

STRINGS IN NONTRIVIAL GRAVITINO AND RAMOND-RAMOND BACKGROUNDS

Ioannis Giannakis¹

*Physics Department
Rockefeller University
1230 York Avenue
New York, NY 10021*

Abstract

In this paper we discuss deformations of the BRST operator of the fermionic string. These deformations preserve inpotency of the BRST operator and correspond to turning on infinitesimal Gravitino and Ramond-Ramond spacetime fields.

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¹giannak@summit.rockefeller.edu

One of the outstanding problems of string theory is to understand the equations of motion for the fields of the theory (massless and massive) and the higher symmetries that relate them [1],[2]. Progress towards this direction can be achieved by studying infinitesimal deformations of the SuperVirasoro algebras that preserve superconformal invariance [3]. The problem of finding superconformal deformations is an interesting problem in its own right, but it also provides us with insights into the symmetry structure of string theory since spacetime symmetry transformations are particular superconformal deformations [4]. In a recent paper [5] we constructed a class of superconformal deformations, termed *canonical deformations*, in terms of superfields (see also [6]). Although properly speaking we need to discuss deformations of two copies of the SuperVirasoro algebra in the remainder of this paper we shall concentrate only on one copy. More specifically we found that a deformation of the form

$$\delta\mathcal{T}(\sigma) = \delta T_F(\sigma) + \theta\delta T(\sigma) = \Phi_F(\sigma) + \theta\Phi_B(\sigma) \quad (1)$$

where $\Phi_B(\Phi_F)$ is the bosonic (fermionic) component of a superfield of dimension $(\frac{1}{2}, \frac{1}{2})$, preserves superconformal invariance.

Canonical deformations have a number of interesting features: superprimary fields of dimension $(\frac{1}{2}, \frac{1}{2})$ are in natural correspondence with the physical states of string theory, being the vertex operators. As such they have a nice spacetime interpretation in terms of turning on spacetime fields. Appealing though they are canonical deformations have also significant drawbacks. They do not appear to describe spacetime fermions and R-R bosonic fields which are written in terms of spin fields. Spin fields cannot be written as superfields. These string backgrounds have attracted interest recently due to the conjectured AdS/CFT equivalence [7]. We might attempt to identify the bosonic component $\Phi_B(\sigma)$ of the canonical deformation with the appropriate spacetime gravitino vertex operator

$$\begin{aligned} \delta T(\sigma) = \Phi_B(\sigma) &= \Psi_\mu^\alpha(X)S_\alpha e^{-\frac{\phi}{2}}\bar{\partial}X^\mu + \tilde{\Psi}_\mu^\alpha(X)(X)\tilde{S}_\alpha e^{-\frac{\phi}{2}}\partial X^\mu \\ &+ \partial_\lambda\Psi_\mu^\alpha(X)S_\alpha e^{-\frac{\phi}{2}}\tilde{\psi}^\lambda\tilde{\psi}^\mu + \partial_\lambda\tilde{\Psi}_\mu^\alpha(X)\psi^\lambda\psi^\mu\tilde{S}_\alpha e^{-\frac{\phi}{2}}. \end{aligned} \quad (2)$$

In order to calculate δT_F we need to calculate the commutator of $\Phi_B(\sigma)$ with the supercurrent $T_F(\sigma)$. The commutator of the vertex operator which is written in terms of spin fields with the supercurrent T_F is not well-defined since the corresponding OPE in the complex plane involves branch cut singularities

$$\begin{aligned} T_F(z)\Phi_B(w) &= \frac{\gamma^\lambda_{\alpha\dot{\beta}}\partial_\lambda\Psi_\mu^\alpha(X)S^{\dot{\beta}}e^{-\frac{\phi}{2}}\bar{\partial}X^\mu}{(z-w)^{\frac{3}{2}}} + \frac{\gamma^\lambda_{\alpha\dot{\beta}}\tilde{\Psi}_\mu^\alpha(X)S^{\dot{\beta}}e^{-\frac{\phi}{2}}\partial X_\lambda\bar{\partial}X^\mu}{(z-w)^{\frac{1}{2}}} \\ &+ \frac{\gamma^\rho_{\alpha\dot{\beta}}\partial_\rho\partial_\lambda\Psi_\mu^\alpha(X)S^{\dot{\beta}}e^{-\frac{\phi}{2}}\tilde{\psi}^\lambda\tilde{\psi}^\mu}{(z-w)^{\frac{3}{2}}} + \frac{\gamma^\rho_{\alpha\dot{\beta}}\partial_\lambda\Psi_\mu^\alpha(X)S^{\dot{\beta}}e^{-\frac{\phi}{2}}\partial X_\lambda\tilde{\psi}^\lambda\tilde{\psi}^\mu}{(z-w)^{\frac{1}{2}}} \end{aligned} \quad (3)$$

where we have omitted terms that are either regular or have poles as singularities. This suggests then that the canonical deformations we have constructed in terms of superfields are not the most general solution to the deformation equations.

The failure to derive an expression for δT_F is puzzling. It is not clear if the presence of the spin fields breaks superconformal invariance or if superconformal invariance is realised in an apparent nonlocal manner.

Instead of deforming the stress energy superfield, we could have deformed the BRST charges Q and \bar{Q} . Nilpotency then requires

$$\{Q, \delta Q\} = 0, \quad \{\bar{Q}, \delta \bar{Q}\} = 0, \quad \{Q, \delta \bar{Q}\} + \{\bar{Q}, \delta Q\} = 0 \quad (4)$$

under the infinitesimal deformations

$$Q \rightarrow Q + \delta Q, \quad \bar{Q} \rightarrow \bar{Q} + \delta \bar{Q}. \quad (5)$$

Although the two approaches are equivalent in the presence of NS-NS backgrounds, they are not necessarily equivalent in the presence of gravitino and Ramond-Ramond backgrounds. In fact, given a deformed BRST charge, the components of the deformed stress energy superfield can be extracted by calculating the commutator or anticommutator of Q with the ghost field b or β , assuming that the commutator or anticommutator exists. In the presence of spin fields the commutator of β with the deformed BRST charge does not exist.

Next we shall derive the form of the deformation of the BRST operator which corresponds to turning on a spacetime gravitino. We shall employ a particular formalism which relates superconformal deformations and spacetime symmetries. In string theory, the stress energy superfield $\mathcal{T}_\Phi = T_{F(\Phi)} + \theta T_\Phi$ depends on the spacetime fields Φ . Spacetime symmetries are superconformal deformations that induce changes in the spacetime fields:

$$\begin{aligned} \delta T &= i[h, T_\Phi] = T_{\Phi+\delta\Phi} - T_\Phi \\ \delta T_F &= i[h, T_{F(\Phi)}] = T_{F(\Phi+\delta\Phi)} - T_{F(\Phi)}. \end{aligned} \quad (6)$$

The operator h is the generator of the spacetime symmetry; it is the zero mode of a sum of dimension $(1, 0)$ and $(0, 1)$ currents. The previous discussion can also be carried through in terms of the BRST formalism. Let us suppose that Q_Φ is nilpotent BRST charge, function of the spacetime fields. Then $\Phi \rightarrow \Phi + \delta\Phi$ is a spacetime symmetry if

$$\delta Q_\Phi = i[h, Q_\Phi] = Q_{\Phi+\delta\Phi} - Q_\Phi \quad (7)$$

In order to generate a gravitino background we need to perform a supersymmetry transformation about flat spacetime. The operator h that generates $N = 2$ spacetime supersymmetry transformations is [8]

$$h = \int d\sigma \left[\epsilon^\alpha(X) S_\alpha e^{-\frac{\phi}{2}} + \tilde{\epsilon}^\alpha(X) \tilde{S}_\alpha e^{-\frac{\tilde{\phi}}{2}} \right] \quad (8)$$

where S^α , $e^{-\frac{\phi}{2}}$, \tilde{S}^α and $e^{-\frac{\tilde{\phi}}{2}}$ are the spin fields for the two-dimensional fermions $\psi_\mu(\sigma)$, $\tilde{\psi}_\mu(\sigma)$ and the superconformal ghosts $\beta(\sigma)$, $\gamma(\sigma)$, $\tilde{\beta}(\sigma)$, $\tilde{\gamma}(\sigma)$ respectively. The integrand is again superprimary of dimension $(1, 0)$ only if the parameters $\epsilon^\alpha(X)$ and $\tilde{\epsilon}^\alpha(X)$ satisfy

$$\square \epsilon^\alpha(X) = \square \tilde{\epsilon}^\alpha(X) = 0, \quad \gamma^\mu \partial_\mu \epsilon^\alpha = \tilde{\gamma}^\mu \partial_\mu \tilde{\epsilon}^\alpha = 0. \quad (9)$$

Let's calculate $i[h, Q]$. The result is

$$\begin{aligned} \delta Q = i[h, Q] &= \int d\sigma \left(c\partial_\mu \epsilon^\alpha(X) S_\alpha e^{-\frac{\phi}{2}} \bar{\partial} X^\mu + c\partial_\mu \tilde{\epsilon}^\alpha(X) \tilde{S}_\alpha e^{-\frac{\tilde{\phi}}{2}} \partial X^\mu \right) (\sigma) \\ &+ \frac{1}{2} \int d\sigma e^\phi \eta \partial_\mu \tilde{\epsilon}^\alpha(X) \psi^\mu \tilde{S}_\alpha e^{-\frac{\tilde{\phi}}{2}} (\sigma). \end{aligned} \quad (10)$$

Although these backgrounds are pure gauges, we gain an insight into the form of the deformation which corresponds to turning on the appropriate spacetime fields. The obvious ansatz for the canonical deformation then corresponding to gravitino propagation about flat spacetime is

$$\delta Q = \int d\sigma \left[c\Psi_\mu^\alpha(X) S_\alpha e^{-\frac{\phi}{2}} \bar{\partial} X^\mu + c\tilde{\Psi}_\mu^\alpha(X) \tilde{S}_\alpha e^{-\frac{\tilde{\phi}}{2}} \partial X^\mu + \frac{1}{2} e^\phi \eta \tilde{\Psi}_\mu^\alpha(X) \psi^\mu \tilde{S}_\alpha e^{-\frac{\tilde{\phi}}{2}} \right] (\sigma) \quad (11)$$

since under a supersymmetry transformation the gravitinos transform about flat spacetime as, $\delta\Psi_\mu^\alpha = \partial_\mu \epsilon^\alpha$ and $\delta\tilde{\Psi}_\mu^\alpha = \partial_\mu \tilde{\epsilon}^\alpha$. This particular ansatz does not obey the deformation equations

$$\{Q, \delta Q\} = 0, \quad \{\bar{Q}, \delta\bar{Q}\} = 0, \quad \{Q, \delta\bar{Q}\} + \{\delta Q, \bar{Q}\} = 0 \quad (12)$$

and it has to be supplemented with extra terms. We find that

$$\begin{aligned} \delta Q &= \int d\sigma \left[c\Psi_\mu^\alpha(X) S_\alpha e^{-\frac{\phi}{2}} \bar{\partial} X^\mu + c\tilde{\Psi}_\mu^\alpha(X) \tilde{S}_\alpha e^{-\frac{\tilde{\phi}}{2}} \partial X^\mu + c\partial_\lambda \Psi_\mu^\alpha(X) S_\alpha e^{-\frac{\phi}{2}} \tilde{\psi}^\lambda \tilde{\psi}^\mu \right. \\ &+ \left. c\partial_\lambda \tilde{\Psi}_\mu^\alpha(X) \psi^\lambda \psi^\mu \tilde{S}_\alpha e^{-\frac{\tilde{\phi}}{2}} + \frac{1}{2} e^\phi \eta \tilde{\Psi}_\mu^\alpha(X) \psi^\mu \tilde{S}_\alpha e^{-\frac{\tilde{\phi}}{2}} \right] (\sigma) \end{aligned} \quad (13)$$

satisfy equations (12) if the gravitino wave function satisfies the following equations

$$\square\Psi_\mu^\alpha(X) = 0, \quad \gamma^\mu \partial_\mu \Psi_\nu^\alpha(X) = 0, \quad \partial^\mu \Psi_\mu^\alpha(X) = 0. \quad (14)$$

We observe the emergence of an equation of motion and a gauge condition. Of course the first equation is redundant since it follows from the Dirac equation. The gauge condition imposes transversality in X -space and eliminates part of the spin $\frac{1}{2}$ component of $\Psi_\mu^\alpha(X)$. The gauge condition does not fix the gauge completely and subsequently the fermionic wavefunction describes the emission of a gravitino (spin $\frac{3}{2}$ part) and a dilatino (spin $\frac{1}{2}$ part). In order to separate the gravitino and dilatino parts we write $\Psi_\mu^\alpha(X) = \chi_\mu^\alpha(X) + \gamma_\mu \lambda^\alpha(X)$ and demand that $\gamma^\mu \chi_\mu^\alpha = 0$. The gravitino and dilatino wavefunctions can be expressed in terms of $\Psi_\mu^\alpha(X)$

$$\lambda^\alpha(X) = \frac{1}{D} \gamma^\mu \Psi_\mu^\alpha(X), \quad \chi_\mu^\alpha(X) = \Psi_\mu^\alpha(X) - \frac{1}{D} \gamma_\mu \gamma^\lambda \Psi_\lambda^\alpha(X) \quad (15)$$

and the equations (16) imply

$$\gamma^\mu \partial_\mu \chi_\nu^\alpha(X) = 2\partial_\nu \lambda^\alpha, \quad \gamma^\mu \chi_\mu^\alpha(X) = 0, \quad \partial^\mu \chi_\mu^\alpha(X) = 0, \quad \gamma^\mu \partial_\mu \lambda^\alpha(X) = 0. \quad (16)$$

It is obvious again that the most general superconformal deformation is not canonical since it corresponds to turning on spacetime fields in a particular gauge. In a subsequent

publication we shall discuss how to relax the gauge condition and go beyond canonical deformations.

Finally we will discuss deformations that correspond to turning on Ramond-Ramond spacetime fields. These fields appear in type *II* superstrings and their vertex operators are written in terms of bispinors $F^{\alpha\beta}(X)$. The spinor indices are contracted by the left-right spin fields [9]. Strings in Ramond-Ramond backgrounds have been discussed in [10]. In order to generate a Ramond-Ramond background about flat spacetime we need to perform two consecutive supersymmetry transformations. This leads us (11) to the following ansatz for the superconformal deformation that corresponds to turning on Ramond-Ramond backgrounds

$$\delta Q = \int d\sigma c(\sigma) F^{\alpha\beta}(X) S_\alpha \tilde{S}_\beta e^{-\frac{1}{2}(\phi+\tilde{\phi})}. \quad (17)$$

This ansatz obeys the deformation equations if the bispinor $F^{\alpha\beta}(X)$ obeys the following equations

$$\square F^{\alpha\beta}(X) = 0, \quad \gamma_{\alpha\dot{\beta}}^\mu \partial_\mu F^{\alpha\beta}(X) = 0 \quad \gamma_{\beta\dot{\alpha}}^\mu \partial_\mu F^{\alpha\beta}(X) = 0. \quad (18)$$

The bispinors $F^{\alpha\beta}$ describe a collection of massless antisymmetric tensors $F^{\mu_1\mu_2\cdots\mu_d}$ as can be seen by expanding them in a complete basis of all gamma matrix antisymmetric products

$$F^{\alpha\beta}(X) = \sum_{n=0}^{10} \frac{i^n}{n!} F^{\mu_1\mu_2\cdots\mu_n}(X) (\gamma_{\mu_1\mu_2\cdots\mu_n})^{\alpha\beta}. \quad (19)$$

The chirality conditions that the bispinor obeys limits the number of the antisymmetric tensors present in the spectrum of the theory. In type *IIA* string theory $S_\alpha e^{-\frac{\phi}{2}}$ and $\tilde{S}_\beta e^{-\frac{\tilde{\phi}}{2}}$ have opposite chirality while in type *IIB* they have the same

$$(\gamma_{11})_\delta^\alpha F^{\delta\beta}(X) = \pm F^{\alpha\delta}(X) (\gamma_{11})_\delta^\alpha = F^{\alpha\beta}(X). \quad (20)$$

Furthermore we can convert the equations of motion for the bispinor wavefunction onto equations for the antisymmetric tensor wavefunctions by using γ matrix identities

$$\partial^{[\lambda} F^{\mu_1\cdots\mu_d]} = 0 \quad \partial_\lambda F^{\lambda\mu_2\cdots\mu_d} = 0 \quad (21)$$

which are the Bianchi identity and the massless equation of motion for an antisymmetric tensor field strength.

In this paragraph we shall summarize what we have done in this paper. We discussed deformations of superconformal field theories by varying the BRST operator Q such that $(Q + \delta Q)^2 = 0$, thus preserving nilpotency of the BRST operator to first order in δQ . These deformations describe strings propagating in gravitino and Ramond-Ramond backgrounds.

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