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Research Article

Geometry Optimization of Self-Similar Transport Network

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We optimize geometries of various self-similar transport networks using a three-step strategy based on the entransy theory. Using this optimization method, we obtained optimal relationships of geometric parameters of T-shape networks for fluid flow, heat conduction, convective heat transfer, and other transport phenomena. Some optimization results agree well with the existing theories or experimental data. The optimized transport network structure depends strongly on the optimization objective and the constraints, so that both the maximum heat transfer effect and minimum flow resistance cannot be satisfied at the same time.

1. Introduction

Self-similar transport networks exist in natural world and human life extensively. For instance, the leaf venations of trees in nature, the windpipe network in lungs and the blood vessel network in human bodies, and the water, gas, oil, and power supplies of a city or even a country. The optimization of transport networks has gained increasing attentions in recent years due to its importance with great challenging [1–8]. It has been widely used that the mechanical/electrical energy dissipation rate is minimized for an optimal hydraulic/electrical network [5, 6, 9–11]. Ordonez et al. [2] studied the optimal structure of flow network which connected one point to a number of points by minimizing the fluid power losses, and Durand [6, 12] obtained the optimal flow networks in terms of minimizing the mechanical dissipative energy with respect to two constraints: certain total channel volume and certain total channel surface area.Bohn and Magnasco [5] introduced an electrical energy dissipation rate function, which should be minimized for an optimal electrical transport network.Rodriguez-Iturabe [13] explained the tree-like structure and some empirical relationships of the river drainage network by the principles of minimum energy expenditure. However, the minimum energy dissipation principle is hardly applicable directly to heat and mass transportation networks, because the concept of energy is unsuitable (for a mass transport network) or conserved (for the thermal energy in a heat conduction network) rather than dissipated. New principles had to be developed for the optimization of heat and mass transfer processes. A constructal theory has been proposed to construct an optimal network for volume-point heat conduction problem [14].

Recently, a physical quantity, "*entransy*", was proposed by Guo et al. [15] to characterize the heat transfer capability of an object. In analogy with the theory system of electricity or mechanics, the entransy of an object is featured as the "potential energy" of the internal energy (U) at the temperature (T). Though the thermal energy is conserved, the entransy dissipates in heat transfer processes and the entransy dissipation rate can be used as a criterion for optimization of heat transfer [15]. Chen et al. combined the entransy theory with the constructal theory for optimal network geometries of heat transfer [16–20]. More recently, it has been proved that the entransy is consistent with the macroscopic appearance of potential energy of "thermomass" in heat transfer [21–24]. Examples have shown successes of minimum entransy dissipation rate principle for optimization of heat and mass transfer in complex systems [15, 25–30]. Inspired by Guo's theory, Liu et al. [31] extended the concept of entransy into heat and fluid flow networks. The concept of entransy dissipation rate in heat/mass transport network is comparable to the concept of energy dissipation rate. They therefore derived the formulations of the entransy dissipation rate for transport networks, and the analysis indicated that the minimum entransy dissipation rate leads to the optimal transfer performance of transport network subject to a given constraint. Their optimization analyses agreed well with the existing experimental data and optimization theories for transport networks [31].

In this paper, we are focusing on the geometry optimization of self-similar transport networks using the Entransy theory. The rest parts of this paper are organized as follows. In Section 2, we introduce the basic concept and theory of Entransy, and the optimization strategy using the Entransy theory. In Section 3, we will first optimize structures of T-shape transport networks for laminar flow, turbulence, heat conduction, convective heat transfer, and species diffusion. Finally we summarize the optimization geometric parameters of transport networks.

2. Entransy Theory and Minimum Entransy Dissipation Principle

2.1. Fundamentals of Entransy Theory

Transport processes can be generally treated as a generalized "mass" movement driven by a "potential" difference. Thus some common characteristics can be abstracted among various transport networks such as electrical, hydraulic, and heat or mass transport networks. Therefore, once the constructed electric network was optimized with the minimum energy dissipation principle, the corresponding hydraulic network may be optimized as well. Alternatively, one can obtain the structure of optimal heat conduction networks by identifying and minimizing the physical quantity of a thermal system which corresponds to the concept of energy in the electric/hydraulic systems.

Entransy is such a quantity, which describes the capability of heat conduction in continuum and was originally used for optimization of heat transfer devices and defined as the integral of a half of product between the internal energy (U) and the thermal potential (T) of a small element (Ω) over the system [15, 31]

$$G = \int_{\text{system}} \left(\frac{1}{2}UT\right)_{\Omega} d\Omega. \tag{2.1}$$

The entransy dissipation rate per unit volume (J) in heat transfer process, which measures the heat transfer irreversibility, can be calculated by

$$J = -\vec{q} \cdot \nabla T, \tag{2.2}$$

where $\vec{q}$ is the heat flux and ∇T is the temperature gradient. Guo and his colleagues have proved that the heat transfer process is optimized when the entransy dissipation rate reaches the minimum or the maximum depending on the constraints [15, 26, 27, 31]. In a recent work, Liu et al. [31] generalized the entransy concept of heat transfer to other transport processes, such as mass diffusion. The generalized entransy can be calculated by the product between a generalized flux and a generalized potential gradient.

Furthermore, Liu et al. [31] deduced the entransy dissipation rate for a discrete network from the continuum system, which is calculated by

$$J_{\text{network}} = \sum_n (P_n S_n) = F^* \left(\sum_{n:\{S_n>0\}} \frac{S_n}{F^*} P_n - \sum_{n:\{S_n<0\}} \frac{-S_n}{F^*} P_n \right) = F^* \left(\overline{P}_{\text{in}} - \overline{P}_{\text{out}} \right) = F^* \overline{\Delta P}, \tag{2.3}$$

where F^*, S, P represent the injecting flux, the generalized source, and the generalized potential for transport, respectively. For a given constraint, a minimized entransy dissipation rate leads to a minimum average potential difference in system, which means the transport network is optimal.

2.2. Strategy of Optimization Process

To optimize the geometry of a transport network, a three-step process can be followed. First, we need to find the entransy dissipation rate of transport network as a function of the network geometry:

$$J_{\text{network}} = \sum_i J_i(x_i), \tag{2.4}$$

where x_i represents the characteristic length of the transport channel at the ith level. It may be the length, radius, or area of cross-section of the channels. For one-dimensional linear transport in each level, a general form of the entransy dissipation rate was given in the early work [31, equations (7)-(13)].

The second step is to construct the optimization function. Our final goal of optimization is to find the right geometry leading the minimum entransy dissipation in the transport process. In real systems, this goal has to be constrained to some conditions, which are called "constraints". For example, when we want to maxmize the transport flow rate through a network for a given pumping power, this pumping power is the constraint. Once the optimization constraint (C) is determined, we build the optimization function (Π) by introducing a Lagrange multiplier (λ) as [5, 31]

$$\Pi = J_{\text{network}} - \lambda C = \sum_i J_i(x_i) + \lambda \sum_i C_i(x_i). \tag{2.5}$$

Finally to get the minimum entransy dissipation rate, the partial derivative of the optimization function with respect to x_i has to be zero:

$$\frac{\partial \Pi}{\partial x_i} = \frac{\partial J_{\text{network}}}{\partial x_i} - \lambda \frac{\partial C}{\partial x_i} = 0. \tag{2.6}$$

Equation (2.6) together with the conservation equation will lead to the optimal geometry of transport network.

3. Optimize the Transport Network

In this section we will demonstrate the application of optimization of the transport network with generalized minimum entransy dissipation principle. The transport phenomena and governing equations are so different for various transport networks. For a river network, we have to consider at least two basic conditions: laminar or turbulence flows. For a high-efficient heat transfer network, it may be through heat conduction, convection, or both. For simplification, we consider only one means of transport in each case in this work.

We first focus on the T-shape network which has been widely used in engineering. A typical unit is shown in Figure 1, which is one-in-two outstructured network with circular cross-sections. We are to optimize the geometric structures of transport networks for different cases, including laminar flow, turbulence flow, heat conduction, convective heat transfer, and diffusions.

3.1. Laminar Flow

When considering fluid flows through the network, we may want to get the maximum volume flow rate (Q_v) through the networks with a given total volume (V). For a fully developed incompressible laminar flow, the Hagen-Poiseuille's law gives

$$Q_v = \frac{\pi r^4}{8\mu} \frac{\Delta p}{L}, \tag{3.1}$$

where μ is the viscosity of the fluid and Δp the pressure drop within the channel of a radius $r = d/2$ and a length L.

The entransy dissipation rate of the ith level channels is calculated by

$$J_i^{\text{mass}} = \frac{\pi r_i^4}{8\mu L_i} (\Delta p_i)^2. \tag{3.2}$$

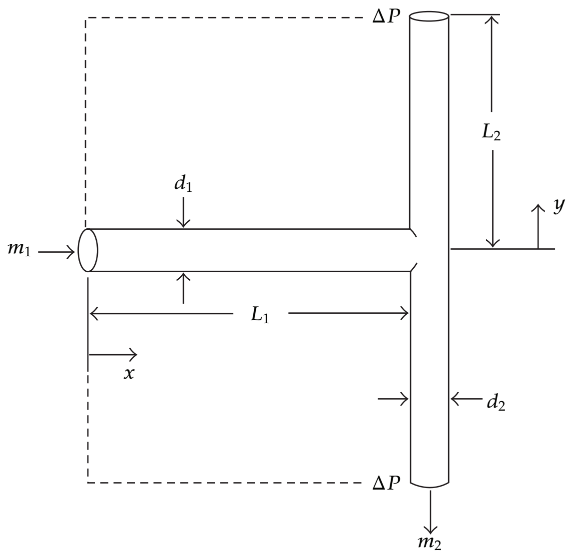

Figure 1: T-shape-round-tube unit of transport network.

Therefore the optimization function can be written by introducing a Lagrange multiplier λ as

$$\Pi = J_{\text{network}}^{\text{mass}} - \lambda V = \sum_i \frac{\pi r_i^4 (\Delta p_i)^2}{8\mu L_i} - \lambda \sum_i \pi r_i^2 L_i. \tag{3.3}$$

In order to minimize the entransy dissipation rate, let the partial derivative of the Lagrange function with respect to the radius r_i be zero and we can get a generalized solution of (3.3)

$$\Delta p_i = \frac{\chi L_i}{r_i}, \tag{3.4}$$

where $\chi = \sqrt{4\mu/\lambda}$ is mostly a constant.

The conservation of mass at a junction gives $m_1 = 2m_2$, which leads to

$$\frac{\pi r_1^4}{8\mu L_1} \Delta p_1 = \sum_{i=2}^{n} \frac{\pi r_i^4}{8\mu L_i} \Delta p_i. \tag{3.5}$$

Therefore we can get a relation between the cross-sectional radius of a parent channel and that of the daughter channels

$$r_1^3 = \sum_{i=2}^{n} r_i^3. \tag{3.6}$$

This result agrees with Murray's law. There is no other constraint in the transport process through a self-similar structure, so that the result is accessible for asymmetric and multibranch transport networks as well.

The geometry can be further optimized with the length of tubes. Hence, we need to introduce another constraint. For instance both constant volume and constant structure area of the entire network were used as constraints to optimize a T-shape network structure [32]. Inspired by this, here we give another constraint: a constant total structure area, which means the area the structure occupies but not the surface area. For a double-level network, the 2nd constraint leads to $2L_1L_2$ = const, so that substituting (3.4) into (3.2) results in the entransy dissipation rate as

$$J_{\text{network}} = \frac{\pi L_1 r_1^2}{2\lambda} + \frac{\pi L_2 r_2^2}{\lambda}. \tag{3.7}$$

The optimization function can be written by introducing another Lagrange multiplier η as

$$\Pi_{\text{mass}} = \frac{\pi L_1 r_1^2}{2\lambda} + \frac{\pi L_2 r_2^2}{\lambda} - \eta L_1 L_2. \tag{3.8}$$

Leting partial derivative of (3.8) with respect to r_1 and r_2 be zero, we can get

$$r_1^3 = \left(\frac{2\lambda\eta}{\pi}L_1\right)^{3/2}, \qquad r_2^3 = \left(\frac{\lambda\eta}{\pi}L_0\right)^{3/2} \tag{3.9}$$

which leads to

$$\frac{L_1}{L_2} = 2^{1/3}. \tag{3.10}$$

This result is very consistent with the previous optimizations [32].

3.2. Turbulence Flow

Different from the laminar flow, the pressure drop in turbulence flows is related to the geometric parameters by the Darcy-Weisbach equation:

$$\Delta p = f\rho\frac{L}{d}\frac{\overline{v}^2}{2}, \tag{3.11}$$

where f is the Darcy friction factor, ρ is the density of fluid, and $\overline{v}$ is the average velocity. The entransy dissipation rate of the ith channel is

$$J_i^{\text{mass}} = \frac{\pi}{4}\sqrt{\frac{2}{fL_i\rho}}d_i^{5/2}(\Delta p_i)^{3/2}. \tag{3.12}$$

The objective is the maximum flow rate and the constraint is the given total volume. The optimization function is then

$$\Pi_{\text{mass}} = J_{\text{network}}^{\text{mass}} - \lambda V = \sum_i \frac{\pi}{4}\sqrt{\frac{2}{fL_i\rho}}d_i^{5/2}(\Delta p_i)^{3/2} - \lambda \sum_i \frac{\pi}{4} d_i^2 L_i. \tag{3.13}$$

Let the partial derivative of the Lagrange function with respect to d_i be zero. Together with the conservation of mass at a junction, we can get a generalized solution as

$$d_1^{7/3} = \sum_{i=2}^{n} d_i^{7/3}. \tag{3.14}$$

For a double-level network, with another constraint of given total structure area, similar with the optimization process in Section 3.1, we can further get the optimal length relationship as

$$\frac{L_1}{L_2} = 2^{1/7}. \tag{3.15}$$

3.3. Heat Conduction

In engineering scale heat flux is proportional to the area of cross-section and the temperature gradient. The transport process generally follows the Fourier's Law: $Q = kA\nabla T$, where A denotes the cross-section area of the channel with a thermal conductivity k.

When distributing high-thermal-conductivity materials into a system, we want to achieve the best heat conduction effect of the high-conductivity network with a given total volume. Thus the heat flow rate can be related to the geometry of the network by

$$Q_i = kA_\text{i}\frac{\Delta T_i}{L_i}. \tag{3.16}$$

The entransy dissipation rate of the heat conduction is

$$J_{\text{network}} = \sum_i kA_i \frac{(\Delta T_i)^2}{L_i}. \tag{3.17}$$

The optimization function can be written by introducing a Lagrange multiplier λ to minimize the entransy dissipation rate:

$$\Pi_{\text{heat}} = J_{\text{network}} - \lambda \sum_i V_i = \sum_i kA_i \frac{(\Delta T_i)^2}{L_i} - \lambda \sum_i A_i L_i. \tag{3.18}$$

A generalized solution of this function is

$$\Delta T_i = \alpha L_i, \tag{3.19}$$

where $\alpha = \sqrt{\lambda/k}$ is a constant. The conservation of thermal flux at a junction gives $Q_1 = \sum_i Q_i$ which leads to

$$r_1^2 = \sum_{i=2}^{n} r_i^2. \tag{3.20}$$

This agrees well with the result of constructal theory [14]. Still a further optimal length relationship with another constraint as in Section 3.1 is

$$L_1 = L_2. \tag{3.21}$$

3.4. Convective Heat Transfer

Convective heat transfer contains both heat and mass transfer process. When optimizing such networks, we need to consider both minimizing flow resistance and maximizing heat flux. For simplification, we assume the flow to be a fully developed laminar flow with a constant Nusselt number. The thermal conductivity of walls and the temperature difference between the wall and the fluid are also assumed constant.

Based on the Newton's law of cooling, the entransy dissipation rate of the ith channel can be written as

$$J_i = 2\pi h_i r_i L_i (\Delta T)^2, \tag{3.22}$$

where h is the convective heat transfer coefficient. For a given volume of network, the optimization equation is

$$J_{\text{network}} = \sum_i 2\pi h_i r_i L_i (\Delta T)^2 - \lambda \sum_i \pi r_i^2 L_i. \tag{3.23}$$

The generalized solution of the optimization function is

$$r_i = \frac{h_i (\Delta T)^2}{\lambda}. \tag{3.24}$$

Therefore we get the relationship of radius:

$$r_1 = r_2 = r_3 = \cdots = r_n. \tag{3.25}$$

For another given total structure area constraint, the length of channels for a double-level network can be further optimized by

$$\frac{L_1}{L_2} = 2. \tag{3.26}$$

Comparing these results with those for the pure laminar flow network we can conclude that both the maximum heat transfer effect and minimum flow resistance cannot be satisfied at the same time.

3.5. Other Transport Phenomena

Because of the similarity of physics and governing equations, this optimization strategy can be easily extended to electrical conduction and mass diffusion. If the constraints are also equivalent, the optimization results of the heat conduction network are even available to be used for these two transports, as shown in (3.20) and (3.21). We also noticed that when coupled with fluid flow, the mass diffusion may have a different optimization objective. For examples, the objective for pure fluid flow is to maximize the flow rate, while that for coupling fluid flow with mass diffusion may be to maximize the mass diffusion effect, such as the transmural nutrients transport in blood vessel network. For such cases, the entransy dissipation rate for the *i*th vessel is proportional to the vessel surface area by

$$J_i^{\text{mass}} = 2\pi r_i d_i \cdot D_m (\Delta c)^2, \tag{3.27}$$

where D_m is the mass diffusion coefficient and c is the concentration. The optimization function for a given pumping power is therefore

$$\Pi_{\text{mass}} = J_{\text{network}}^{\text{mass}} - \lambda W = D_m (\Delta c)^2 \sum_i 2\pi r_i d_i - \lambda \sum_i \frac{\pi r_i^4}{8\mu} \frac{(\Delta p_i)^2}{d_i}, \tag{3.28}$$

which leads to the optimal network geometry requiring

$$r_1^{2.5} \propto \sum_{i=2}^{n} r_i^{2.5}. \tag{3.29}$$

This result is different from the Murray's law and has been validated by some experimental data from small intestine of dogs [31]. This indicates again that the optimized transport network structure depends on the optimization objective and the constraint.

T-shape transport network may be the simplest artificial geometry of network. However in practice, other kinds of bifurcation geometry, such as the Y-shape, are more popularly used. For these bifurcation geometries, the angle is the other optimization target. Such optimization analysis can be found in the literature in [33]. The results of angle optimization can be used together with the size optimizations in this work to construct the optimal structure of transport networks.

4. Conclusions

We have optimized geometries of various transport networks using a three-step strategy based on the entransy theory. We first find the entransy dissipation rate of transport network as a function of the network geometry based on the transport governing equations, then build the optimization function by introducing a Lagrange multiplier with the optimization constraint and finally get the optimized geometric parameters by minimizing the entransy dissipation rate. Using this optimization strategy and method, we obtained optimal relationships of geometric parameters for T-shape networks for fluid flow, heat conduction, convective heat transfer, and other transport phenomena. Some optimization results agree well with the existing theories or experimental data. We also find that the optimized transport network structure depends strongly on the optimization objective and the constraints. For example, for one transport network both the maximum heat transfer effect and minimum flow resistance cannot be satisfied at the same time. The angle optimization of other kinds of bifurcation geometric network by microelement analysis can be used together with the size optimizations in this work to construct the optimal structure of transport networks.

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