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Nonlinear Electrostatic Ion-Acoustic Waves in the Solar Atmosphere

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Abstract

Basing on recent solar models, the excitation of ion-acoustic turbulence in the weakly-collisional, fully and partially-ionized regions of the solar atmosphere is investigated. Within the frame of hydrodynamics, conditions are found under which the heating of the plasma by ion-acoustic type waves is more effective than the Joule heating. Taking into account wave and Joule heating effects, a nonlinear differential equation is derived, which describes the evolution of nonlinear ion-acoustic waves in the collisional plasma.

1 Introduction

The solar atmosphere is a rather complicated inhomogeneous plasma structure where the values of the main parameters, e.g. of the temperature and densities of charged and neutral particles, vary within orders of magnitude [1]. According to accepted numerical models, one may conclude that the solar atmosphere consists of three different layers: of the lower-laying weakly-ionized photospheric plasma with relatively strong particle collisions, of the weakly-collisional chromospheric plasma, and, above a very thin transition region, of the fully-ionized high-temperature coronal plasma. In the present paper, hydrodynamic models of linear and non-linear ion-acoustic waves in the weakly-collisional solar atmosphere are developed. In this region, the electron density n_e is about 10^{17} m^{-3}, the neutral-particle density n_n is $10^{16} - 10^{20}$ m^{-3}, the electron temperature equals $T_e \approx 4200 - 6300$ K, the electron plasma frequency ω_{pe} and the electron cyclotron frequency ω_{ce} are of the order of $3 \cdot 10^{10}$ Hz, electron Debye and electron cyclotron radius amout to $3 \cdot 10^{7}$ m^{3}, and the ratio of the electron-neutral collision frequency to the coulomb collision frequency ν_{ei} varies between 10^{-4} and 10^{-1}.

2 Hydrodynamic model

The investigation of the ion-acoustic waves is done starting with the hydrodynamic system of equations for electrons and ions taking thermal conductivity effects into account. The momentum and energy balances as well as continuity equations used read

$$m_e n_e \frac{d\vec{v}_e}{dt} = -\nabla(n_e T_e) - e n_e \vec{E} - \nu_{en} m_e n_e \vec{v}_e + \vec{R}_e, \tag{1}$$

$$m_i n_i \frac{d\vec{v}_i}{dt} = -\nabla(n_i T_i) + e n_i \vec{E} - \nu_{in} m_i n_i \vec{v}_i - \vec{R}_e + \eta_i \Delta \vec{v}_i, \tag{2}$$

$$\frac{3}{2} n_a \frac{dT_a}{dt} + n_a T_a \nabla \vec{v}_a = -\mathrm{div}\vec{q}_a + Q_a, \quad a = e, i \tag{3}$$

$$\frac{\partial n_a}{\partial t} + \nabla(n_a \vec{v}_a) = 0, \quad a = e, i. \tag{4}$$

Here the index $a = e$ designates the electrons and $a = i$ determines the ions, $d/dt = \partial/\partial t + v_a \nabla$. n_a, T_a, v_a are the density, temperature and velocity of the particle of kind a, respectively. Deriving Eq. (1, 2) the ion viscosity $\eta_i = n_i T_i/\nu_i$ and the collision frequencies of the charged particles with the neutrals ν_{an} were taken into account. $\nu_i = \nu_{in} + \nu_{ie} + \nu_{ii}$. The friction force $\vec{R}_e = -a_1 \nu_{ei} n_e m_e (\vec{v}_e - \vec{v}_i) - a_2 n_e \nabla T_e$ consists of two parts. One part occurs because of the coulomb collisions between the electrons and ions having a non-zero relative velocity. The second contribution is the thermoforce which is caused by the temperature gradient. The electron heat current $\vec{q}_e = a_2 n_e T_e (\vec{v}_e - \vec{v}_i) - a_3 n_e T_e \nabla T_e/(m_e \nu_{ei})$ is a consequence of the relative velocity of the electrons with respect to the ions, and of the gradient of the electron temperature. The ion heat current reads $q_i = -a_i n_i T_i \nabla T_i / m_i \nu_{ie}$. The amount of heat contained in the electron system because of the electron-ion collisions and the existence of a temperature gradient is $Q_e = a_1 m_e n_e \nu_{ei} (\vec{v}_e - \vec{v}_i)^2 + a_2 n_e (\vec{v}_e - \vec{v}_i) \nabla T_e - 3 m_e n_e \nu_{ei} (T_e - T_i)/m_i$, and the heat of the ion component because of the collisions with the electrons may be estimated by $Q_i = 3 m_e n_e \nu_{ei} (T_e - T_i)/m_i$. The coefficients a_1, a_2, a_3, and a_i depend on the type of the collision integral chosen for the description of the plasma. Within the Braginsky model [2], the coefficients equal $a_1 = 0.51$, $a_2 = 0.71$, $a_3 = 3.16$, and $a_i = 3.9$.

3 Linear ion-acoustic waves

In the following the changes of the electron and ion temperatures because of the wave and Joule heating by collisions will be considered. It is assumed that the equilibrium temperatures of the electrons and ions equal T_{eo} and T_{io} respectively. Further it is suggested that in the plasma weak disturbances of the density n_{a1}, velocity $\vec{v}_{a1}$ and temperature T_{a1} of the charged particles exist. The disturbances are approximately proportional $\exp(-i\omega t + ikr)$, and the quasineutrality condition $n_{io} = n_{eo} = n_o$ is satisfied. Then, from the continuity equation Eq. (4) follows

$$n_{e1} = \frac{\vec{k}\vec{v}_{e1}}{\omega_e^*}, \quad n_{i1} = \frac{\vec{k}\vec{v}_{i1}}{\omega_i^*}, \quad \omega_e^* = \omega - \vec{k}\vec{v}_{eo}, \quad \omega_i^* = \omega - \vec{k}\vec{v}_{io}. \tag{5}$$

$\vec{v}_{oe}$ and $\vec{v}_{oi}$ are constant drift velocities of the electrons and ions, which may be caused by constant electric fields or by the existence of beams of charged particles. From the energy balances Eqs. (3) and Eq. (5), it follows in first order of the wave disturbances ($v_{oi} = 0$, v_{te}-thermal velocity of particle a)

$$\frac{T_{e1}}{T_o} = \frac{n_{e1}\omega_e^*}{n_o \hat{\omega}_e} \left(1 + i\left[2a_1 \frac{\nu_e k v_o}{k^2 v_{te}^2} + 3\frac{m_e \nu_e \omega_i^*}{m_i \omega_e^* \hat{\omega}_i}\right]\right) \left(1 + 9\frac{m_e^2 \nu_e^2}{m_i^2 \hat{\omega}_i \hat{\omega}_e}\right)^{-1}, \tag{6}$$

$$\frac{T_{i1}}{T_o} = \frac{\omega}{d\hat{\omega}_i} \left\{ \frac{n_{i1}\omega_i^*}{n_o \hat{\omega}} + \frac{3i m_e \nu_e n_{e1} \Omega_e}{m_i \omega \hat{\omega}_e n_o} - \frac{3i m_e \nu_e n_{i1}}{m_i \hat{\omega}_e n_o} \left(a_2 + \frac{2i a_1 \nu_e k v_o}{k^2 v_{te}^2}\right) \right\}, \tag{7}$$

$\Omega_e = \omega_e^* + a_2\omega + i2a_1\nu_e k v_o/(k^2 v_{te}^2)(\omega - \vec{k}\vec{v}_o/2)$, $d = 1 + 9m_e^2\nu_e^2/(m_i^2\hat{\omega}_i\hat{\omega}_e)$, $\hat{\omega}_e = 3\omega_e^*/2 + i a_3 k^2 v_{te}^2/\nu_e + 3i m_e \nu_e/m_i$, $\hat{\omega}_i = 3\omega_i^*/2 + i a_i k^2 v_{ti}^2/\nu_e + 3i m_e \nu_e/m_i$. Under the condition $|d-1| \ll 1$, the heating of the plasma by the waves is stronger than the Joule heating.

In the following, it is assumed that the plasma is almost fully ionized and consists of electrons and heavy ions. Ion viscosity plays the main role in the wave damping, and the electron temperature gradients cause an electron heat current. From Eqs. (1, 2) then follows ($m_e \ll m_i$, $\vec{v} \approx \vec{v}_i$, and $n_e = n_i = n$)

$$\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \nabla)\vec{v}_i = -\frac{1}{n m_i}\nabla(T_e + T_i)n + \frac{\eta_i}{m_i}\Delta\vec{v}_i - \nu_{in}\vec{v}_i. \tag{8}$$

Linearizing Eq. (8), one may find for the ion velocity

$$\vec{v}_i = \frac{\vec{k}c_s^2}{\omega_i^{**} + ik^2\eta}\left(\frac{n}{n_o} + \frac{1}{1+\tau}\left[\tau\frac{T_{e1}}{T_{eo}} + \frac{T_{i1}}{T_{io}}\right]\right), \quad T_e = T_{eo} + T_{e1}, \quad T_i = T_{io} + T_{i1}, \tag{9}$$

$$\omega_i^{**} = \omega - kv_{io} + i\nu_{in}, \quad c_s^2 = \frac{(T_{eo} + T_{io})k_B}{m_i}, \quad \tau = \frac{T_{eo}}{T_{io}}. \tag{10}$$

In the case $\mid d - 1 \mid \ll 1$, from Eqs. (5-7) the dispersion equation

$$\omega_i^*\omega_i^{**} + ik^2\eta\omega_i^* = k^2c_s^2(1+a+ib), \quad a = \tau\mathrm{Re}r_e + \mathrm{Re}r_i, \quad b = \tau\mathrm{Im}r_e + \mathrm{Im}r_i, \quad r_e = \frac{\Omega_e}{\hat{\omega}_e}, \quad r_i = \frac{\omega_i^*}{\hat{\omega}_i} \tag{11}$$

may be found. Introducing as usual the real ω_r and imaginary γ parts of the wave frequency, $\omega = \omega_r + i\gamma$, one has

$$\omega_r^2 = k^2c_s^2(1+a), \quad \gamma = \frac{1}{2}\left[bkc_s - \left(\nu_{in} + \frac{k^2v_{ti}^2}{\nu_i}\right)\right]. \tag{12}$$

Thus, under the condition $bkc_s > \nu_{in} + k^2v_{ti}^2/\nu_i$, an instability occurs. If in the plasma the collisions between charged and neutral particles are negligible and the coulomb collisions play the main role, this condition may be represented by $v_o/c_s > k^2v_{te}^2/(2a_1\nu_{ei}^2(1+\tau)) = \beta_{cr}$. Obtaining the last inequality, it was used that $b \approx 2a_1\nu_{ei}\vec{k}\vec{v}_o/k^2v_{te}^2$. Thus, for the solar atmosphere at altitudes of $h = 500$ km, with $T_e \approx 4.2 \cdot 10^3$ K, $v_{te} \approx 2.5 \cdot 10^5$m/s and $\nu_e \approx 5 \cdot 10^7$ Hz, one finds $\beta_{cr} \approx 5 \cdot 10^{-3} \cdot k^2$. At altitudes $h = 1000$ km, one has $T_e \approx 6 \cdot 10^3$ K, $v_{te} \approx 3 \cdot 10^5$m/s and $\nu_{ei} \approx 2.2 \cdot 10^6$ Hz, $\beta_{cr} \approx 2 \cdot 10^{-2} \cdot k^2$. Then it follows from $k < \sqrt{v_o/c_s}\nu_e/v_{te}$, that $k < 200$ m^{-1} at $h = 500$ km, and $k < 7.3$ m^{-1} at $h = 1000$ km. At the temperature minimum with $T_{min} = 4.2 \cdot 10^3$ K ($h = 515$ km), $v_{te} \approx 2.5 \cdot 10^5$m/s and $\nu_e \approx 8 \cdot 10^6$ Hz, one has wave numbers below 32 m^{-1} and wave lengths above 19 cm. Thus the condition for the plasma heating by wave-particle collisions $\mid d - 1 \mid \ll 1$ is satisfied, $kc_s \gg 3 \cdot 5 \cdot 10^{-4} \cdot \nu_e = 1.2 \cdot 10^{-2}$, and thus $k \gg 2 \cdot 10^{-6}$ m^{-3} as $c_s \approx 5.9 \cdot 10^3$ m/s. From the above estimates follows, that in the case $3m_e\nu_e/m_i \ll a_2k^2v_{te}^2/\nu_e$ the plasma of the solar atmosphere is essentially heated by the waves. If there would not be any heating by the waves, the considered oscillations would be ion-acoustic ones. If $a \ll 1$ is satisfied, the heating process modifies the wave dispersion only a little. The amplitudes of the linear ion-acoustic waves increase ($\gamma > 0$) if the heating by the waves (first term in the brackets of Eq. (12)) is more effective than the damping of the waves by the collisions of the ions with the neutral particles and by the ion viscosity. The waves increase because of the (assumed to be) constant electron drift velocity, which may be a consequence of almost constant electric fields or of the appearance of electron beams.

4 Nonlinear ion-acoustic waves

As follows from the linear theory, under supercritical conditions in the solar atmosphere occur ion-acoustic waves with increasing amplitudes. Thus here the system of equations Eqs. (1-4) will be solved again, but this time the wave disturbances will not be considered as small ones. Taking into account the main terms describing the increase of the wave amplitudes (heat currents because of temperature gradients) and their saturation (ion viscosity), Eqs. (1-4) may be transformed into

$$\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \nabla)\vec{v}_i = -c_s^2(\nabla n + 0.5\nabla T_e) + \eta \Delta \vec{v}_i - \nu_{in}\vec{v}_i, \quad (13)$$

$$\frac{\partial}{\partial t}\ln n + \vec{v}_i \nabla n + \mathrm{div}\vec{v}_i = 0, \quad (14)$$

$$nT_e \frac{d}{dt}\ln\left(\frac{T_e^{3/2}}{n}\right) = \mathrm{div}\kappa_e \nabla T_e + nT_e \nabla \ln n. \quad (15)$$

For simplicity, in Eqs. (13, 15) it was assumed $T_{eo} = T_{io} = T_o$, $c_s^2 = 2T_o/m_i$ and $\kappa_e = a_2 n_e v_{te}^2/\nu_e$, and the ion heating was neglected. The physical sense of the energy balance Eq. (15) consists in the fact, that the changes of the entropy ($S = T_e^{3/2}/N$ is the entropy transferred from the system by one electron, $N = n_e V$ represents the number of electrons) are caused by dissipative processes which are the result of the gradients of the temperature and the density of the charged particles. In this case, Eq. (15) may be approximately represented by

$$\kappa_o \Delta T_e + (\vec{v}_o \nabla)\ln N = 0, \quad \kappa_o = a_2 v_{te}^2/\nu_e, \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \quad (16)$$

In the case of weak disturbances, Eq. (16) gives a relation between the electron temperature disturbances and the density disturbances, $T_{e1}/T_o \sim ikv_o\nu_e/(a_2k^2v_{te}^2) = bn_{e1}/n_o$. Further, combining Eqs. (13, 14, 16) one finds

$$\frac{\partial^2 \vec{v}_i}{\partial t^2} - c_s^2 \Delta \vec{v}_i = -\frac{c_s^2}{2\kappa_o} v_o \nabla \vec{v}_i + \eta_o \frac{\partial}{\partial t}\Delta \vec{v}_i - \nu_{in}\frac{\partial \vec{v}_i}{\partial t} - \frac{\partial}{\partial t}(\vec{v}_i \nabla)\vec{v}_i. \quad (17)$$

Taking the relation (14) between the density and the velocity of the charged particles into account, an analogeous equation may also be derived for N.

The nonlinear partial differential equation describes the nonlinear evolution of ion-acoustic waves. The left side of Eq. (17) determines the propagation velocity of a nonlinear structure (of the nonlinear wave). The first term on the right side of Eq. (17) describes the excitation of the wave by the current (caused by a constant electric field), the second term is responsible for the high-frequency harmonics because of the ion viscosity, and the third contribution describes the damping by the collisions with the neutral particles. The last term takes the nonlinear effect into account.
It is extremely difficult to find an analytical solution of the nonlinear differential equation Eq. (17). Thus, here the evolution of one-dimensional nonlinear structures will be investigated. The one-dimensional form of Eq. (17) reads

$$\frac{\partial^2 v_i}{\partial t^2} - c_s^2 \frac{\partial^2 v_i}{\partial x^2} = -\frac{c_s^2}{2\kappa_o} v_o \frac{\partial v_i}{\partial x} + \eta_o \frac{\partial^3}{\partial t \partial x^2} v_i - \nu_{in}\frac{\partial v_i}{\partial t} - \frac{1}{2}\frac{\partial^2}{\partial t \partial x} v_i^2. \quad (18)$$

To look for possible stationary nonlinear structures in the plasma, in Eq. (18) a new spatial coordinate $s = x - vt$ will be introduced, where v is the velocity of the stationary structure. Thus, one obtains the equation

$$\beta_o \frac{\partial^3 v_i}{\partial s^3} + (M^2 - 1)\frac{\partial^2 v_i}{\partial s^2} + \alpha \frac{\partial v_i}{\partial s} - \sigma \frac{\partial^2 v_i^2}{\partial s^2} = 0. \tag{19}$$

$M = v/c_s$ equals the propagation velocity of the structure which is normalized by the sound velocity, $\beta = v_i \eta_o / c_s^2$, $\alpha = v_o/(2\kappa_o) - M\nu_i/c_s$, and $\sigma = 1/c_s^2$. Integrating Eq. (19) once over s, one observes (v_i, $\partial v_i/\partial s$ and $\partial^2 v_i/\partial s^2$ vanish at $s \to \infty$)

$$\beta_o \frac{\partial^2 v_i}{\partial s^2} + (M^2 - 1)\frac{\partial v_i}{\partial s} + \alpha v_i - \sigma \frac{\partial v_i^2}{\partial s} = C. \tag{20}$$

Solving Eq. (20) in first order with respect to $\mid M^2 - 1 \mid$ (using the Krylov-Bogoljubov method [3]), one gets for the ion velocity disturbances

$$v_i = r(s)\cos\varphi(s) = r_o \exp\left(-\frac{(M^2-1)s}{2\beta_o}\right)\cos\left(\varphi_o + \sqrt{\frac{\alpha}{\beta_o}}s\right), \tag{21}$$

where $r_o = r(s = 0)$ and $\varphi_o = \varphi(s = 0)$ are the amplitude and the phase of the nonlinear wave at $s = 0$.

5 Conclusions

• The exciation of ion-acoustic turbulence in the weakly-collisional, fully and partially ionized regions of the solar atmosphere is studied taking into account temperature gradients.
• Therefore the variations of the electron and ion temperatures because of effective particle collisions - including heating by the waves - are considered.
• Nonlinear structures in the solar atmosphere are studied using the hydrodynamic system of equations for electrons and ions taking thermal conductivity effects into account.
• A nonlinear partial differential equation is derived describing the behaviour of nonlinear solar ion-acoustic waves.

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