

Research Article

Coupled Scalar Fields Oscillons and Breathers in Some Lorentz Violating Scenarios

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We discuss the impact of the breaking of the Lorentz symmetry on the usual oscillons, the so-called flat-top oscillons, and the breathers. Our analysis is performed by using a Lorentz violation scenario rigorously derived in the literature. We show that the Lorentz violation is responsible for the origin of a kind of deformation of the configuration, where the field configuration becomes oscillatory in a localized region near its maximum value. Furthermore, we show that the Lorentz breaking symmetry produces a displacement of the oscillon along the spatial direction; the same feature is present in the case of breathers. We also show that the effect of a Lorentz violation in the flat-top oscillon solution is responsible by the shrinking of the flat-top. Furthermore, we find analytically the outgoing radiation; this result indicates that the amplitude of the outgoing radiation is controlled by the Lorentz breaking parameter, in such a way that this oscillon becomes more unstable than its symmetric counterpart; however, it still has a long living nature.

1. Introduction

The study of nonlinear systems is becoming an area of increasing interest along the last few decades [1, 2]. In fact, such nonlinear behavior of physical systems is found in a broad part of physical systems nowadays. This includes condensed matter systems, field theoretical models, modern cosmology, and a large number of other domains of the physical science [3–28]. One of the reasons of this increasing interest is the fact that many of those systems present a countable number of distinct degenerate minimal energy configurations. In many cases that degenerate structure can be studied through simple models of scalar fields possessing a potential with two or more degenerate minima. For instance, in two or more spatial dimensions, one can describe the so-called domain walls [4] connecting different portions of the space where the field is at different values of the degenerate minima of the field potential. In other words, the field configuration interpolates between two of those potential minima. At this point, it is important to remark that a powerful insight to solve nonlinear problems analytically

was introduced by Bolgomol'nyi, Prasad, and Sommerfield [5, 6]. In this case, the method shown by Bolgomol'nyi, Prasad, and Sommerfield is now called BPS approach, and it is based on obtaining a first-order differential equation from the energy functional. By using this method, it is possible to find solutions that minimize the energy of the configuration and that ensure their stability.

In the context of the field theory it is quite common the appearance of solitons [7], which are field configurations presenting a localized and shape-invariant aspect, having a finite energy density as well as being capable of keeping their shape unaltered after a collision with another solitons. The presence of those configurations is nowadays well understood in a wide class of models, presenting or not topological nature. As examples one can cite the monopoles, textures, strings and kinks [8].

An important feature of a large number of interesting nonlinear models is the presence of topologically stable configurations, which prevents them from decaying due to small perturbations. Among other types of nonlinear field configurations, there is a specially important class of time-dependent

stable solutions, the breathers appearing in the Sine-Gordon like models. Another time-dependent field configuration whose stability is granted for by charge conservation are the Q-balls as baptized by Coleman [9] or nontopological solitons [10]. However, considering the fact that many physical systems interestingly may present a metastable behavior, a further class of nonlinear systems may present a very long-living configuration, usually known as oscillons. This class of solutions was discovered in the seventies of the last century by Bogolyubsky and Makhankov [29], and rediscovered posteriorly by Gleiser [30]. Those solutions, appeared in the study of the dynamics of first-order phase transitions and bubble nucleation. Since then, more and more works were dedicated to the study of these objects [30–48].

Oscillons are quite general configurations and are found in inflationary cosmological models [30], in the Abelian-Higgs $U(1)$ models [31], in the standard model $SU(2) \times U(1)$ [32], in axion models [33], in expanding universe scenarios [34], and in systems involving phase transitions [35].

The usual oscillon aspect is typically that of a bell shape which oscillates sinusoidally in time. Recently, Amin and Shirokoff [36] have shown that, depending on the intensity of the coupling constant of the self-interacting scalar field, it is possible to observe oscillons with a kind of plateau at its top. In fact, they have shown that these new oscillons are more robust against collapse instabilities in three spatial dimensions.

At this point, it is interesting to remark that Segur and Kruskal [37] have shown that the asymptotic expansion do not represent in general an exact solution for the scalar field; in other words, it simply represents an asymptotic expansion of first order in ϵ , and it is not valid at all orders of the expansion. They have also shown that in one spatial dimension they radiate [37]. In a recent work, the computation of the emitted radiation of the oscillons was extended to the case of two and three spatial dimensions [38]. Another important result was put forward by Hertzberg [39]. In that work, he was able to compute the decaying rate of quantized oscillons, and it was shown that its quantum rate decay is very distinct of the classical one.

On the other hand, some years ago, Kostelecký and Samuel [49] started to study the problem of the Lorentz and CPT (charge conjugation-parity-time reversal) symmetry breaking. This was motivated by the fact that the superstring theories suggest that Lorentz symmetry should be violated at higher energies. After that seminal work, a theoretical framework about Lorentz and CPT symmetry breaking has been rigorously developed. As an example, the effects on the standard model due to the CPT violation and Lorentz breaking were presented by Colladay and Kostelecký [50]. Recently, a large amount of works considering the impact of some kind of Lorentz symmetry breaking has appeared in the literature [51–66]. As another example, recently Belich et al. [52] studied the Aharonov-Bohm-Casher problem with a nonminimal Lorentz-violating coupling. In that reference, the authors have shown that the Lorentz violation is responsible by the lifting of the original degeneracies in the absence of magnetic field, even for a neutral particle.

By introducing a dimensional reduction procedure to $(1 + 2)$ dimensions presented in [53], Casana, Carvalho, and

Ferreira applied the approach to investigate the dimensional reduction of the CPT -even electromagnetic sector of the standard model extension. Another important work was presented by Boldo et al. [54], where the problem of Lorentz symmetry violation gauge theories in connection with gravity models was analyzed. In a very recent work, Kostelecký and Mewes [55] also analyzed the effects of Lorentz violation in neutrinos.

In recent years, investigations about topological defects in the presence of Lorentz symmetry violation have been addressed in the literature [56–58]. Works have also been done on monopole and vortices in Lorentz violation scenarios [59]. For instance, in [59], a question about the Lorentz symmetry violation on BPS vortices was investigated. In that paper, the Lorentz violation allows a control of the radial extension and of the magnetic field amplitude of the Abrikosov-Nielsen-Olesen vortices.

In fact, Lorentz invariance is the most fundamental symmetry of the standard model of particle physics and it has been very well verified in several experiments. But, it is important to remark that we cannot be sure that this, or any other, symmetry is exact. Their consequences shall be verified through experimental data. This affirmation is encouraged due to the fact that there exist some experimental tests of the Lorentz invariance being carried in low energies, in other words, energies smaller than 14 TeV. Thus, from this fact, we can suspect that at high energies the Lorentz invariance could not be preserved. As an example, in the string theory there is a possibility that we could be living in a universe which is governed by noncommutative coordinates [67]. In this scenario it was shown in [68] that the Lorentz invariance is broken.

Furthermore, in a cosmological scenario, the occurrence of high energy cosmic rays above the Greisen-Zatsepin-Kuzmin (GZK) cutoff [69] or super GZK events has been found in astrophysical data [70]. This event indicates the possibility of a Lorentz violation [71].

The impact of Lorentz violation on the cosmological scenario is very important, because several of its weaknesses could be easily explained by the Lorentz violation. For instance, it was shown by Bekenstein [72] that the problem of the dark matter is associated with the Lorentz violating gravity and in [73] Lorentz violation also is used to clarify the dark energy problem. Nowadays, the breaking of the Lorentz symmetry is a fabulous mechanism for description of several problems and conflicts in cosmology, such as the baryogenesis, primordial magnetic field, nucleosynthesis, and cosmic rays [74].

In the inflationary scenario with Lorentz violation, Kanno and Soda [75] have shown that Lorentz violation affects the dynamics of the inflationary model. In this case, those authors showed that, using a scalar-vector-tensor theory with Lorentz violation, the exact Lorentz violation inflationary solutions are found in the absence of the inflaton potential. Therefore, the inflation can be connected with the Lorentz violation.

Here, it is convenient to us to emphasize that the inflation is the fundamental ingredient to solve both the horizons as the flatness problems of the standard model of the very early universe. Approximately 10^{-33} seconds after the inflation,

the inflaton decays to radiation, where quarks, leptons, and photons were coupled to each other. In this case, the baryonic matter was prevented from forming. Therefore, approximately 1.388×10^{12} seconds after the Big Bang, the universe has cooled enough to allow photons to freely travel through the universe. After that, matter has become dominant in the universe.

At this point, it is important to remark that the postinflationary universe is governed by real scalar fields where nonlinear interactions are present. Thus, it was shown in [76] that oscillons can easily dominate the postinflationary universe. In that work, it was demonstrated that the postinflationary universe can contain an effective matter-dominated phase, during which it is dominated by localized concentrations of scalar field matter. Furthermore, in a very recent work [77], a class of inflationary models was introduced, giving rise to oscillons configurations. In this case, it was argued that these oscillons could dominate the matter density of the universe for a given time. Thus, one could naturally wonder about the effect of Lorentz violation over this scenario.

Thus, in this work we are interested in answering the following issues. Can oscillons and breathers exist in scenarios with Lorentz violation symmetry? If oscillons and breathers exist in these scenarios, how is their profile changed? Furthermore, what happens with the lifetime of the oscillons?

Therefore, in this paper, we will show that oscillons and breathers can be found in Lorentz violation scenarios; our study is performed by using Lorentz violation theories rigorously derived in the literature [50, 78]. As a consequence, the principal goal here is to analyze the case of two nonlinearly coupled scalar fields case. However, we use a constructive approach, so that we start by studying the cases of one scalar field models and, then, use those results in the study we are primarily interested in.

This paper is organized as follows. In Section 2 we present the description of the Lagrangian density for a real scalar field in the presence of a Lorentz violation scenario. In Section 3 we calculate the respective commutation relations of the Poincaré group in the standard-model extension (SME) in a 1+1-dimensional flat Minkowski space-time. The approach of the equation of motion is given in Section 4. Usual oscillons in the background of the Lorentz violation is analyzed in Section 5. In Section 6 we will find the flat-top oscillons which violate the Lorentz symmetry. The breathers solutions are presented in Section 7. We discuss the outgoing radiation by oscillons in Section 8. In Section 9 we will present the oscillons in a two-scalar field theory. Finally, we summarize our conclusions in Section 10.

2. Standard-Model Extension Lagrangian

In this section, we present a scalar field theory in a 3 + 1-dimensional flat Minkowski space-time, but here we consider a break of the Lorentz symmetry. In low energy, Lorentz, and *CPT* symmetries the standard model (SM) of particle physics is experimentally well supported, but in high energies the superstring theories suggest that Lorentz symmetry should be violated, and in this context, the framework to study Lorentz and *CPT* violation is the so-called standard-model extension.

In the description of the SME, the Lagrangian density for a real scalar field containing Lorentz violation (LV), which can be read as a simplified version of the Higgs model, is given by [50, 78]

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{1}{2} k^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi), \quad (1)$$

where φ is a real scalar field, $k^{\mu\nu}$ is a dimensionless tensor which controls the degree of Lorentz violation, and $V(\varphi)$ is the self-interaction potential. It is important to remark that, some years ago [56], this Lagrangian density was used to study defect structures in Lorentz and *CPT* violating scenarios. In that case, the authors showed that the violation of Lorentz and *CPT* symmetries is responsible by the appearance of an asymmetry between defects and antidefects. This was generalized in [56]. Furthermore, one similar Lagrangian density has been applied in the study on the renormalization of the scalar and Yukawa field theories with Lorentz violation. In that case, it was shown that a LV theory with N scalar fields, interacting through a ϕ^4 interaction, can be written as

$$\begin{aligned} \mathcal{L}_K = & \frac{1}{2} (\partial_\mu \varphi_i) (\partial^\mu \varphi_i) + \frac{1}{2} \sum_{i=1}^N K_{\mu\nu}^i \partial^\mu \varphi_i \partial^\nu \varphi_i - \frac{1}{2} \lambda^2 \varphi_i^2 \\ & + \sum_{i=1}^N u_i^\beta \varphi_i \partial_\beta \varphi_i + \sum_{j=1}^N \varphi_i^2 v_j^\beta \partial_\beta \varphi_j - \frac{g}{4!} (\varphi_i^2)^2. \end{aligned} \quad (2)$$

As a simple example, those authors showed for $K_{\mu\nu}^i = K_{00}^i \delta_\mu^0 \delta_\nu^0$ that the dispersion relation is given by $E = \sqrt{p^2 - K_{00}^i (p^0)^2 + \lambda^2}$, which is implied in a LV. Therefore, using explicit calculations, the quantum corrections in the above LV theory was studied, and these results show that the theory is renormalizable.

Now, returning to (1), we can write the Lagrangian density in the following form:

$$\mathcal{L} = \frac{1}{2} (\eta^{\mu\nu} + k^{\mu\nu}) \partial_\mu \varphi \partial_\nu \varphi - V(\varphi). \quad (3)$$

In this case, the Minkowski metric is modified from $g^{\mu\nu}$ to $\eta^{\mu\nu} + k^{\mu\nu}$, which is responsible for the breaking of the Lorentz symmetry [50, 78, 79]. At this point, it is possible to apply an appropriate linear transformation of the space-time variable x^μ , in order to map the above Lagrangian density into a Lorentz-like covariant form, but this leads to changes in the fields and coupling constants of the potential. Thus, the coupling constants and the fields are rescaled in function of the $k^{\mu\nu}$ parameters.

Clearly, as a final product, the LV and Lorentz invariant Lagrangians have the same equation of motion. The fundamental difference between these two equations comes from the fact that the new variables x^μ carry information of the Lorentz violations through the $k^{\mu\nu}$ parameters. In other words, in the transformed variables, the system looks to be covariant (under boosts of the transformed space-time variables). However, as a consequence of the fact that the resulting couplings become not invariant when one changes from a reference frame to another, there is no real Lorentz invariance.

For instance, such behavior would be analogous to a change of the value of the electrical charge when one moves from an inertial reference frame to another one, which is forbidden.

In the Lagrangian density (1), $k^{\mu\nu}$ is a constant tensor represented by a 4×4 matrix. It is the term which can be responsible for the breaking of the Lorentz symmetry. Thus, we write the tensor $k^{\mu\nu}$ in the following form:

$$k^{\mu\nu} = \begin{pmatrix} k^{00} & k^{01} & k^{02} & k^{03} \\ k^{10} & k^{11} & k^{12} & k^{13} \\ k^{20} & k^{21} & k^{22} & k^{23} \\ k^{30} & k^{31} & k^{32} & k^{33} \end{pmatrix}. \quad (4)$$

In general $k^{\mu\nu}$ has arbitrary parameters, but it is important to remark that if this matrix is real, symmetric, and traceless, the *CPT* symmetry is kept [50, 78]. Here, we comment that under *CPT* operation, $\partial_\mu \rightarrow -\partial_\mu$, the term $k^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ goes as $k^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \rightarrow +k^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$. Thus, one notices that $k^{\mu\nu}$ is always *CPT*-even, regardless of its properties. Furthermore, the tensor $k^{\mu\nu}$ should be symmetric in order to avoid a vanishing contribution.

In a recent work, Anacleto et al. [80] also analyzed a similar process to break the Lorentz symmetry, where the tensor $k^{\mu\nu}$ was used to study the problem of acoustic black holes in the Abelian-Higgs model with Lorentz symmetry breaking. In another work by Anacleto et al. [80], the tensor $k^{\mu\nu}$ was used to study the superresonance effect from a rotating acoustic black hole with Lorentz symmetry breaking. Finally, in a very recent work [57], a generalized two-field model in 1+1-dimensions which presents a constant tensor and vector functions was introduced. In that case, a class of traveling solitons in Lorentz and *CPT* breaking systems was found.

However, we can find systems with Lorentz symmetry break which has an additional scalar field [79]:

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}\partial_\mu\varphi_1\partial^\mu\varphi_1 + \frac{1}{2}\partial_\mu\varphi_2\partial^\mu\varphi_2 + \frac{1}{2}k^{\mu\nu}\partial_\mu\varphi_1\partial_\nu\varphi_1 \\ & - \frac{m_1\varphi_1^2}{2} - \frac{m_2\varphi_2^2}{2} - V(\varphi_1, \varphi_2). \end{aligned} \quad (5)$$

In the above Lagrangian density, we have a different coefficient correcting the metric, but the coefficients for Lorentz violation cannot be removed from the Lagrangian density using variables or fields redefinitions and observable effects of the Lorentz symmetry break can be detected in the above theory. Therefore, theories with fewer fields and fewer interactions allow more redefinitions and observable effects.

3. SME Lagrangian: One Field Theory (OFT)

In this section, we will work in a 1+1-dimensional Minkowski space-time. Here, we study a scalar field theory in the presence of a Lorentz violating scenario. The theory that we study is given by the Lagrangian density (1). Thus, in this case, the corresponding Lagrangian density must be as follows:

$$\mathcal{L}_{1+1} = \frac{1}{2}\alpha_1(\partial_t\varphi)^2 - \frac{1}{2}\alpha_2(\partial_x\varphi)^2 + \frac{1}{2}\alpha_3\partial_t\varphi\partial_x\varphi - V(\varphi), \quad (6)$$

where

$$\begin{aligned} \alpha_1 &\equiv (1 + k^{00}), & \alpha_2 &\equiv (1 - k^{11}), \\ \alpha_3 &\equiv (k^{01} + k^{10}), \\ \partial_t &\equiv \frac{\partial}{\partial t}, & \partial_x &\equiv \frac{\partial}{\partial x}. \end{aligned} \quad (7)$$

At this point, it is important to remark that the Lagrangian density clearly has not manifest covariance. Furthermore, it is possible to observe that the covariance is recovered by choosing $k^{00} = k^{11} = 0$ and $k^{01} = -k^{10}$ (or $k^{01} = k^{10} = 0$). Other possibilities that do not represent a LV are $k^{00} = -k^{11}$ and $k^{01} = -k^{10}$ (or $k^{01} = k^{10} = 0$).

Now, from the above, we can easily construct the corresponding Hamiltonian density:

$$\mathcal{H} = \beta_1\Pi^2 + \beta_2(\partial_x\varphi)^2 + \beta_3\Pi(\partial_x\varphi) + V(\varphi), \quad (8)$$

where $\beta_1 = 1/(2\alpha_1)$, $\beta_2 = [2\alpha_1\alpha_2 + \alpha_3(\alpha_3 - 1)]/(4\alpha_1)$, $\beta_3 = -\alpha_3/(2\alpha_1)$, and Π is the conjugate momentum, which is given by

$$\Pi = \alpha_1\partial_t\varphi + \left(\frac{\alpha_3}{2}\right)\partial_x\varphi. \quad (9)$$

Let us now see how the Poincaré algebra is modified in this scenario. The idea of the present analysis is to see how the Poincaré invariance is broken. In other words, verify how this scenario has the Lorentz symmetry violated. Therefore, for this we write down the three Poincaré generators, the Hamiltonian H , the total momentum P , and the Lorentz boost M :

$$\begin{aligned} H &= \int dx \mathcal{H}, \\ P &= \int dx \left[\frac{\Pi(\partial_x\varphi)}{\alpha_1} - \frac{\alpha_3(\partial_x\varphi)^2}{2\alpha_1} \right], \\ M &= \int dx \left\{ t \left[\frac{\Pi(\partial_x\varphi)}{\alpha_1} - \frac{\alpha_3(\partial_x\varphi)^2}{2\alpha_1} \right] - x\mathcal{H} \right\}. \end{aligned} \quad (10)$$

With this, we can calculate the commutation relations of the Poincaré group. Thus, after straightforward calculations of the usual commutation relations, it is not difficult to conclude that

$$\begin{aligned} [H, P] &= -i \left(\frac{\alpha_3}{\alpha_1^2} \right) \int dx (\partial_x\varphi)(\partial_x\Pi), \\ [M, H] &= -i \int dx \left((4\beta_1\beta_2 + \beta_3^2)\Pi(\partial_x\varphi) \right. \\ &\quad \left. - \frac{\alpha_3}{\alpha_1^2}(\partial_x\varphi)(\partial_x\Pi) \right. \\ &\quad \left. + 2\beta_2\beta_3(\partial_x\varphi)^2 + 2\beta_1\beta_3(\Pi)^2 \right), \end{aligned}$$

$$\begin{aligned}
[M, P] = & -i\frac{H}{\alpha_1} + i\frac{\alpha_3}{2\alpha_1^2} \\
& \cdot \int dx \left(\Pi (\partial_x \varphi) \right. \\
& \left. + x (\partial_x \varphi) (\partial_x \Pi) + \frac{\beta_3}{4\beta_1} (\partial_x \varphi)^2 \right). \quad (11)
\end{aligned}$$

From the above relations, we can see that the Poincaré algebra is not closed, since the usual commutations are not recovered. As a consequence, in this scenario we have one violation of the Lorentz symmetry. However, it is possible to recover the complete commutation relations by taking $k^{00} = k^{11} = 0$ and $k^{01} = -k^{10}$ (or $k^{01} = k^{10} = 0$). For instance, making $k^{00} = k^{11} = 0$ and $k^{01} = -k^{10}$, we have

$$[H, P] = 0, \quad [M, H] = -iP, \quad [M, P] = -iH. \quad (12)$$

At this point we can verify that, for the cases $k^{00} = -k^{11}$ and $k^{01} = -k^{10}$ (or $k^{01} = k^{10} = 0$), the commutation relations (11) lead to

$$[H, P] = 0, \quad [M, H] = -i\alpha_1 P, \quad [M, P] = -\frac{iH}{\alpha_1}. \quad (13)$$

However, in the above case, we can recover the usual Poincaré algebra using the rescale $P = \bar{P}/\alpha_1$. Thus, such commutation relations indicate that there is no LV in this tensor configuration.

In summary, the Lagrangian density (6) has explicit dependence on the parameters k^{00} , k^{11} , k^{01} , and k^{10} , which is responsible for the violation of the Lorentz symmetry. This happens due to the fact that the Poincaré invariance is not preserved, as one can see from (11).

4. Equation of Motion in Lorentz Violation Scenarios: OFT

In this section, we will study the equation of motion in the presence of the scenario with Lorentz violation of the previous section. Here, our aim is to study the case in the 1 + 1-dimensional Minkowski space-time. As a consequence, we will study the theory that is governed by the Lagrangian density (6). Consequently, the corresponding classical equation of motion can be written as

$$\alpha_1 \frac{\partial^2 \varphi(x, t)}{\partial t^2} - \alpha_2 \frac{\partial^2 \varphi(x, t)}{\partial x^2} + \alpha_3 \frac{\partial^2 \varphi(x, t)}{\partial x \partial t} + V_\varphi = 0, \quad (14)$$

where $V_\varphi \equiv \partial V / \partial \varphi$. Note that the above equation is carrying information about the symmetry breaking of the theory.

Here, if one applies the transformation involving the Lorentz boost in the above equation of motion, one gets

$$q_1 \frac{\partial \varphi^2(x', t')}{\partial t'^2} - q_2 \frac{\partial \varphi^2(x', t')}{\partial x'^2} + q_3 \frac{\partial \varphi^2(x', t')}{\partial x' \partial t'} + V_\varphi = 0, \quad (15)$$

where

$$\begin{aligned}
x' &= \gamma(x - vt), & t' &= \gamma\left(t - \frac{vx}{c^2}\right), \\
\gamma &= \frac{1}{\sqrt{1 - (v/c)^2}}, \\
q_1 &= \gamma^2 \left(\frac{\alpha_1 c^2 - \alpha_2 v^2 - \alpha_3 cv}{c^4} \right), \\
q_2 &= \gamma^2 \left(\frac{-\alpha_1 v^2 + \alpha_2 c^2 + \alpha_3 cv}{c^2} \right), \\
q_3 &= \gamma^2 \left(\frac{-2v\alpha_1 c + 2c\alpha_2 v - \alpha_3 (c^2 + v^2)}{c^3} \right). \quad (16)
\end{aligned}$$

Following the above demonstration, we can see clearly that this equation is not invariant under boost transformations. For instance, we can conclude that the possibilities [$k^{00} = -k^{11}$, $k^{01} = -k^{10}$] or [$k^{00} = -k^{11}$, $k^{01} = k^{10} = 0$] lead to the following equations:

$$\begin{aligned}
\frac{\alpha_1}{c^2} \frac{\partial \varphi^2(x, t)}{\partial t^2} - \alpha_1 \frac{\partial \varphi^2(x, t)}{\partial x^2} + V_\varphi &= 0, \\
\frac{\alpha_1}{c^2} \frac{\partial \varphi^2(x', t')}{\partial t'^2} - \alpha_1 \frac{\partial \varphi^2(x', t')}{\partial x'^2} + V_\varphi &= 0. \quad (17)
\end{aligned}$$

Note that there is no modification of the equations; in other words, the possibilities [$k^{00} = -k^{11}$, $k^{01} = -k^{10}$] or [$k^{00} = -k^{11}$, $k^{01} = k^{10} = 0$] do not represent a genuine factor for LV.

In order to solve analytically the differential equation (14) and simultaneously keep the breaking of the Lorentz symmetry, we must decouple the equation. For this, we apply the rotation

$$\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} X \\ T \end{pmatrix}, \quad (18)$$

where θ is an arbitrary rotation angle. Thus, (14) in the new variables is rewritten as

$$h_1 \frac{\partial^2 \varphi(X, T)}{\partial T^2} - h_2 \frac{\partial^2 \varphi(X, T)}{\partial X^2} + V_\varphi = 0, \quad (19)$$

with the definitions

$$\begin{aligned}
\theta &\equiv -\frac{1}{2} \arctan\left(\frac{\alpha_3}{\alpha_1 + \alpha_2}\right), \\
h_1 &\equiv \frac{\alpha_1^2 - \alpha_2^2 + [\alpha_3^2 + (\alpha_1 + \alpha_2)^2] \cos(2\theta)}{2(\alpha_1 + \alpha_2)}, \\
h_2 &\equiv \frac{\alpha_2^2 - \alpha_1^2 + [\alpha_3^2 + (\alpha_1 + \alpha_2)^2] \cos(2\theta)}{2(\alpha_1 + \alpha_2)}. \quad (20)
\end{aligned}$$

Note that the rotation angle θ has been chosen in order to eliminate the dependence in the term $\partial^2\varphi/\partial X\partial T$. Now, performing the dilations $T = \sqrt{h_1}\Upsilon$ and $X = \sqrt{h_2}Z$, one gets

$$\frac{\partial^2\varphi(Z, \Upsilon)}{\partial\Upsilon^2} - \frac{\partial^2\varphi(Z, \Upsilon)}{\partial Z^2} + V_\varphi = 0. \quad (21)$$

From now on we will use the above equation to describe the profile of oscillons and breathers. It is of great importance to remark that the above equation has all the information about the violation of the Lorentz symmetry. In fact, the field $\varphi(Z, \Upsilon)$ carries on the dependence of the parameters that break the Lorentz symmetry; this information arises from the fact that new variables Z and Υ have explicit dependence on the $k^{\mu\nu}$ elements.

5. Usual Oscillons with Lorentz Violation: OFT

Now, we study the case of a scalar field theory which supports usual oscillons in the presence of Lorentz violating scenarios. The profile of the usual oscillons is one in which the spatial structure is localized in the space and, in the most cases, is governed by a function of the type $\text{sech}(x)$. On the other hand, the temporal structure is like $\cos(t)$, which is periodic. The theory that we will study is given by the Lagrangian density (6). In this case, we showed in the last section that the corresponding classical equation of motion, after some manipulations, can be represented by (21). Thus, in order to analyze usual oscillons in this situation, we choose the potential that was used in [36], which is written as

$$V(\varphi) = \frac{1}{2}\varphi^2 - \frac{1}{4}\varphi^4 + \frac{g}{6}\varphi^6, \quad (22)$$

where g represents a free coupling constant and we will consider a regime where $g \gg 1$.

Since our primordial interest is to find periodic and localized solutions, it is useful, as usual in the study of the oscillons, to introduce the following scale transformations in t and x :

$$\tau = \omega\Upsilon, \quad y = \epsilon Z, \quad (23)$$

with $\omega = \sqrt{1 - \epsilon^2}$. Thus, the equation of the motion (21) becomes

$$\omega^2 \frac{\partial^2\varphi(y, \tau)}{\partial\tau^2} - \epsilon^2 \frac{\partial^2\varphi(y, \tau)}{\partial y^2} + \varphi - \varphi^3 + g\varphi^5 = 0. \quad (24)$$

Now we are in a position to investigate the usual oscillons. But it is important to remark that the fundamental point is that here we have the effects of the Lorentz symmetry breaking. We can see this by inspecting the above equation of motion, which is carrying information about the terms of the Lorentz breaking through the variables y and τ . We observe that it is possible to recover the original equation of motion for usual oscillons choosing $k^{00} = k^{11} = 0$ and $k^{01} = -k^{10}$ (or $k^{01} = k^{10} = 0$). In this case, the Lorentz symmetry is recovered.

Next we expand φ as

$$\varphi(y, \tau) = \epsilon\varphi_1(y, \tau) + \epsilon^3\varphi_3(y, \tau) + \epsilon^5\varphi_5(y, \tau) + \dots \quad (25)$$

Note that the above expansion has only odd powers of ϵ ; this occurs because the equation is odd in φ . Let us now substitute this expansion of the scalar field into the equation of motion (24). This leads to

$$\frac{\partial^2\varphi_1}{\partial\tau^2} + \varphi_1 = 0, \quad (26)$$

$$\frac{\partial^2\varphi_3}{\partial\tau^2} + \varphi_3 - \frac{\partial^2\varphi_1}{\partial\tau^2} - \frac{\partial^2\varphi_1}{\partial y^2} - \varphi_1^3 = 0. \quad (27)$$

Therefore, the solution of (26) is of the following form:

$$\varphi_1(y, \tau) = \Phi(y) \cos(\tau). \quad (28)$$

Here we call attention to the fact that the solution must be smooth at the origin and vanishing when y becomes infinitely large.

In order to find the solution of $\Phi(y)$, let us substitute the solution obtained for $\varphi_1(y, \tau)$ into (27). Thus, it is not hard to conclude that

$$\frac{\partial^2\varphi_3}{\partial\tau^2} + \varphi_3 = \left(\frac{d^2\Phi}{dy^2} - \Phi + \frac{3}{4}\Phi^3 \right) \cos(\tau) + \frac{1}{4}\Phi^3 \cos(3\tau). \quad (29)$$

Solving the above equation, we find a term which is linear in the time-like variable τ , resulting in a nonperiodical solution, and we are interested in solutions which are periodical in time. Then, to avoid this we will impose that

$$\frac{d^2\Phi}{dy^2} - \Phi + \frac{3}{4}\Phi^3 = 0. \quad (30)$$

At this point, one can verify that the above equation can be integrated to give

$$\left(\frac{d\Phi}{dy} \right)^2 + U(\Phi) = E, \quad (31)$$

where $U(\Phi) = -\Phi^2 + (3/8)\Phi^4$. Note that in the above equation, the arbitrary constant E should be set to zero in order to get solitonic solution. This condition allows the field configuration to go asymptotically to the vacua of the field potential $U(\Phi)$. Now, we must solve (31) with $E = 0$. In this case one gets

$$\Phi(y) = \sqrt[4]{\frac{8}{3}} [\text{sech}(y)]^{1/2}. \quad (32)$$

As one can see, up to the order $\mathcal{O}(\epsilon)$, the corresponding solution for the field in the original variables is given by

$$\begin{aligned} \varphi_{\text{osc}}(x, t) = & \epsilon \sqrt[4]{\frac{8}{3}} \left(\sqrt{\text{sech} \left[\frac{\epsilon [x \cos(\theta) + t \sin(\theta)]}{\sqrt{h_2}} \right]} \right) \\ & \times \cos \left[\frac{\omega [-x \sin(\theta) + t \cos(\theta)]}{\sqrt{h_1}} \right] + \mathcal{O}(\epsilon^3). \end{aligned} \quad (33)$$

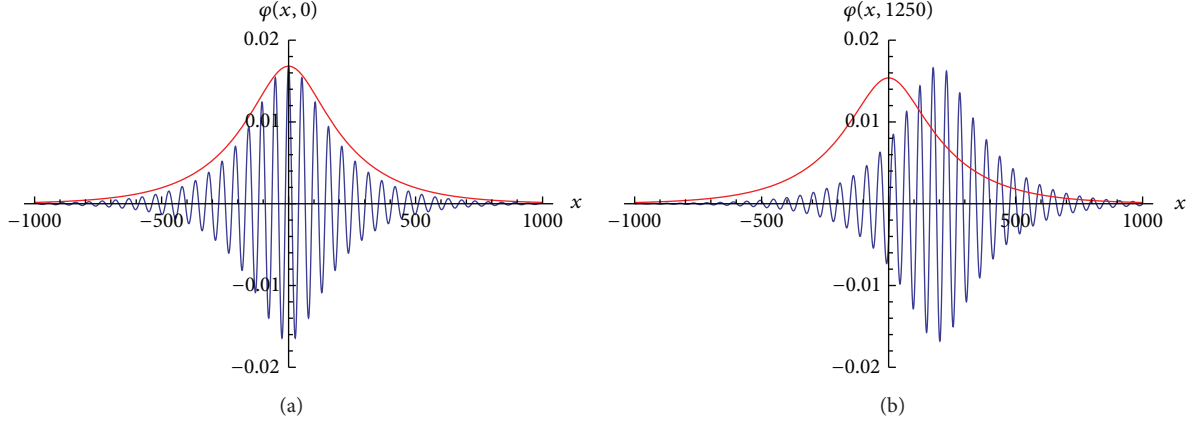


FIGURE 1: Profile of the usual oscillons in 1 + 1-dimensions with Lorentz and *CPT* breaking for $t = 0$ (left) and $t = 1250$ (right) with $\epsilon = 0.01$. The thin line corresponds to the case with $k_{00} = 0.12, k_{11} = 0.30, k_{01} = 0.27,$ and $k_{10} = 0.21$ and the thick line to the case with $k_{\mu\nu} = 0$.

The profile of the above solution is plotted in Figure 1 for some values of the $k^{\mu\nu}$ parameters. In Figure 1 we see the profile of the usual oscillon in the presence of the background of the Lorentz breaking symmetry. In this case, one can check that the dependence of the solution on the Lorentz breaking parameters is responsible for a kind of deformation of the configuration, where the field configuration becomes oscillatory in a localized region near its maximum value. Furthermore, in the course of the time, it is possible to observe that the Lorentz breaking symmetry produces a displacement of the oscillon along the spatial direction. In this case we will call these configurations as ‘‘enveloped oscillons’’, since in $t = 0$ the new configuration is enveloped by the oscillon with Lorentz symmetry.

Moreover, one can note that if one wants to recover the Lorentz symmetry, it is necessary to impose that $k^{00} = k^{11} = 0$ and $k^{01} = -k^{10}$ (or $k^{01} = k^{10} = 0$).

6. Flat-Top Oscillons with Lorentz Violation: OFT

Some years ago, a new class of oscillons, which is characterized by a kind of plateau at its top, was presented by Amin and Shirokoff [36]. In that work, the authors have shown that this configuration has an important impact on an expanding universe. Thus, in this section, we will describe the impacts of the Lorentz violation over the flat-top oscillons. We will study the case in 1 + 1-dimensional Minkowski space-time where the classical equation of motion is given by (21). Also, in order to analyze the flat-top oscillons in this scenario, we choose the potential that was used in [36], which is represented in (22).

Now, we begin a direct attack to the problem of finding the flat-top oscillons. Likewise to the procedure presented in [36], we introduce a re-scaled scalar field by $\phi(Z, Y) = \phi(y, \tau)/\sqrt{g}$, where $Z = \sqrt{g}y, \tau = \omega Y$ and $\omega = \sqrt{1 - \alpha^2/g}$. It is important to remark that the constant α^2 is responsible by the change in the frequency, its presence comes from the

nonlinear potential. Thus, it is not difficult to conclude that the classical equation of motion can be rewritten as

$$(\partial_\tau^2 \phi + \phi) + g^{-1} [-\alpha^2 \partial_\tau^2 \phi - \partial_y^2 \phi - \phi^3 + \phi^5] = 0. \quad (34)$$

So, we are in a position to investigate the so-called flat-top oscillons. But it is important to remark that the fundamental point is that all the effects of the Lorentz symmetry breaking are present implicitly in the classical field. Of course, it is possible to recover the original equation of motion presented by Amin and Shirokoff [36] through a suitable choice of $k^{\mu\nu}$.

Let us go further on our search for flat-top oscillons. For this, we expand ϕ as

$$\phi(y, \tau) = \phi_1(y, \tau) + g^{-1} \phi_3(y, \tau) + \dots \quad (35)$$

If we substitute the above expansion of the scalar field into the equation of motion (34), and collect the terms in order $\mathcal{O}(1)$ and $\mathcal{O}(g^{-1})$, we find

$$\frac{\partial^2 \phi_1}{\partial \tau^2} + \phi_1 = 0, \quad (36)$$

$$\frac{\partial^2 \phi_3}{\partial \tau^2} + \phi_3 - \alpha^2 \frac{\partial^2 \phi_1}{\partial \tau^2} - \frac{\partial^2 \phi_1}{\partial y^2} - \phi_1^3 + \phi_1^5 = 0. \quad (37)$$

Therefore, the solution of (36) is of the form

$$\phi_1(y, \tau) = \Psi(y) \cos(\tau). \quad (38)$$

In order to find the solution of $\Psi(y)$ let us substitute the solution obtained for $\phi_1(y, \tau)$ into (37). Thus, it is not hard to conclude that

$$\begin{aligned} \frac{\partial^2 \phi_3}{\partial \tau^2} + \phi_3 = & \left(\frac{d^2 \Psi}{dy^2} - \alpha^2 \Psi + \frac{3}{4} \Psi^3 - \frac{5}{8} \Psi^5 \right) \cos(\tau) \\ & + \left(\frac{3}{4} \Psi^3 - \frac{5}{16} \Psi^5 \right) \cos(3\tau) - \frac{\Psi^5}{16} \cos(5\tau), \end{aligned} \quad (39)$$

whose solution can be written as

$$\begin{aligned} \phi_3(y, \tau) = & \frac{1}{8} [4G(y) - 2H(y) + 8c_1] \cos(\tau) \\ & - H(y) \cos(3\tau) + 4[G(y)\tau + 2c_2] \sin(\tau), \end{aligned} \quad (40)$$

where we defined that $G(y) \equiv ((d^2\Psi/dy^2) - \alpha^2\Psi + (3/4)\Psi^3 - (5/8)\Psi^5)$ and $H(y) \equiv ((3/4)\Psi^3 - (5/16)\Psi^5)$. Furthermore, c_1 and c_2 are arbitrary integration constants.

Since that the solution of the function ϕ_3 has a term which is linear in the variable τ , resulting into a non-periodical solution, and we are interested in solutions which are periodical in time, we will impose that $G(y)$ vanishes. As a consequence, we get

$$\frac{d^2\Psi}{dy^2} = \left(\alpha^2\Psi - \frac{3}{4}\Psi^3 + \frac{5}{8}\Psi^5 \right). \quad (41)$$

At this point, one can verify that the above equation has the same profile of the equation presented in [36]. Therefore, this equation can be integrated to give

$$\frac{1}{2} \left(\frac{d\Psi}{dy} \right)^2 + U(\Psi) = E, \quad (42)$$

where $U(\Psi) = -(1/2)\alpha^2\Psi^2 + (3/16)\Psi^4 - (5/48)\Psi^6$. Note that, in the above equation, the arbitrary constant E should be set

to zero in order to get solitonic solution. This condition allows the field configuration to go asymptotically to the vacua of the field potential $U(\Psi)$. On the other hand, it is usual to impose that the profile of $\Psi(y)$ should be smooth at $y = 0$, and then it is necessary to make $d\Psi(0)/dy = 0$. As a consequence $E = U(\Psi_0) = 0$, which implies

$$\alpha^2 = \frac{3}{8}\Phi_0^2 - \frac{5}{24}\Phi_0^4, \quad (43)$$

with $\Psi_0 \equiv \Psi(0)$. Thus, solving the above equation in Ψ_0 , we have a critical value $a \leq \alpha_c = \sqrt{27/160}$. Above this critical value, Ψ_0 becomes imaginary.

Now, we must solve (42) with $E = 0$. In this case, we have

$$\frac{d\Psi}{\sqrt{\alpha^2\Psi^2 - (3/8)\Psi^4 + (5/24)\Psi^6}} = dy. \quad (44)$$

From this, it follows that

$$\Psi(y) = \frac{(u\sqrt[4]{4vu})}{\sqrt{2\sqrt{v} + \cosh\left[2y\sqrt{uv(\alpha_c^2 - \alpha^2)}\right]}}, \quad (45)$$

where $v = 27/[160(\alpha_c^2 - \alpha^2)]$ and $u = (v - 1)/v$.

As one can see, up to the order $\mathcal{O}(1)$, the corresponding solution for the field in the original variables is given by

$$\begin{aligned} \varphi_{FT}(x, t) = & \frac{u\sqrt[4]{4vu}}{\sqrt{2g\sqrt{v} + g \cosh\left\{2[x \cos(\theta) + t \sin(\theta)] \sqrt{uv(\alpha_c^2 - \alpha^2)}/\sqrt{gh_2}\right\}}} \\ & \times \cos\left\{\frac{\omega[-x \sin(\theta) + t \cos(\theta)]}{\sqrt{h_1}}\right\} + \mathcal{O}(g^{-3/2}). \end{aligned} \quad (46)$$

The profile of the above solution is plotted in Figure 2. In Figure 2, we see the profile of the flat-top oscillon in the presence of the background of the Lorentz breaking symmetry. In this case, one can check that the dependence of the solution on the Lorentz breaking parameters is responsible for a control of the size of the oscillon plateau. Thus, by measuring the width of the oscillon one could be able to verify the existence and the degree of the breaking of the symmetry. In Figure 3, we see the typical profile of the flat-top oscillon.

There one can note that the energy density becomes more localized close to the origin when the Lorentz breaking increases.

7. Breathers with Lorentz Violation: OFT

We will now construct the profile of a breather in a 1 + 1-dimensional Minkowski space-time. Again, we will use the

classical equation of motion (21). The breather solutions arise from the sine-Gordon model:

$$V(\varphi) = \frac{\gamma}{\beta} [1 - \cos(\beta\varphi)]. \quad (47)$$

The sine-Gordon model is invariant under $\varphi \rightarrow \varphi + 2n\pi$, where n is an integer number. In this case, the classical equation of motion is

$$\frac{\partial^2\varphi(Z, Y)}{\partial Y^2} - \frac{\partial^2\varphi(Z, Y)}{\partial Z^2} + \gamma \sin(\beta\varphi) = 0. \quad (48)$$

The above equation can be solved by the inverse-scattering method [81]. Thus, after straightforward calculations, we conclude that the breather solution is given by

$$\varphi_B(Z, Y) = \frac{4}{\beta} \arctan \left[\frac{\sqrt{\gamma - w^2} \sin(wY)}{w \cosh\left(Z\sqrt{\gamma - w^2}\right)} \right], \quad (49)$$

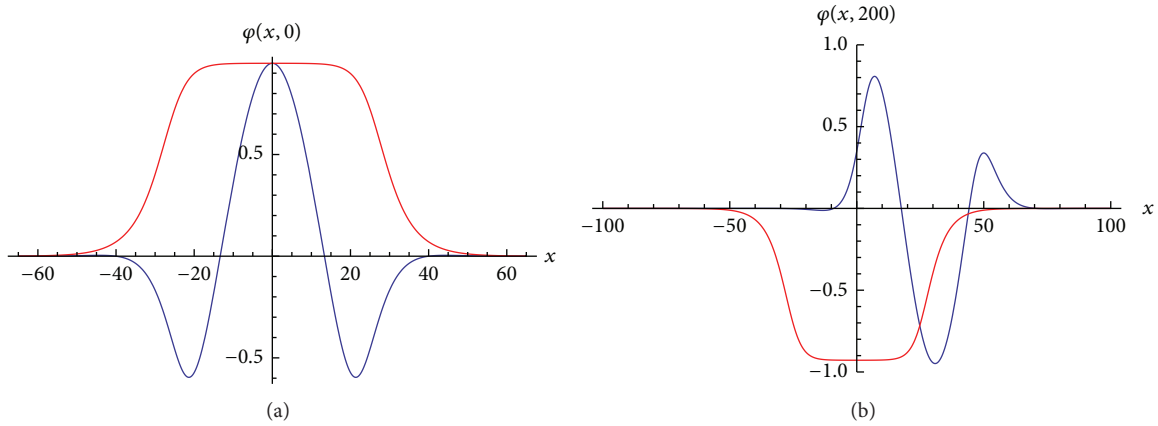


FIGURE 2: Profile of the flat-top oscillons in 1 + 1-dimensions with Lorentz symmetry breaking for $t = 0$ (left) and $t = 200$ (right) with $g = 5$. The thin line corresponds to the case with $k_{00} = 0.12$, $k_{11} = 0.30$, $k_{01} = 0.27$, and $k_{10} = 0.21$ and the thick line to the case with $k_{\mu\nu} = 0$.

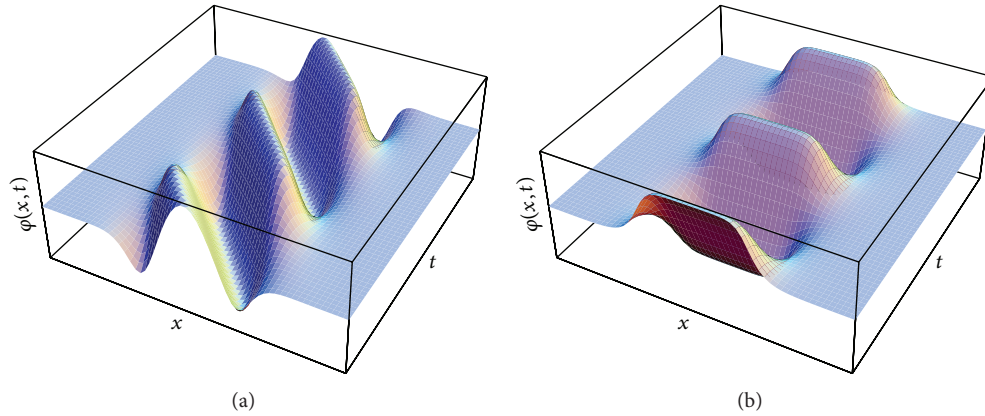


FIGURE 3: Typical profile of the flat-top oscillon. The left-hand figure corresponds to the case with Lorentz breaking symmetry and the right-hand figure to the one with Lorentz symmetry.

where w is the frequency of oscillation and describes different breathers. In Figures 4 and 5 we show the behavior of the above solution.

8. Radiation of Oscillons with Lorentz Violation Symmetry: OFT

An important characteristic of the oscillons is its radiation emission. In a seminal work by Segur and Kruskal [37], it was shown that oscillons in one spatial dimension decay emitting radiation. Recently, the computation of the emitted radiation in two and three spatial dimensions was done in [38]. On the other hand, in a recent paper by Hertzberg [39], it was found that the quantum radiation is very distinct of the classic one. It is important to remark that the author has shown that the amplitude of the classical radiation emitted can be found using the amplitude of the Fourier transform of the spatial structure of the oscillon.

Thus, in this section, we describe the outgoing radiation in scenarios with Lorentz violation symmetry. Here, we will establish a method in 1 + 1-dimensional Minkowski space-time that allows computing the classical radiation of oscillons in scenarios with Lorentz symmetry breaking. This is done by following the method presented in [39]. This method suggests that we can write the solution of the classical equation of motion in the following form:

$$\varphi_{\text{sol}}(x, t) = \varphi_{\text{osc}}(x, t) + \eta(x, t), \quad (50)$$

where $\varphi_{\text{osc}}(x, t)$ is the oscillon solution and $\eta(x, t)$ represents a small correction. Let us substitute this decomposition of the scalar field into the equation of motion (14). This leads to

$$\begin{aligned} \alpha_1 \frac{\partial^2 \varphi_{\text{osc}}}{\partial t^2} - \alpha_2 \frac{\partial^2 \varphi_{\text{osc}}}{\partial x^2} + \alpha_3 \frac{\partial^2 \varphi_{\text{osc}}}{\partial x \partial t} + \alpha_1 \frac{\partial^2 \eta}{\partial t^2} \\ - \alpha_2 \frac{\partial^2 \eta}{\partial x^2} + \alpha_3 \frac{\partial^2 \eta}{\partial x \partial t} + U(\varphi_{\text{osc}}, \eta) = 0, \end{aligned} \quad (51)$$

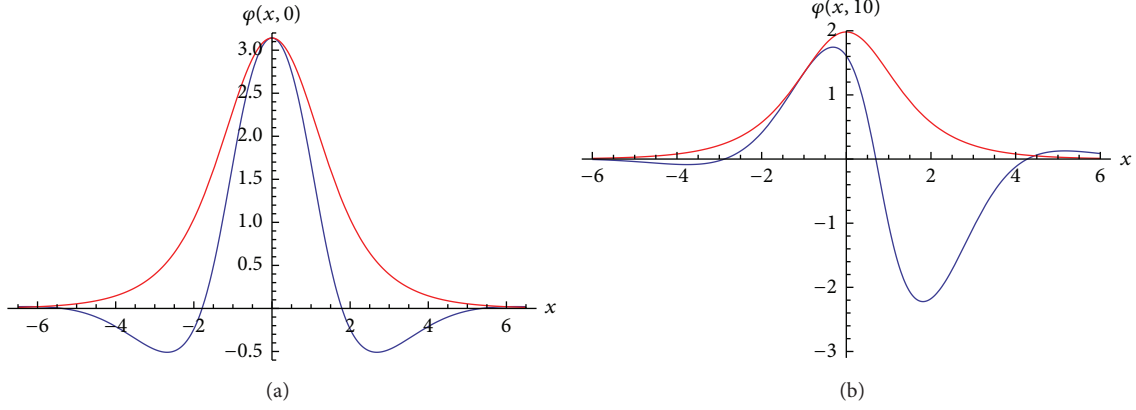


FIGURE 4: Profile of the breathers 1 + 1-dimensions with Lorentz symmetry breaking for $t = 0$ (left) and $t = 10$ (right) with $\nu = 2$, $w = 1$, and $\beta = 1$. The thin line corresponds to the case with $k_{00} = 0.28$, $k_{11} = 0.30$, $k_{01} = 0.27$, and $k_{10} = 0.37$ and the thick line to the case with $k_{\mu\nu} = 0$.

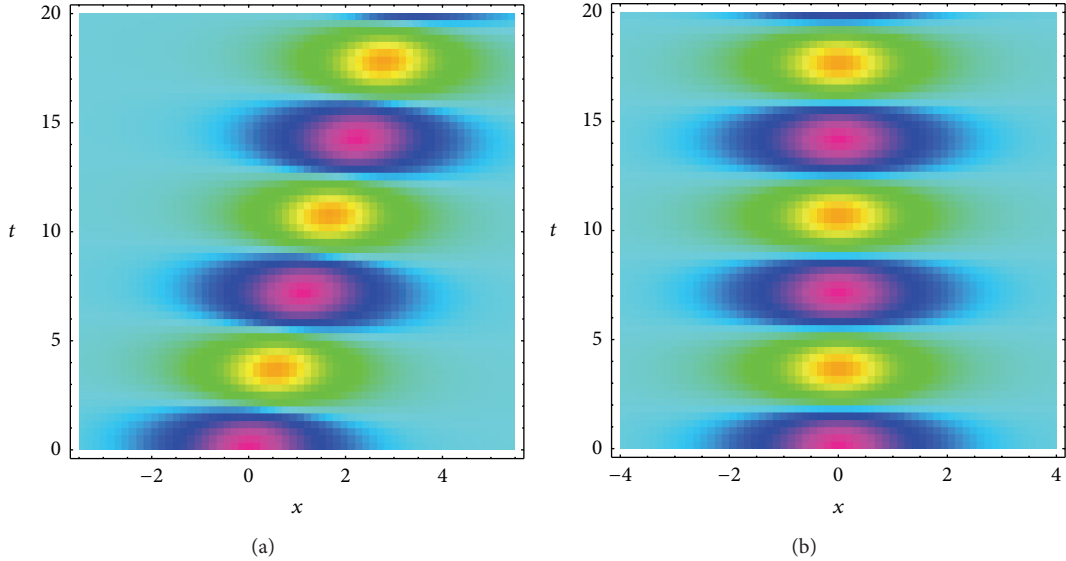


FIGURE 5: Density plot of a Breather. Solution with Lorentz symmetry breaking (left) and to the one Lorentz symmetry (right).

where $U(\varphi_{\text{osc}}, \eta)$ is a function which depends on the form of $V_{\varphi_{\text{sol}}}(\varphi_{\text{sol}})$. In order to decouple the above equation, we apply the rotation (18) and the dilations $T = \sqrt{h_1}\Upsilon$ and $X = \sqrt{h_2}Z$. Thus, we find

$$\begin{aligned} \frac{\partial^2 \varphi_{\text{osc}}(Z, \Upsilon)}{\partial \Upsilon^2} - \frac{\partial^2 \varphi_{\text{osc}}(Z, \Upsilon)}{\partial Z^2} + \frac{\partial^2 \eta(Z, \Upsilon)}{\partial \Upsilon^2} \\ - \frac{\partial^2 \eta(Z, \Upsilon)}{\partial Z^2} + U(\varphi_{\text{osc}}, \eta) = 0. \end{aligned} \quad (52)$$

From the above equation, it is possible to find the solution for $\eta(Z, \Upsilon)$ which carries the dependence on the parameters

that break the Lorentz symmetry. We want to investigate the model given by (22); then, we have

$$\begin{aligned} U(\varphi_{\text{osc}}, \eta) = \varphi_{\text{osc}} + \eta - \varphi_{\text{osc}}^3 - \eta^3 + 3\varphi_{\text{osc}}^2\eta + 3\varphi_{\text{osc}}\eta^2 \\ + g(\varphi_{\text{osc}}^5 + \eta^5 + 10\varphi_{\text{osc}}^2\eta^3 + 10\varphi_{\text{osc}}^3\eta^2 \\ + 5\varphi_{\text{osc}}\eta^4 + 5\varphi_{\text{osc}}^4\eta). \end{aligned} \quad (53)$$

As η represents a small correction, we assume that the nonlinear terms η^2 , η^3 , η^4 , and η^5 and the parametric driving terms $3\eta\varphi_{\text{osc}}^2$ and $5g\eta\varphi_{\text{osc}}^4$ can be neglected. At this point, it is important to remark that the parametric driven terms were not considered because we are working in an asymptotic

regime where φ_{osc} is also small. In this case, (52) takes the form

$$\frac{\partial^2 \eta(Z, Y)}{\partial Y^2} - \frac{\partial^2 \eta(Z, Y)}{\partial Z^2} + \eta(Z, Y) = -J(Z, Y), \quad (54)$$

where

$$J(Z, Y) = \frac{\partial^2 \varphi_{\text{osc}}(Z, Y)}{\partial Y^2} - \frac{\partial^2 \varphi_{\text{osc}}(Z, Y)}{\partial Z^2} + \varphi_{\text{osc}}(Z, Y) - \varphi_{\text{osc}}^3(Z, Y) + g\varphi_{\text{osc}}^5(Z, Y). \quad (55)$$

We can use the Fourier transform for solving differential equation (54) where $J(Z, Y)$ acts as a source. With this in mind, we write down the Fourier integral transforms

$$\eta(R, w) = \frac{1}{\sqrt{2\pi}} \int dZ dY \eta(Z, Y) \times \exp[-i(RZ - wY)], \quad (56)$$

$$J(R, w) = \frac{1}{\sqrt{2\pi}} \int dZ dY J(Z, Y) \times \exp[-i(RZ - wY)].$$

Then, we have the corresponding solution

$$\eta(Z, Y) = \frac{1}{\sqrt{2\pi}} \int dR dw \eta(R, w) \times \exp[i(RZ - wY)], \quad (57)$$

where

$$\eta(R, w) = -\frac{J(R, w)}{R^2 - (w^2 + 1)}. \quad (58)$$

From the above approach, it is possible to find the radiation field for the oscillons. As a consequence of the method, the oscillons expansion must be truncated.

8.1. SME Usual Oscillons Radiation: OFT. In this subsection, we will study the outgoing radiation of the usual oscillons in a Lorentz violation scenario. In this case, the oscillon expansion truncated in order N is given by

$$\varphi(y, \tau) = \epsilon \varphi_1(y, \tau) + \epsilon^3 \varphi_3(y, \tau) + \epsilon^5 \varphi_5(y, \tau) + \dots + \epsilon^N \varphi_N(y, \tau). \quad (59)$$

As an example, we will consider $N = 1$. This is the case where the field configuration corresponds to the oscillon

$$\varphi_{\text{osc}}(y, \tau) = \epsilon \varphi_1(y, \tau). \quad (60)$$

Substituting (60) in (55), we obtain

$$J(Z, Y) = \left(\sqrt[4]{\frac{8}{3}} \right) \epsilon^3 [\text{sech}(\epsilon Z)]^{3/2} \cos(3\omega Y). \quad (61)$$

Thus, for $N = 1$ we can solve easily the integral (57) which allows finding $\eta(Z, Y)$. Therefore, we can generalize the result to N substituting the expansion (59) in (55) and using the differential equation (24). After the calculations, the result is

$$J(Z, Y) = C_N \epsilon^{N+2} [\text{sech}(\epsilon Z)]^{N+1/2} \cos(\bar{n}\omega Y) + \dots, \quad (62)$$

where C_N are constant coefficients. For instance, for $N = 1$ we have $C_1 = \sqrt[4]{8/3}$. Next we calculate $\eta(Z, Y)$ as given by (57). After straightforward computations, one can conclude that

$$\eta(Z, Y) = \frac{\pi \sqrt{\pi} C_N \epsilon^{N+2}}{k_{\text{rad}}} \cos(\omega_{\text{rad}} Y) \times \sin(k_{\text{rad}} Z) \int dZ [\text{sech}(\epsilon Z)]^{N+1/2} \cos(k_{\text{rad}} Z), \quad (63)$$

where

$$\omega_{\text{rad}} = \bar{n}\omega, \quad k_{\text{rad}} = \sqrt{\omega_{\text{rad}}^2 - 1}. \quad (64)$$

On expression (63), we note that there is an outgoing radiation which has an amplitude described by the integral

$$A(k_{\text{rad}}) = \frac{\pi \sqrt{\pi} C_N \epsilon^{N+2}}{k_{\text{rad}}} \times \int dZ [\text{sech}(\epsilon Z)]^{N+1/2} \cos(k_{\text{rad}} Z), \quad (65)$$

and we also note that the radiation has frequency ω_{rad} and wave number k_{rad} . We can make use of the above generalization to calculate the amplitude of radiation of the usual oscillons in Lorentz violation scenario. For instance, for $N = 1$, we have

$$A(k_{\text{rad}}) = \frac{4\pi \sqrt{2\pi} C_1 \epsilon^3}{k_{\text{rad}}} \times [b_1 F(a_1, a_2, a_3, -1) + b_1^* F(a_1, a_2^*, a_3^*, -1)], \quad (66)$$

where $F(a_1, a_2, a_3, -1)$ and $F(a_1, a_2^*, a_3^*, -1)$ are hypergeometric functions with

$$b_1 = \frac{1}{3\epsilon - 2ik_{\text{rad}}}, \quad a_1 = \frac{3}{2}, \quad (67)$$

$$a_2 = \frac{3}{4} - \frac{ik_{\text{rad}}}{2\epsilon}, \quad a_3 = \frac{7}{4} - \frac{ik_{\text{rad}}}{2\epsilon}.$$

In Figure 6 we see how the amplitude of the outgoing radiation changes with the parameters of $k^{\mu\nu}$. From Figure 1, it can be seen that the amplitude of the outgoing radiation of the oscillons is controlled by the terms of the Lorentz breaking of the model, in such a way that the radiation amplitude will decay faster when the Lorentz breaking increases.

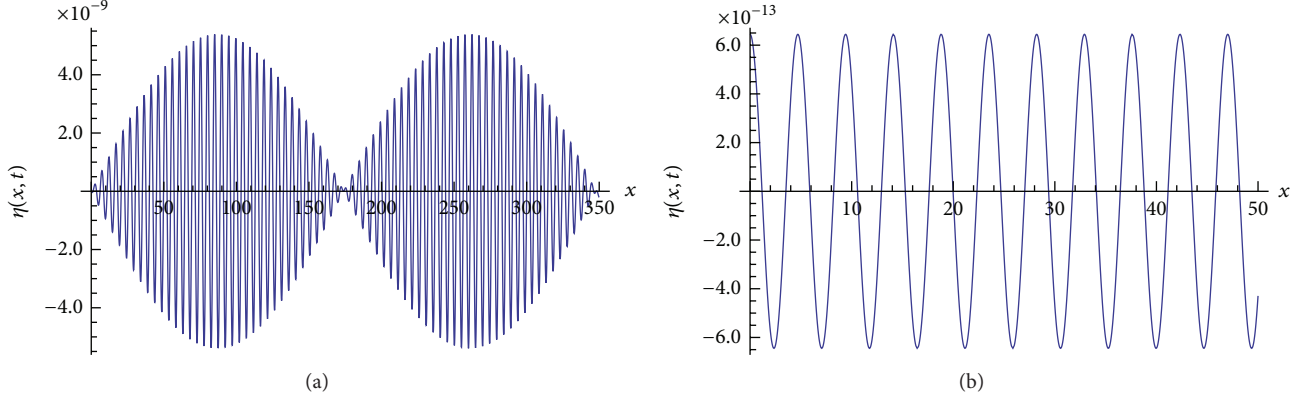


FIGURE 6: Amplitude of the outgoing radiation determined by the Fourier transform. The left-hand figure corresponds to the case with Lorentz breaking symmetry and the right-hand to the case with Lorentz symmetry.

8.2. *SME Flat-Top Oscillons Radiation: OFT.* We will now present the outgoing radiation by the flat-top oscillons in Lorentz violation scenario. Here, the associated oscillon expansion truncated in N is defined as

$$\begin{aligned} \varphi(y, \tau) = & \varphi_1(y, \tau) + \frac{1}{g} \varphi_3(y, \tau) \\ & + \frac{1}{g^2} \varphi_5(y, \tau) + \dots + \frac{1}{g^{N-1}} \varphi_{2N-1}(y, \tau). \end{aligned} \quad (68)$$

Substituting the above expansion in (55), we have that

$$\begin{aligned} J(Z, \Upsilon) &= \bar{C}_N \\ &\times \bar{C}_N \left\{ \frac{(u\sqrt[4]{4vu})}{\sqrt{2g\sqrt{v} + g \cosh[2Z\sqrt{uv(\alpha_c^2 - \alpha^2)}/\sqrt{g}]}} \right\}^{N+2} \\ &\times \cos(\bar{n}\omega\Upsilon) + \dots, \end{aligned} \quad (69)$$

where \bar{C}_N are constant coefficients. Now we calculate $\eta(Z, \Upsilon)$ as given by (57). After straightforward computations, one can conclude that

$$\begin{aligned} \eta(Z, \Upsilon) &= \frac{\pi\sqrt{\pi}\bar{C}_N}{\bar{k}_{\text{rad}}} \cos(\bar{\omega}_{\text{rad}}\Upsilon) \sin(\bar{k}_{\text{rad}}Z) \\ &\times \int d\bar{Z} \left\{ \frac{(u\sqrt[4]{4v\bar{u}})}{\sqrt{2g\sqrt{\bar{v}} + g \cosh[2\bar{Z}\sqrt{u\bar{v}g(\alpha_c^2 - \alpha^2)}]}} \right\}^{N+2} \\ &\times \cos(\bar{k}_{\text{rad}}\bar{Z}), \end{aligned} \quad (70)$$

where

$$\bar{\omega}_{\text{rad}} = \bar{n}\bar{\omega}, \quad \bar{k}_{\text{rad}} = \sqrt{\bar{\omega}_{\text{rad}}^2 - 1}. \quad (71)$$

From the above expression, we see that there is an outgoing radiation which has its amplitude described by the integral

$$\begin{aligned} A(k_{\text{rad}}) &= \frac{\pi\sqrt{\pi}\bar{C}_N}{\bar{k}_{\text{rad}}} \int dZ \cos(\bar{k}_{\text{rad}}Z) \\ &\times \left\{ \frac{(u\sqrt[4]{4vu})}{\sqrt{2\sqrt{v} + \cosh[2Z\sqrt{uv(\alpha_c^2 - \alpha^2)}/\sqrt{g}]}} \right\}^{N+2}. \end{aligned} \quad (72)$$

We can make use of the above generalization to calculate the amplitude of radiation of the flat-top oscillons in Lorentz violation scenario. For instance, for $N = 1$, we have

$$A(k_{\text{rad}}) = \frac{4\pi\bar{C}_N}{A_0\bar{k}_{\text{rad}}} \left(\frac{u\sqrt[4]{4vu}}{\sqrt{g}} \right)^3 (\xi_1 \mathcal{F}_a + \xi_1^* \mathcal{F}_b), \quad (73)$$

where $\mathcal{F}_a = \mathcal{F}(\Omega_1; \Omega_2; \Omega_2; \Omega_3, \Omega_4, \Omega_5)$ and $\mathcal{F}_b = \mathcal{F}(\Omega_1^*; \Omega_2; \Omega_2; \Omega_3^*, \Omega_4, \Omega_5)$ are the Appell hypergeometric functions of two variables and

$$\begin{aligned} A_0 &= 2\sqrt{\frac{uv(\alpha_c^2 - \alpha^2)}{\sqrt{g}}}, \quad \xi_1 = 3 + \frac{2ik_{\text{rad}}}{A_0}, \\ \Omega_1 &= \frac{3}{2} - \frac{ik_{\text{rad}}}{A_0}, \quad \Omega_2 = \frac{3}{2}, \quad \Omega_3 = \frac{5}{2} - \frac{ik_{\text{rad}}}{A_0}, \\ \Omega_4 &= \sqrt{A_0^2 - 1} - A_0, \quad \Omega_5 = \frac{1}{\sqrt{A_0^2 - 1} - A_0}. \end{aligned} \quad (74)$$

In this case, we see that the amplitude of the outgoing radiation changes with the parameters $k^{\mu\nu}$. We can see that the amplitude of the outgoing radiation of the oscillons is

controlled by the terms of the Lorentz breaking of the model, in such way that the radiation amplitude will decay faster when the Lorentz breaking increases.

9. Oscillons with LV: Two-Field Theory (TFT)

We have seen in Section 2 that the most important scenario with LV is that described by a theory with two scalar fields, because it is possible to find observable effects of the LV. Then, in this section, we study a two-scalar field theory in the presence of a LV scenario. The theory that we will study is similar to that given by Potting [79]. Here, we will work with the corresponding Lagrangian density

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \partial_\mu \varphi_1 \partial^\mu \varphi_1 + \frac{1}{2} \partial_\mu \varphi_2 \partial^\mu \varphi_2 \\ & + \frac{1}{2} k^{\mu\nu} \partial_\mu \varphi_1 \partial_\nu \varphi_2 - V(\varphi_1, \varphi_2), \end{aligned} \quad (75)$$

where $V(\varphi_1, \varphi_2)$ is the interaction potential. For example, in order to find oscillons solutions, we can choose the potential in the form

$$\begin{aligned} V(\varphi_1, \varphi_2) = & \frac{g}{3} (\varphi_1^6 + \varphi_2^6) - \frac{1}{2} (\varphi_1^4 + \varphi_2^4) \\ & + \varphi_1^2 + \varphi_2^2 + 5g (\varphi_1^4 \varphi_2^2 + \varphi_1^2 \varphi_2^4) - 3\varphi_1^2 \varphi_2^2. \end{aligned} \quad (76)$$

In order to decouple the Lagrangian density (75), we apply the rotation

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}. \quad (77)$$

After straightforward computations, one can conclude that

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \partial_\mu \sigma_1 \partial^\mu \sigma_1 + \frac{1}{2} k_1^{\mu\nu} \partial_\mu \sigma_1 \partial_\nu \sigma_1 \\ & + \frac{1}{2} \partial_\mu \sigma_2 \partial^\mu \sigma_2 + \frac{1}{2} k_2^{\mu\nu} \partial_\mu \sigma_2 \partial_\nu \sigma_2 - V(\sigma_1, \sigma_2), \end{aligned} \quad (78)$$

where

$$k_1^{\mu\nu} = \frac{1}{4} k^{\mu\nu}, \quad k_2^{\mu\nu} = -\frac{1}{4} k^{\mu\nu}, \quad (79)$$

and the potential is

$$V(\sigma_1, \sigma_2) = V(\sigma_1) + V(\sigma_2), \quad (80)$$

with

$$V(\sigma_i) = \frac{g}{6} \sigma_i^6 - \frac{1}{4} \sigma_i^4 + \frac{1}{2} \sigma_i^2, \quad i = 1, 2. \quad (81)$$

It is important to note that applying the rotations in the fields, the Lagrangian density was decoupled into two independent Lagrangians $\mathcal{L} = \sum_{i=1}^2 \mathcal{L}_i$, where

$$\mathcal{L}_i = \frac{1}{2} \partial_\mu \sigma_i \partial^\mu \sigma_i + \frac{1}{2} k_i^{\mu\nu} \partial_\mu \sigma_i \partial_\nu \sigma_i - V(\sigma_i). \quad (82)$$

We can see that all the preceding approaches and results can be used here to find the fields σ_1 and σ_2 . Another important point that it is convenient to remark at this point comes from the fact that any variable x^μ redefinition will carry information of the parameter $k^{\mu\nu}$ which is responsible by LV.

As we are working in 1 + 1-dimensions, the Lagrangian (82) becomes

$$\begin{aligned} \mathcal{L}_i^{(1+1)} = & \frac{1}{2} a_i (\partial_t \sigma_i)^2 - \frac{1}{2} b_i (\partial_x \sigma_i)^2 \\ & + \frac{1}{2} d_i \partial_t \sigma_i \partial_x \sigma_i - V(\sigma_i), \quad i = 1, 2. \end{aligned} \quad (83)$$

In this case, we have

$$a_i \equiv (1 + k_i^{00}), \quad b_i \equiv (1 - k_i^{11}), \quad d_i \equiv (k_i^{01} + k_i^{10}). \quad (84)$$

Now it is quite clear why the Lagrangian density (75) is more important and general than the one described by (1) first because the commutation relations of the Poincaré group is not closed, indicating a Lorentz violation and second because it is impossible to perform coordinate changes to eliminate the LV parameters in (78), because if we apply a coordinate change in order to write the Lagrangian in an covariant form, only one of the sectors will stay invariant.

Now, by using the approaches described in Section 4, we find the following equations:

$$\frac{\partial^2 \sigma_i(Z_i, \Upsilon_i)}{\partial \Upsilon_i^2} - \frac{\partial^2 \sigma_i(Z_i, \Upsilon_i)}{\partial Z_i^2} + V_{\sigma_i} = 0, \quad (85)$$

where

$$\begin{aligned} Z_i = & \frac{x \cos(\theta_i) + t \sin(\theta_i)}{\sqrt{L_i}}, \\ \Upsilon_i = & \frac{-x \sin(\theta_i) + t \cos(\theta_i)}{\sqrt{H_i}}. \end{aligned} \quad (86)$$

With the set

$$\begin{aligned} \theta_i = & -\frac{1}{2} \arctan\left(\frac{d_i}{a_i + b_i}\right), \\ L_i = & \frac{b_i^2 - a_i^2 + [d_i^2 + (a_i + b_i)^2] \cos(2\theta_i)}{2(a_i + b_i)}, \\ H_i = & \frac{a_i^2 - b_i^2 + [d_i^2 + (a_i + b_i)^2] \cos(2\theta_i)}{2(a_i + b_i)}. \end{aligned} \quad (87)$$

Fortunately, we can find periodical solutions for the fields σ_1 and σ_2 from (85). In this case, we are looking for oscillon-like solutions. These solutions were presented in Sections 5 and 6. Thus, from those sections we can show that

$$\begin{aligned}
& \sigma_i^{(\text{USUAL})}(x, t) \\
&= \epsilon_i \sqrt[4]{\frac{8}{3}} \left(\sqrt{\operatorname{sech} \left\{ \frac{\epsilon_i [x \cos(\theta_i) + t \sin(\theta_i)]}{\sqrt{L_i}} \right\}} \right) \\
&\quad \times \cos \left\{ \frac{\omega_i [-x \sin(\theta_i) + t \cos(\theta_i)]}{\sqrt{H_i}} \right\} + \mathcal{O}(\epsilon_i^3), \\
& \sigma_i^{(\text{FLAT-TOP})}(x, t) \\
&= \frac{u_i \sqrt[4]{4v_i u_i}}{\sqrt{2g\sqrt{v_i} + g \cosh \left\{ 2 [x \cos(\theta_i) + t \sin(\theta_i)] \sqrt{u_i v_i (\alpha_c^2 - \alpha_i^2)} / \sqrt{gL_i} \right\}}} \\
&\quad \times \cos \left\{ \frac{\omega_i [-x \sin(\theta_i) + t \cos(\theta_i)]}{\sqrt{H_i}} \right\} + \mathcal{O}(g^{-3/2}).
\end{aligned} \tag{88}$$

In the above solutions, $\sigma_i^{(\text{USUAL})}$ represents the usual oscillons and $\sigma_i^{(\text{FLAT-TOP})}$ are the flat-top ones. Furthermore, we have

$$\begin{aligned}
\omega_i &= \sqrt{1 - \epsilon_i^2}, & \bar{\omega}_i &= \sqrt{1 - \frac{\alpha_i^2}{g}}, \\
v_i &= \frac{27}{[160(\alpha_c^2 - \alpha_i^2)]}, & u_i &= \frac{(v_i - 1)}{v_i}.
\end{aligned} \tag{89}$$

As above asserted, the original scalar fields φ_1 and φ_2 are obtained from the fields σ_1 and σ_2 in the following form:

$$\varphi_1 = \frac{\sigma_1 + \sigma_2}{2}, \quad \varphi_2 = \frac{\sigma_1 - \sigma_2}{2}. \tag{90}$$

It is important to remark that the resulting solutions do not present merely algebraic relation between σ_i and the original parameters of the theory but essentially lead to physical consequences. As one can see, there are two kinds of frequencies which can be combined for each scalar field φ_i . This means that their solutions can be considered as a superposition of two independent fields and, as a consequence, we can have interference phenomena in the structure of the oscillon.

An important question concerns the stability of the solutions; given that each field φ_i is a combination of the fields σ_i , the stability and longevity of the oscillons are guaranteed. From a mathematical point of view, one can think that the original fields consist of linear combinations of σ_i . The same occurs when we calculated the outgoing radiation, and in that case we have two radiation fields η_1 and η_2 , which are independent solutions with small resulting amplitudes. As a consequence, their linear combinations, $\bar{\eta}_1 = \eta_1 + \eta_2$ and $\bar{\eta}_2 = \eta_1 - \eta_2$, will give the radiation field of solutions φ_i . Therefore, as η_i are very small solutions, we still have the stability and longevity of the solutions guaranteed.

10. Conclusions

In this work we have investigated the so-called flat-top oscillons in the case of Lorentz breaking scenarios. We have shown that the Lorentz violation symmetry is responsible for the appearance of a kind of deformation of the configuration. On the other hand, from inspection of the results coming from the flat-top oscillons in 1 + 1-dimensions with Lorentz breaking in comparison with the flat-top given in [36], one can see that the oscillons are carrying information about the terms of the Lorentz breaking of the model; in this case by taking $k^{00} = k^{11} = 0$ and $k^{01} = -k^{10}$ (or $k^{01} = k^{10} = 0$), one recovers the solution presented in [36]. Furthermore, this can lead one to obtain the degree of symmetry breaking by measuring the width of the oscillon in 1 + 1-dimensions. One important question about the nonlinear solution is related to its stability. Thus, we studied the solutions found here by using the procedure introduced by Hertzberg [36, 39]. We concluded that the radiation emitted by these oscillons is controlled by the terms of the Lorentz breaking of the model, in such a way that the radiation will decay more quickly as the terms become larger. Finally, all the results obtained for the case of one scalar field model are promptly extended for the case of doublets of nonlinearly coupled scalar fields.

Moreover, it is important to highlight that the bounds in Lorentz violation theories in the standard model are very small and are compatible with the stability observed for the oscillons here introduced. On the other hand, observable effects of these oscillons in the real world are possible, for instance, in a cosmological context. In that case, the life time of these oscillons can be decisive in the generation of coherent structures after cosmic inflation [82, 83], where it was shown that oscillons can contribute up to 20% of the energy density of the Universe. Thus, in this scenario, one should find bounds on the Lorentz violation which will open a new window to detect observable effects of breaking Lorentz symmetry. This possibility is encouraged by the fact that the break of the Lorentz symmetry induces a kind of beat phenomenon

in the structure of the outgoing radiation, in contrast with the Lorentz invariant case (see Figure 6). In this way, in a real world, one can detect the difference in the frequency of the outgoing radiation, effect that would indicate the presence of a violation of the Lorentz symmetry. Therefore, in order to deal with these questions, we are presently working in a future work where oscillons in cosmological backgrounds with Lorentz symmetry breaking are presented.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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