Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2014, Article ID 708252, 14 pages
http://dx.doi.org/10.1155/2014/708252

Research Article

Distributed Wireless Networked H_∞ Control for a Class of Lurie-Type Nonlinear Systems

Wen Ren and Bugong Xu

College of Automation Science and Engineering, South China University of Technology, Guangzhou 510641, China

Correspondence should be addressed to Bugong Xu; aubgxu@scut.edu.cn

Received 27 December 2013; Accepted 28 March 2014; Published 5 May 2014

Academic Editor: Ge Guo

Copyright © 2014 W. Ren and B. Xu. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

A new approach to solving the distributed control problem for a class of discrete-time nonlinear systems via a wireless neural control network (WNCN) is presented in this paper. A unified Lurie-type model termed delayed standard neural network model (DSNNM) is used to describe these nonlinear systems. We assume that all neuron nodes in WNCN which have limited energy, storage space, and computing ability can be regarded as a subcontroller, then the whole WNCN is characterized by a mesh-like structure with partially connected neurons distributed over a wide geographical area, which can be considered as a fully distributed nonlinear output feedback dynamic controller. The unreliable wireless communication links within WNCN are modeled by fading channels. Based on the Lyapunov functional and the S-procedure, the WNCN is solved and configured for the DSNNM to absolutely stabilize the whole closed-loop system in the sense of mean square with a H_∞ disturbance attenuation index using LMI approach. A numerical example shows the effectiveness of the proposed design approaches.

1. Introduction

Artificial neural networks (ANNs) are one of the effective technologies in modeling and controlling complex nonlinear systems due to the universal nonlinear function approximation property of ANNs. Because all biological neural networks (BNNs) have the recursive properties and most industrial processes are nonlinear dynamic system, the recurrent neural networks (RNNs) which have internal feedback loops and are suitable for dynamic mapping have attracted increasing attention in the control field [1]. Many researchers have extensively investigated RNNs-based design methods for nonlinear control systems. For example, in [2], diagonal recurrent neural networks (DRNNs) are constructed to identify and control, respectively, for both BIBO and non-BIBO nonlinear plants. Lin et al. [3] studied an FPGA-based computed force control system based on the Elman neural network (ENN) considered as a particular class of RNN to achieve the high-accuracy position control of linear ultrasonic motor. A neural controller using recurrent learning (RTRL) network updating algorithm for nonlinear plants with unknown dynamics is presented in [4]. Guaranteed cost control for exponential synchronization of cellular neural networks (CNNs) with various activation functions and mixed time-varying delays is investigated in [5]. As a dynamic system, the stability analysis of RNNs and stabilization synthesis of RNNs-based control systems are a primary consideration. One of the main characteristics of RNNs is that the nonlinear activation functions in RNNs are of the sigmoidal type. Since the various sigmoidal functions in RNNs belong to a subset of nonlinear functions of Lurie-type system [6], during the past two decades, there have been a large number of research contributions concerning the absolute stability of RNNs such as [7–14]. It is worth noting that a new neural network model termed by the standard neural network model (SNNM) is proposed in [12]. Most nonlinear control systems based on delayed (or nondelayed) RNNs can be converted into the SNNMs, the absolute stability of which can be analyzed using a *unified approach* in the sense of Lurie [6, 13, 14]. However, the traditional static and dynamic control methods [12–14] for SNNMs are centralized and do not apply to distributed networked control systems.

Boosted by advances in computing, communications, and sensing technologies, cyber-physical systems (CPSs) in which computational and physical components are closely conjoined and coordinated are becoming increasingly ubiquitous [15, 16]. A large number of embedded devices (such as sensors, actuators, and controllers) distributed over a vast geographical area in CPSs will depend more and more on communications networks to achieve information interaction and manipulate physical entities; therefore, wireless networked control systems (WNCSs) represent a new research frontier of CPSs and have recently received a great deal of attention [17]. Employing wireless networks for CPSs will enhance the flexibility and expandability of system (e.g., network nodes are easy to move or be deployed in scenes which have difficulties in wiring) whilst reducing installation, maintenance, debugging, and labour costs. However, the unreliable communication channels, resource constraints, and limited bandwidth that characterize the wireless technology require special care and raise new challenges to communication, signal processing, closed-loop control, and so forth. Recently, many researchers have investigated these issues and some significant results were obtained and many are in progress. Shi and Zhang [18] investigate the remote state estimation and optimal schedule for two sensors under bandwidth constraint. Guo et al. [19] consider the control and actuators/sensors scheduling problem for linear system and then propose a novel stability criterion based on the modes of Markov chains and the transmission delays. The problem of joint design of an output feedback controller and the medium access scheduling policy are investigated for networked control systems in [20, 21]. The analysis and design of state feedback controllers for linear systems where there are limitations on the number of active actuators and transmission delays are studied in [22]. A decentralized event-triggered control method over wireless sensor and actuator network (WSAN) of centralized controllers is discussed in [23]. Furthermore, with network scale unceasingly expanding, any of the sensing/actuating nodes cannot access/act to the full state of the physical plant, so development and design of distributed control methods for a large-scale WNCSs are still hotspot issues in both engineering and academic fields [24].

At present, the wireless network (WN) is considered primarily as a communication medium in most of the research results [17, 23–26] for WNCSs. It means that the nodes in WN will only achieve the data communication and transmission tasks among sensing/actuating nodes and one or more dedicated controllers. However, these works have potential drawbacks such as that the WNCS is susceptible to the failures of those dedicated controllers and the packet losses and delays over unreliable wireless communication links among nodes. Pajic et al. [27] propose the basic concept of wireless control network (WCN), a new fully distributed control method for WNCSs, in which the control function is achieved over a multihop WN. For WCN, the entire multihop network fulfills itself as a distributed controller where every node can be regarded as a local (small) linear dynamical controller for linear physical plants [28].

In this paper, we focus on the distributed networked H_∞ control and absolute stability analysis of delayed standard neural network model (DSNNM) based on a wireless neuron control network (WNCN) introduced in [29], which is an improved nonlinear WCN. In summary, the aim for introducing WNCN stems from the need of a distributed control approach for WNCSs. There are many practical application requirements that also motivate this study. Typical examples include industrial humidity, ventilation, air conditioning (HVAC) control systems in [24], the networked process control for the distillation column in [30], the drip irrigation control for agriculture using wireless sensor and actuator network (WSAN) [31], and so forth.

Compared with normal RNN being the fully connected among neurons and having a layered architecture as shown in Figure 1(a), the WNCN, as a special kind of control-oriented RNN, is characterized by a mesh-like structure with partially connected neurons distributed over a wide geographical area. Consider a scenario where several neural nodes forming with limited computation and wireless communication capabilities are deployed around an industrial plant and can exchange information with immediate neighbor neuron nodes to form a wireless mesh network, some of which can also receive state values of the plant from neighbor sensors or send control signals to neighbor actuators, respectively, as shown in Figure 1(b). Compared with WCN behaving as a linear dynamical system, WNCN is essentially a nonlinear wireless mesh RNN system. To the best of our knowledge, the problem formulation is novel.

The remainder of this paper is organized as follows. In Section 2, we first briefly cover the delayed standard neural network model (DSNNM) and then describe the nonlinear dynamic behaviors of WNCN. Section 3 investigates the absolute stability and the H_∞ performance of the closed-loop system. The criteria to synthesis of the optimal H_∞ controller based on WNCN without stochastic packet dropping are first presented in Section 4, and then the result is extended to study the robust case based on stochastic WNCN with fading communication channels in Section 5. In Section 6, a numerical example is given to demonstrate the effectiveness of the derived results. And finally, conclusions are drawn in Section 7.

Notation. $\mathbb{R}^n$ is the n-dimensional Euclidean space. $\mathbb{R}^{n\times m}$ is the set of real $n \times m$ matrices. A^T denotes the transpose of matrix A. $\mathbf{Tr}(A)$ denotes the trace of a square matrix A. $\mathbb{S}^n$ denotes the set of symmetric $n \times n$ matrices. $\mathbb{S}^n_+$ denotes the set of positive semidefinite $n \times n$ matrices. $\mathbb{S}^n_{++}$ denotes the set of positive definite $n \times n$ matrices. The curled inequality symbol $\succeq$ $(\succ, \preceq, \prec)$ is used to denote generalized inequality: $A, B \in \mathbb{S}^n$, the matrix inequality $A \succeq (\succ, \preceq, \prec) 0$ means that $A \in \mathbb{S}^n_+ (\mathbb{S}^n_{++}, -\mathbb{S}^n_+, -\mathbb{S}^n_{++})$, and $A \succeq (\succ, \preceq, \prec) B \Leftrightarrow A - B \succeq (\succ, \preceq, \prec) 0$. $k(J)$ denotes the cardinality of set J. I denotes an identity matrix of appropriate order. e_i denotes the ith vector of the standard basis of $\mathbb{R}^n$. $\mathbb{E}(\cdot)$ denotes the estimation operator. diag$(\cdot)$ denotes a diagonal matrix. $*$ is used as an ellipsis for terms induced by symmetry. If $X \in \mathbb{R}^p$ and $Y \in \mathbb{R}^q$, $C(X;Y)$ denotes the space of all continuous functions mapping $\mathbb{R}^p \to \mathbb{R}^q$. $l_2\ [0;\infty)$ is the space of square integrable vectors. $\|\cdot\|$ denotes the Euclidean norm for vectors or the spectral norm of matrices.

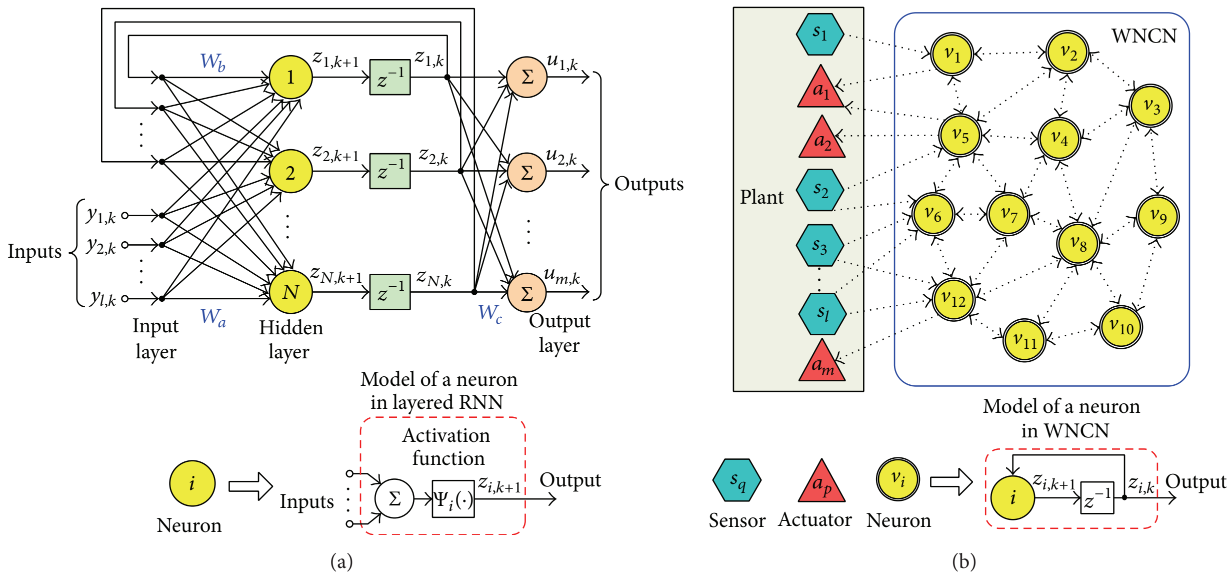


Figure 1: (a) A three-layer fully connected RNN. (b) Example of WNCN with 10 neurons consisting of a wireless mesh network (WMN) where dashed lines represent radio connectivity.

2. Problem Formulation

2.1. Delayed Standard Neural Network Model. Consider the following discrete-time DSNNM with input-output:

$$\mathscr{P}\begin{cases} x(k+1) = Ax(k) + A_d x(k-d) + B_\phi \phi(\varepsilon(k)) \\ \qquad + B_\omega \omega(k) + B_u u(k), \\ \varepsilon(k) = C_\varepsilon x(k) + C_d x(k-d) + D_\phi \phi(\varepsilon(k)) \\ \qquad + D_\omega \omega(k) + D_u u(k), \\ y(k) = C_y x(k), \end{cases} \quad (1)$$

with the initial condition function $x(k) = \varpi(k), \forall k \in [-d, 0]$, where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the control input vector, $y(k) \in \mathbb{R}^l$ is the measured output vector, $\omega(k) \in \mathbb{R}^r$ is the disturbance that belongs to $l_2\,[0, \infty)$, $\phi(\varepsilon(k)) \in C(\mathbb{R}^L; \mathbb{R}^L)$ is the activation function with the input vector $\varepsilon(k) \in \mathbb{R}^L$, $L \in \mathbb{R}$ is the number of nonlinear activation functions, $d > 0$ is the time delay, $A \in \mathbb{R}^{n\times n}$, $A_d \in \mathbb{R}^{n\times n}$, $B_\phi \in \mathbb{R}^{n\times L}$, $B_\omega \in \mathbb{R}^{n\times r}$, $B_u \in \mathbb{R}^{n\times m}$, $C_\varepsilon \in \mathbb{R}^{L\times n}$, $C_d \in \mathbb{R}^{L\times n}$, $D_\phi \in \mathbb{R}^{L\times L}$, $D_\omega \in \mathbb{R}^{L\times r}$, $D_u \in \mathbb{R}^{L\times m}$, and $C_y \in \mathbb{R}^{l\times n}$. Assume that the activation function ϕ satisfies $\phi(0) = 0$ and belongs to a type of set $\Omega(K)$ as follows:

$$\Omega(K) \triangleq \left\{\phi \mid 0 \le \frac{\phi_i(\varepsilon_i(k))}{\varepsilon_i(k)} \le k_i, i = 1, \dots, L\right\}, \quad (2)$$

which means that ϕ is sector restricted to the interval $[0, K]$, where $K = \operatorname{diag}(k_1, \dots, k_L) \succ 0$.

2.2. Wireless Neural Control Network. The traditional design approaches of dynamic controllers based on SNNMs are centralized and the dimension of controller and plant must remain consistent [12–14]. However, without losing system stability, the WNCN can be structured as a distributed recurrent neurocontroller (RNC) with arbitrary dimension which will be in favor of controlling the complex nonlinear systems with multiple geographically distributed sensors (multi-output) and actuators (multi-input). So, the motivation for introducing WNCN stems from the need for distributed control approaches for WNCSs.

Assume that we use a WNCN consisting of N neuron nodes to control the aforementioned DSNNM $\mathscr{P}$. The wireless network in the whole system can be described by a directed graph as follows:

$$\mathscr{G} \triangleq \left\{\underbrace{\mathscr{A} \cup \mathscr{V} \cup \mathscr{S}}_{\text{vertex set}}, \underbrace{\mathscr{E}^{\mathscr{I}} \cup \mathscr{E}^{\mathscr{C}} \cup \mathscr{E}^{\mathscr{O}}}_{\text{edge set}}\right\}, \quad (3)$$

where $\mathscr{V} = \{v_1, \dots, v_N\}$ is the set of N neuron nodes, $\mathscr{A} = \{a_1, \dots, a_m\}$ is the set of m actuators which can execute the input vector $u(k) = [u_1(k), \dots, u_m(k)]^T$, $\mathscr{S} = \{s_1, \dots, s_l\}$ is the set of l sensors used to measure the output vector $y(k) = [y_1(k), \dots, y_l(k)]^T$, and edge sets $\mathscr{E}^{\mathscr{I}} = \{(v_i, a_p) \mid v_i \in \mathscr{V}, a_p \in \mathscr{A}\}$, $\mathscr{E}^{\mathscr{C}} = \{(v_i, v_j) \mid v_j, v_i \in \mathscr{V}\}$, and $\mathscr{E}^{\mathscr{O}} = \{(s_q, v_i) \mid s_q \in \mathscr{S}, v_i \in \mathscr{V}\}$ correspond to the physical radio communication links in the wireless network. Define the following three sets:

the *neighbor sensors* of v_i, $\forall i \in \{1, \dots, N\}$

$$\mathscr{S}^{v_i} \triangleq \left\{s_q \mid s_q \in \mathscr{S}, \exists \left(s_q, v_i\right) \in \mathscr{E}^{\mathscr{O}}\right\} = \left\{s_q \mid w_{iq}^{\mathscr{O}} \ne 0\right\}, \quad (4)$$

the *neighbor neurons* of a_p, $\forall p \in \{1, \dots, m\}$

$$\mathscr{V}^{a_p} \triangleq \left\{v_i \mid v_i \in \mathscr{V}, \exists \left(v_i, a_p\right) \in \mathscr{E}^{\mathscr{I}}\right\} = \left\{v_i \mid w_{pi}^{\mathscr{I}} \ne 0\right\}, \quad (5)$$

the *neighbor neurons* of v_i, $\forall i \in \{1, \dots, N\}$

$$\mathscr{V}^{v_i} \triangleq \left\{v_j \mid v_j \in \mathscr{V}, \exists \left(v_j, v_i\right) \in \mathscr{E}^{\mathscr{C}}\right\} = \left\{v_j \mid w_{ij}^{\mathscr{C}} \ne 0\right\}, \quad (6)$$

where $w_{pi}^{\mathcal{I}}$, $w_{ij}^{\mathcal{C}}$, $w_{iq}^{\mathcal{O}}$ are the weights of edge (v_i, a_p), (v_j, v_i), and (s_q, v_i), respectively. This implies that $s_q \in \mathcal{S}^{v_i}$ if v_i can receive data directly from s_q, $v_i \in \mathcal{V}^{a_p}$ if a_p can receive data directly from v_i, and $v_j \in \mathcal{V}^{v_i}$ if v_i can receive data directly from v_j.

The dynamic behavior of the neuron node v_i may be represented by the following pair of nonlinear equations:

$$z_i(k+1) = \psi_i(\xi_i(k)),$$
$$\xi_i(k) = w_{ii}^{\mathcal{C}} z_i(k) + \sum_{v_j \in \mathcal{V}^{v_i}} w_{ij}^{\mathcal{C}} z_j(k) + \sum_{s_q \in \mathcal{S}^{v_i}} w_{iq}^{\mathcal{O}} y_q(k), \quad (7)$$

where $z_i(k)$ is the state of neuron node v_i, $z_j(k)$ is the state of neuron node v_j, $v_j \in \mathcal{V}^{v_i}$, $y_q(k)$ is the measurement value of sensor s_q, $s_q \in \mathcal{S}^{v_i}$, $\xi_i(k)$ is the weighted linear combination of v_i's present state and exogenous input signals (from neuron nodes in $\mathcal{V}^{v_i}$ or sensors in $\mathcal{S}^{v_i}$), and $\psi_i(\cdot) \in \Omega(\overline{K})$ is the activation function of neuron node v_i, where $\overline{K} = \mathrm{diag}(k_{L+1}, \ldots, k_{L+N}) \succ 0$. Each plant input $u_p(k)$, $p \in \{1, \ldots, m\}$ is a weighted linear combiner output due to neighbor neuron nodes of the actuator a_p as follows:

$$u_p(k) = \sum_{v_i \in \mathcal{V}^{a_p}} w_{pi}^{\mathcal{I}} z_i(k). \quad (8)$$

If each neuron node is regarded as a nonlinear dynamical subcontroller, the whole WNCN consisting of N neuron nodes may act as a fully distributed RNC whose dynamic behavior may be described as

$$\mathcal{K} \begin{cases} z(k+1) = \psi(\xi(k)), \\ \xi(k) = W^{\mathcal{C}} z(k) + W^{\mathcal{O}} y(k), \\ u(k) = W^{\mathcal{I}} z(k), \end{cases} \quad (9)$$

where $z \in \mathbb{R}^N$ is the state vector of WNCN, $\psi \in C(\mathbb{R}^N; \mathbb{R}^N)$, $\xi \in \mathbb{R}^N$, $W^{\mathcal{C}} \in \mathbb{R}^{N\times N}$, $W^{\mathcal{O}} \in \mathbb{R}^{N\times l}$, and $W^{\mathcal{I}} \in \mathbb{R}^{m\times N}$. In the above-mentioned equations, $\forall i \in \{1, \ldots, N\}$, $w_{ij}^{\mathcal{C}} = 0$ if $v_j \notin \mathcal{V}^{v_i} \cup \{v_i\}$, $w_{iq}^{\mathcal{O}} = 0$ if $s_q \notin \mathcal{S}^{v_i}$, and $w_{pi}^{\mathcal{I}} = 0$ if $v_i \notin \mathcal{V}^{a_p}$. Therefore, the weight matrices $W^{\mathcal{C}}$, $W^{\mathcal{O}}$, and $W^{\mathcal{I}}$ have the sparsity constraints. This means that the WNCN has considerably fewer weights (accounting for little computational overhead) than the fully connected neural network, which is conducive to industrial real-time control.

In this paper, a MAC synchronized network protocol based on time division multiple access (TDMA) architecture is used to schedule neuron nodes in WNCN to accomplish the cooperative control for the system (1). Under the scheme, every neuron node v_i, $i \in \{1, \ldots, N\}$ transmits its state information once per time frame. In the beginning, v_i has an arbitrary initial state value and then successively receives information from its neighbors in $\mathcal{V}^{v_i}$ and $\mathcal{S}^{v_i}$ in each time slot of frame. After v_i has received all the information from its neighbors, v_i will update its state by (7). Furthermore, in a similar way, every actuator a_p, $p \in \{1, \ldots, m\}$ can receive the combination of control signals from neighbor neuron nodes in $\mathcal{V}^{a_p}$ and then act to system (1) by (8).

Define vectors $\tilde{x} = [x^T, z^T]^T \in \mathbb{R}^{n+N}$, $\tilde{\varepsilon} = [\varepsilon^T, \xi^T]^T \in \mathbb{R}^{L+N}$, and $\tilde{\phi} = [\phi^T, \psi^T]^T \in \Omega(\widetilde{K})$, $\widetilde{K} = \mathrm{diag}(k_1, \ldots, k_{L+N}) \succ 0$. Then the overall closed-loop system $\mathcal{G}$ of the DSNNM $\mathcal{P}$ and the WNCN $\mathcal{K}$ is described as

$$\mathcal{G} \begin{cases} \tilde{x}(k+1) = \underbrace{\begin{bmatrix} A & B_u W^{\mathcal{I}} \\ 0 & 0 \end{bmatrix}}_{\widetilde{A}} \underbrace{\begin{bmatrix} x(k) \\ z(k) \end{bmatrix}}_{\tilde{x}(k)} + \underbrace{\begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix}}_{\widetilde{A}_d} \underbrace{\begin{bmatrix} x(k-d) \\ z(k-d) \end{bmatrix}}_{\tilde{x}(k-d)} \\ \qquad + \underbrace{\begin{bmatrix} B_\phi & 0 \\ 0 & I \end{bmatrix}}_{\widetilde{B}_\phi} \underbrace{\begin{bmatrix} \phi(\varepsilon(k)) \\ \psi(\xi(k)) \end{bmatrix}}_{\tilde{\phi}(\tilde{\varepsilon}(k))} + \underbrace{\begin{bmatrix} B_\omega \\ 0 \end{bmatrix}}_{\widetilde{B}_\omega} \omega(k), \\ \tilde{\varepsilon}(k) = \underbrace{\begin{bmatrix} C_\varepsilon & D_u W^{\mathcal{I}} \\ W^{\mathcal{O}} C_y & W^{\mathcal{C}} \end{bmatrix}}_{\widetilde{C}_\varepsilon} \underbrace{\begin{bmatrix} x(k) \\ z(k) \end{bmatrix}}_{\tilde{x}(k)} + \underbrace{\begin{bmatrix} C_d & 0 \\ 0 & 0 \end{bmatrix}}_{\widetilde{C}_d} \underbrace{\begin{bmatrix} x(k-d) \\ z(k-d) \end{bmatrix}}_{\tilde{x}(k-d)} \\ \qquad + \underbrace{\begin{bmatrix} D_\phi & 0 \\ 0 & 0 \end{bmatrix}}_{\widetilde{D}_\phi} \underbrace{\begin{bmatrix} \phi(\varepsilon(k)) \\ \psi(\xi(k)) \end{bmatrix}}_{\tilde{\phi}(\tilde{\varepsilon}(k))} + \underbrace{\begin{bmatrix} D_\omega \\ 0 \end{bmatrix}}_{\widetilde{D}_\omega} \omega(k). \end{cases} \quad (10)$$

Consider that the performance output of the closed-loop system $\mathcal{G}$ is described as $\tilde{y}(k) = \widetilde{C}_y \tilde{x}(k)$, where $\widetilde{C}_y = [I \ 0]$, then the following definition is introduced.

Definition 1 (see [6, 32, 33]). Given a scalar $\gamma > 0$, the closed-loop system $\mathcal{G}$ is said to be absolutely stable with a H_∞-norm bound γ if there exists a distributed dynamic neural controller WNCN $\mathcal{K}$ such that the following conditions are satisfied (Algorithm 1).

(1) With zero disturbance, that is, $\omega(k) = 0$, the zero solution of the closed-loop system $\mathcal{G}$ is globally asymptotically stable, $\forall \tilde{x}(0)$, $\forall \tilde{\phi} \in \Omega(\widetilde{K})$.

(2) Under the zero-initial condition, the performance output $\tilde{y}(k)$ satisfies

$$\sum_{k=0}^{\infty} \|\tilde{y}(k)\|^2 \le \gamma^2 \sum_{k=0}^{\infty} \|\omega(k)\|^2, \quad \forall \text{ nonzero } \omega(k). \quad (11)$$

Then the WNCN $\mathcal{K}$ is said to be a H_∞ controller for the DSNNM $\mathcal{P}$. Furthermore, if we can find a minimal γ^* to satisfy the above conditions, the WNCN $\mathcal{K}$ is an optimal H_∞ controller.

Our aim is to design the WNCN $\mathcal{K}$ for DSNNM $\mathcal{P}$ such that the closed-loop system $\mathcal{G}$ satisfies the requirements (1) and (2)in Definition 1.

3. H_∞ Performance Analysis of the Closed-Loop System

In this section, we will investigate the absolute stability and H_∞ performance of the closed-loop system $\mathscr{G}$. Before deducing the main results, we need to make use of the following two lemmas.

Lemma 2 (S-procedure [34]). *Let $T_0, T_1, \ldots, T_p \in \mathbb{S}^n$. If there exists $\tau_i \geq 0$, $i = 1, \ldots, p$ such that*

$$T_0 - \sum_{i=1}^{p} \tau_i T_i \prec 0 \tag{12}$$

then $\zeta^T T_0 \zeta < 0$ for all $\zeta \neq 0$ such that $\zeta^T T_i \zeta \leq 0$, $i = 1, \ldots, p$.

Lemma 3 (Schur complement [35]). *Consider a matrix $X \in \mathbb{S}^n$ partitioned as*

$$X = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}, \tag{13}$$

where $A \in \mathbb{S}^k$. If A is nonsingular, the matrix $S = C - B^T A^{-1} B$ is called the Schur complement of A in X. Then, the following characterizations of positive definiteness or semidefiniteness of the block matrix X hold:

$$\begin{aligned} &(1)\ X \succ 0, \quad iff\ A \succ 0, S \succ 0, \\ &(2)\ If\ A \succ 0, \quad then\ X \succeq 0\ iff\ S \succeq 0. \end{aligned} \tag{14}$$

Theorem 4. *Given $\gamma > 0$ and WNCN $\mathscr{K}$ with parameter set $\mathscr{K} = \{W^{\mathscr{O}}, W^{\mathscr{C}}, W^{\mathscr{I}}\}$, if there exist appropriate dimension matrices $P \succ 0$, $R \succ 0$, and $T \succeq 0$, such that the following matrix inequality holds:*

$$\Xi_1 = \begin{bmatrix} \begin{pmatrix} \widetilde{A}^T P \widetilde{A} - P \\ +R + \widetilde{C}_y^T \widetilde{C}_y \end{pmatrix} & \widetilde{A}^T P \widetilde{A}_d & \widetilde{A}^T P \widetilde{B}_\phi + \widetilde{C}_\varepsilon^T T \widetilde{K} & \widetilde{A}^T P \widetilde{B}_\omega \\ * & \widetilde{A}_d^T P \widetilde{A}_d - R & \widetilde{A}_d^T P \widetilde{B}_\phi + \widetilde{C}_d^T T \widetilde{K} & \widetilde{A}_d^T P \widetilde{B}_\omega \\ * & * & \begin{pmatrix} \widetilde{B}_\phi^T P \widetilde{B}_\phi + \widetilde{D}_\phi^T T \widetilde{K} \\ +T \widetilde{K} \widetilde{D}_\phi - 2T \end{pmatrix} & \widetilde{B}_\phi^T P \widetilde{B}_\phi + T \widetilde{K} \widetilde{D}_\omega \\ * & * & * & \widetilde{B}_\omega^T P \widetilde{B}_\omega - \gamma^2 I \end{bmatrix} \prec 0, \tag{15}$$

where $\widetilde{K} = \operatorname{diag}(k_1, \ldots, k_{L+N})$, then the zero solution of closed-loop system $\mathscr{G}$ is absolutely stable and the H_∞-norm constraint (11) *is achieved for all nonzero $\omega(k)$.*

Proof. From system $\mathscr{G}$ with $\omega(k) = 0$, one can obtain

$$\overline{\mathscr{G}} \begin{cases} \widetilde{x}(k+1) = \widetilde{A}\widetilde{x}(k) + \widetilde{A}_d \widetilde{x}(k-d) + \widetilde{B}_\phi \widetilde{\phi}(\widetilde{\varepsilon}(k)), \\ \widetilde{\varepsilon}(k) = \widetilde{C}_\varepsilon \widetilde{x}(k) + \widetilde{C}_d \widetilde{x}(k-d) + \widetilde{D}_\phi \widetilde{\phi}(\widetilde{\varepsilon}(k)). \end{cases} \tag{16}$$

Assume that the $\widetilde{x}(k) = 0$ is the only equilibrium of $\overline{\mathscr{G}}$. Consider the following Lyapunov-Krasovskii functional for systems $\overline{\mathscr{G}}$ as

$$V(\widetilde{x}(k)) = \widetilde{x}^T(k) P \widetilde{x}(k) + \sum_{i=1}^{d} \widetilde{x}^T(k-i) R \widetilde{x}(k-i). \tag{17}$$

According to the sector bound set $\Omega(\widetilde{K})$ of $\widetilde{\phi}$, we have

$$\begin{aligned} &\widetilde{\phi}_i(\widetilde{\varepsilon}_i(k)) \cdot \tau_i \cdot \left[\widetilde{\phi}_i(\widetilde{\varepsilon}_i(k)) - k_i \widetilde{\varepsilon}_i(k)\right] \\ &\quad = \tau_i \widetilde{\phi}_i^2(\widetilde{\varepsilon}_i(k)) - \tau_i k_i \widetilde{\varepsilon}_i(k) \widetilde{\phi}_i(\widetilde{\varepsilon}_i(k)) \\ &\quad \leq 0, \end{aligned} \tag{18}$$

where $\varepsilon_i \geq 0$, $i = 1, \ldots, L+N$.

Now, by defining the difference of $V(\widetilde{x}(k))$ along $\overline{\mathscr{G}}$ as $\Delta V(\widetilde{x}(k)) \triangleq V(\widetilde{x}(k+1)) - V(\widetilde{x}(k))$ and using Lemma 2 (S-procedure), we can obtain

$$\begin{aligned} &\Delta V(\widetilde{x}(k)) \\ &= \widetilde{x}^T(k+1) P \widetilde{x}(k+1) - \widetilde{x}^T(k) P \widetilde{x}(k) \\ &\quad + \widetilde{x}(k)^T R \widetilde{x}(k) - \widetilde{x}^T(k-d) R \widetilde{x}(k-d) \\ &\leq \left[\widetilde{A}\widetilde{x}(k) + \widetilde{A}_d \widetilde{x}(k-d) + \widetilde{B}_\phi \widetilde{\phi}(\widetilde{\varepsilon}(k))\right]^T \\ &\quad \times P\left[\widetilde{A}\widetilde{x}(k) + \widetilde{A}_d \widetilde{x}(k-d) + \widetilde{B}_\phi \widetilde{\phi}(\widetilde{\varepsilon}(k))\right] \\ &\quad - \widetilde{x}^T(k) P \widetilde{x}(k) + \widetilde{x}(k)^T R \widetilde{x}(k) \\ &\quad - \widetilde{x}^T(k-d) R \widetilde{x}(k-d) - 2\sum_{i=1}^{L+N} \tau_i \widetilde{\phi}_i^2(\widetilde{\varepsilon}_i(k)) \\ &\quad + 2\sum_{i=1}^{L+N} \tau_i k_i \widetilde{\varepsilon}_i(k) \widetilde{\phi}_i(\widetilde{\varepsilon}_i(k)) \\ &= \begin{bmatrix} \widetilde{x}(k) \\ \widetilde{x}(k-d) \\ \widetilde{\phi}(\widetilde{\varepsilon}(k)) \end{bmatrix}^T \end{aligned}$$

$$\times\underbrace{\begin{bmatrix}\widetilde{A}^TP\widetilde{A}-P+R & \widetilde{A}^TP\widetilde{A}_d & \widetilde{A}^TP\widetilde{B}_\phi+\widetilde{C}_\varepsilon^TT\widetilde{K}\\ * & \widetilde{A}_d^TP\widetilde{A}_d-R & \widetilde{A}_d^TP\widetilde{B}_\phi+\widetilde{C}_d^TT\widetilde{K}\\ * & * & \begin{pmatrix}\widetilde{B}_\phi^TP\widetilde{B}_\phi+\widetilde{D}_\phi^TT\widetilde{K}\\ +TK\widetilde{D}_\phi-2T\end{pmatrix}\end{bmatrix}}_{\Xi_0}\times\begin{bmatrix}\widetilde{x}(k)\\ \widetilde{x}(k-d)\\ \widetilde{\phi}(\widetilde{\varepsilon}(k))\end{bmatrix}, \tag{19}$$

where $T=\operatorname{diag}(\tau_1,\tau_2,\ldots,\tau_{L+N})\geq 0$. By Lemma 3 (Schur complement), if $\Xi_1\prec 0$ (15) holds, $\Xi_0\prec 0$ also holds. So, if $\Xi_1\prec 0$, system $\mathcal{G}$ with $\omega(k)=0$, that is, system $\overline{\mathcal{G}}$, is globally asymptotically stable, $\forall\widetilde{\phi}\in\Omega(\widetilde{K})$.

Next, for $\forall\ell>0$, define

$$\begin{aligned}J_\ell&=\sum_{k=0}^{\ell}\|\widetilde{y}(k)\|^2-\gamma^2\sum_{k=0}^{\ell}\|\omega(k)\|^2\\&=\sum_{k=0}^{\ell}\left[\widetilde{y}^T(k)\,\widetilde{y}(k)-\gamma^2\omega^T(k)\,\omega(k)\right].\end{aligned}\tag{20}$$

Consider the zero initial condition $V(\widetilde{x}(0))=0$, that is

$$\begin{aligned}V(\widetilde{x}(\ell+1))&=\sum_{k=0}^{\ell}\left[V(\widetilde{x}(k+1))-V(\widetilde{x}(k))\right]\\&=\sum_{k=0}^{\ell}\Delta V(\widetilde{x}(k))>0.\end{aligned}\tag{21}$$

Therefore, for system $\mathcal{G}$, defining vector $\zeta(k)=[\widetilde{x}^T(k)\ \widetilde{x}^T(k-d)\ \widetilde{\phi}^T(\widetilde{\varepsilon}(k))\ \omega^T(k)]^T$ and according to (19) and (20), we have

$$\begin{aligned}J_\ell&=\sum_{k=0}^{\ell}\left[\widetilde{y}^T(k)\,\widetilde{y}(k)-\gamma^2\omega^T(k)\,\omega(k)+\Delta V(\widetilde{x}(k))\right]\\&\quad-V(\widetilde{x}(\ell+1))\\&\leq\sum_{k=0}^{\ell}\left[\widetilde{y}^T(k)\,\widetilde{y}(k)-\gamma^2\omega^T(k)\,\omega(k)+\Delta V(\widetilde{x}(k))\right]\\&=\sum_{k=0}^{\ell}\zeta^T(k)\,\Xi_1\zeta(k).\end{aligned}\tag{22}$$

If $\Xi_1\prec 0$ (15) holds, $\lim_{\ell\to\infty}J_\ell=\lim_{\ell\to\infty}\sum_{k=0}^{\ell}\zeta^T(k)\,\Xi_1\zeta(k)<0$. Thus, $\forall$ nonzero $\omega(k)\in l_2\,[0,\infty)$ and the H_∞-norm constraint (11) is achieved. This completes the proof. □

4. H_∞ Controller Design Based on WNCN

In the previous stage, the matrix inequality condition $\Xi_1\prec 0$ (15) is not an LMI, which cannot be solved by LMI tools. In what follows, we first convert the matrix inequality condition $\Xi_1\prec 0$ (15) into a cone complementarity problem (CCP) and then use the cone complementarity linearization (CCL) algorithm introduced in [36] to formulate a convex optimization problem with LMI constraints to obtain the appropriate parameters of WNCN (i.e., interconnection weight matrices set $\mathcal{K}=\{W^{\mathcal{O}},W^{\mathcal{C}},W^{\mathcal{I}}\}$).

Lemma 5 (see [27]). *There exist matrices $P,Q\in\mathbb{S}_{++}^n$ satisfying the constraint $Q=P^{-1}$ if and only if they are optimal points for the problem*

$$\begin{aligned}&\min\quad\mathbf{Tr}\{QP\}\\&\textit{subject to}\quad\begin{bmatrix}Q & I\\ I & P\end{bmatrix}\geq 0,\quad Q,P\in\mathbb{S}_{++}^n,\end{aligned}\tag{23}$$

and the optimal cost of the problem is n.

Theorem 6. *Given a scalar $\gamma>0$, the closed-loop system $\mathcal{G}$ is said to be absolutely stabilizable by using a WNCN $\mathcal{K}$ and the H_∞-norm constraint* (11) *is achieved for all nonzero $\omega(k)$ if there exist appropriate dimension matrices $P\succ 0$, $Q\succ 0$, $R\succ 0$, $\Sigma\geq 0$, and $W^{\mathcal{O}},W^{\mathcal{C}},W^{\mathcal{I}}\in\mathcal{K}$ such that the following optimization problem:*

$$\min\quad\mathbf{Tr}\{QP\}\tag{24}$$

$$\text{s.t.}\quad\Xi_2=\begin{bmatrix}-P+R & 0 & \widetilde{C}_\varepsilon^T & 0 & \widetilde{A}^T & \widetilde{C}_y^T\\ * & -R & \widetilde{C}_d^T & 0 & A_d^T & 0\\ * & * & \widetilde{D}_\phi\Sigma\widetilde{K}+\Sigma\widetilde{K}\widetilde{D}_\phi^T-2\Sigma & \widetilde{D}_\omega & \Sigma\widetilde{K}\widetilde{B}_\phi^T & 0\\ * & * & * & -\gamma^2I & \widetilde{B}_\omega^T & 0\\ * & * & * & * & -Q & 0\\ * & * & * & * & * & -I\end{bmatrix}\prec 0,\tag{25}$$

$$\begin{bmatrix}Q & I\\ I & P\end{bmatrix}\geq 0,\quad Q,P\in\mathbb{S}_{++}^{n+N},\tag{26}$$

where

$$\widetilde{A} = \begin{bmatrix} A & B_u W^{\mathcal{I}} \\ 0 & 0 \end{bmatrix}, \qquad \widetilde{C}_\varepsilon = \begin{bmatrix} C_\varepsilon & D_u W^{\mathcal{I}} \\ W^{\mathcal{O}} C_y & W^{\mathcal{C}} \end{bmatrix}, \tag{27}$$

is feasible with optimal cost $n + N$.

Proof. Inequality $\Xi_1 \prec 0$ (15) in Theorem 4 can be rewritten as

$$\begin{bmatrix} -P+R & 0 & \widetilde{C}_\varepsilon^T T\widetilde{K} & 0 \\ * & -R & \widetilde{C}_d^T T\widetilde{K} & 0 \\ * & * & \widetilde{D}_\phi^T T\widetilde{K} + T\widetilde{K}\widetilde{D}_\phi - 2T & T\widetilde{K}\widetilde{D}_\omega \\ * & * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \widetilde{A}^T \\ \widetilde{A}_d^T \\ \widetilde{B}_\phi^T \\ \widetilde{B}_\omega^T \end{bmatrix} P \begin{bmatrix} \widetilde{A} & \widetilde{A}_d & \widetilde{B}_\phi & \widetilde{B}_\omega^T \end{bmatrix} + \begin{bmatrix} \widetilde{C}_y^T \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \widetilde{C}_y & 0 & 0 & 0 \end{bmatrix} \prec 0. \tag{28}$$

Then, using Lemma 3 (Schur complement), the inequality (28) is equivalent to

$$\begin{bmatrix} -P+R & 0 & \widetilde{C}_\varepsilon^T T\widetilde{K} & 0 & \widetilde{A}^T & \widetilde{C}_y^T \\ * & -R & \widetilde{C}_d^T T\widetilde{K} & 0 & \widetilde{A}_d^T & 0 \\ * & * & \widetilde{D}_\phi^T T\widetilde{K} + T\widetilde{K}\widetilde{D}_\phi - 2T & T\widetilde{K}\widetilde{D}_\omega & \widetilde{B}_\phi^T & 0 \\ * & * & * & -\gamma^2 I & \widetilde{B}_\omega^T & 0 \\ * & * & * & * & -Q & 0 \\ * & * & * & * & * & -I \end{bmatrix} \prec 0, \tag{29}$$

where $Q = P^{-1}$. By defining

$$S = \left(T\widetilde{K}\right)^{-1}, \qquad \Sigma = \left(T\widetilde{K}\right)^{-1} T \left(T\widetilde{K}\right)^{-1} \tag{30}$$

and pre- and postmultiplying the left-hand side matrix of (29) by $\operatorname{diag}(I, I, S, I, I, I)$, respectively, the inequality (29) is equivalent to

$$\begin{bmatrix} -P+R & 0 & \widetilde{C}_\varepsilon^T & 0 & \widetilde{A}^T & \widetilde{C}_y^T \\ * & -R & \widetilde{C}_d^T & 0 & \widetilde{A}_d^T & 0 \\ * & * & S\widetilde{D}_\phi^T + \widetilde{D}_\phi S - 2\Sigma & \widetilde{D}_\omega & S\widetilde{B}_\phi^T & 0 \\ * & * & * & -\gamma^2 I & \widetilde{B}_\omega^T & 0 \\ * & * & * & * & -Q & 0 \\ * & * & * & * & * & -I \end{bmatrix} \prec 0. \tag{31}$$

According to (30), the following equations hold:

$$S^{-1} = T\widetilde{K}, \qquad T = S^{-1}\Sigma S^{-1}. \tag{32}$$

Form (32), we know

$$S = \Sigma\widetilde{K}. \tag{33}$$

Substituting (33) into (31), one can obtain $\Xi_2 \prec 0$ (25). By using Lemma 5, the nonconvex constraint $Q = P^{-1}$ is approximated with an optimization problem. This completes the proof. □

So far the WNCN has been designed to guarantee the absolute stability with a given H_∞-norm bound γ of the closed-loop system. In what follows, we give the Algorithm 2 based on the bisection method to design WNCN for the optimal H_∞ control problem: min γ s.t. (15), $P \succ 0$, $R \succ 0$, $T \succeq 0$.

5. Robust H_∞ Controller Design Based on Fading WNCN

Due to the large geographical nature of the closed-loop system $\mathcal{G}$ over a WN, a realistic distributed control design approach for WNCN should take the communication packet losses into account.

According to [37], we adopt the *fading* channel models to simulate the unreliable wireless communication links in WNCN as shown in Figure 2(a). First, define a bijective mapping $\Omega : \{(a,b)\} \rightarrow \{\tau\}$, $(a,b) \in \mathscr{E}^{\mathcal{O}} \cup \mathscr{E}^{\mathcal{C}} \cup \mathscr{E}^{I}$, where $\tau = \{1,\dots,\rho\}$ and $\rho = k(\mathscr{E}^{\mathcal{O}}) + k(\mathscr{E}^{\mathcal{C}}) + k(\mathscr{E}^{I})$ is the total number of wireless links in WNCN, to concisely enumerate all links in the network. Therefore, the weights of links $\{(a,b)\}$ can be mapped to $w_\tau^{\mathcal{O}}, w_\tau^{\mathcal{C}}, w_\tau^{\mathcal{I}}$, $\forall\tau = \Omega(a,b)$ and then compacted into the following weight vector as

$$\begin{aligned} w &= \left[\left(w^{\mathcal{O}}\right)^T, \left(w^{\mathcal{C}}\right)^T, \left(w^{\mathcal{I}}\right)^T\right]^T \\ &= \left[\left\{w_\tau^{\mathcal{O}}\right\}_{\tau=\Omega(s_q,v_i)}, \left\{w_\tau^{\mathcal{C}}\right\}_{\tau=\Omega(v_j,v_i)}, \left\{w_\tau^{\mathcal{I}}\right\}_{\tau=\Omega(v_i,a_p)}\right]^T, \end{aligned} \tag{34}$$

Step 1. Set $k=0$. If there exists an initial feasible solution set $\Upsilon_0=\{P,Q,R,\Sigma,W^{\mathcal{O}},W^{\mathcal{C}},W^{\mathcal{I}}\}$ satisfying the constraints (25)-(26), let $\mathcal{X}_0=P$, $\mathcal{Y}_0=Q$. Otherwise, exit.
Step 2. If $k\le\kappa$, go to Step 3 where κ is the assumed maximum number of iteration. Otherwise, exit.
Step 3. At $k\ge 0$, obtain the feasible solution set $\Upsilon_{k+1}=\{P,Q,R,\Sigma,W^{\mathcal{O}},W^{\mathcal{C}},W^{\mathcal{I}}\}$ by solving the following LMI problem: $\min \mathbf{Tr}\{\mathcal{X}_kQ+\mathcal{Y}_kP\}$ s.t. (25)-(26).
Step 4. Substitute Υ_{k+1} into (15). If (15) holds, stop the algorithm. Otherwise, set $k=k+1$, $\mathcal{X}_k=P$, $\mathcal{Y}_k=Q$ and go to Step 2.

Algorithm 1: Given a scalar $\gamma>0$, solving the H_∞ WNCN for closed-loop system $\widetilde{\mathcal{G}}$.

Step 1. Set $k=0$. Let γ_-^0 and γ_+^0 be the initial lower and upper bounds of γ, that is, $\gamma_0\in[\gamma_-^0,\gamma_+^0]$ where $\gamma_-^0=0$, γ_+^0 can be assigned an arbitrarily sufficiently large value to make inequalities (25)-(26) have initial feasible solution set $\Upsilon_0=\{P,Q,R,\Sigma,W^{\mathcal{O}},W^{\mathcal{C}},W^{\mathcal{I}}\}$.
Step 2. At $k\ge 0$, compute $\gamma_k=(\gamma_-^k+\gamma_+^k)/2$.
Step 3. Use Algorithm 1 to check whether the feasible solution set Υ_{k+1} satisfying inequality (15). If Υ_{k+1} exists, set $\gamma_+^k=\gamma^k$. Otherwise, set $\gamma_-^k=\gamma^k$.
Step 4. If $\gamma_+^k-\gamma_-^k\le\epsilon$, where ϵ is the assumed calculation accuracy, set optimal H_∞ performance index $\gamma^*=\gamma_+^k$ and exit. Otherwise, set $k=k+1$, go to Step 2.

Algorithm 2: Minimizing γ to solve the optimal H_∞ WNCN for closed-loop system $\widetilde{\mathcal{G}}$.

where $w^{\mathcal{O}}\in\mathbb{R}^{\rho^{\mathcal{O}}}$, $w^{\mathcal{C}}\in\mathbb{R}^{\rho^{\mathcal{C}}}$, $w^{\mathcal{I}}\in\mathbb{R}^{\rho^{\mathcal{I}}}$, and $w\in\mathbb{R}^{\rho}$. Let $t_{k,\tau}$ denote the data packet transmitted over the τth communication link at time k. Then, aggregating all of $t_{k,\tau}$ in a vector $t_k\in\mathbb{R}^{\rho}$, we can obtain

$$t(k)=\underbrace{\left[\begin{array}{c|c} W_t^{\mathcal{O}} & 0 \\ \hline 0 & \begin{matrix} W_t^{\mathcal{C}} \\ W_t^{\mathcal{I}}\end{matrix}\end{array}\right]}_{W^{\text{or}}}\left[\begin{array}{c|c} C_y & 0 \\ \hline 0 & I_N\end{array}\right]\left[\begin{array}{c} x(k) \\ \hline z(k)\end{array}\right]=\widetilde{W}^{\text{or}}\widetilde{x}(k), \tag{35}$$

where $W_t^{\mathcal{O}}=\operatorname{diag}(w^{\mathcal{O}})\,[e_q^T]_{q|\tau=\Omega(s_q,v_i)}$, $e_q\in\mathbb{R}^l$, $W_t^{\mathcal{C}}=\operatorname{diag}(w^{\mathcal{C}})[e_j^T]_{j|\tau=\Omega(v_j,v_i)}$, $e_j\in\mathbb{R}^N$, and $W_t^{\mathcal{I}}=\operatorname{diag}(w^{\mathcal{I}})\,[e_i^T]_{i|\tau=\Omega(v_i,a_p)}$, $e_i\in\mathbb{R}^N$.

Remark 7. $W^{\text{or}}\in\mathbb{R}^{\rho\times(N+l)}$ is a *row selection matrix* whose each row contains a single nonzero element which equals to a corresponding weight $w_\tau^{\mathcal{O}}$, $w_\tau^{\mathcal{C}}$ or $w_\tau^{\mathcal{I}}$.

Next, let $r(k)$ denote the received date from $t(k)$ via the unreliable wireless communication links. $\gamma_\tau(k)$, $\tau\in\{1,\dots,\rho\}$ is independent and identically distributed (I.I.D) Bernoulli random variable with mean $\mu_\tau=\mathbb{E}[\gamma_\tau(k)]$ and variance $\sigma_\tau^2=\mathbb{E}[(\gamma_\tau(k)-\mu_\tau)^2]$. $\gamma_\tau(k)$ indicates whether packet $t_\tau(k)$ is successfully received by $r_\tau(k)$; that is, $\gamma_\tau(k)=1$ if packet arrives and $\gamma_\tau(k)=0$ otherwise. If $\Delta_\tau(k)$, $\tau\in\{1,\dots,\rho\}$ denotes an I.I.D Bernoulli random variable with zero-mean and variance σ_τ^2, $\gamma_\tau(k)$ can be transformed into a robust form such that $\gamma_\tau(k)=\mu_\tau+\Delta_\tau(k)$, where μ_τ is nominal value and $\Delta_\tau(k)$ is random perturbation value. Thus, the fading channel model is described by the following bijective mapping:

$$\Gamma: t(k)\longrightarrow r(k)=\Gamma(k)\,t(k)=(\mathrm{M}+\Delta(k))\,t(k), \tag{36}$$

where $\Gamma(k)=\operatorname{diag}(\gamma_1(k),\dots,\gamma_\rho(k))$, $\mathrm{M}=\operatorname{diag}(\mu_1,\dots,\mu_\rho)$, $\Delta(k)=\operatorname{diag}(\Delta_1(k),\dots,\Delta_\rho(k))$, $\mathbb{E}[\Delta(k)\Delta(k)^T]=\operatorname{diag}(\sigma_1^2,\dots,\sigma_\rho^2)$, and $\mathbb{E}[\Delta(k)]=0$.

Thus, the dynamic behavior of the *fading* WNCN with stochastic packet losses can be described as follows:

$$\widetilde{\mathcal{K}}\begin{cases} z(k+1)=\psi(\xi(k)), \\ \xi(k)=W_\mu^{\mathcal{C}}z(k)+W_\mu^{\mathcal{O}}y(k)+W_\Delta^{\mathcal{CO}}\Delta(k)\,t(k), \\ u(k)=W_\mu^{\mathcal{I}}z(k)+W_\Delta^{\mathcal{I}}\Delta(k)\,t(k), \end{cases} \tag{37}$$

where $W_\Delta^{\mathcal{CO}}=W_\Delta^{\mathcal{C}}+W_\Delta^{\mathcal{O}}$ and

$$W_\mu^{\mathcal{O}}=\left[\left(w_\mu^{\mathcal{O}}\right)_{iq}\right]_{N\times l},$$

$$\text{where }\left(w_\mu^{\mathcal{O}}\right)_{iq}=\begin{cases}\mu_\tau w_\tau^{\mathcal{O}}, & \exists v_i\in\mathcal{V}, s_q\in\mathcal{S}, \\ & \Omega\left(s_q,v_i\right)=\tau \\ 0, & \text{else},\end{cases}$$

$$W_\mu^{\mathcal{C}}=\left[\left(w_\mu^{\mathcal{C}}\right)_{ij}\right]_{N\times N},$$

$$\text{where }\left(w_\mu^{\mathcal{C}}\right)_{ij}=\begin{cases}\mu_\tau w_\tau^{\mathcal{C}}, & \text{if } i\ne j, \exists v_i,v_j\in\mathcal{V}, \\ & \Omega\left(v_j,v_i\right)=\tau \\ w_{ii}^{\mathcal{C}}, & \text{if } i=j, \exists v_i\in\mathcal{V} \\ 0, & \text{else},\end{cases}$$

$$W_\mu^{\mathcal{I}}=\left[\left(w_\mu^{\mathcal{I}}\right)_{pi}\right]_{m\times N},$$

$$\text{where }\left(w_\mu^{\mathcal{I}}\right)_{pi}=\begin{cases}\mu_\tau w_\tau^{\mathcal{I}}, & \exists v_i\in\mathcal{V}, a_p\in\mathcal{A}, \\ & \Omega\left(v_i,a_p\right)=\tau \\ 0, & \text{else},\end{cases}$$

$$W_{\Delta}^{\mathcal{O}} = \left[\left(w_{\Delta}^{\mathcal{O}}\right)_{i\tau}\right]_{N\times\rho},$$
$$\text{where } \left(w_{\Delta}^{\mathcal{O}}\right)_{i\tau} = \begin{cases} 1, & \exists v_i \in \mathcal{V}, s_q \in \mathcal{S}, \\ & \Omega\left(s_q, v_i\right) = \tau \\ 0, & \text{else}, \end{cases}$$
$$W_{\Delta}^{\mathcal{C}} = \left[\left(w_{\Delta}^{\mathcal{C}}\right)_{i\tau}\right]_{N\times\rho},$$
$$\text{where } \left(w_{\Delta}^{\mathcal{C}}\right)_{i\tau} = \begin{cases} 1, & \exists v_i, v_j \in \mathcal{V}, \\ & \Omega\left(v_j, v_i\right) = \tau \\ 0, & \text{else}, \end{cases}$$
$$W_{\Delta}^{\mathcal{J}} = \left[\left(w_{\Delta}^{\mathcal{J}}\right)_{p\tau}\right]_{m\times\rho},$$
$$\text{where } \left(w_{\Delta}^{\mathcal{J}}\right)_{p\tau} = \begin{cases} 1, & \exists v_i \in \mathcal{V}, a_p \in \mathcal{A}, \\ & \Omega\left(v_i, a_p\right) = \tau \\ 0, & \text{else}. \end{cases} \tag{38}$$

Remark 8. Similar to W^{or}, matrices $W_{\Delta}^{\mathcal{C}\mathcal{O}}$ and $W_{\Delta}^{\mathcal{J}}$ are used to select which elements of $\Delta(k)t(k)$ are added to $\xi(k)$ and $u(k)$, respectively.

As shown in Figure 2(b), DSNNM (1) is controlled by a fading network composed by the mean WNCN (MWNCN) and the stochastic perturbation Δ. Consider the following stochastic closed-loop system:

$$\widetilde{\mathcal{G}} \begin{cases} \widetilde{x}(k+1) = \widetilde{A}_{\mu}\widetilde{x}(k) + \widetilde{A}_{d}\widetilde{x}(k-1) + \widetilde{B}_{\phi}\widetilde{\phi}(\widetilde{\varepsilon}(k)) \\ \qquad + \widetilde{B}_{\omega}\omega(k) + \widetilde{W}_{1}^{\text{dst}}\Delta(k)t(k), \\ \widetilde{\varepsilon}(k) = \widetilde{C}_{\varepsilon}^{\mu}\widetilde{x}(k) + \widetilde{C}_{d}\widetilde{x}(k-1) + \widetilde{D}_{\phi}\widetilde{\phi}(\widetilde{\varepsilon}(k)) \\ \qquad + \widetilde{D}_{\omega}\omega(k) + \widetilde{W}_{2}^{\text{dst}}\Delta(k)t(k), \end{cases} \tag{39}$$

where $\widetilde{A}_{\mu} = \begin{bmatrix} A & B_u W_{\mu}^{\mathcal{J}} \\ 0 & 0 \end{bmatrix}$, $\widetilde{C}_{\varepsilon}^{\mu} = \begin{bmatrix} C_{\varepsilon} & D_u W_{\mu}^{\mathcal{J}} \\ W_{\mu}^{\mathcal{O}} C_y & W_{\mu}^{\mathcal{C}} \end{bmatrix}$, and $\widetilde{W}_{1}^{\text{dst}} = \begin{bmatrix} B_u W_{\Delta}^{\mathcal{J}} \\ 0 \end{bmatrix}$, $\widetilde{W}_{2}^{\text{dst}} = \begin{bmatrix} D_u W_{\Delta}^{\mathcal{J}} \\ W_{\Delta}^{\mathcal{C}\mathcal{O}} \end{bmatrix}$.

Definition 9 (see [6, 32, 33]). Given scalar $\gamma > 0$ and $M(k)$, the state correlation matrix, as $M(k) \triangleq \mathbb{E}\{\widetilde{x}(k)\widetilde{x}(k)^T\}$, the stochastic closed-loop system $\widetilde{\mathcal{G}}$ is said to be absolutely stable in mean-square with a H_{∞}-norm bound γ, if there exists a distributed dynamic neural controller WNCN $\mathcal{K}$ such that the following conditions are satisfied.

(1) With zero disturbance, that is, $\omega(k) = 0$, $\lim_{k\to\infty} M(k) = 0$, $\forall \widetilde{x}(0)$, and $\forall \widetilde{\phi} \in \Omega(\widetilde{K})$.

(2) Under the zero-initial condition, the performance output $\widetilde{y}(k)$ satisfies

$$\sum_{k=0}^{\infty}\mathbb{E}\left\{\left\|\widetilde{y}(k)\right\|^2\right\} \leq \gamma^2 \sum_{k=0}^{\infty}\mathbb{E}\left\{\left\|\omega(k)\right\|^2\right\}, \quad \forall \text{ nonzero } \omega(k). \tag{40}$$

Then the WNCN $\mathcal{K}$ is said to be a robust H_{∞} controller for the DSNNM $\mathcal{P}$. Furthermore, if we can find a minimal γ^* to satisfy the above conditions, the WNCN $\mathcal{K}$ is an optimal robust H_{∞} controller.

Theorem 10. *Given a scalar $\gamma > 0$, the stochastic closed-loop system $\widetilde{\mathcal{G}}$ is said to be absolutely stabilizable in mean-square by using a fading WNCN $\widetilde{\mathcal{K}}$ and the H_{∞}-norm constraint (40) is achieved for all nonzero $\omega(k)$ if there exist appropriate dimension matrices $P \succ 0$, $Q \succ 0$, $R \succ 0$, $\Sigma \geq 0$, $W_{\mu}^{\mathcal{O}}$, $W_{\mu}^{\mathcal{C}}$, $W_{\mu}^{\mathcal{J}}$, and scalar θ_i, $i = 1, \ldots, \rho$, such that the following optimization problem:*

$$\min \quad \mathbf{Tr}\{QP\} \tag{41}$$

$$\text{s.t.} \quad \Xi_3 = \begin{bmatrix} -P + R + \Pi & 0 & \left(\widetilde{C}_{\varepsilon}^{\mu}\right)^T & 0 & \widetilde{A}_{\mu}^T & \widetilde{C}_{y}^T \\ * & -R & \widetilde{C}_{d}^T & 0 & \widetilde{A}_{d}^T & 0 \\ * & * & \widetilde{D}_{\phi}\Sigma\widetilde{K} + \Sigma\widetilde{K}\widetilde{D}_{\phi}^T - 2\Sigma & \widetilde{D}_{\omega} & \Sigma\widetilde{K}\widetilde{B}_{\phi}^T & 0 \\ * & * & * & -\gamma^2 I & \widetilde{B}_{\omega}^T & 0 \\ * & * & * & * & -Q & 0 \\ * & * & * & * & * & -I \end{bmatrix} \prec 0, \tag{42}$$

$$\theta_i \geq \sigma_i^2\left(\widetilde{W}_1^{dst}\right)_i^T P\left(\widetilde{W}_1^{dst}\right)_i, \quad \forall i = 1, \ldots, \rho, \tag{43}$$

$$\begin{bmatrix} Q & I \\ I & P \end{bmatrix} \geq 0, \quad Q, P \in \mathbb{S}_{++}^{n+N}, \tag{44}$$

where $\Pi = (\widetilde{W}^{or})^T\Theta\widetilde{W}^{or}$, $\Theta = \operatorname{diag}(\theta_1, \ldots, \theta_{\rho})$, and $\theta_i = \sigma_i^2(\widetilde{W}_1^{dst})_i^T P(\widetilde{W}_1^{dst})_i$, $(\widetilde{W}_1^{dst})_i$ denote the ith column of the matrix $\widetilde{W}_1^{dst}$, is feasible with optimal cost $n + N$.

Proof. Consider a Lyapunov candidate as follows:

$$V[M(k)] = \mathbf{Tr}[M(k)P]. \tag{45}$$

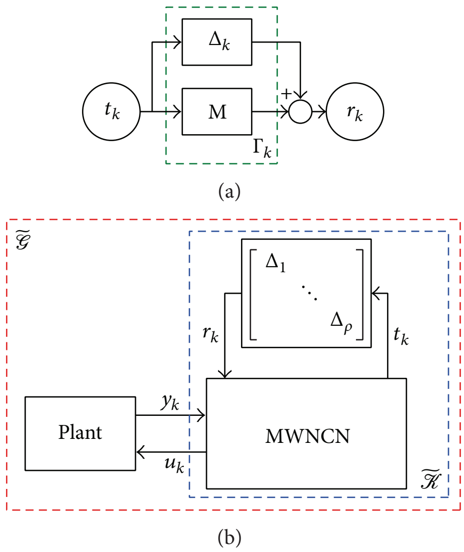


FIGURE 2: (a) The wireless communication links simulated as fading channels and (b) WNCN is transformed into a robust sense.

The difference of $V\ [M(k)]$ along the trajectory of stochastic closed-loop system $\widetilde{\mathscr{G}}$ with $\omega(k) = 0$ is given by

$$\begin{aligned}\Delta V[M(k)] &= V[M(k+1)] - V[M(k)] \\ &= \mathbf{Tr}[M(k+1)P] - \mathbf{Tr}[M(k)P] \\ &= \mathbf{Tr}\left\{\mathbb{E}\left[\widetilde{x}^T(k+1)P\widetilde{x}(k+1) - \widetilde{x}^T(k)P\widetilde{x}(k)\right]\right\} \\ &= \mathbb{E}\left[\widetilde{x}^T(k+1)P\widetilde{x}(k+1) - \widetilde{x}^T(k)P\widetilde{x}(k)\right].\end{aligned} \tag{46}$$

By considering $\Delta(k)$ with $\mathbb{E}[\Delta(k)] = 0$ and $\mathbb{E}[\Delta(k)\Delta^T(k)] = \operatorname{diag}(\sigma_1^2, \ldots, \sigma_\rho^2)$ is independent from $\widetilde{x}(k)$ and $\widetilde{\phi}(\widetilde{\varepsilon}(k))$ and using Lemma 2 (S-procedure) one can obtain

$$\begin{aligned}&\Delta V[M(k)] \\ &= \begin{bmatrix}\widetilde{x}(k) \\ \widetilde{x}(k-d) \\ \widetilde{\phi}(\widetilde{\varepsilon}(k))\end{bmatrix}^T \\ &\times \underbrace{\begin{bmatrix}\begin{pmatrix}\widetilde{A}_\mu^T P\widetilde{A}_\mu - P \\ +R+\Pi\end{pmatrix} & \widetilde{A}_\mu^T P\widetilde{A}_d & \widetilde{A}_\mu^T P\widetilde{B}_\phi + \left(\widetilde{C}_\varepsilon^\mu\right)^T T\widetilde{K} \\ * & \widetilde{A}_d^T P\widetilde{A}_d - R & \widetilde{A}_d^T P\widetilde{B}_\phi + \widetilde{C}_d^T T\widetilde{K} \\ * & * & \begin{pmatrix}\widetilde{B}_\phi^T P\widetilde{B}_\phi + \widetilde{D}_\phi^T T\widetilde{K} \\ +T\widetilde{K}\widetilde{D}_\phi - 2T\end{pmatrix}\end{bmatrix}}_{\widetilde{\Xi}_0} \\ &\times \begin{bmatrix}\widetilde{x}(k) \\ \widetilde{x}(k-d) \\ \widetilde{\phi}(\widetilde{\varepsilon}(k))\end{bmatrix},\end{aligned} \tag{47}$$

where $T = \operatorname{diag}(\tau_1, \tau_2, \ldots, \tau_{L+N})$, $\widetilde{K} = \operatorname{diag}(k_1, \ldots, k_{L+N})$, $\Pi = (\widetilde{W}^{\text{or}})^T\Theta\widetilde{W}^{\text{or}}$, $\Theta = \operatorname{diag}(\theta_1, \ldots, \theta_\rho)$, $\theta_i = \sigma_i^2(\widetilde{W}_1^{\text{dst}})_i^T P(\widetilde{W}_1^{\text{dst}})_i$, $(\widetilde{W}_1^{\text{dst}})_i$ denote the ith column of the matrix $\widetilde{W}_1^{\text{dst}}$.

According to (20)–(22), $\forall \ell > 0$, we have

$$\begin{aligned}\mathbb{E}\{J_\ell\} &= \sum_{k=0}^{\ell}\mathbb{E}\left\{\|\widetilde{y}(k)\|^2\right\} - \gamma^2\sum_{k=0}^{\ell}\mathbb{E}\left\{\|\omega(k)\|^2\right\} \\ &= \sum_{k=0}^{\ell}\mathbb{E}\left\{\widetilde{y}^T(k)\widetilde{y}(k) - \gamma^2\omega^T(k)\omega(k)\right\} \\ &\le \sum_{k=0}^{\ell}\zeta^T(k)\widetilde{\Xi}_1\zeta(k),\end{aligned} \tag{48}$$

where

$$\widetilde{\Xi}_1 = \begin{bmatrix}\begin{pmatrix}\widetilde{A}_\mu^T P\widetilde{A}_\mu - P + R \\ +\widetilde{C}_y^T\widetilde{C}_y + \Pi\end{pmatrix} & \widetilde{A}_\mu^T P\widetilde{A}_d & \widetilde{A}_\mu^T P\widetilde{B}_\phi + \left(\widetilde{C}_\varepsilon^\mu\right)^T T\widetilde{K} & \widetilde{A}_\mu^T P\widetilde{B}_\omega \\ * & \widetilde{A}_d^T P\widetilde{A}_d - R & \widetilde{A}_d^T P\widetilde{B}_\phi + \widetilde{C}_d^T T\widetilde{K} & \widetilde{A}_d^T P\widetilde{B}_\omega \\ * & * & \begin{pmatrix}\widetilde{B}_\phi^T P\widetilde{B}_\phi + \widetilde{D}_\phi^T T\widetilde{K} \\ +T\widetilde{K}\widetilde{D}_\phi - 2T\end{pmatrix} & \widetilde{B}_\phi^T P\widetilde{B}_\phi + T\widetilde{K}\widetilde{D}_\omega \\ * & * & * & \widetilde{B}_\omega^T P\widetilde{B}_\omega - \gamma^2 I.\end{bmatrix}. \tag{49}$$

Since $\widetilde{\Xi}_0$ is the principal minor of $\widetilde{\Xi}_1$, if $\widetilde{\Xi}_1 \prec 0$, then $\widetilde{\Xi}_0 \prec 0$ such that $M(k)$ converges to zeros as $k \to \infty$ for system $\widetilde{\mathscr{G}}$ with $\omega(k) = 0$, $\forall\widetilde{x}(0)$, and $\forall\widetilde{\phi} \in \Omega(\widetilde{K})$. Further, if $\widetilde{\Xi}_1 \prec 0$ holds, $\lim_{\ell\to\infty}\mathbb{E}\{J_\ell\} \le \lim_{\ell\to\infty}\sum_{k=0}^{\ell}\zeta^T(k)\widetilde{\Xi}_1\zeta(k) < 0$. Thus, $\forall$ nonzero $\omega(k) \in l_2\ [0,\infty)$ and the H_∞-norm constraint (40) is achieved. Similar to the proof process of Theorem 6, by using Lemma 3 (Schur complement), the inequality $\widetilde{\Xi}_1 \prec 0$ is equivalent to optimization problems (41)–(44) described in Theorem 10. This completes the proof. □

As in the previous section, we present Algorithms 3 and 4.

6. Numerical Simulation

Consider the following nonlinear system [38]:

$$\begin{aligned}x_1(k+1) = {}& -x_1^2(k) + 0.3x_2(k) + 0.1x_1^2(k-2) \\ & - 0.2x_1(k-2)x_2(k-2) + u_1(k),\end{aligned}$$

Step 1. Set $k = 0$. If there exists an initial feasible solution set $\Upsilon_0 = \{P, Q, R, \Sigma, W_\mu^{\mathcal{O}}, W_\mu^{\mathcal{C}}, W_\mu^{\mathcal{I}}\}$ satisfying the constraints (42)–(44), let $\mathcal{X}_0 = P$, $\mathcal{Y}_0 = Q$. Otherwise, exit.
Step 2. If $k \le \kappa$, go to Step 3 where κ is the assumed maximum number of iteration. Otherwise, exit.
Step 3. At $k \ge 0$, obtain the feasible solution set $\Upsilon_{k+1} = \{P, Q, R, \Sigma, W_\mu^{\mathcal{O}}, W_\mu^{\mathcal{C}}, W_\mu^{\mathcal{I}}\}$ by solving the following LMI problem: $\min \mathbf{Tr}\{\mathcal{X}_k Q + \mathcal{Y}_k P\}$ s.t. (42)–(44).
Step 4. Substitute Υ_{k+1} into matrix $\widetilde{\Xi}_1$. If inequality $\widetilde{\Xi}_1 \prec 0$ holds, stop the algorithm. Otherwise, set $k = k + 1$, $\mathcal{X}_k = P$, $\mathcal{Y}_k = Q$ and go to Step 2.

Algorithm 3: Given a scalar $\gamma > 0$, solving the H_∞ *fading* WNCN with unreliable communication links for closed-loop system $\widetilde{\mathcal{G}}$.

Step 1. Set $k = 0$. Let γ_-^0 and γ_+^0 be the initial lower and upper bounds of γ, that is, $\gamma_0 \in [\gamma_-^0, \gamma_+^0]$ where $\gamma_-^0 = 0$, γ_+^0 can be assigned an arbitrarily sufficiently large value to make inequalities (42)–(44) have initial feasible solution set $\Upsilon_0 = \{P, Q, R, \Sigma, W_\mu^{\mathcal{O}}, W_\mu^{\mathcal{C}}, W_\mu^{\mathcal{I}}\}$.
Step 2. At $k \ge 0$, computer $\gamma_k = (\gamma_-^k + \gamma_+^k)/2$.
Step 3. Use Algorithm 3 to check whether the feasible solution set Υ_{k+1} satisfying $\widetilde{\Xi}_1 \prec 0$. If Υ_{k+1} exists, set $\gamma_+^k = \gamma^k$. Otherwise, set $\gamma_-^k = \gamma^k$.
Step 4. If $\gamma_+^k - \gamma_-^k \le \epsilon$, where ϵ is the assumed calculation accuracy, set optimal H_∞ performance index $\gamma^* = \gamma_+^k$ and exit. Otherwise, set $k = k + 1$, go to Step 2.

Algorithm 4: Minimizing γ to solve the optimal H_∞ *fading* WNCN with unreliable communication links for closed-loop system $\widetilde{\mathcal{G}}$.

$$\begin{aligned} x_2(k+1) &= 0.1x_1(k) + x_2(k) + 0.5u_2(k), \\ y(k) &= 0.6x_1(k). \end{aligned} \tag{50}$$

According to [39, 40], when we consider the disturbance $w(k)$, the nonlinear system (50) can be transformed into the discrete-time DSNNM (1), where $A = \begin{bmatrix} -0.5 & 0.3 \\ 0.1 & 1 \end{bmatrix}$, $A_d = \begin{bmatrix} 0.05 & -0.1 \\ 0 & 0 \end{bmatrix}$, $B_\phi = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $B_u = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$, $C_\varepsilon = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $C_d = \begin{bmatrix} -0.1 & 0.2 \end{bmatrix}$, $D_\phi = 0$, $D_u = 0_{1\times 2}$, $C_y = \begin{bmatrix} 0.6 & 0 \end{bmatrix}$, $B_\omega = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}$, $D_\omega = 0.1$, and $K = I$.

Consider that the double-input-single-output (DISO) discrete-time DSNNM described above is synthesized by a WNCN which consists of 6 wireless neuron nodes shown in Figure 3. In WNCN, each wireless communication link is modeled as a fading channel with same packet arrival rate (mean) δ and variance $\sigma^2 = \delta(1-\delta)$. For $\delta = 0.95\%$, Algorithm 4 can be solved by CVX, a package for specifying and solving convex programs [41]. Then, we obtain the minimum optimal H_∞ performance index $\gamma^* = 0.7921$, the solutions of (41)–(44), and the interconnection weight matrix parameters of WNCN as follows:

$$P = \begin{bmatrix} 3.6031 & -1.0546 & -0.0825 & -0.0825 & 0.0047 & 0.0047 & -0.0617 & -0.0617 \\ -1.0546 & 11.3398 & 1.2546 & 1.2546 & -1.1665 & -1.1665 & -1.1256 & -1.1256 \\ -0.0825 & 1.2546 & 1.1610 & 0.0840 & -0.0821 & -0.0821 & -0.0785 & -0.0785 \\ -0.0825 & 1.2546 & 0.0840 & 1.1610 & -0.0821 & -0.0821 & -0.0785 & -0.0785 \\ 0.0047 & -1.1665 & -0.0821 & -0.0821 & 1.1495 & 0.0725 & 0.0708 & 0.0708 \\ 0.0047 & -1.1665 & -0.0821 & -0.0821 & 0.0725 & 1.1495 & 0.0708 & 0.0708 \\ -0.0617 & -1.1256 & -0.0785 & -0.0785 & 0.0708 & 0.0708 & 1.1454 & 0.0684 \\ -0.0617 & -1.1256 & -0.0785 & -0.0785 & 0.0708 & 0.0708 & 0.0684 & 1.1454 \end{bmatrix},$$

$$Q = \begin{bmatrix} 0.2931 & 0.0501 & -0.0192 & -0.0192 & 0.0378 & 0.0378 & 0.0545 & 0.0545 \\ 0.0501 & 0.1789 & -0.1421 & -0.1421 & 0.1359 & 0.1359 & 0.1342 & 0.1342 \\ -0.0192 & -0.1421 & 0.9929 & 0.0644 & -0.0579 & -0.0579 & -0.0576 & -0.0576 \\ -0.0192 & -0.1421 & 0.0644 & 0.9929 & -0.0579 & -0.0579 & -0.0576 & -0.0576 \\ 0.0378 & 0.1359 & -0.0579 & -0.0579 & 0.9884 & 0.0599 & 0.0594 & 0.0594 \\ 0.0378 & 0.1359 & -0.0579 & -0.0579 & 0.0599 & 0.9884 & 0.0594 & 0.0594 \\ 0.0545 & 0.1342 & -0.0576 & -0.0576 & 0.0594 & 0.0594 & 0.9890 & 0.0605 \\ 0.0545 & 0.1342 & -0.0576 & -0.0576 & 0.0594 & 0.0594 & 0.0605 & 0.9890 \end{bmatrix},$$

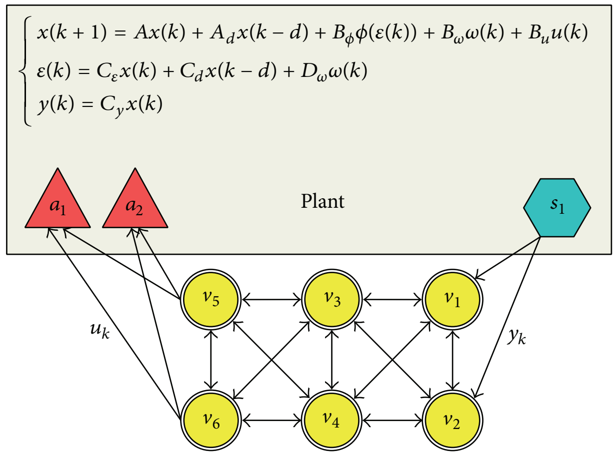


FIGURE 3: A discrete-time DSNNM synthesized by a WNCN with 6 wireless neuron nodes.

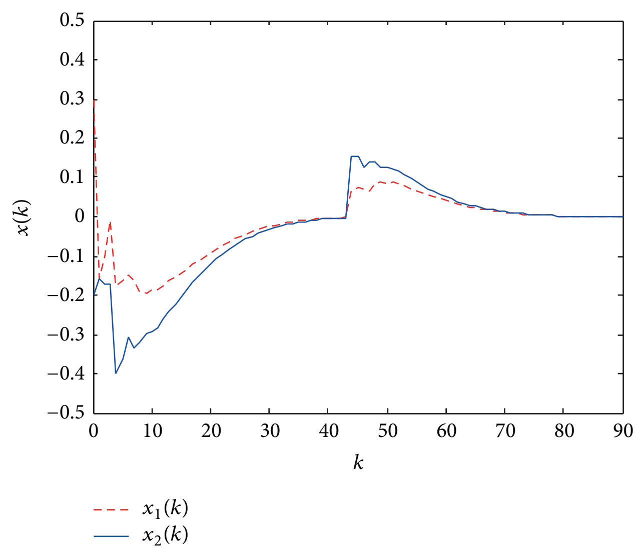


FIGURE 4: State response of the closed-loop system under H_∞ controller WNCN with $\gamma^* = 0.7921$.

$$R = \begin{bmatrix} 1.1713 & -0.1182 & 0.0122 & 0.0122 & -0.0342 & -0.0342 & -0.1895 & -0.1895 \\ 0 & 0.5427 & -0.0197 & -0.0197 & -0.0580 & -0.0580 & 0.0405 & 0.0405 \\ 0 & 0 & 0.7390 & -0.7183 & 0.0078 & 0.0078 & -0.0614 & -0.0614 \\ 0 & 0 & 0 & 0.1738 & 0.0657 & 0.0657 & -0.5153 & -0.5153 \\ 0 & 0 & 0 & 0 & 0.7494 & -0.6879 & 0.0713 & 0.0713 \\ 0 & 0 & 0 & 0 & 0 & 0.2973 & 0.3447 & 0.3447 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.7422 & -0.7089 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2198 \end{bmatrix},$$

$$\Sigma = \operatorname{diag}(0.5313, 1.2461, 0.1357, 0.4233, 1.4452, 0.7652, 1.2891),$$

$$W_\mu^{\mathscr{C}} = \begin{bmatrix} 0.0484 & 0.0484 & 0.0003 & 0.0003 & 0 & 0 \\ 0.0484 & 0.0484 & 0.0003 & 0.0003 & 0 & 0 \\ -0.4925 & -0.4925 & 0.0110 & 0.0110 & -0.1193 & -0.1193 \\ -0.4925 & -0.4925 & 0.0110 & 0.0110 & -0.1193 & -0.1193 \\ 0 & 0 & 0.4870 & 0.4870 & -0.0879 & -0.0879 \\ 0 & 0 & 0.4870 & 0.4870 & -0.0879 & -0.0879 \end{bmatrix}, \qquad W_\mu^{\mathscr{O}} = \begin{bmatrix} -1.3588 \\ -1.3588 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$W_\mu^{\mathscr{I}} = \begin{bmatrix} 0 & 0 & 0 & 0 & -0.0646 & -0.0646 \\ 0 & 0 & 0 & 0 & -0.2989 & -0.2989 \end{bmatrix}. \tag{51}$$

Figure 4 shows the simulation results of the state trajectories of controlled discrete-time DSNNM, where state $x(k)$ is initialized arbitrarily in interval $[-0.5, 0.5]$ at $k = 0$ and $k = 40$, respectively, and the disturbance input is nonlinear load $1/k^2$. It is easily seen that the WNSN with a distributed architecture solved by Algorithm 4 using CVX toolbox can ensure the absolute stability of the closed-loop system in the mean-square sense with optimal H_∞ performance.

7. Conclusions

A novel wireless networked H_∞ control approach based on WNCN has been considered for a class of Lurie-type nonlinear systems named DSNNM. The WNCN which can absolutely stabilize the closed-loop system in mean-square with a desired H_∞ disturbance rejection level can be obtained by solving LMIs using a CVX toolbox (release 2.0 (beta)). Simulation results have illustrated the feasibility of the distributed control methods presented in this paper.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (61174070) and the Specialized Research Fund for the Doctoral Program (20110172110033).

References

[1] Z. Q. Sun, Z. D. Deng, and Z. X. Zhang, *Intelligent Control Theory and Technology*, Tsinghua University Press, Beijiing, China, 2nd edition, 2011.

[2] C. C. Ku and K. Y. Lee, "Diagonal recurrent neural networks for dynamic systems control," *IEEE Transactions on Neural Networks*, vol. 6, no. 1, pp. 144–156, 1995.

[3] F. J. Lin, Y. C. Hung, and S. Y. Chen, "FPGA-based computed force control system using Elman neural network for linear ultrasonic motor," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 4, pp. 1238–1253, 2009.

[4] S. Zerkaoui, F. Druaux, E. Leclercq, and D. Lefebvre, "Stable adaptive control with recurrent neural networks for square MIMO non-linear systems," *Engineering Applications of Artificial Intelligence*, vol. 22, no. 4-5, pp. 702–717, 2009.

[5] T. Botmart and W. Weera, "Guaranteed cost control for exponential synchronization of cellular neural networks with mixed time-varying delays via hybrid feedback control," *Abstract and Applied Analysis*, vol. 2013, Article ID 175796, 12 pages, 2013.

[6] X. Liao and P. Yu, *Absolute Stability of Nonlinear Control Systems*, Springer, New York, NY, USA, 2008.

[7] L. Jin, P. N. Nikiforuk, and M. M. Gupta, "Absolute stability conditions for discrete-time recurrent neural networks," *IEEE Transactions on Neural Networks*, vol. 5, no. 6, pp. 954–964, 1994.

[8] D. Danciu, V. Rasvan, and D. Popescu, "On the absolute stability for recurrent neural networks with time delays," in *Proceedings of the 13th International Carpathian Control Conference (ICCC '12)*, pp. 97–102, High Tatras, Slovakia, May 2012.

[9] S. Hu and J. Wang, "Absolute exponential stability of a class of continuous-time recurrent neural networks," *IEEE Transactions on Neural Networks*, vol. 14, no. 1, pp. 35–45, 2003.

[10] H. Lu, R. Shen, and F. Chung, "Absolute exponential stability of a class of recurrent neural networks with multiple and variable delays," *Theoretical Computer Science*, vol. 344, no. 2-3, pp. 103–119, 2005.

[11] J. A. K. Suykens, J. Vandewalle, and B. de Moor, "Lur'e systems with multilayer perceptron and recurrent neural networks: absolute stability and dissipativity," *IEEE Transactions on Automatic Control*, vol. 44, no. 4, pp. 770–774, 1999.

[12] M. Liu, S. Zhang, G. Yan, and S. Wang, "A new neural network model and its application," in *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics (SMC '04)*, pp. 5864–5869, Hague, The Netherlands, October 2004.

[13] M. Liu, "Delayed standard neural network models for control systems," *IEEE Transactions on Neural Networks*, vol. 18, no. 5, pp. 1376–1391, 2007.

[14] M. Liu and H. Wang, "A novel stabilizing control for neural nonlinear systems with time delays by state and dynamic output feedback," *International Journal of Control, Automation and Systems*, vol. 6, no. 1, pp. 24–34, 2008.

[15] Q. Wang and S. Gopalakrishnan, "Adapting a main-stream internet switch architecture for multihop real-time industrial networks," *IEEE Transactions on Industrial Informatics*, vol. 6, no. 3, pp. 393–404, 2010.

[16] W. Wolf, "Cyber-physical systems," *Computer*, vol. 42, no. 3, pp. 88–89, 2009.

[17] X. Cao, P. Cheng, J. Chen, and Y. Sun, "An online optimization approach for control and communication codesign in networked cyber-physical systems," *IEEE Transactions on Industrial Informatics*, vol. 9, no. 1, pp. 439–450, 2013.

[18] L. Shi and H. Zhang, "Scheduling two gauss-markov systems: an optimal solution for remote state estimation under bandwidth constraint," *IEEE Transactions on Signal Processing*, vol. 60, no. 4, pp. 2038–2042, 2012.

[19] G. Guo, Z. Lu, and Q. Han, "Control with markov sensors/actuators assignment," *IEEE Transactions on Automatic Control*, vol. 57, no. 7, pp. 1799–1804, 2012.

[20] G. Guo, "A switching system approach to sensor and actuator assignment for stabilisation via limited multi-packet transmitting channels," *International Journal of Control*, vol. 84, no. 1, pp. 78–93, 2011.

[21] Z. Lu and G. Guo, "Communications and control co-design: a combined dynamic-static scheduling approach," *Science China: Information Sciences*, vol. 55, no. 11, pp. 2495–2507, 2012.

[22] G. Guo, "Linear systems with medium access constraint and Markov actuator assignment," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 57, no. 11, pp. 2999–3010, 2010.

[23] M. Mazo Jr. and P. Tabuada, "Decentralized event-triggered control over wireless sensor/actuator networks," *IEEE Transactions on Automatic Control*, vol. 56, no. 10, pp. 2456–2461, 2011.

[24] J. M. Chen, X. H. Cao, P. Cheng, Y. Xiao, and Y. X. Sun, "Distributed collaborative control for industrial automation with wireless sensor and actuator networks," *IEEE Transactions on Industrial Electronics*, vol. 57, no. 12, pp. 4219–4230, 2010.

[25] O. C. Imer, S. Yüksel, and T. Başar, "Optimal control of LTI systems over unreliable communication links," *Automatica*, vol. 42, no. 9, pp. 1429–1439, 2006.

[26] E. Garone, B. Sinopoli, A. Goldsmith, and A. Casavola, "LQG control for MIMO systems over multiple erasure channels with perfect acknowledgment," *IEEE Transactions on Automatic Control*, vol. 57, no. 2, pp. 450–456, 2012.

[27] M. Pajic, S. Sundaram, J. L. Ny, G. J. Pappas, and R. Mangharam, "The wireless control network: synthesis and robustness," in *Proceedings of the 49th IEEE Conference on Decision and Control (CDC '10)*, pp. 7576–7581, Atlanta, Ga, USA, December 2010.

[28] M. Pajic, S. Sundaram, G. J. Pappas, and R. Mangharam, "The wireless control network: a new approach for control over networks," *IEEE Transactions on Automatic Control*, vol. 56, no. 10, pp. 2305–2318, 2011.

[29] W. Ren and B. Xu, "Wireless neural control network for nonlinear systems," in *Proceedings of the 32nd Chinese Control Conference (CCC '13)*, pp. 6537–6542, Xi'an, China, July 2013.

[30] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design*, John Wiley & Sons, New York, NY, USA, 1996.

[31] Z. Li, N. Wang, T. S. Hong, A. Franzen, and J. N. Li, "Closed-loop drip irrigation control using a hybrid wireless sensor and actuator network," *Science China Information Sciences*, vol. 54, no. 3, pp. 577–588, 2011.

[32] L. Yu, J. Chu, and H. Su, "Robust memoryless H_∞ controller design for linear time-delay systems with norm-bounded time-varying uncertainty," *Automatica*, vol. 32, no. 12, pp. 1759–1762, 1996.

[33] F. Yang, Z. Wang, Y. S. Hung, and M. Gani, "H_∞ control for networked systems with random communication delays," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 511–518, 2006.

[34] S. P. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, Pa, USA, 1994.

[35] S. P. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, New York, NY, USA, 2004.

[36] L. El Ghaoui, F. Oustry, and M. AitRami, "A cone complementarity linearization algorithm for static output-feedback and related problems," *IEEE Transactions on Automatic Control*, vol. 42, no. 8, pp. 1171–1176, 1997.

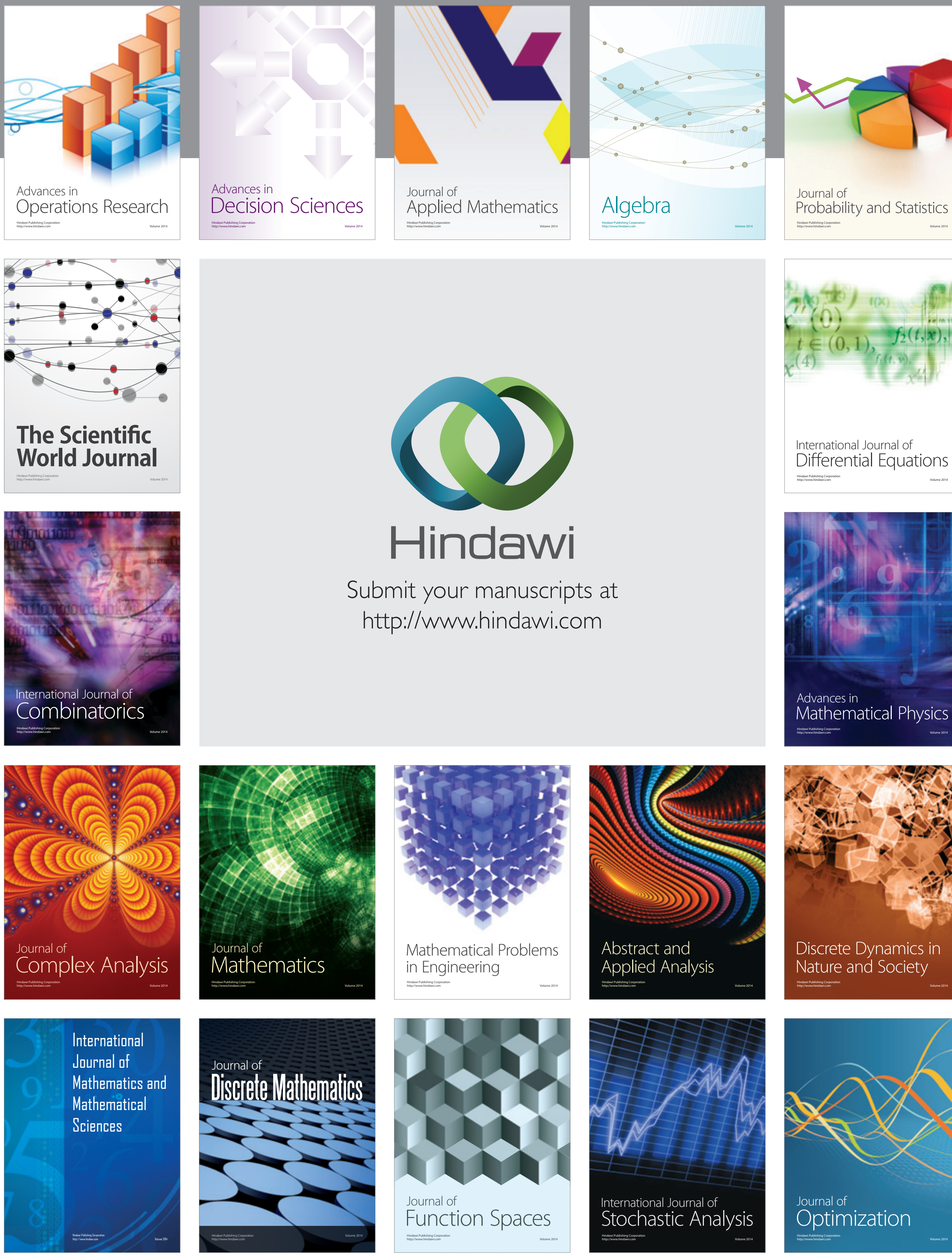
[37] N. Elia, "Remote stabilization over fading channels," *Systems and Control Letters*, vol. 54, no. 3, pp. 237–249, 2005.

[38] S. Y. Xu and J. Lam, "Robust H_∞ control for uncertain discrete-time-delay fuzzy systems via output feedback controllers," *IEEE Transactions on Fuzzy Systems*, vol. 13, no. 1, pp. 82–93, 2005.

[39] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, John Wiley & Sons, New York, NY, USA, 2001.

[40] M. Liu, "Unified stabilizing controller synthesis approach for discrete-time intelligent systems with time delays by dynamic output feedback," *Science in China F: Information Sciences*, vol. 50, no. 4, pp. 636–656, 2007.

[41] M. Grant and S. Boyd, *CVX: Matlab Software for Disciplined Convex Programming*, Version 2.0, CVX Research, Inc., 2011, http://cvxr.com/cvx.


Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
Algebra
Journal of Probability and Statistics
The Scientific World Journal
International Journal of Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
Mathematical Problems in Engineering
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
International Journal of Stochastic Analysis
Journal of Optimization