Internat. J. Math. & Math. Sci.
VOL. 20 NO. 3 (1997) 451-456

COMMON FUZZY HYBRID FIXED POINT THEOREMS FOR A SEQUENCE OF FUZZY MAPPINGS

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(Received September 28, 1995 and in revised form May 20, 1996)

ABSTRACT. In this paper, we discuss the concepts of fuzzy hybrid fixed points, of g-Φ-contractive type fuzzy mappings and common fuzzy hybrid fixed point theorems of a sequence of fuzzy mappings. Our theorems improve and generalize the corresponding recent important results.

KEY WORDS AND PHRASES: Fuzzy hybrid fixed point, common fuzzy hybrid fixed point, g-Φ-contractive type fuzzy mapping, g-contractive type fuzzy mapping, sequence of fuzzy mappings.
1991 AMS SUBJECT CLASSIFICATION CODES: 54H25, 47H10.

1. INTRODUCTION

Heilpern [1] first introduced the concept of fuzzy mappings and proved fixed point theorems for contraction fuzzy mappings. Chang [2] introduced the concept of Φ-contraction type fuzzy mappings, and proved a fixed point theorem, which is an extension of the result of Heilpern. Also, he obtained common fixed point theorems for a sequence of fuzzy mappings. Lee, et al [3-4] introduced the concept of g-contractive type fuzzy mappings, and proved a common fixed point theorem for sequence of fuzzy mappings on a complete metric linear space.

In this paper, we introduced g-Φ-contractive type fuzzy mappings and defined the concept of the fuzz hybrid fixed point for fuzzy mappings, proved common fuzzy hybrid fixed point theorems for a sequence of fuzzy mappings on a complete metric space. Our theorems improve and generalize the recent important results of [1-4].

2. PRELIMINARIES

Throughout this paper let (E,d) be a complete metric space, $CB(E)$ be a collection of all non-empty bounded closed subsets of E and $C(E)$ be a collection of all non-empty compact subsets of E Let Z^+ be the set of all positive integers. A mapping $B : B \to [0,1]$ is called a fuzzy subset over E We denote by $W(E)$ the family of all fuzzy subsets over E. Let $A \in W(E)$, $\forall\, \alpha \in [0,1]$. Set $(A)_\alpha = \{x \in E : A(x) \geq \alpha\}$ is called the α-cut set of A. A mapping $T : E \to W(E)$ is called fuzzy mapping over E.

DEFINITION 2.1. Let the function $\Phi : [0, +\infty)^5 \to [0, +\infty)$. We say Φ satisfies the condition (Φ_1), (Φ_2) or (Φ_3), if $(\Phi_1)\Phi$ is upper semi-continuous and non-decreasing for each variable $(\Phi_2)\Phi(t,t,t,at,bt) \leq Q(t)$, $\forall\, t \geq 0, a, b = 0,1,2,$ and $a+b=2$, where $Q(t) : [0, +\infty) \to [0, +\infty)$, $Q(0) = 0, Q(t) < t$, $\forall\, t > 0$. $(\Phi_3)\Phi(t,t,t,at,bt) \leq rt$, where $r \in (0,1)$ is a constant, $a, b = 0,1,2$ and $a+b=2$.

DEFINITION 2.2. Let $T : E \to W(E)$. We say that $T : E \to W(E)$ satisfies the condition A_1 (A_2). If there exists $\alpha(x) : E \to (0,1]$ such that $\forall x \in E, (Tx)_{\alpha(x)} \in CB(E)$ ($C(E)$).

Let $T_i : E \to W(E)(i = 1,2,...)$. We say $T_i : E \to W(E)(i = 1,2,...)$ satisfies the condition A_1 (A_2). If there exists a sequence of functions $a_i(x) : E \to (0,1]$ $(i = 1,2,...)$ such that $\forall x \in E$, $(Tix)_{\alpha_i(x)} \in CB(E)$ (or $C(E)$).

Let $T : E \to W(E)$ satisfies the condition A_1 (or A_2), $\forall\ x \in E, \tilde{T}x = (Tx)_{\alpha(x)} \in CB(E)$. $\tilde{T} : E \to CB(E)$ is called the set-valued mapping induced by T.

DEFINITION 2.3. Let $g : E \to E$ be a single-valued mapping, $F : E \to W(E)$ and $G : E \to W(E)$ be two fuzzy mappings satisfying condition A_1. If, $\forall\ x, y \in E$, $u_x \in \tilde{F}x$ ($\tilde{G}x$) there exists $y_y \in \tilde{G}y$ ($\tilde{F}y$) such that

$$d(u_x, v_y) \le \Phi(d(g(x), g(y)), d(g(x), g(u_x)), d(g(y), g(v_y)) \\ d(g(x), g(v_y)), d(g(y), g(u_x))). \tag{2.1}$$

Then, we say that F and G satisfy the condition B.

DEFINITION 2.4. Let $F : E \to W(E)$, $G : E \to W(E)$ be two fuzzy mappings satisfying the condition A_1. If for any $x, y \in E$, $u_x \in \tilde{F}x(\tilde{G}x)$ there exists $v_y \in \tilde{G}y(\tilde{F}y)$ such that

$$d(u_x, v_y) \le \Phi(d(x,y), d(x,u_x), d(y,v_y), d(x,v_y), d(y,u_x)). \tag{2.2}$$

Then, we say that F, G satisfy the condition C.

DEFINITION 2.5. Let $F : E \to W(E)$ and $G : E \to W(E)$ be two fuzzy mappings satisfying the condition A_1. If for any $x, y \in E$, $u_x \in \tilde{F}x(\tilde{G}x)$ there exists $v_y \in \tilde{G}y(\tilde{F}y)$ such that

$$H(\tilde{F}_x, \tilde{G}_y) \le \Phi(d(x,y), d(x,\tilde{F}_x), d(y,\tilde{G}y), d(x,\tilde{G}y), d(y,\tilde{F}x)), \tag{2.3}$$

where $d(x, \tilde{F}x) = \min_{p \in \tilde{F}x} d(x,p)$ and H is the Hausdorff metric induced by d, then, we say that F and G satisfy the condition D.

DEFINITION 2.6. Let $g : E \to E$ be a single-valued mapping, $F_i : E \to W(E)$ $(i = 1,2,...)$ be a sequence of fuzzy mappings, if for any $i, j \in Z^+$, F_i and F_j satisfy conditions A_1 and B. Moreover, Φ in condition B satisfies condition (Φ_1) and (Φ_2). Then we say $F_i : E \to W(E)(i = 1,2,...)$ be a g-Φ-contractive type sequence of fuzzy mappings. In particular, when $F_i = F_j = F$ ($\forall\ i, j \in Z^+$) we say $F : E \to W(E)$ be a g-Φ-contractive type fuzzy mapping.

DEFINITION 2.7. Let $F : E \to W(E)$. If $P \in E$ such that $Fp(p) = \max_{x \in E} Fp(u)$, then P is called a fixed point of F. Let $F_i : E \to W(E)$ $(i = 1,2,...)$. If $P \in E$ such that $\left(\bigcap_{i=1}^{+\infty} Fip\right)(p) = \max_{x \in E} \left(\bigcap_{i=1}^{+\infty} Fip\right)(u)$ then P is called a common fixed point of $\{F_i\}$.

DEFINITION 2.8. Let $T : E \to E$ be a single-valued mapping and $F : E \to W(E)$ be a fuzzy mapping. If $P \in E$ such that $P = Tp$ and $Fp(p) = \max_{x \in E} Fp(u)$, then P is called a fuzzy hybrid fixed point of T and F.

Let $T : E \to E$ be a single-valued mapping and $F_i : E \to W(E)$ $(i = 1,2,...)$ be a sequence of fuzzy mappings. If $p \in E$ such that $p = T_p$ and $\left(\bigcap_{i=1}^{+\infty} Fip\right)(p) = \max_{x \in E} \left(\bigcap_{i=1}^{+\infty} Fip\right)(u)$, then p is called a common fuzzy hybrid fixed point of T and $\{F_i\}$.

3. MAIN RESULTS

THEOREM 3.1. Let (E, d) be a complete metric space. Let:

(1) $T : E \to E$ be a single-valued continuous mapping such that $\forall x, y \in E$

$$d(Tx, Ty) \le d(x, Ty) \tag{3.1}$$

(2) $F_i : E \to W(E)\,(i = 1, 2, ...)$ be a g-Φ-contractive type sequence of fuzzy mappings, where $g : E \to E$ is a non-expansive mapping, $\alpha_i(x) : E \to (0, 1]\,(i = 1, 2, ...)$ such that $\forall\, x \in E, T(Fix)_{\alpha_i(x)} = (FiTx)_{\alpha_i(Tx)}\,(i = 1, 2, ...)$.

(3) Let $\delta > 1$, $x_0 \in T(E)$, $x_1 \in (F_1x_0)_{\alpha_1(x_0)}$, $\{t_x\}_{x=0}^{+\infty}$ be a sequence of nonnegative real numbers which is defined as follows

$$to = 0, t_1 > d(x_0, x_1), t_{k+1} = t_k + Q(\delta(t_k - t_{k-1})),\ k = 1, 2, ... \tag{3 2}$$

If $\lim_{K\to\infty} t_k = t_* < +\infty$, then there exists $P \in E$ such that $P = Tp$ and $\left(\bigcap_{i=1}^{+\infty} Fip\right)(p) \geq \min_{i\geq 1}\{\alpha_i(P)\}$, when $\alpha_i(x) = \max_{x\in E} Fix(u)\,(i = 1, 2, ...)$ be a sequence of functions satisfying the condition (2). Then there exists $P \in E$ such that $P = Tp$ and $\left(\bigcap_{i=1}^{+\infty} Fip\right)(p) = \max_{x\in E}\left(\bigcap_{i=1}^{+\infty} Fip\right)(u)$, i.e P be a common fuzzy hybrid fixed point of T and $\{F_i\}$.

PROOF. Let $T(E) = \{x\,|\,x = Tu, u \in E\}, F(E) = \{x\,|\,x = Tx, x \in E\}$. It is obvious that $F(E) \subseteq T(E)$. Next we prove that $T(E) \subseteq F(E)$, $\forall\, Z_1 \in T(E)$, $\exists\, u_1 \in E$ with $Z_1 = Tu_1$, by (3.1), $0 \leq d(Tz_1, Tu_1) \leq d(z_1, Tu_1) = d(z_1, z_1) = 0$, $\therefore Tz_1 = Tu_1 = z_1, z_1 \in F(E)$. Thus $T(E) \subseteq F(E)$, $T(E) = F(E)$.

We prove that $\forall\, x \in T(E)$, $\tilde{F}ix \subseteq T(E)\,(i = 1, 2, ...)$. In fact, for $x \in T(E) = F(E)$ by $x = Tx$, $T(Fix)_{\alpha_i(x)} = (FiTx)_{\alpha_i(Tx)}(i = 1, 2, ...)$, we have $\tilde{F}ix = \tilde{F}iTx = (FiTx)_{\alpha_i(Tx)} = T(Fix)_{\alpha_i(x)} = T\tilde{F}ix \subseteq T(E)(i = 1, 2, ...)$ take $x_0 \in T(E), x_1 \in \tilde{F}_1x_0 \subseteq T(E)$, by the condition B and $g : E \to E$ be a non-expansive mapping, $\exists\, x_2 \in \tilde{F}_2x_1$ such that

$$\begin{aligned} d(x_1, x_2) &\leq \Phi(d(g(x_0), g(x_1)), d(g(x_0), g(x_1)), d(g(x_1), g(x_2)) \\ &\qquad d(g(x_0), g(x_2)), \alpha(g(x_1), g(x_1))) \\ &\leq \Phi(d(x_0, x_1), d(x_0, x_1), d(x_1, x_2), d(x_0, x_2), d(x_1, x_1)) \end{aligned}$$

for $x_2 \in \tilde{F}_2x_1$, $\exists\, x_3 \in \tilde{F}_3x_2$ such that

$$d(x_2, x_3) \leq \Phi(d(x_1, x_2), d(x_1, x_2), d(x_2, x_3), d(x_1, x_3)d(x_2, x_2)).$$

Taking this procedure repeatedly, we can define a sequence $\{x_s\}$ in $T(E)$, satisfying $x_s \in \tilde{F}_{sx_{s-1}} \subseteq T(E)$, $x_{s+1} \in \tilde{F}_{s+1}x_s \subseteq T(E)$, and

$$d(x_s, x_{s+1}) \leq \Phi(d(x_{s-1}, x_s), d(x_{s-1}, x_s), d(x_s, x_{s+1}), d(x_{s-1}, x_{s+1}), d(x_s, x_s)). \tag{3.3}$$

We prove that $\{x_s\}_{s=0}^{+\infty}$ be convergent. First we prove the following inequality

$$d(x_n, x_{n-1}) \leq \delta(t_n - t_{n-1})(n = 1, 2, ...) \tag{3.4}$$

for $n = 1, d(x_1, x_0) < t_1 = t_1 - t_0 < \delta(t_1 - t_0)$, (3.4) is true. Suppose that $n = k$. (3.4) is true, i.e. $d(x_k, x_{k-1}) \leq \delta(t_k - t_{k-1})$ We prove that it remains true for $n = k + 1$, when $n = k + 1$, by $(\Phi_1), (\Phi_2)$, (3.2), (3.3), $d(x_{k-1}, x_{k+1}) \leq d(x_{k-1}, x_k) + d(x_k, x_{k+1})$, and it is easy to prove that $d(x_{k+1}, x_k) \leq d(x_{k-1}, x_k)$, we have

$$\begin{aligned} d(x_{k+1}, x_k) &\leq \Phi(d(x_k, x_{k-1}), d(x_k, x_{k-1}), d(x_k, x_{k+1}), d(x_{k-1}, x_{k+1}), d(x_k, x_k)) \\ &\leq \Phi(d(x_k, x_{k-1}), d(x_k, x_{k-1}), d(x_{k-1}, x_k), 2d(x_{k-1}, x_k), 0) \\ &\leq \Phi(\delta(t_k - t_{k-1}), \delta(t_k - t_{k-1}), \delta(t_k - t_{k-1}), 2\delta(t_k - t_{k-1}), 0) \\ &\leq Q(\delta(t_k - t_{k-1})) = t_{k+1} - t_k < \delta(t_{k+1} - t_k). \end{aligned}$$

Thus (3.4) remains true for $n = k + 1$. This completes the proof of (3.4).

By $\lim_{k\to\infty} = t_* < +\infty$ and (3.4) $d(x_{k+m}, x_k) \leq \sum_{j=k}^{k+m-1} d(x_{j+1}, x_j) \leq \delta \sum_{j=k}^{k+m-1} (t_{j+1} - t_j) = \delta(t_{k+m} - t_k)$.

Thus $\{xs\}_{s=0}^{+\infty}$ be a Cauchy sequence in $T(E)$. Since $(T(E), d)$ is a complete metric space, therefore $\exists$ $P \in E$ such that $\lim_{s\to\infty} x_s = P$, $\because P \in T(E) = F(E)$, $\therefore P = Tp$. Next, we prove that $P \in \bigcap_{i=1}^{+\infty} \tilde{F}ip$, $\forall$

$m \in Z^+$, $\because x_s \in \tilde{F}sx_{s-1}(n=1,2,\ldots)$ by the condition B and $g : E \to E$ be a nonexpansive mapping, $\exists\, v_s \in \tilde{F}mp$ such that

$$d(x_s, v_s) \le \phi(d(x_{s-1}, p), d(x_{s-1}, x_s), d(p, v_s), d(x_{s-1}, v_s), d(p, x_s)) \tag{3.5}$$

by the condition A_1, $\tilde{F}mp = (Fmp)_{\alpha_m(p)} \in CB(E), \tilde{F}mp$ be a non-empty bounded closet set of $E, v_s \in \tilde{F}mp$. Thus $\{d(v_s,p)\}$ be a bounded sequence of real numbers. Therefore, there exists $\{d(v_{s_j},p)\} \subseteq \{d(v_s,p)\}$ satisfies $\lim_{j\to\infty} d(v_{s_j},p) = d$, by (3.5) and $d(v_{s_j}, x_{s_j-1}) \le d(v_{s_j},p) + d(x_{s_j-1},p)$, we have

$$d(v_{s_j},p) \le d(p,x_{s_j}) + \Phi(d(x_{s_j-1},p), d(x_{s_j-1},x_{s_j}), d(p,v_{s_j}), d(x_{s_j-1},p) + d(p,v_{s_j}), d(p,x_{s_j})).$$

Let $j \to +\infty$, by $d(v_{s_j},p) \to d$, $x_s \to p$, (Φ_1), (Φ_2), we have, when $d \neq 0$

$$d \le +\Phi(0,0,d,0+d,0) \le Q(d) < d.$$

This is a contradiction, therefore, $d = 0$, i.e. $\lim_{j\to\infty} v_{s_j} = p$, by $v_{s_j} \in \tilde{F}mp$ and $v_{s_j} \to p$, $\therefore\ p \in \tilde{F}mp = (Fmp)_{\alpha_m(p)} (\forall\, m \in z^+)$ i.e. $Fmp(p) \ge \alpha_m(p)(m=1,2,\ldots)$. Thus $Fmp(p) \ge \min_{i\ge1}\{\alpha_i(p)\}(m=1,2,\ldots)$, $\left(\bigcap_{m=1}^{+\infty} Fmp\right)(p) = \min_{i\ge1}\{\alpha_i(p)\}$.

When $\alpha_i(x) = \max_{x\in E} Fix(u)(i=1,2,\ldots)$. Then $\left(\bigcap_{i=1}^{+\infty} Fip\right)(p) \ge \min_{i\ge1}\{\alpha_i(p)\} \ge \min_{i\ge1}\max_{\mu\in E} Fip(u) \ge \min_{i\ge1} Fip(u) = \left(\bigcap_{i=1}^{+\infty} Fip\right)(u)$, $\forall\, u \in E$. Thus $\left(\bigcap_{i=1}^{+\infty} Fip\right)(p) \ge \max_{\mu\in E}\left(\bigcap_{i=1}^{+\infty} Fip\right)(u) \ge \left(\bigcap_{i=1}^{+\infty} Fip\right)(p)$. $\therefore\ \left(\bigcap_{i=1}^{+\infty} Fip\right)(p) = \max_{\mu\in E}\left(\bigcap_{i=1}^{+\infty} Fip\right)(u)$, i.e. p be common fixed point of $\{F_i\}$, by $p \in T(E) = F(E)$, $p = Tp$. Thus p be a common fuzzy hybrid fixed point of T and $\{F_i\}$.

COROLLARY 3.1. Let $(E,d), T : E \to E$ and $F_i : E \to W(E)\,(i = 1,2,\ldots)$ satisfy the conditions of Theorem 3.1. Moreover Φ satisfies the condition (Φ_3), then the conclusion of Theorem 3.1 remains true.

PROOF. Taking $t_0 = 0$, $x_0 \in T(E)$, $x_1 \in \tilde{F}_1 x_0$, $t_1 > d(x_0, x_1)$. We define a sequence of non-negative real numbers $\{t_k\}_{k=0}^{+\infty}$ as follows:

$$t_{k+1} = t_k + r\delta(t_k - t_{k-1}),\ k = 1,2,\ldots \tag{3.6}$$

where $\delta > 1$ and $\delta r < 1$, r be a constant in the condition (Φ_3). It follows from (3.6)

$$t_{k+1} - t_k = r\delta(t_k - t_{k-1}) = \cdots = (r\delta)^k t_1.$$

Therefore we have $\lim_{k\to\infty} t_k = \lim_{k\to\infty} \sum_{i=1}^{k} (t_i - t_{i-1}) = \frac{t_1}{1-r\delta} < +\infty$. The conclusion of Corollary 3 1 follows from Theorem 3.1 immediately.

COROLLARY 3.2. Let $(E,d), T : E \to E$ satisfies the condition of Theorem 3.1 Let $F_i : E \to W(E)\,(i = 1,2,\ldots)$ for $\alpha_i(x) : E \to (0,1]\,(i = 1,2,\ldots)$ satisfies the condition A_i and $\forall$ $x \in E$, $T(Fix)_{\alpha_i(x)} = (FiTx)_{\alpha_i(Tx)}(i = 1,2,\ldots)$. Moreover, for any $i, j \in z^+, x, y, \in E, u_x \in \tilde{F}ix$, $\exists$ $v_y \in Fjy$ such that

$$\begin{aligned} d(u_x, v_y) \le q \max\{&d(g(x),g(y)), d(g(x),g(u_x)) \\ &d(g(y),g(v_y)), \frac{1}{2}[d(g(x),g(v_y)) + d(g(y),g(u_x))]\} \end{aligned} \tag{3.7}$$

where $q \in (0,1)$ is a constant, $g : E \to E$ be a non-expansive mapping. Then the conclusion of Theorem 3.1 remains true.

PROOF. Taking $\Phi(t_1,t_2,t_3,t_4,t_5) = q\max\{t_1,t_2,t_3,\frac{1}{2}(t_4+t_5)\}$, we have $\Phi(t,t,t,at,bt) = qt$, where $a,b = 0,1,2$ and $a+b=2$. It is easy to see that Φ satisfies the condition (Φ_1) and (Φ_3), therefore the conclusion follows from Corollary 3.1 directly.

THEOREM 3.2. Let (E,d) and $T: E \to E$ satisfy the condition of Theorem 3.1. Let $F_i : E \to W(E)\,(i=1,2,\dots)$ for $\alpha_i(x) : E \to (0,1]\,(i=1,2,\dots)$ satisfy the condition A_1 and $\forall$ $x \in E, T(Fix)_{\alpha_i(x)} = (FiTx)_{\alpha_i(Tx)}(i=1,2,\dots)$. Moreover for any $i,j \in Z^+, x,y \in E, u_x \in \tilde{F}ix$, $\exists$ $v_y \in \tilde{F}jy$ such that

$$d(u_x,v_y) \le \alpha_1 d(g(x),g(u_x)) + \alpha_2 d(g(y),g(v_y)) + \alpha_3 d(g(y),g(u_x)) \\ + \alpha_r d(g(x),g(v_y)) + \alpha_5 d(g(x),g(y)) \tag{3 8}$$

where $g: E \to E$ be a non-expansive mapping, $\alpha_i > 0\,(i=1,2,\dots,5), \alpha_1+\alpha_2+\dots+\alpha_5 < 1$ and $\alpha_3 \ge \alpha_4$. Then the conclusion of Theorem 3.1 remains true.

PROOF. By proof of Theorem 3.1, $T(E) = F(E)$, and $\forall\ x \in T(E), \tilde{F}ix \subseteq T(E)(i=1,2,\dots)$, by (3.8) and $g: E \to E$ be a non-expansive mapping, the same as the proof of Theorem 3.1 We can define a sequence $\{x_s\} \subseteq T(E)$, such that $x_{s+1} \subseteq \tilde{F}_{s+1}x_s \subseteq T(E)$. Moreover

$$\begin{aligned} d(x_s,x_{s+1}) &\le \alpha_1 d(x_{s-1},x_s) + \alpha_2(x_s,x_{s+1}) + \alpha_3 d(x_s,x_s) \\ &\quad + \alpha_4 d(x_{s-1},x_{s+1}) + \alpha_5 d(x_{s-1},x_s) \\ &\le \alpha_1 d(x_{s-1},x_s) + \alpha_2 d(x_s,x_{s+1}) + \alpha_4 d(x_{s-1},x_s) \\ &\quad + \alpha_4 d(x_s,x_{s+1}) + \alpha_5 d(x_{s-1},x_s). \end{aligned}$$

Therefore

$$\alpha(x_s,x_{s+1}) \le \frac{\alpha_1+\alpha_4+\alpha_5}{1-\alpha_2-\alpha_4} d(x_{s-1},x_s) \tag{3.9}$$

$\because\ \alpha_3 \ge \alpha_4 > 0,\ \alpha_1+\dots+\alpha_5 < 1$. Thus $r = \frac{\alpha_1+\alpha_4+\alpha_5}{1-\alpha_2-\alpha_4} < 1$, we have

$$d(x_s,x_{s+1}) \le rd(x_{s-1},x_s) \le r^2 d(x_{s-2},x_s) \le \dots \le r^s d(x_0,x_1). \tag{3.10}$$

By (3.10), it is easy to see that $\{x_s\}_{s=1}^{+\infty}$ is a Cauchy sequence in $T(E)$. Thus $\exists\ p \in T(E)$, such that $\lim_{s\to\infty} x_s = p$. Next, we prove that $p \in \bigcap_{m=1}^{+\infty} \tilde{F}_{mp}$, $\forall\ m \in Z^+$, for $x_s \in \tilde{F}_s x_{s-1}(n=1,2,\dots)$, by assumption, $\exists\ v_s \in \tilde{F}mp$ such that

$$\begin{aligned} d(x_s,v_s) &\le \alpha_1 d(x_{s-1},x_s) + \alpha_2 d(p,v_s) + \alpha_3 d(p,x_s) \\ &\quad + \alpha_4 d(x_{s-1},v_s) + \alpha_5 d(x_{s-1},p) \\ &\le \alpha_1 d(x_{s-1},x_s) + \alpha_2 d(p,x_s) + \alpha_2 d(x_s,v_s) \\ &\quad \alpha_3 d(p,x_s) + \alpha_4 d(x_{s-1},x_s) + \alpha_4 d(x_s,v_s) + \alpha_5 d(x_{s-1},p). \end{aligned}$$

Thus we have

$$\begin{aligned} (1-\alpha_2-\alpha_4)d(x_s,v_s) &\le \alpha_1 d(x_{s-1},x_s) + \alpha_2 d(p,x_s) \\ &\quad + \alpha_3 d(p,x_s) + \alpha_4 d(x_{s-1},x_s) + \alpha_5 d(x_{s-1},p). \end{aligned}$$

We have $d(x_s,v_s) \to 0(n \to +\infty)$. Thus $d(v_s,p) \le d(v_s,x_s) + d(x_s,p) \to 0(n \to +\infty)$, $\therefore \lim_{s\to\infty} v_s = p$, by $v_s \in \tilde{F}mp \in CB(E)$. Therefore $p \in \tilde{F}mp(\forall\ m \in z^+)$. By $p = Tp$ and $p \in \bigcap_{m=1}^{+\infty} \tilde{F}mp$, the same as the proof of Theorem 3.1, we obtain the conclusion of Theorem 3.1.

When $T = I$ is the identity operator on E, we obtain the following result.

COROLLARY 3.3. Let (E,d) and $F_i : E \to W(E)\,(i=1,2,\dots)$ satisfy the conditions of Theorem 3.2. Then there exists $p \in E$ such that $\left(\bigcap_{i=1}^{+\infty} Fip\right)(p) \ge \min_{i\ge1}\{\alpha_i(p)\}$, when $\alpha_i(x) = \max_{x\in E} Fix(u)$ $(i=1,2,\dots)$ satisfies corresponding conditions, p is a common fixed point of $\{F_i\}$.

REMARK 3.1. When $\alpha_3 = \alpha_4$ in the condition (3.8) of Theorem 3.2, Theorem 3.2 is a special case of Corollary 3.2. Corollary 3.3 is an improvement and generalized version of Theorem 3.1 of [4] and Theorem 3.10 of [3]. In Theorem 3 of [2], if $\{F_i\}$ for $\{\alpha_i(x)\}$ satisfy condition A_2, then Theorem 3 of [2] is a special case of Theorem 3.1 of this paper. In fact, when $T = I$ and $g = I$ are identity operators on E, by the theorem of Nadler [5], it is easy to see the condition D implies the condition C.

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