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Weak coupling instabilities of two-dimensional lattice electrons

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Abstract

We present a consistent and systematic study of a two-dimensional extended Hubbard model in the weak coupling limit. Quite generally, the electron gas is unstable towards a superconducting state even in the absence of phonons, since high-energy spin fluctuations create an effective attraction between the quasi-particles. However in the special case of a half-filled nearest-neighbor tight-binding band, the Fermi surface is nested and the system is at a Van Hove singularity. In this situation, there are six competing instabilities: s- and d-wave superconductivity, spin- and charge-density waves and two phases with circulating charge and spin currents, respectively. The required renormalization group formalism can be presented on a most elementary level, connecting the idea of the "parquet summation" to the more modern concept of Wilson's effective action. As a result, a rich phase diagram is obtained as a function of the model interaction. We also argue that the one-loop approximation is insufficient in a case where the Fermi surface is at a Van Hove singularity but not nested.

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1. Superconductivity in the repulsive Hubbard model

The possibility of a superconducting instability for electron systems with purely repulsive interactions and without phonons was already pointed out by Kohn and Luttinger [1]. In fact, superconductivity turns out to be the most general instability in the weak coupling limit. It is driven by low energy particle–particle excitations of the Fermi sea, which lead to infrared divergences in perturbation theory.

The main theoretical quantity of our investigation is the two-particle scattering vertex $\Gamma(k_1,k_2,k_3)$. It depends on the momenta and (Matsubara-) frequencies of the incoming (k_1,k_2) and outgoing $(k_3,k_4=k_1+k_2-k_3)$ particles. We calculate this quantity for an extended Hubbard model on a square lattice, in the limit of weak interactions and small energy scales. The knowledge of the vertex function allows for the computation of various generalized susceptibilities.

A consistent summation of the divergent terms in the perturbative calculation of Γ is given by the series of ladder diagrams depicted in Fig. 1, where the bare interaction is replaced by the p–p irreducible part $I_{\rm pp}(k_1,k_2,k_3)$ of the vertex. This means that the bare interaction is dressed by particle–hole (p–h) corrections, which can be interpreted as spin fluctuations. In the general case, when the Fermi surface is not nested and contains no Van Hove singularity, $I_{\rm pp}$ can be calculated within naïve perturbation theory, since the infrared divergences appear exclusively in the p–p channel. The next step is to diagonalize the kernel $I^{\rm BCS}(k,k')=I_{\rm pp}(k,-k,-k')$, where $k=(k_0,\mathbf{k})$ and $k'=(k'_0,\mathbf{k}')$ vary along the Fermi surface, i.e. $k_0=k'_0=0$ and $\varepsilon_{\mathbf{k}}=\varepsilon_{\mathbf{k}'}=\varepsilon_F$. The eigenvalue equation

$$\sum_{\mathbf{k}'}\delta(\varepsilon_{\mathbf{k}'}-\varepsilon_{\rm F})I^{\rm BCS}(\mathbf{k},\mathbf{k}')\Delta_{\mathbf{k}'}=\lambda\cdot\Delta_{\mathbf{k}} \qquad (1)$$

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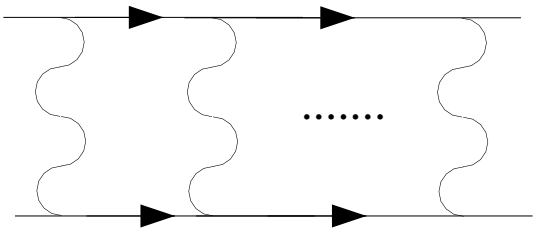

Fig. 1. Ladder diagrams.

is equivalent to the BCS gap equation in the limit of weak coupling and for $T \to T_c$. Attractive eigenvalues of I^{BCS} immediately introduce a pole in the vertex function Γ, a feature generally interpreted as the signature of a two-particle pairing state.

Even if the bare interaction is purely repulsive, p–h corrections produce momentum dependent attractive channels, thus leading to superconductivity. Calculations for the 2D Hubbard model show d_{xy}-wave pairing for fillings lower than 0.5 electrons per site and $d_{x^2-y^2}$-wave above 0.6 [2–4]. The energy scale of the instability increases drastically as half filling is approached.

2. The nearest-neighbor tight-binding band at half filling

For the half-filled nearest-neighbor tight-binding band, the Fermi surface is perfectly nested and contains two saddle points (Van Hove singularities). In this situation there are infrared divergences both in the p–p and p–h channels. A consistent treatment of these divergences amounts to a summation of the parquet series within logarithmic precision or, equivalently, the solution of a functional renormalization group (RG) equation to one-loop order [3,4]. Three slightly different variations of this equation have been derived and studied in recent years [6–8]. The three schemes agree within the leading logarithmic approximation, and differ only in subleading terms which are in any case not controlled within a one-loop approach.

A mainly analytic investigation of the RG equation in the limit of low energy scales yields a rich phase diagram as a function of the model interaction [5]. Apart from s- and $d_{x^2-y^2}$-wave superconductivity, spin- and charge-density waves as well as two phases with circulating charge or spin currents (d-density waves) appear as possible instabilities. This phase diagram is shown to be exact in the weak-coupling limit using symmetry arguments.

Unlike numerical studies of the functional RG equation [6–8], which rely on a discretization, we do not obtain a simultaneous divergence of different, say superconducting and spin-density wave susceptibilities as the energy scale is lowered, but only of the leading susceptibility.

3. More general Van Hove fillings

The case of a Fermi surface which is at Van Hove filling but not nested is particularly interesting because it potentially contains new physics such as non Fermi liquid behavior, itinerant ferromagnetism or Pomeranchuk instabilities. We argue that, unfortunately, the one-loop RG equation is not sufficient to provide reliable information in this case. The reason is that the one-loop equation generates a correct summation merely of the *leading* divergent terms in the perturbation series, but the above mentioned phenomena depend on subleading logarithmic divergences. A consistent weak-coupling treatment should at least include a self-consistent determination of the Fermi surface, self-energy corrections to both the quasi-particle weight and the Fermi velocity as well as two-loop corrections to the RG equation of the vertex. The required techniques to achieve this are not yet fully developed.

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