f(y_n)\|^2} \langle \nabla f(y_n), y_n - v \rangle + \tau_n^2 \frac{f^2(y_n)}{\|\nabla f(y_n)\|^2} \\ &\leq \|y_n - v\|^2 - 2\tau_n \frac{f^2(y_n)}{\|\nabla f(y_n)\|^2} + \tau_n^2 \frac{f^2(y_n)}{\|\nabla f(y_n)\|^2} \\ &= \|y_n - v\|^2 - \tau_n(2 - \tau_n) \frac{f^2(y_n)}{\|\nabla f(y_n)\|^2}, \end{aligned} \quad (3.3)$$

$$\|y_n - v\|^2 = \|\alpha_n(u - v) + (1 - \alpha_n)(x_n - v)\|^2 \leq \alpha_n \|u - v\|^2 + (1 - \alpha_n) \|x_n - v\|^2.$$

It follows that

$$\begin{aligned} \|x_{n+1} - v\|^2 &\leq \alpha_n \|u - v\|^2 + (1 - \alpha_n) \|x_n - v\|^2 - \tau_n(2 - \tau_n) \frac{f^2(y_n)}{\|\nabla f(y_n)\|^2} \\ &\leq \alpha_n \|u - v\|^2 + (1 - \alpha_n) \|x_n - v\|^2 \\ &\leq \max \{ \|u - v\|^2, \|x_n - v\|^2 \}. \end{aligned} \quad (3.4)$$

By induction, we deduce

$$\|x_{n+1} - v\| \leq \max \{ \|u - v\|, \|x_0 - v\| \}. \quad (3.5)$$

Hence,  $\{x_n\}$  is bounded.

At the same time, we note that

$$\|y_n - v\|^2 = \|\alpha_n(u - v) + (1 - \alpha_n)(x_n - v)\|^2 \leq (1 - \alpha_n) \|x_n - v\|^2 + 2\alpha_n \langle u - v, y_n - v \rangle. \quad (3.6)$$

Therefore,

$$\begin{aligned}
\|x_{n+1} - v\|^2 &\leq (1 - \alpha_n)\|x_n - v\|^2 + 2\alpha_n\langle u - v, y_n - v \rangle - \tau_n(2 - \tau_n)\frac{f^2(y_n)}{\|\nabla f(y_n)\|^2} \\
&= (1 - \alpha_n)\|x_n - v\|^2 + 2\alpha_n\langle u - v, \alpha_n(u - v) + (1 - \alpha_n)(x_n - v) \rangle \\
&\quad - \tau_n(2 - \tau_n)\frac{f^2(y_n)}{\|\nabla f(y_n)\|^2} \\
&= (1 - \alpha_n)\|x_n - v\|^2 + 2\alpha_n^2\|u - v\|^2 + 2\alpha_n(1 - \alpha_n)\langle u - v, x_n - v \rangle \\
&\quad - \tau_n(2 - \tau_n)\frac{f^2(y_n)}{\|\nabla f(y_n)\|^2}.
\end{aligned} \tag{3.7}$$

It follows that

$$\begin{aligned}
\|x_{n+1} - v\|^2 - \|x_n - v\|^2 + \alpha_n\left(\|x_n - v\|^2 - 2\alpha_n\|u - v\|^2 + 2\alpha_n\langle u - v, x_n - v \rangle\right) \\
+ \tau_n(2 - \tau_n)\frac{f^2(y_n)}{\|\nabla f(y_n)\|^2} \leq 2\alpha_n\langle u - v, x_n - v \rangle.
\end{aligned} \tag{3.8}$$

Next, we will prove that  $x_n \rightarrow v$ . Set  $\omega_n = \|x_n - v\|^2$  for all  $n \geq 0$ . Since  $\alpha_n \rightarrow 0$  and  $\inf_n \tau_n(2 - \tau_n) > 0$ , we may assume without loss of generality that  $\tau_n(2 - \tau_n) \geq \sigma$  for some  $\sigma > 0$ . Thus, we can rewrite (3.8) as

$$\omega_{n+1} - \omega_n + \alpha_n U_n + \frac{\sigma f^2(y_n)}{\|\nabla f(y_n)\|^2} \leq 2\alpha_n\langle u - v, x_n - v \rangle, \tag{3.9}$$

where  $U_n = \|x_n - v\|^2 - 2\alpha_n\|u - v\|^2 + 2\alpha_n\langle u - v, x_n - v \rangle$ .

Now, we consider two possible cases.

*Case 1.* Assume that  $\{\omega_n\}$  is eventually decreasing; that is, there exists  $N > 0$  such that  $\{\omega_n\}$  is decreasing for  $n \geq N$ . In this case,  $\{\omega_n\}$  must be convergent and from (3.9) it follows that

$$\begin{aligned}
0 \leq \frac{\sigma f^2(y_n)}{\|\nabla f(y_n)\|^2} &\leq \omega_n - \omega_{n+1} - \alpha_n U_n + 2\alpha_n\|u - v\|\|x_n - v\| \\
&\leq \omega_n - \omega_{n+1} + M\alpha_n,
\end{aligned} \tag{3.10}$$

where  $M > 0$  is a constant such that  $\sup_n \{2\|u - v\|\|x_n - v\| + \|U_n\|\} \leq M$ . Letting  $n \rightarrow \infty$  in (3.10), we get

$$\lim_{n \rightarrow \infty} f(y_n) = 0. \tag{3.11}$$

Since  $\{y_n\}$  is bounded, there exists a subsequence  $\{y_{n_k}\}$  of  $\{y_n\}$  converging weakly to  $\tilde{x} \in C$ . Since,  $x_n - y_n \rightarrow 0$ , we also have  $\{x_{n_k}\}$  of  $\{x_n\}$  converging weakly to  $\tilde{x} \in C$ . From the weak lower semicontinuity of  $f$ , we have

$$0 \leq f(\tilde{x}) \leq \liminf_{k \rightarrow \infty} f(y_{n_k}) = \lim_{n \rightarrow \infty} f(y_n) = 0. \quad (3.12)$$

Hence,  $f(\tilde{x}) = 0$ ; that is,  $A\tilde{x} \in Q$ . This indicates that

$$\omega_w(y_n) = \omega_w(x_n) \subset \Gamma. \quad (3.13)$$

Furthermore, by using the property of the projection (c), we deduce

$$\limsup_{n \rightarrow \infty} \langle u - v, x_n - v \rangle = \max_{\tilde{x} \in \omega_w(x_n)} \langle u - P_\Gamma(u), \tilde{x} - P_\Gamma(u) \rangle \leq 0. \quad (3.14)$$

From (3.8), we obtain

$$\omega_{n+1} \leq (1 - \alpha_n)\omega_n + \alpha_n \left( 2\alpha_n \|u - v\|^2 + 2(1 - \alpha_n) \langle u - v, x_n - v \rangle \right). \quad (3.15)$$

This together with Lemma 2.1 imply that  $\omega_n \rightarrow 0$ .

*Case 2.* Assume that  $\{\omega_n\}$  is not eventually decreasing. That is, there exists an integer  $n_0$  such that  $\omega_{n_0} \leq \omega_{n_0+1}$ . Thus, we can define an integer sequence  $\{\tau_n\}$  for all  $n \geq n_0$  as follows:

$$\tau(n) = \max\{k \in \mathbb{N} \mid n_0 \leq k \leq n, \omega_k \leq \omega_{k+1}\}. \quad (3.16)$$

Clearly,  $\tau(n)$  is a nondecreasing sequence such that  $\tau(n) \rightarrow +\infty$  as  $n \rightarrow \infty$  and

$$\omega_{\tau(n)} \leq \omega_{\tau(n)+1}, \quad (3.17)$$

for all  $n \geq n_0$ . In this case, we derive from (3.10) that

$$\frac{\sigma f^2(y_{\tau(n)})}{\|\nabla f(y_{\tau(n)})\|^2} \leq M\alpha_{\tau(n)} \rightarrow 0. \quad (3.18)$$

It follows that

$$\lim_{n \rightarrow \infty} f(y_{\tau(n)}) = 0. \quad (3.19)$$

This implies that every weak cluster point of  $\{y_{\tau(n)}\}$  is in the solution set  $\Gamma$ ; that is,  $\omega_w(y_{\tau(n)}) \subset \Gamma$ . So,  $\omega_w(x_{\tau(n)}) \subset \Gamma$ . On the other hand, we note that

$$\begin{aligned} \|y_{\tau(n)} - x_{\tau(n)}\| &= \alpha_{\tau(n)} \|u - x_{\tau(n)}\| \longrightarrow 0, \\ \|x_{\tau(n)+1} - y_{\tau(n)}\| &\leq \frac{\tau_{\tau(n)} f(y_{\tau(n)})}{\|\nabla f(y_{\tau(n)})\|} \longrightarrow 0. \end{aligned} \quad (3.20)$$

From which we can deduce that

$$\limsup_{n \rightarrow \infty} \langle u - v, x_{\tau(n)} - v \rangle = \max_{\tilde{x} \in \omega_w(x_{\tau(n)})} \langle u - P_{\Gamma}(u), \tilde{x} - P_{\Gamma}(u) \rangle \leq 0. \quad (3.21)$$

Since  $\omega_{\tau(n)} \leq \omega_{\tau(n)+1}$ , we have from (3.9) that

$$\omega_{\tau(n)} \leq (1 - 2\alpha_{\tau(n)}) \langle u - v, x_{\tau(n)} - v \rangle + 2\alpha_{\tau(n)} \|u - v\|^2. \quad (3.22)$$

Combining (3.21) and (3.22) yields

$$\limsup_{n \rightarrow \infty} \omega_{\tau(n)} \leq 0, \quad (3.23)$$

and hence

$$\lim_{n \rightarrow \infty} \omega_{\tau(n)} = 0. \quad (3.24)$$

From (3.15), we have

$$\limsup_{n \rightarrow \infty} \omega_{\tau(n)+1} \leq \limsup_{n \rightarrow \infty} \omega_{\tau(n)}. \quad (3.25)$$

Thus,

$$\lim_{n \rightarrow \infty} \omega_{\tau(n)+1} = 0. \quad (3.26)$$

From Lemma 2.2, we have

$$0 \leq \omega_n \leq \max\{\omega_{\tau(n)}, \omega_{\tau(n)+1}\}. \quad (3.27)$$

Therefore,  $\omega_n \rightarrow 0$ . That is,  $x_n \rightarrow v$ . This completes the proof.  $\square$

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