

Research Article

Fault Tolerant Control for Civil Structures Based on LMI Approach

Chunxu Qu, Linsheng Huo, and Hongnan Li

Faculty of Infrastructure Engineering, Dalian University of Technology, Dalian 116024, China

Correspondence should be addressed to Linsheng Huo; lishuo@dlut.edu.cn

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The control system may lose the performance to suppress the structural vibration due to the faults in sensors or actuators. This paper designs the filter to perform the fault detection and isolation (FDI) and then reforms the control strategy to achieve the fault tolerant control (FTC). The dynamic equation of the structure with active mass damper (AMD) is first formulated. Then, an estimated system is built to transform the FDI filter design problem to the static gain optimization problem. The gain is designed to minimize the gap between the estimated system and the practical system, which can be calculated by linear matrix inequality (LMI) approach. The FDI filter is finally used to isolate the sensor faults and reform the FTC strategy. The efficiency of FDI and FTC is validated by the numerical simulation of a three-story structure with AMD system with the consideration of sensor faults. The results show that the proposed FDI filter can detect the sensor faults and FTC controller can effectively tolerate the faults and suppress the structural vibration.

1. Introduction

The structural control system is commonly classified into four types: passive, active, hybrid, and semiactive control [1–9]. An active control system contains sensors, controllers, and actuators. Sensors can measure the structural vibration data that is sent to the controllers. The controllers compute the desired force according to the control strategy and sensor data and then command the control devices to generate control force for vibration reduction. Active control can achieve many control decisions. Therefore, active control is an efficient method to resist external excitation, such as earthquake and strong wind. Control strategy is the key of active control, which is widely studied in recent decades [10–16]. However, there are some controller design problems in civil engineering due to the characteristics of the civil structures, such as large number of degrees of freedom and uncertainties of system parameters, in which many researchers are interested [17–21].

In practical engineering, sensors and actuators may be destroyed due to the lack of maintenance or extreme loading such as strong wind or intense earthquake. These faulty sensors and control devices may degrade the performance of vibration suppressing and even enlarge the structural

response. Therefore, the robustness is important for the control system. Fault tolerant control (FTC) technology is an efficient method to promote the system reliability. FTC can tolerate faults caused by the components of control system, such as sensors and actuators, and then guarantee the system performance. FTC can be divided into two categories, passive FTC and active FTC. The difference between them exists if the faults are identified online. Passive FTC cannot detect faults online and cannot be recognized as traditional robust control [22]. The faults in passive FTC are assumed as system model uncertainties. Accordingly, passive FTC has limited tolerance ability because its robustness is only for the estimated faults, and the faults cannot be detected online. On the other hand, active FTC can detect faults online, which is called the fault detection and isolation (FDI) technology, and suppress excessive structural response.

This paper designs a fault detection and isolation (FDI) filter and reforms a fault tolerant controller (FTC) through LMI approach considering sensor faults in civil building structures with active mass damper (AMD). An estimated system is built to facilitate the design of FDI and FTC. In order to make the difference between the estimated and the practical system smaller, an FDI filter is designed, which

can minimize the H_{∞} norm of the transfer function from all the inputs to the residual of the estimated and practical state. According to the bounded real lemma, solving the FDI filter problem is transformed to solve a matrix inequality, which can be deformed to a linear matrix inequality (LMI) by variable substitution method [23]. LMI problem is convex and can be solved easily. After the FDI filtering, the new data is used to isolate the sensor faults and transmitted to the previous designed controllers so that the control strategy is reformed. The process executes the function of fault tolerant control (FTC). Finally, the numerical example of a three-story building is provided to validate the performance of FDI and FTC.

2. Formulation of AMD Control System

Considering n degrees of freedom civil structural system which is subjected to a unidirectional earthquake with $\ddot{q}_g(t)$ acceleration, the dynamic equation is described as follows:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{T}_g\ddot{q}_g(t) + \mathbf{T}_u\mathbf{u}(t), \quad (1)$$

where \mathbf{M} , \mathbf{C} , and $\mathbf{K} \in \mathbb{R}^{n \times n}$ are, respectively, mass, damping, and stiffness matrix; $\mathbf{q}(t)$, $\dot{\mathbf{q}}(t)$ and $\ddot{\mathbf{q}}(t) \in \mathbb{R}^{n \times 1}$ are the relative displacement, relative velocity, and relative acceleration vector, respectively; $\mathbf{u}(t) \in \mathbb{R}^{m_u \times 1}$ is the control force vector generated by actuators and $\mathbf{T}_u \in \mathbb{R}^{n \times m_u}$ and $\mathbf{T}_g \in \mathbb{R}^{n \times 1}$ are the location matrices of the control force and earthquake excitation, respectively.

For a structure with an AMD on the top floor as shown in Figure 1, the control force is exerted only on the top floor. The control force location matrix \mathbf{T}_u and earthquake excitation location matrix \mathbf{T}_g are represented as

$$\mathbf{T}_u = \begin{bmatrix} 0, 0, \dots, 0, 1 \\ \vdots \\ 0, 0, \dots, 0, 1 \end{bmatrix}_{1 \times n}, \quad (2)$$

$$\mathbf{T}_g = -\mathbf{M}\{\mathbf{1}\}_{n \times 1}.$$

The dynamic equation of the AMD system can be formulated as

$$m_a(\ddot{q}_a(t) + \ddot{q}_g(t)) + c_a(\dot{q}_a(t) - \dot{q}_n(t)) + k_a(q_a(t) - q_n(t)) = F_a(t), \quad (3)$$

where m_a , c_a , and $k_a \in \mathbb{R}^{1 \times 1}$ are, respectively, mass, damping, and stiffness of AMD system and $q_a(t)$, $\dot{q}_a(t)$, and $\ddot{q}_a(t) \in \mathbb{R}^{1 \times 1}$ are the relative displacement, relative velocity, and relative acceleration of AMD system. $q_n(t)$ and $\dot{q}_n(t) \in \mathbb{R}^{1 \times 1}$ are relative displacement and relative velocity of the n th floor, which is also the top floor and can be represented as

$$q_n(t) = \mathbf{T}_n\mathbf{q}(t), \quad \dot{q}_n(t) = \mathbf{T}_n\dot{\mathbf{q}}(t), \quad (4)$$

$$\mathbf{T}_n = \begin{bmatrix} 0, 0, \dots, 0, 1 \\ \vdots \\ 0, 0, \dots, 0, 1 \end{bmatrix}_{1 \times n}.$$

$F_a(t) \in \mathbb{R}^{1 \times 1}$ is the force between structural system and AMD mass. The motion equation of the structural system with AMD can be derived as

$$\mathbf{M}_s\ddot{\mathbf{q}}_s(t) + \mathbf{C}_s\dot{\mathbf{q}}_s(t) + \mathbf{K}_s\mathbf{q}_s(t) = \mathbf{T}_{gs}\ddot{q}_g(t) + \mathbf{T}_{us}F_a(t). \quad (5)$$

The matrices \mathbf{M}_s , \mathbf{C}_s , \mathbf{K}_s , \mathbf{q}_s , \mathbf{T}_{gs} , and \mathbf{T}_{us} are denoted as

$$\mathbf{M}_s = \begin{bmatrix} \mathbf{M} \\ m_a \end{bmatrix} = \begin{bmatrix} m_1 & & & \\ & \ddots & & \\ & & m_n & \\ & & & m_a \end{bmatrix}_{(n+1) \times (n+1)},$$

$$\mathbf{q}_s = \begin{bmatrix} \mathbf{q} \\ q_a \end{bmatrix} = \begin{bmatrix} q_1 \\ \vdots \\ q_n \\ q_a \end{bmatrix}_{(n+1) \times 1},$$

$$\mathbf{T}_{gs} = \begin{bmatrix} \mathbf{T}_g \\ -m_a \end{bmatrix} = -\mathbf{M}_s \begin{Bmatrix} 1 \\ \vdots \\ 1 \\ 1 \end{Bmatrix}_{(n+1) \times 1},$$

$$\mathbf{T}_{us} = \begin{bmatrix} -\mathbf{T}_u \\ 1 \end{bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ -1 \\ 1 \end{Bmatrix}_{(n+1) \times 1}, \quad (6)$$

$$\mathbf{K}_s = \begin{bmatrix} \mathbf{K} + \mathbf{T}_u k_a \mathbf{T}_n & -\mathbf{T}_u k_a \\ -k_a \mathbf{T}_n & k_a \end{bmatrix}$$

$$= \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & \ddots & -k_n & 0 \\ 0 & -k_n & k_n + k_a & -k_a \\ 0 & 0 & -k_a & k_a \end{bmatrix}_{(n+1) \times (n+1)},$$

$$\mathbf{C}_s = \begin{bmatrix} \mathbf{C} + \mathbf{T}_u c_a \mathbf{T}_n & -\mathbf{T}_u c_a \\ -c_a \mathbf{T}_n & c_a \end{bmatrix}$$

$$= \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 \\ -c_2 & \ddots & -c_n & 0 \\ 0 & -c_n & c_n + c_a & -c_a \\ 0 & 0 & -c_a & c_a \end{bmatrix}_{(n+1) \times (n+1)},$$

where m_i , c_i , and k_i are the i th floor mass, damping, and stiffness. Equation (5) can be represented in state space as follows:

$$\dot{\mathbf{X}}_s(t) = \mathbf{A}_s\mathbf{X}_s(t) + \mathbf{B}_{1s}\ddot{q}_g(t) + \mathbf{B}_{2s}F_a(t), \quad (7)$$

where $\mathbf{A}_s \in \mathbb{R}^{2(n+1) \times 2(n+1)}$, $\mathbf{B}_{1s} \in \mathbb{R}^{2(n+1) \times 1}$, and $\mathbf{B}_{2s} \in \mathbb{R}^{2(n+1) \times 1}$ are system matrix, excitation matrix, and control matrix, respectively.

One can see that

$$\mathbf{A}_s = \begin{bmatrix} [\mathbf{0}]_{(n+1) \times (n+1)} & [\mathbf{I}]_{(n+1) \times (n+1)} \\ -\mathbf{M}_s^{-1}\mathbf{K}_s & -\mathbf{M}_s^{-1}\mathbf{C}_s \end{bmatrix},$$

$$\mathbf{B}_{1s} = \begin{bmatrix} [\mathbf{0}]_{(n+1) \times 1} \\ \mathbf{M}_s^{-1}\mathbf{T}_{gs} \end{bmatrix}, \quad (8)$$

$$\mathbf{B}_{2s} = \begin{bmatrix} [\mathbf{0}]_{(n+1) \times 1} \\ \mathbf{M}_s^{-1}\mathbf{T}_{us} \end{bmatrix}.$$

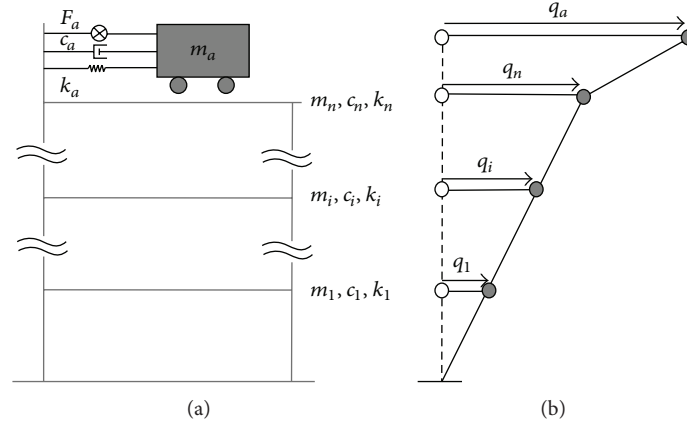


FIGURE 1: The model of AMD structure system.

$\mathbf{X}_s(t) = [\mathbf{q}_s(t); \dot{\mathbf{q}}_s(t)] \in \mathbb{R}^{2(n+1) \times 1}$ is the state vector, which contains relative displacement and relative velocity. This

relative state vector can be transformed to another state vector which consists of interstory drift and interstory velocity as,

$$\mathbf{X}(t) = [q_1 \quad q_2 - q_1 \quad \cdots \quad q_n - q_{n-1} \quad q_a - q_n, \quad \dot{q}_1 \quad \dot{q}_2 - \dot{q}_1 \quad \cdots \quad \dot{q}_a - \dot{q}_n]^T \in \mathbb{R}^{2(n+1) \times 1}. \quad (9)$$

Define a transformation matrix $\Gamma \in \mathbb{R}^{2(n+1) \times 2(n+1)}$ to obtain $\mathbf{X}(t)$ such that $\mathbf{X}(t) = \Gamma \mathbf{X}_s(t)$. Substituting $\mathbf{X}_s(t) = \Gamma^{-1} \mathbf{X}(t)$ into (7) and left-multiplying this equation with Γ , the state equation expressed by $\mathbf{X}(t)$ becomes

$$\dot{\mathbf{X}}(t) = \mathbf{A} \mathbf{X}(t) + \mathbf{B}_1 \ddot{q}_g(t) + \mathbf{B}_2 F_a(t), \quad (10)$$

where $\mathbf{A} = \Gamma \mathbf{A}_s \Gamma^{-1} \in \mathbb{R}^{2(n+1) \times 2(n+1)}$, and $\mathbf{B}_1 = \Gamma \mathbf{B}_{1s} \in \mathbb{R}^{2(n+1) \times 1}$, $\mathbf{B}_2 = \Gamma \mathbf{B}_{2s} \in \mathbb{R}^{2(n+1) \times 1}$, and Γ is shown as

$$\Gamma = \begin{bmatrix} \Gamma_s & [\mathbf{0}]_{(n+1) \times (n+1)} \\ [\mathbf{0}]_{(n+1) \times (n+1)} & \Gamma_s \end{bmatrix}, \quad (11)$$

$$\Gamma_s = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & -1 & 1 \end{bmatrix}_{(n+1) \times (n+1)}.$$

3. Fault Detection and Isolation and Fault Tolerant Control

A simple and efficient way to design a controller for this structural control system with AMD is to design the H_∞ state feedback gain $\mathbf{G}_\infty \in \mathbb{R}^{1 \times 2(n+1)}$ [24], such that

$$F_a(t) = \mathbf{G}_\infty \mathbf{X}(t). \quad (12)$$

However, when there are some faults in sensors or actuators, the performance of vibration suppressing may be degraded and even enlarge the structural response. So, it is

necessary to detect and isolate faults such that the controller can tolerate the faults. This paper only considers the faults in sensors and designs a fault detection and isolation (FDI) filter. The designed FDI filter is intended to estimate structural system state, which can be used to reappear the structural sensor data with fewer faults. The difference between the measured output data and the estimated output data is the faults. The estimated output data can be used to isolate the faults transmitted to the designed H_∞ state feedback gain \mathbf{G}_∞ . So, the controller strategy is reformed and can perform the fault tolerant control performance.

For the faults in sensors, the sensor data including faults can be represented as

$$\mathbf{y}_v(t) = \mathbf{C}_y \mathbf{X}(t) + \mathbf{v}(t), \quad (13)$$

where $\mathbf{v}(t) \in \mathbb{R}^{q \times 1}$ is the faults part in sensor data $\mathbf{y}_v(t) \in \mathbb{R}^{q \times 1}$ and $\mathbf{C}_y \mathbf{X}(t)$ is the real part of sensor data, which is the system real output. The faults can be recognized as the additional data into the real sensor data. Combined with (10), the following equation can be derived:

$$\dot{\mathbf{X}}(t) = \mathbf{A} \mathbf{X}(t) + \mathbf{B}_1 \ddot{q}_g(t) + \mathbf{B}_2 F_a(t), \quad (14)$$

$$\mathbf{y}_v(t) = \mathbf{C}_y \mathbf{X}(t) + \mathbf{v}(t).$$

The faults estimator is shown in Figure 2 and its state space formulation can be described as

$$\dot{\hat{\mathbf{X}}}(t) = \mathbf{A} \hat{\mathbf{X}}(t) + \mathbf{B}_2 F_a(t) + \mathbf{r}(t),$$

$$\hat{\mathbf{y}}(t) = \mathbf{C}_y \hat{\mathbf{X}}(t), \quad (15)$$

$$\mathbf{r}(t) = \mathbf{K} (\hat{\mathbf{y}}(t) - \mathbf{y}_v(t)),$$

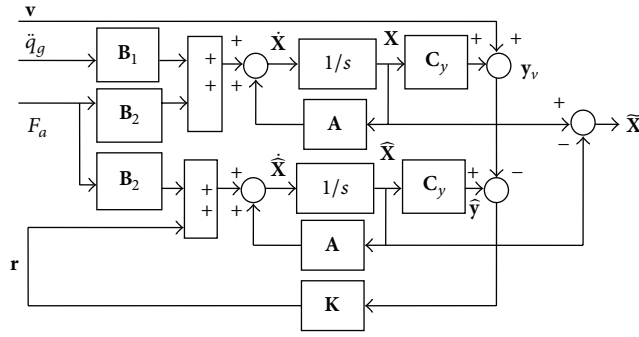


FIGURE 2: FDI diagram of AMD structure system.

where $\widehat{\mathbf{X}}(t) \in \mathbb{R}^{2(n+1) \times 1}$ is the estimated state for the structural state $\mathbf{X}(t)$, $\widehat{\mathbf{y}}(t) \in \mathbb{R}^{q \times 1}$ is the estimated measurement output for the real part of $\mathbf{y}_v(t) \in \mathbb{R}^{q \times 1}$, that is, $\mathbf{C}_y \mathbf{X}(t)$, $\mathbf{r}(t) \in \mathbb{R}^{2(n+1) \times 1}$ is the adjustment vector for estimated state vector, and $\mathbf{K} \in \mathbb{R}^{2(n+1) \times q}$ is the FDI filter to make the estimated state vector closed with structural system state vector. In this structural system with the sensor faults, the state is estimated so that the sensor data real part $\mathbf{C}_y \mathbf{X}(t)$ can be estimated as $\widehat{\mathbf{y}}(t) = \mathbf{C}_y \widehat{\mathbf{X}}(t)$. The faults in sensor data $\mathbf{y}_v(t) \in \mathbb{R}^{q \times 1}$ can be calculated by $\mathbf{y}_v(t) - \widehat{\mathbf{y}}(t)$ and then can be isolated by replacing $\mathbf{y}_v(t)$ with $\widehat{\mathbf{y}}(t)$. This is the fault detection and isolation (FDI). According to (14) and (15), the residue equation is obtained as follows:

$$\begin{aligned} \dot{\widetilde{\mathbf{X}}}(t) &= \widetilde{\mathbf{A}}\widetilde{\mathbf{X}}(t) + \widetilde{\mathbf{B}}\mathbf{w}(t), \\ \widetilde{\mathbf{Z}}(t) &= \widetilde{\mathbf{C}}\widetilde{\mathbf{X}}(t) + \widetilde{\mathbf{D}}\mathbf{w}(t). \end{aligned} \quad (16)$$

$\widetilde{\mathbf{A}}$, $\widetilde{\mathbf{B}}$, $\widetilde{\mathbf{C}}$, $\widetilde{\mathbf{D}}$, $\widetilde{\mathbf{X}}(t)$, $\widetilde{\mathbf{Z}}(t)$, and $\mathbf{w}(t)$ are defined as

$$\begin{aligned} \widetilde{\mathbf{A}} &= \mathbf{A} + \mathbf{K}\mathbf{C}_y \in \mathbb{R}^{2(n+1) \times 2(n+1)}, \\ \widetilde{\mathbf{B}} &= [\mathbf{B}_1 \quad \mathbf{K}] \in \mathbb{R}^{2(n+1) \times (q+1)}, \end{aligned}$$

$$\widetilde{\mathbf{C}} = \mathbf{I} \in \mathbb{R}^{2(n+1) \times 2(n+1)},$$

$$\widetilde{\mathbf{D}} = [\mathbf{0}]_{2(n+1) \times (q+1)} \in \mathbb{R}^{2(n+1) \times (q+1)}, \quad (17a)$$

$$\begin{aligned} \widetilde{\mathbf{X}}(t) &= (\mathbf{X}(t) - \widehat{\mathbf{X}}(t)) \in \mathbb{R}^{2(n+1) \times 1}, \\ \widetilde{\mathbf{Z}}(t) &= \widetilde{\mathbf{X}}(t) \in \mathbb{R}^{2(n+1) \times 1}, \end{aligned} \quad (17b)$$

$$\mathbf{w}(t) = [\dot{q}_g(t); \mathbf{v}(t)] \in \mathbb{R}^{(q+1) \times 1},$$

where $\widetilde{\mathbf{Z}}(t) = \widetilde{\mathbf{X}}(t) \in \mathbb{R}^{2(n+1) \times 1}$ is the output of the residue equation and $\widetilde{\mathbf{X}}(t)$ is the state of the residue equation.

In order to estimate the system state $\mathbf{X}(t)$ precisely, this paper designs the FDI filter \mathbf{K} in (15) to minimize the H_∞ norm of the transfer function from input $\mathbf{w}(t)$ to output $\widetilde{\mathbf{Z}}(t)$. This design can be recognized as H_∞ optimization problem. It can be described as designing a static controller \mathbf{K} to stabilize the system (16) such that

$$J = \|\mathbf{H}_{zw}(s)\|_\infty = \sup_{\omega, \|\mathbf{w}(t)\|_2 \neq 0} \left(\frac{\|\widetilde{\mathbf{Z}}(t)\|_2}{\|\mathbf{w}(t)\|_2} \right) < \gamma, \quad (18)$$

where γ is a given scalar. According to the bounded real lemma [25], the following two statements are equivalent:

- (1) H_∞ norm of the system in (16) is less than γ , and $\widetilde{\mathbf{A}}$ is stable;
- (2) There exists a symmetric positive definite matrix $\Phi \in \mathbb{R}^{2(n+1) \times 2(n+1)}$ such that the following matrix inequality holds:

$$\begin{bmatrix} \widetilde{\mathbf{A}}^T \Phi + \Phi \widetilde{\mathbf{A}} & \Phi \widetilde{\mathbf{B}} & \widetilde{\mathbf{C}}^T \\ \widetilde{\mathbf{B}}^T \Phi & -\gamma \mathbf{I} & \widetilde{\mathbf{D}}^T \\ \widetilde{\mathbf{C}} & \widetilde{\mathbf{D}} & -\gamma \mathbf{I} \end{bmatrix} < \mathbf{0}. \quad (19)$$

Substituting (17b) into (19),

$$\begin{bmatrix} (\mathbf{A} + \mathbf{K}\mathbf{C}_y)^T \Phi + \Phi (\mathbf{A} + \mathbf{K}\mathbf{C}_y) & \Phi [\mathbf{B}_1 \quad \mathbf{K}]_{2(n+1) \times (q+1)} & \mathbf{I}_{2(n+1) \times 2(n+1)} \\ [\mathbf{B}_1 \quad \mathbf{K}]_{(q+1) \times 2(n+1)}^T \Phi & -\gamma \mathbf{I}_{(q+1) \times (q+1)} & [\mathbf{0}]_{(q+1) \times 2(n+1)} \\ \mathbf{I}_{2(n+1) \times 2(n+1)} & [\mathbf{0}]_{2(n+1) \times (q+1)} & -\gamma \mathbf{I}_{2(n+1) \times 2(n+1)} \end{bmatrix} < \mathbf{0}. \quad (20)$$

Because both Φ and \mathbf{K} are unknown variables and multiplied together, this problem cannot be solved directly. This paper employs variable substitution method [23] to linearize this constraint.

Define a new variable $\Xi = \Phi \mathbf{K} \in \mathbb{R}^{2(n+1) \times q}$ and take it into

$$\begin{bmatrix} \mathbf{A}^T \Phi + \Phi \mathbf{A} + \mathbf{C}_y^T \Xi^T + \Xi \mathbf{C}_y & [\Phi \mathbf{B}_1]_{2(n+1) \times 1} & \Xi_{2(n+1) \times q} & \mathbf{I}_{2(n+1) \times 2(n+1)} \\ [\mathbf{B}_1^T \Phi]_{1 \times 2(n+1)} & -\gamma \mathbf{I}_{1 \times 1} & [\mathbf{0}]_{1 \times q} & [\mathbf{0}]_{1 \times 2(n+1)} \\ \Xi_{q \times 2(n+1)} & [\mathbf{0}]_{q \times 1} & -\gamma \mathbf{I}_{q \times q} & [\mathbf{0}]_{q \times 2(n+1)} \\ \mathbf{I}_{2(n+1) \times 2(n+1)} & [\mathbf{0}]_{2(n+1) \times 1} & [\mathbf{0}]_{2(n+1) \times q} & -\gamma \mathbf{I}_{2(n+1) \times 2(n+1)} \end{bmatrix} < \mathbf{0}. \quad (21)$$

Equation (21) is a linear matrix inequality with matrix variables Φ and Ξ . The optimal values of Φ and Ξ can be computed through the solver feasp and mincx in MATLAB LMI toolbox. Then, \mathbf{K} can be obtained from $\mathbf{K} = \Phi^{-1}\Xi$. So, the FDI filter in (15) is designed.

Through FDI filter, the estimated state $\widehat{\mathbf{X}}(t)$ and the estimated sensor data $\widehat{\mathbf{y}}(t) = \mathbf{C}_y\widehat{\mathbf{X}}(t)$ can be very closed to the system state $\mathbf{X}(t)$ and the sensor data real part $\mathbf{C}_y\mathbf{X}(t)$, respectively. The estimated state $\widehat{\mathbf{X}}(t)$ can be recognized as the input of H_∞ state feedback gain \mathbf{G}_∞ . This reformed control system can be described in Figure 3. The reformed control strategy is different from the one in (12)

$$\bar{\mathbf{F}}_a(t) = \mathbf{G}_\infty\widehat{\mathbf{X}}(t). \quad (22)$$

As a summary, when the sensor data has faults, the control system cannot guarantee a good performance. Through FDI filter, the sensor data real part and system state can be estimated. The estimated information can be used to isolate the sensor faults. Then, put the estimated state into the previous designed H_∞ state feedback controller to reform the control strategy such that the control system can perform the fault tolerant control (FTC).

4. Numerical Example

Considering a three-story numerical example [26], an AMD control device is set on the top of this structure as shown in

$$\mathbf{G}_\infty = [3.2155 \ 3.1275 \ 6.7705 \ -0.0127 \ -0.1344 \ -0.0877 \ -0.1335 \ -0.0188] \times 10^6. \quad (25)$$

The El Centro NS earthquake acceleration record $\ddot{q}_g(t)$ with its peak scaled to 200 gal (2 m/s^2) is used as the ground excitation. The response suppression comparisons are shown in Figures 5 and 6 for the bare structure without control, passive tuned mass damper (TMD) structure, and active mass damper (AMD) structure without sensor faults. The bare structure is the structure without actuators. The TMD structure has actuators but does not generate control force. The control strategy for the AMD structure is H_∞ state feedback control.

In Figure 6, \mathbf{G}_∞ represents H_∞ state feedback controller. Figures 5 and 6 illustrate the relative displacement response on the 3rd floor. The maximum value of the relative displacement for the TMD structure is 24.1% less than the one for the bare structure, while AMD structure is 67.0% off. Therefore,

$$\mathbf{K} = \begin{bmatrix} -6.4162 & -2.4851 & -0.9160 & 0.4926 & -92.0527 & -118.6123 & -28.5338 & -23.9462 \\ -1.0675 & -2.3501 & -0.6025 & 0.1106 & -42.2075 & 93.6605 & -82.5947 & -27.4098 \\ 0.3623 & -0.1068 & -1.8490 & -0.4097 & 39.9383 & -55.9578 & 105.6514 & -90.3240 \end{bmatrix}^T. \quad (26)$$

Taking \mathbf{K} into (15) and putting the estimated state $\widehat{\mathbf{X}}(t)$ into state feedback gain \mathbf{G}_∞ , the control strategy is reformed to perform the fault tolerate control.

Figure 4. Define the structural mass, damping, and stiffness as follows:

$$\begin{aligned} \mathbf{M} &= \begin{bmatrix} 6 & & \\ & 6 & \\ & & 6 \end{bmatrix} \times 10^3 \text{ kg}, \\ \mathbf{C} &= \begin{bmatrix} 12.4 & -5.16 & \\ -5.16 & 12.4 & -4.59 \\ & -4.59 & 7.2 \end{bmatrix} \times 10^3 \text{ N/(m/s)}, \\ \mathbf{K} &= \begin{bmatrix} 3.4 & -1.8 & \\ -1.8 & 3.4 & -1.6 \\ & -1.6 & 1.6 \end{bmatrix} \times 10^6 \text{ N/m}. \end{aligned} \quad (23)$$

AMD control system mass, damping, and stiffness are as follows:

$$\begin{aligned} m_a &= 360 \text{ kg}, \\ k_a &= 18819 \text{ N/m}, \\ c_a &= 365.3910 \text{ N/(m/s)}. \end{aligned} \quad (24)$$

Suppose the sensor measurement outputs are the system states. H_∞ state feedback control strategy is employed to control this structural system with AMD. The H_∞ state feedback gain \mathbf{G}_∞ in (12) can be obtained as follows:

TMD and AMD can both suppress the structural vibration. Furthermore, AMD structure can reduce the vibration 56.5% more than TMD structure.

If there exist some faults in sensors, the faulty signal data $\mathbf{v}(t)$ is supposed to be located in the third floor displacement sensors. And its mean value and variance are zero and 2.5×10^{-5} , respectively. The comparison of the 3rd interstory drift between the AMD structure and the TMD structure is shown in Figure 7.

From Figure 7, AMD structural control system with H_∞ state feedback control strategy loses its control performance when there exist faults in the sensors. Accordingly, it is necessary to detect and isolate the faults in the sensors. Through proposed FDI filter design method, the gain \mathbf{K} is obtained as follows:

Figure 8 compares the interstory drift in the top story between the real interstory drift and the estimated drift. The related coefficients are shown in Table 1. From Table 1, the

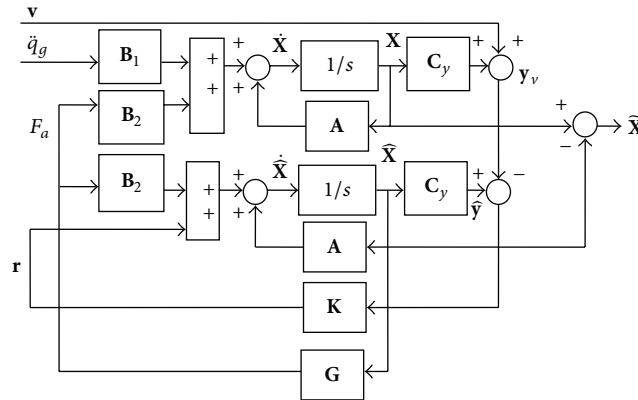


FIGURE 3: FTC diagram of AMD structure system.

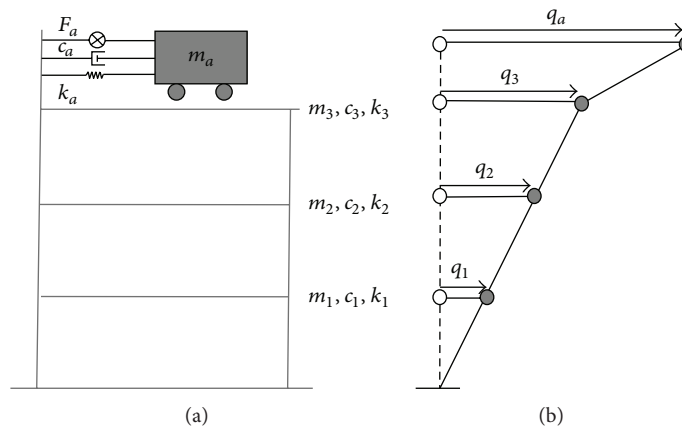


FIGURE 4: Three-story structure model with AMD system.

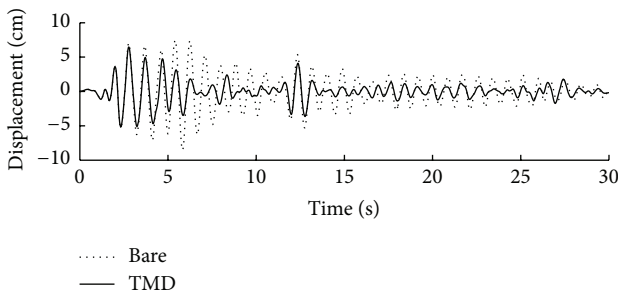


FIGURE 5: Comparison of the 3rd floor relative displacement between no control and TMD passive control.

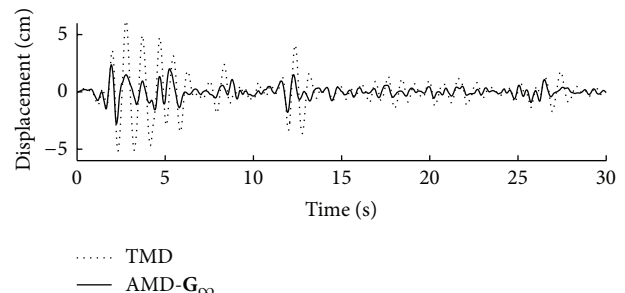


FIGURE 6: Comparison of the 3rd floor relative displacement between TMD passive control and AMD-H ∞ state feedback control.

dash line, which is the real drift, matches the solid line, which is the estimated drift by FDI. From Table 1, the related coefficients (RCs) are almost close to 1, which means that FDI can estimate drift precisely.

Through FTC strategy, the interstory drift is suppressed, and the third story drift is shown in Figures 9 and 10. Table 2 shows some details about vibration suppression effects for other stories.

Figure 9 illustrates the comparison of the interstory drift in the 3rd story between TMD structure and AMD structure considering sensor faults. In AMD structure, the control

strategy is FTC. AMD structure can suppress the structural vibration more effectively than TMD structure does. While only using H ∞ state feedback control strategy, the vibration is worsened as shown in Figure 7. Figure 10 can illustrate the difference more directly.

From Table 2, the maximum values of all the interstory drifts for AMD structure with H ∞ state feedback control strategy are 29.37%, 38.29%, and -46.50% less than the TMD structural drifts, respectively. While using FTC strategy, the drifts are 23.42%, 39.19%, and 54.78% less than the TMD

TABLE 1: Correlation coefficient between FDI estimated state and the real state.

	Comparison between the states estimated by FDI and structure system real states							
	Interstory drift				Interstory velocity			
	q_1 cm	$q_2 - q_1$ cm	$q_3 - q_2$ cm	$q_a - q_3$ cm	\dot{q}_1 cm	$\dot{q}_2 - \dot{q}_1$ cm	$\dot{q}_3 - \dot{q}_2$ cm	$\dot{q}_a - \dot{q}_3$ cm
RC	0.8422	0.8064	0.8584	0.9989	0.7741	0.7483	0.9035	0.9985

TABLE 2: Suppressing effect comparisons among the bared structure, TMD structure, AMD structure without faults, and AMD H ∞ structure with faults.

Systems	q_1 cm	$q_2 - q_1$ cm	$q_3 - q_2$ cm	$q_a - q_3$ cm	$F_a(t)$ kN
Bare system	4.04	2.81	1.68	—	—
TMD system	2.69	2.22	1.57	25.08	—
AMD system without faults using G_{∞}	1.88	1.08	0.86	78.38	15.46
AMD system with faults using G_{∞}	1.90	1.37	2.30	108.7	209.1
AMD system with faults using FTC	2.06	1.35	0.71	83.36	16.00

G_{∞} represents H ∞ state feedback control.

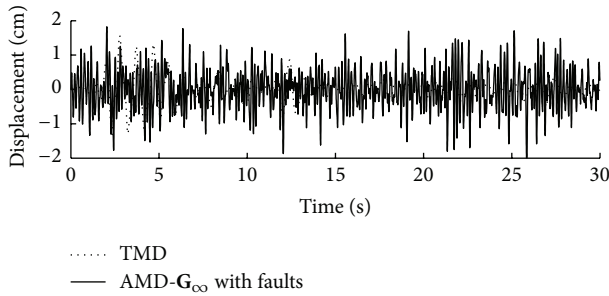


FIGURE 7: Comparison of the 3rd story interstory drift between TMD and AMD with H ∞ state feedback control strategy considering the sensor faults.

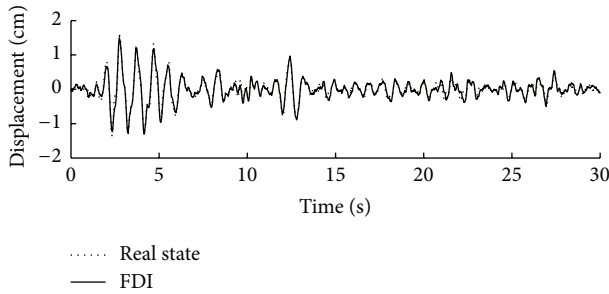


FIGURE 8: Comparison between the real and FDI estimated interstory drift in the top story.

structural drifts, respectively. The actuator force $F_a(t)$ saves a lot of energy using FTC strategy than when only using H ∞ state feedback control strategy. Therefore, FTC can tolerate faults in sensors and suppress structural vibration efficiently.

5. Conclusions

In practical engineering, structural control system may have faults which can degrade control system performance. This

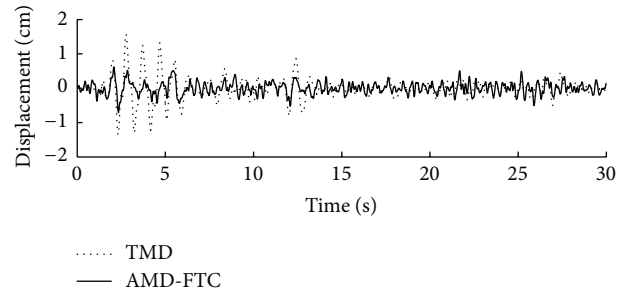


FIGURE 9: Comparison of the interstory drifts in the 3rd story between the TMD and AMD-FTC control.

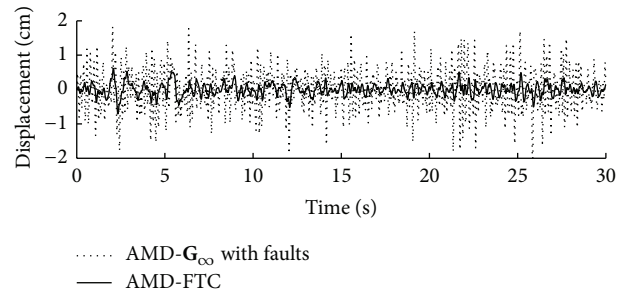


FIGURE 10: Comparison of the 3rd floor interstory drifts between the AMD-H ∞ state feedback control and AMD-FTC control.

paper presents fault tolerant control (FTC) for civil structures. Considering the faults in sensors, fault detection and isolation (FDI) filter is designed. Transmitting the estimated state to H ∞ state feedback controller designed previously, the reformed control strategy is obtained, which is called fault tolerant control (FTC). This paper also presents the derivation of FDI, which transforms the inequality matrix to linear inequality matrix using variable substitution method. A three-story numerical example validates FDI and FTC performance. From this study, there are some conclusions.

- (1) When there is no fault in sensors, H_{∞} state feedback control in AMD structure has better suppressing performance than TMD structure.
- (2) When there are faults in sensors, H_{∞} state feedback control in AMD structure cannot guarantee the control performance and even worsen the vibration.
- (3) FDI designed by the proposed method can detect and isolate the faults. It can estimate the system state precisely. According to the estimated state, the measurement output from sensors can also be estimated.
- (4) FTC designed by the proposed method can tolerate the faults in sensors. So, it can strengthen the robustness of the control system.

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