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Research Article

Research on Submarine Straight-Line Track Control Underwater Based on Nonlinear Proportion Differential

Chenggang Gong,[1] Xia Zhu,[1] Fei Wang,[2] Pan Jiang,[3] and Wei Yu[4]

[1] *School of Statistics and Mathematics, Zhongnan University of Economics and Law, Wuhan 430073, China*
[2] *Business School, Sias International University, Xinzheng 451150, China*
[3] *School of Energy and Power Engineering, Wuhan University of Technology, Wuhan 430070, China*
[4] *State University of New York at Binghamton, Binghamton, NY 13902, USA*

Correspondence should be addressed to Xia Zhu; challengezhuzhu@163.com

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With the development of deep submergence technology, submarine is widely used in many aspects as marine analysis and detection of marine resources. For the reason of strong nonlinearity and coupling in submarine exercise, it is difficult to get satisfactory control effect by conventional control method. In order to control objective of stable straight-line suspension underwater, how to control the change of rudder angle to stabilize attitude and improve the control performance is researched from feature analysis to submarine. Aiming at improving the global stability, kinetic character of straight-line suspension movement underwater is analyzed and modeled firstly, and model of nonlinear relationship about change of rudder angle and attitude is built then. Based on the conditions of global stability asymptotically of submarine tracking control underwater and the physical significance of tracking control by nonlinear proportion differential, a controller is designed for controlling horizontal rudder angle and vertical rudder angle by dynamic feedback, which achieve the balance of tracking controlling both in local and global and guarantee global stable convergence asymptotically. At last, the stability, effectiveness, and global convergence of controller are proved by the simulation experiment.

1. Introduction

During the stable operation under the water, the submarine would move horizontally between two points on purpose for some time. In order to improve the stability and to reduce the energy consumption and cost, the submarine maintains horizontal and straight navigation through operation facilities' control. The autopilot is the indispensable crucial equipment in the system operated by submarine. The traditional course control cannot meet the requirement of straight navigation, so it is quite meaningful to research on the horizontal and straight navigation via autocontrolling. For the feature of inertance, time-lag, and nonlinearity during the process of motion under the water, as well as the impact of environment interference including model parameters and storm disturbing, it is very difficult to control the submarine tracing. Therefore, the research on submarine tracing control has attracted extensive concern from both academia and industries.

The straight track control of the surface ships is a common manner of wake control, which has been widely researched on [1]. At present, the study focuses on the stability of track control via nonlinear feedback control [2], back-stepping technology [3], and feedback linearization [4] to ensure the control effect of global stability of the track. Aiming at straight unstable characteristics of large surface warships, [5] has researched the global asymptotic stability of direction PD control and presented the condition for stability.

Due to the vertical sideway and pitching, as well as inability to correct through GPS under the water, it is more difficult to control the track of the submarine. The controlling equipment of submarine is more complex, and the system variables such as motion and power are nonlinear of each other. So only the nonlinear system control can accurately solve the aforementioned issues. For now, on account of nonlinear controlling system we usually utilize phase-plane technique, Lyapunov method, input/output stable method, approximation linearized method, and describing function method [6]. The research on differential geometry control method overcomes the limits from local linearization and small range of motion and achieves large range of analysis and integration for dynamic system control [7]. Based on the differential geometry theory, the nonlinear control systems theory implements the linearization of the nonlinear system via static state feedback and transformation of coordinates under certain conditions. The nonlinear system is decoupled and linear in the new state space through suitable diffeomorphism, as well as corresponding static or dynamic feedback [8]. The limitation of the research on submarine provides the development ideas of this paper.

Aiming at assurance for the global stability of the control system on submarine, we study the issues of how to stabilize the track control performance via rudder angles transformation during the project practice based on the features of the submarine. This paper primarily analyzes the dynamics features about the linear suspension motion of the submarine, models the nonlinear relationship between the rudder angle and the posture change, and designs the controller to dynamic feedback for the plane angle and the vertical rudder angle according to the conditions of global asymptotic stability controlled by the submarine track and the physical significance controlled by the differential on the nonlinear proportion of the track. Our methods effectively achieve the balance control both locally and globally to ensure the global asymptotic stable convergence.

2. Modeling the Submarine Motion

Since digital computers and devices are used to control ships, it is natural to model ships and their controllers as nonlinear sampled-data systems. In this brief, we extend straight-line trajectory tracking control of continuous-time underactuated ships with state feedback controllers [9] to that of sampled-data underactuated ships with both state and output feedback controllers. We introduce a straight line as a reference trajectory and a reference nonzero forward speed for the ship. On the basis of the Euler approximate models, we design both state and output feedback controllers.

The suspending motion of the submarine under the water is effected by gravity, impetus, and current force, which is complicated and capricious. If the motion of the submarine is to be controlled, it has to analyze the dynamical model of the submarine under the water. For the relative motion of submarine and the fluid, the water power changes constantly as the interaction between its motion state and the marine environment. The main gesture parameters are direction, heading, depth, rolling, and pitching. In the case of low speed and diminutive longitudinal trim, the submarine is disintegrated into the motions of four degrees of freedom in 2 planes.

(1) The motion above the water: we utilize the projection above the water of the submarine to analyze the direction, heading, and the remaining and the change of the horizontal speed without considering the change of floating, diving, or rolling.

(2) The motion of verticality: we study the depth, longitudinal section, and the remaining and change of the snorkeling speed from the view of longitudinal axis.

In the three-dimensional coordinate system under the water, we regard the submarine as a rigid body. The submarine motion with six degrees of freedom (the lateral displacement y, the longitudinal displacement x, the vertical displacement z, the heading angle α, the roll angle β, and the pitching angle γ) is modeled as

$$\begin{aligned}
\widehat{x} &= u\cos\alpha\cos\gamma - v\sin\alpha\cos\gamma, \\
\widehat{y} &= u\sin\alpha\cos\gamma + v\cos\alpha\cos\gamma, \\
\widehat{z} &= u\sin\gamma + v\sin\gamma, \\
\widehat{\alpha} &= \varepsilon, \\
\widehat{\beta} &= \xi, \\
\widehat{\gamma} &= \delta,
\end{aligned} \tag{1}$$

where u denotes the forward speed along the axle wire of rigid body, v denotes the sideway speed perpendicular to the axle wire of rigid body, ε denotes the degree of heading angle, ξ denotes the speed of roll rotation angle, and δ denotes the angular velocity of pitching angle. Considering that transverse speed approaches 0 in general case and the rolling of the submarine does not affect the track, we simplify formula (1) as

$$\begin{aligned}
\widehat{x} &= u\cos\alpha\cos\gamma, \\
\widehat{y} &= u\sin\alpha\cos\gamma, \\
\widehat{z} &= u\sin\gamma, \\
\widehat{\alpha} &= \varepsilon, \\
\widehat{\gamma} &= \delta.
\end{aligned} \tag{2}$$

Considering the effect of nonlinear items in the submarine model, we utilize second-order nonlinear equation of motion to describe the submarine motion and the effect of rudder. The motion equation in the horizontal plane is denoted as

$$T_1\widehat{\alpha} + H_A(\widehat{\alpha}) = K_1\theta_1, \tag{3}$$

where θ_1 denotes the level control rudder angle, T_1 denotes the operation performance parameter of navigation, K_1 denotes the coefficient of hydroplane steering in submarine, and both T_1 and K_1 are greater than 0. H_A reflects the mechanical control and hysteresis feature of the state, which is indicated as

$$H_A(\widehat{\alpha}) = m_1\widehat{\alpha}^3 + n_1\widehat{\alpha}, \tag{4}$$

where m_1 is the coefficient. When $n_1 = 1$, the submarine performs the stability in straight line; when $n_1 = -1$, the submarine performs instability in straight line, and the motion under the water presents instable state in straight line. Therefore we choose $n_1 = -1$.

The motion equation in the vertical plane is denoted as

$$T_2\widehat{\gamma} + H_B(\widehat{\gamma}) = K_2\theta_2, \tag{5}$$

where θ_2 denotes the vertical control rudder angle, T_2 denotes the operation performance parameter of snorkeling and diving, K_2 denotes the coefficient of steering effect in vertical rudder, and both T_2 and K_2 are greater than 0. H_B reflects the mechanical control and hysteresis feature of the state, which is indicated as

$$H_B(\widehat{\gamma}) = m_2\widehat{\gamma}^3 + n_2\widehat{\gamma}. \tag{6}$$

We choose $n_2 = -1$ as well.

Since the longitudinal displacement along the forward direction does not affect the straight navigation, we ignore the analysis of longitudinal displacement x. Integrating (2) into (6), we nonlinearly model the straight instable submarine under the water as follows:

$$\begin{aligned}
\widehat{y} &= u\sin\alpha\cos\gamma,\\
\widehat{z} &= u\sin\gamma,\\
\widehat{\alpha} &= \varepsilon,\\
\widehat{\gamma} &= \delta,\\
\widehat{\varepsilon} &= -\frac{n_1}{T_1}\varepsilon - \frac{m_1}{T_1}\varepsilon^3 + \frac{K_1}{T_1}\theta_1,\\
\widehat{\delta} &= -\frac{n_2}{T_2}\delta - \frac{m_2}{T_2}\delta^3 + \frac{K_2}{T_2}\theta_2,
\end{aligned} \tag{7}$$

where the target of the controller in the automatic pilot in regard to straight navigation under the water is to assure that the offset displacement y, the vertical displacement z, the bow declination α, and the pitching angle γ are all approaching zero. Then we achieve controlling the straight navigation.

On account of the nonlinear model of (7), we design the nonlinear state feedback control rate as follows:

$$\begin{aligned}
\theta_1 &= -k_D'(\alpha + f(y)) - k_D'\cdot\varepsilon,\\
\theta_2 &= -k_P''(\gamma + g(z)) - k_D''\cdot\delta.
\end{aligned} \tag{8}$$

On account of the closed-loop system consisted by (7) and (8), the necessary and sufficient conditions of global asymptotic stability for the submarine under the water are as follows:

(1) when $|y| \to \infty$, $\int_0^y \sin(f(y))dy \to \infty$;

(2) when $y \neq 0$, $\sin(f(y))\cos(g(z))y > 0$;

(3) $f'(y) > 0$, $g'(z) > 0$;

(4) $k_P' > 0$, $k_D' > -n_1/K_1$, $m_1 \geq 0$;

(5) as to arbitrary $y \in R$, we have $(Kk_D'/T_1 + n/T_1) > \sup(uf'(y))$.

3. Local Optimal Control Analytical Solution in Zero Equilibrium Point of Submarine Track

Formula (7) decomposes to a linear expansion in zero equilibrium point and retains one equation term; new formula can be deduced as follows:

$$\begin{aligned}
\widehat{y} &= u\cos(\alpha)\alpha\cos(\gamma),\\
\widehat{z} &= u\cos(\gamma)\gamma,\\
\widehat{\alpha} &= \varepsilon,\\
\widehat{\gamma} &= \delta,\\
\widehat{\varepsilon} &= -\frac{n_1}{T_1}\varepsilon - 3\frac{m_1}{T_1}(0)^2\varepsilon + \frac{K_1}{T_1}\theta_1,\\
\widehat{\delta} &= -\frac{n_2}{T_2}\delta - 3\frac{m_2}{T_2}(0)^2\delta + \frac{K_2}{T_2}\theta_2.
\end{aligned} \tag{9}$$

Various deviation angles are very small when submarine is sailing in straight line, and we can consider $\alpha \approx 0$, $\gamma \approx 0$, $\varepsilon \approx 0$, and $\delta \approx 0$, so formula (9) can be simplified as follows:

$$\begin{aligned}
\widehat{y} &= u\alpha,\\
\widehat{z} &= u\gamma,\\
\widehat{\alpha} &= \varepsilon,\\
\widehat{\gamma} &= \delta,\\
\widehat{\varepsilon} &= -\frac{n_1}{T_1}\varepsilon + \frac{K_1}{T_1}\theta_1,\\
\widehat{\delta} &= -\frac{n_2}{T_2}\delta + \frac{K_2}{T_2}\theta_2.
\end{aligned} \tag{10}$$

Let $y_1 = y/u$ and $z_1 = z/u$; assume $x = [y_1, y_2, \alpha, \gamma, \varepsilon, \delta]^T$ and $\theta = [\theta_1, \theta_2]^T$. It can be gotten from formula (10) as follows:

$$\widehat{x} = Ax + B\theta, \tag{11}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\dfrac{n}{T_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\dfrac{n}{T_2} \end{bmatrix},$$
$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \dfrac{K_1}{T_1} & 0 \\ 0 & \dfrac{K_2}{T_2} \end{bmatrix}. \tag{12}$$

According to mechanical property and unit of different rudder angle, in order to guarantee value invariability of weight coefficient φ in performance criterion function, it can describe relation between state variables y_1 and y_2 and control angles θ_1 and θ_2 by quadratic performance criterion function as follows:

$$J = \int_0^{\infty} \left(y_1^2 + z_1^2 + \frac{\varphi_1 180^2}{\pi^2}\theta_1^2 + \frac{\varphi_2 180^2}{\pi^2}\theta_2^2 \right) dt, \tag{13}$$

where the unit of rudder angles θ_1 and θ_2 is rad, φ_1 is the coefficient of horizontal rudder, and φ_2 is the coefficient of vertical rudder, which is set by experimenter. The smaller φ is, the higher the precision of tracking is, and the range of control angle is larger by this time. The larger φ is, the lower the precision of tracking is, the range of control angle is smaller by this time, and loss of steering engine reduces less. Value range of φ is in the convergence of [0.1, 10] usually.

Algebraic Riccati equation of formula (11) can be translated as follows:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0. \tag{14}$$

So the feedback control rate of optimal state in zero equilibrium point of submarine track can be calculated as follows:

$$\theta_1 = -k'_y y_1 - k'_\alpha \alpha - k'_\varepsilon \varepsilon,$$
$$\theta_2 = -k''_z z_1 - k''_\gamma \gamma - k''_\delta \delta. \tag{15}$$

4. Controller Design

According to eventually uniform boundedness principle, we have the following conclusion: only $\Psi(y_1, z_1) = (u(\sqrt{y_1^2 + z_1^2})/\sqrt{1 + y_1^2 + z_1^2}) - 1/4(\varphi_1^2 + \varphi_2^2) > 0$ is satisfied; the system is uniform and stable boundedness ultimately.

Define $x_1 = y_1$, $x_2 = \sqrt{y_1^2 + z_1^2}$; make

$$V_1 = \frac{1}{2}x_1^2 + \frac{1}{2}\theta^T \alpha \theta + \frac{1}{2}\alpha^{-1}\varphi^2,$$
$$V_2 = V_1 + \frac{1}{2}x_2^2 + \frac{1}{2}\theta^T \gamma \theta + \frac{1}{2}\gamma^{-1}\varphi^2. \tag{16}$$

According to approximation of Mamdani fuzzy system structured by multiple fuzzy inference engine, singleton fuzzifier, centre average defuzzifier, and specific membership functions, define

$$F(x_1) = f'',$$
$$x_2 = \left[y, \frac{\partial \alpha}{\partial y}, \frac{\partial \gamma}{\partial z}, \theta \right]^T. \tag{17}$$

Control parameters of straight-line track underwater can be as follows:

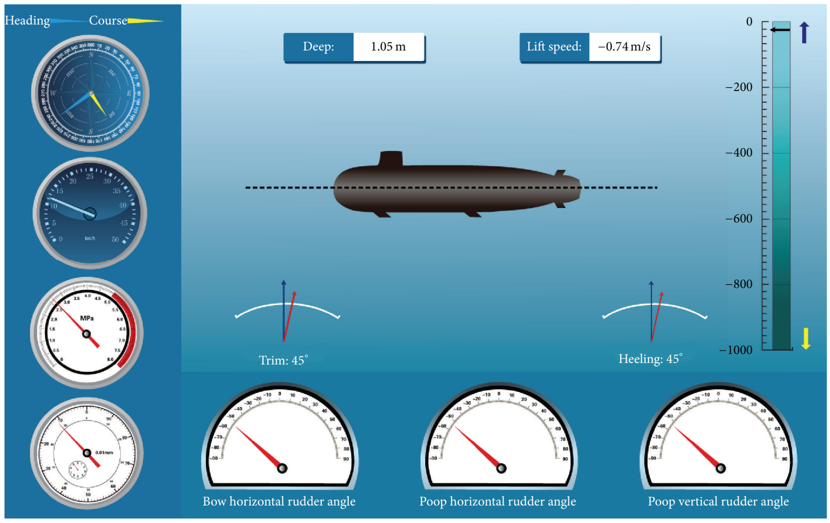
$$\theta_1 = -k'_y \left(\alpha + \arctan\left(\frac{x_1}{k'_\alpha} \right) \right) - k'_\varepsilon \varepsilon,$$
$$\theta_2 = -k''_z z_1 \left(\gamma + \arctan\left(\frac{x_2}{k'_\gamma} \right) \right) - k''_\delta \delta. \tag{18}$$

5. Simulative Experiment

In this section we present an experimental study of our straight-line track control algorithms for detecting effectiveness and correctness of submarine underwater in simulation cabin. The experiment simulates the process of snorkeling and sailing through a virtual submarine system developed by *Dalian JXD Soft Ltd.*, which can realistically simulate 6-DOF motion and influence of ocean current environment of submarine underwater and water surface. In this experiment, the initial conditions of submarine are as follows: length is 149.5 meters, wide is 12.8 meters, draught is 12 meters, max rudder angle is 45°, the inertia link of motion model is used by a time constant as 5 s, the whole structure appears capsule shape, and the initial position is as follows: depth is 30 meters, heading angle is due north, pitching angle is 0.03, and speed is 12 knot. The operation interface of simulator is shown in Figure 1.

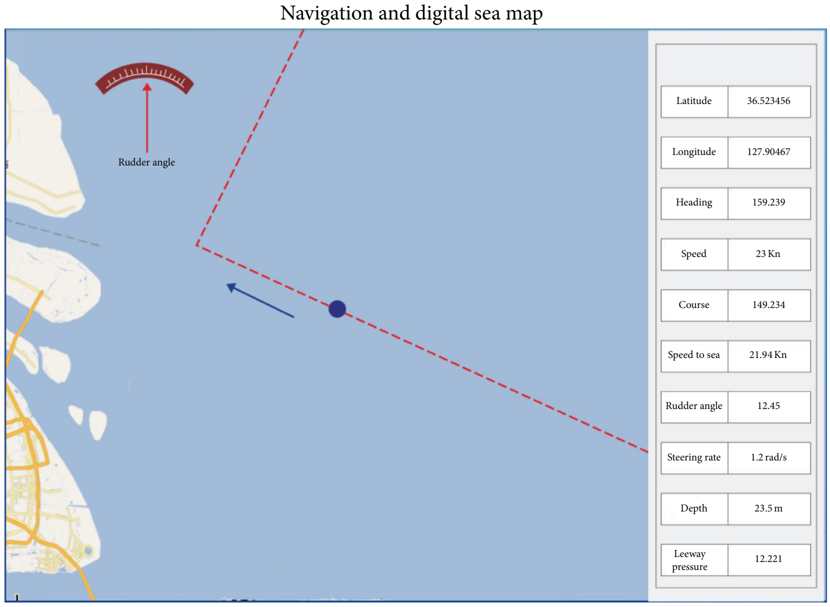
In this experiment, the horizontal plane and vertical rudder are controlled real-timely and dynamically by straight-line track automatic rudder controller designed in Section 4. The all track information as transverse offset, vertical offset, heading angle, heading angular velocity, pitching angle, and pitching angular velocity is recorded anytime real-timely. The sailing time lasts 90 minutes, and the result of records is shown in Figure 2.

The consult is shown in Figure 2, in the process of underwater sailing; the track offset of submarine is convergence in a small neighborhood by straight-line track automatic rudder controller as in Section 4. During the 90 minutes, the mean and variance of transverse offset are −0.02 and 0.45; the mean and variance of vertical offset are 0.07 and 0.72; the mean and variance of heading angle are 0.17 and 0.50; the mean and



(a) Sagittal operation interface of snorkeling

Navigation and digital sea map



(b) Horizontal operation interface of surface

Figure 1: Operation interface of simulator.

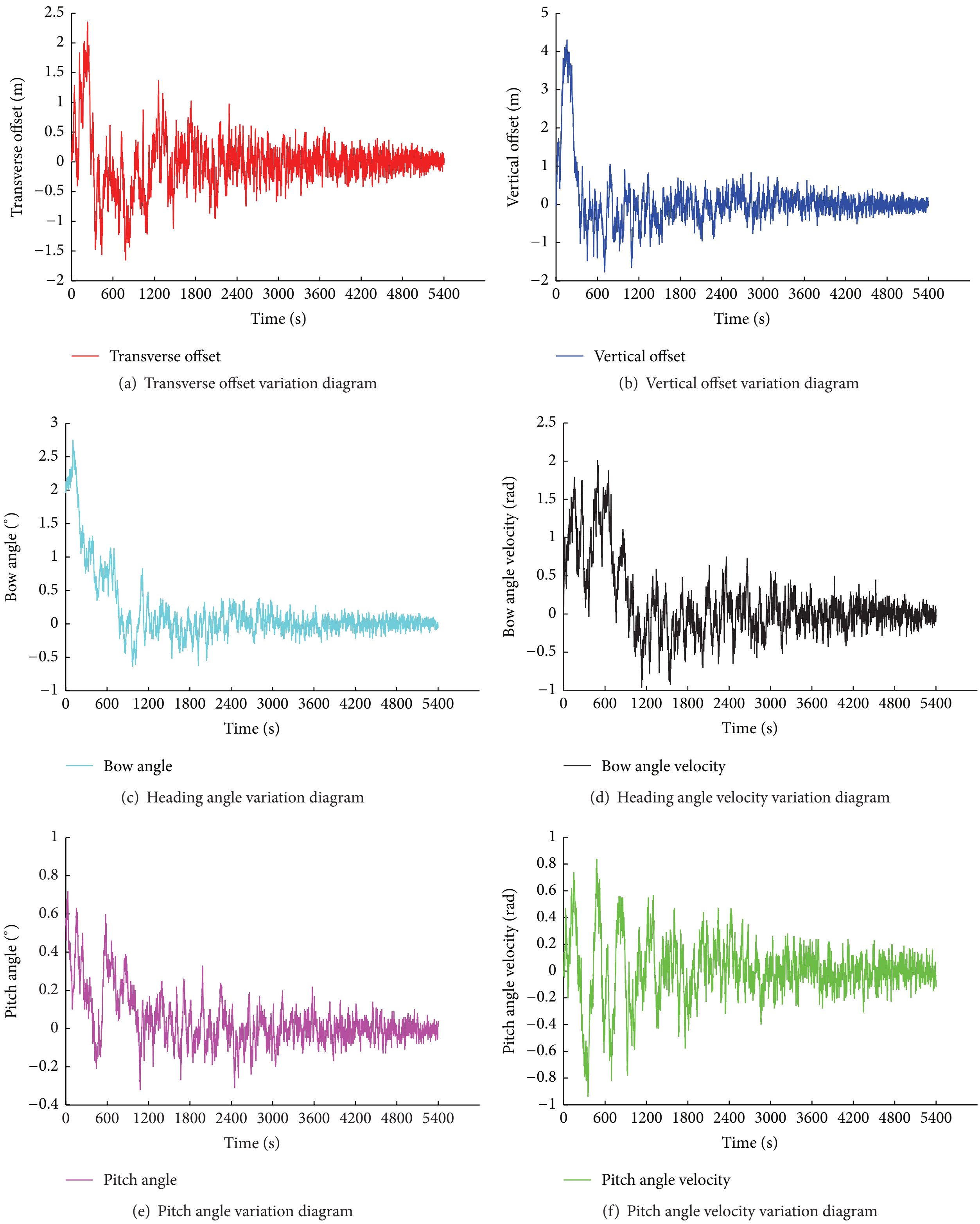
variance of heading angle velocity are 0.13 and 0.46, the mean and variance of pitch angle are −0.16 and 0.64; the mean and variance of pitch angle velocity are −0.15 and 1.14. The result shows that all the parameters are fluctuated in less offset and error precision requirement is satisfied.

In the result of simulation experiment, even if the offset to planned sea route in initial state is large, the submarine can be global asymptotic stability to planned sea route and sailing scheduled by straight-line track automatic rudder controller in this paper.

6. Conclusions

Affected by multiple factors such as gravity, power, ocean current, and external environment, the underwater trajectory of submarine shows such complicated characteristics of movement path and controlled feedback. Hence, the control problems of submarine are nonlinear in delayed and gradual conditions. As an essential ingredient of the submarine underwater operating system, the submarine control system achieved linear-suspended automatic navigation control in complicated surroundings, which enables the efficient and stable motion of submarine.

Against the calculating problem of rudder angle variation per instantaneous time interval under the global convergence of control varying, this paper demonstrates a nonlinear proportional differential based underwater controller to control the movement of submarine. By the dynamic analysis of the internal relations between controlling parameters, movement characteristics, and trajectory performance according to the



(a) Transverse offset variation diagram

(b) Vertical offset variation diagram

(c) Heading angle variation diagram

(d) Heading angle velocity variation diagram

(e) Pitch angle variation diagram

(f) Pitch angle velocity variation diagram

Figure 2: Track parameters during voyage period.

historical and real-time attitude data, the controller can solve the rudder angle control instruction per instantaneous time interval and adjust the instruction in real time, which enables the self-adaption control of the straight-line navigation of submarine.

Our experiment shows that the nonlinear proportional differential method achieved the equilibrium and stability of vertical and horizontal surfaces and enables fast global stabilization in some time, which reached the aim of underwater straight-line navigation. Our research provides the theoretical base for underwater automatic driving of submarine, which has certain theory value and practical significance.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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