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Research Article

H_∞ Control of Singular Markovian Jump Systems with Bounded Transition Probabilities

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This paper discusses H_∞ control problems of continuous-time and discrete-time singular Markovian jump systems (SMJSs) with bounded transition probabilities. Improved sufficient conditions for continuous-time SMJSs to be regular, impulse free, and stochastically stable with γ-disturbance attenuation are established via less conservative inequality to estimate the transition jump rates, so are the discrete-time SMJSs. With the obtained conditions, the design of a state feedback controller which ensures the resulting closed-loop system to be stochastically admissible and with H_∞ performance is given in terms of linear matrix inequalities (LMIs). Finally, illustrative examples are presented to show the effectiveness and the benefits of the proposed approaches.

1. Introduction

Singular systems, also referred to as generalized systems, descriptor systems, implicit systems, differential-algebraic systems, or semistate systems [1], have been extensively studied in the past years due to the fact that singular systems better describe physical systems than regular ones. Many practical problems, such as those in Leontief's dynamic system [2] and electrical and mechanical systems [3, 4], are modeled as singular systems. Considerable attention has been paid to investigate such continuous-time singular systems [5–8] and discrete-time singular systems [9–11], respectively. On the other hand, Markovian jump systems (MJSs) can be regarded as a special class of hybrid systems with finite operation modes whose structures are subject to random abrupt changes. The studies of MJSs are important in practical applications such as manufacturing systems, aircraft control, target tracking, robotics, solar receiver control, and power systems. Therefore, a great deal of attention has been devoted to the study of this class of systems in recent years such as [12–19].

When singular systems experience abrupt changes in their structure and parameters, it is natural to model them as SMJSs. Reference [20] considered the stochastic stability and robust stochastic stability conditions for continuous-time SMJSs. The delay-dependent H_∞ control problem of continuous-time SMJSs was studied in [21] via Jensen's integral inequality approach after introducing free weighting matrices, while [22] gave sufficient conditions for uncertain discrete-time SMJSs with mode-dependent time delays through transforming them into a standard linear system. Particularly, [23] proposed more general condition of H_∞ control for discrete-time SMJSs, which does not require any decomposition of the original system, and the construction of H_∞ controller gain does not need system decomposition. However, all the above-mentioned criteria on MJSs require the values of transition probabilities to be exactly know. Recently, Boukas discussed H_∞ control of discrete-time normal state-space MJSs with bounded transition probabilities in [24] and stabilization of continuous-time SMJSs with full or partial knowledge of transition rates in [25].

Following the work of [24, 25], in this paper, the H_∞ control problem is considered for both continuous-time and discrete-time SMJSs with bounded transition probabilities. Sufficient criteria guaranteeing SMJSs being stochastically admissible with an H_∞ performance are presented in terms of LMIs, which are obtained by using an improved inequality to obtain less conservative estimation on transition probabilities. Based on these, a state feedback controller such that

the resulting closed-loop system is stochastically admissible and possesses a prescribed H_∞ performance is given. Numerical examples are provided to demonstrate the validity of developed methods.

Notation. $\mathbb{R}^n$ denotes the n dimensional Euclidean space. $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices. $|\cdot|$ denotes the Euclidean norm. $\varepsilon\{\cdot\}$ is the expectation operator with respect to some probability measure. In symmetric block matrices, we use "$*$" as an ellipsis for the terms induced by symmetry, diag$\{\cdots\}$ for a block-diagonal matrix, and $(M)^\star \triangleq M + M^T$.

2. Preliminaries

2.1. Continuous-Time Markovian Jump Linear Systems. Consider the following continuous-time SMJS described by

$$\begin{aligned} E\dot{x}(t) &= A(\eta(t))x(t) + B(\eta(t))u(t) + F(\eta(t))w(t) \\ y(t) &= C(\eta(t))x(t) + D(\eta(t))w(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^p$ is the disturbance input, and $y(t) \in \mathbb{R}^q$ is the control output. Matrix E may be singular assumed rank$(E) = r \le n$. Parameter $\eta(t)$ is the continuous-time Markov processes with right continuous trajectories taking values in a finite set $\mathbb{S} = \{1, 2, \ldots, N\}$ with transition probabilities:

$$\Pr(\eta(t+\Delta t) = j \mid \eta(t) = i) = \begin{cases} \lambda_{ij}\Delta t + o(\Delta t) & i \ne j \\ 1 + \lambda_{ii}\Delta t + o(\Delta t) & i = j, \end{cases} \tag{2}$$

where $\Delta t > 0$, $\lim_{\Delta t\to 0}(o(\Delta t)/\Delta t) = 0$, and the transition probability rate satisfies $\lambda_{ij} \ge 0$, for $i, j \in \mathbb{S}, i \ne j$, and

$$\lambda_{ii} = -\sum_{j=1, j\ne i}^{N} \lambda_{ij}. \tag{3}$$

For notational simplicity, in the sequel, for each possible $\eta(t) = i$, $i \in \mathbb{S}$, the matrix $A_{\eta(t)}$ will be denoted by A_i, and so on.

Assumption 1. The transition probabilities for system (1) are assumed to be unknown but vary between two known bounds satisfying the following:

$$0 \le \underline{\lambda}_{ij} \le \lambda_{ij} \le \overline{\lambda}_{ij} \quad \forall i, j \in \mathbb{S}, \tag{4}$$

where $i \ne j$.

From Assumption 1, when we only know that $\underline{\lambda}_i = \min_{j\ne i\in\mathbb{S}}\underline{\lambda}_{ij}$ and $\overline{\lambda}_i = \min_{j\ne i\in\mathbb{S}}\overline{\lambda}_{ij}$, we have $\underline{\lambda}_i \le \lambda_{ij} \le \overline{\lambda}_i$, which is the same as in [25]. Thus, we may say that Assumption 1 is more natural.

Definition 2 (see [26]). (1) The nominal system (1) is said to be regular if $\det(sE - A_i)$ is not identically zero for every $i \in \mathbb{S}$.

(2) The nominal system (1) is said to be impulse free if $\deg(\det(sE - A_i)) = \text{rank}(E)$ for every $i \in \mathbb{S}$.

(3) The nominal system in (1) is said to be stochastically stable if, when $u(t) = 0$ and $w(t) = 0$, there exists a constant $M(x_0, \eta_0)$ such that

$$\mathscr{E}\left\{\int_0^\infty \|x(t)\|^2 dt \mid x_0, \eta_0\right\} \le M(x_0, \eta_0). \tag{5}$$

(4) The nominal system in (1) is said to be stochastically admissible if it is regular, impulse free, and stochastically stable.

Definition 3. Given $\gamma > 0$, the nominal system in (1) is said to be stochastically admissible with γ-disturbance attenuation if it is stochastically admissible and satisfying the following such that

$$\mathscr{E}\left\{\int_0^\infty \|y(t)\|^2 dt\right\} < \gamma^2 \mathbb{E}\left\{\int_0^\infty \|w(t)\|^2 dt\right\} \tag{6}$$

holds for zero-initial condition and any nonzero $w(t) \in \mathscr{L}_2[0, \infty)$.

In this paper, the H_∞ controller such that the resulting closed-loop system is stochastically admissible with (6) is

$$u(t) = K_i x(t), \tag{7}$$

where K_i is to be determined.

Lemma 4. *Given a scalar $\gamma > 0$, the unforced system* (1) *is stochastically admissible with H_∞ performance if there exist matrices P_i such that*

$$E^T P_i = P_i^T E \ge 0,$$

$$\begin{bmatrix} A_i^T P_i + P_i^T A_i + \sum_{j=1}^{N} \lambda_{ij} E^T P_j & P_i^T F_i & C_i^T \\ * & -\gamma^2 I & D_i^T \\ * & * & -I \end{bmatrix} < 0. \tag{8}$$

2.2. Discrete-Time Markovian Jump Linear Systems. Consider the following discrete-time SMJS described by

$$\begin{aligned} Ex(k+1) &= A(\theta(k))x(t) + B(\theta(k))u(k) + F(\theta(k))w(k) \\ y(k) &= C(\theta(k))x(k) + D(\theta(k))w(k), \end{aligned} \tag{9}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the control input, $w(k) \in \mathbb{R}^p$ is the disturbance input, and $y(k) \in \mathbb{R}^q$ is the control output. Matrix E may be singular assumed rank$(E) = r \le n$. Parameter $\theta(k)$ is a discrete-time, discrete-state Markovian chain taking values in a finite set $\mathbb{S}$ with transition probabilities:

$$\Pr(\theta(k+1) = j \mid \theta(k) = i) = \pi_{ij}, \tag{10}$$

where $\pi_{ij} \ge 0$, for $i, j \in \mathbb{S}$, and

$$\sum_{j=1}^{N} \pi_{ij} = 1. \tag{11}$$

Assumption 5. The transition probabilities for (10) are assumed to be unknown but vary between two known bounds satisfying the following:

$$0 \le \underline{\pi}_{ij} \le \pi_{ij} \le \overline{\pi}_{ij} \quad \forall i, j \in \mathbb{S}. \tag{12}$$

From [24], we know that $\sum_{j=1}^{N} \underline{\pi}_{ij}$ and $\sum_{j=1}^{N} \overline{\pi}_{ij}$ may not equal 1. It is not necessary to exactly know the transition probabilities but only the bounds of π_{ij}, which are $\underline{\pi}_{ij}$ and $\overline{\pi}_{ij}$.

Definition 6 (see [26]). (1) The nominal system in (9) is said to be regular if $\det(sE - A_i)$ is not identically zero for every $i \in \mathbb{S}$.

(2) The nominal system in (9) is said to be causal if $\deg(\det(sE - A_i)) = \operatorname{rank}(E)$ for every $i \in \mathbb{S}$.

(3) The nominal system in (9) is said to be stochastically stable if, when $u(k) = 0$ and $w(k) = 0$, there exists a constant $M(x_0, \theta_0)$ such that

$$\mathscr{E}\left\{\sum_{k=0}^{N} \|x(k)\|^2 \mid x_0, \theta_0\right\} \le M(x_0, \theta_0). \tag{13}$$

(4) The nominal system in (9) is said to be stochastically admissible if it is regular, causal, and stochastically stable.

Definition 7. Given $\gamma > 0$, the nominal system in (9) is said to be stochastically admissible with γ-disturbance attenuation such that

$$\mathscr{E}\left\{\sum_{k=0}^{N} \|y(k)\|^2\right\} < \gamma^2 \mathbb{E}\left\{\sum_{k=0}^{N} \|w(k)\|^2\right\} \tag{14}$$

holds for zero-initial condition and any nonzero $w(k) \in \mathscr{L}_2[0, \infty)$.

The corresponding H_∞ controller for system (9) is

$$u(k) = K_i x(k), \tag{15}$$

where K_i is to be determined.

Lemma 8. *Given a scalar $\gamma > 0$, the unforced system (9) is stochastically admissible with H_∞ performance if there exist matrices $P_i = P_i^T$ such that*

$$E^T P_i E \ge 0 \tag{16}$$

$$\begin{bmatrix} A_i^T \widehat{P}_i A_i - E^T P_i E & A_i^T \widehat{P}_i F_i & C_i^T \\ * & -\gamma^2 I + F_i^T \widehat{P}_i F_i & D_i^T \\ * & * & -I \end{bmatrix} < 0, \tag{17}$$

where $\widehat{P}_i = \sum_{j=1}^{N} \pi_{ij} P_j$.

Lemma 9 (see[27]). *Given any real matrices X, Y, and Z with appropriate dimensions and such that $Y > 0$ and is symmetric, then, one has*

$$X^T Y X + X^T Z + Z^T X + Z^T Y^{-1} Z \ge 0. \tag{18}$$

3. Main Results

3.1. Continuous-Time Markovian Jump Linear Systems

Theorem 10. *Given a scalar $\gamma > 0$, the unforced system (1) with constraint (4) is stochastically admissible with H_∞ performance if there exist matrices X_i and $T_{ij} > 0$ such that*

$$X_i^T E^T = E X_i \ge 0$$

$$\begin{bmatrix} (A_i X_i)^* - \sum_{j=1, j\ne i}^{N} \underline{\lambda}_{ij} X_i^T E^T & F_i & X_i^T C_i^T & \overline{S}_i^T(X) & \widetilde{S}_i^T(X) \\ * & -\gamma^2 I & D_i^T & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & \overline{R}_i & 0 \\ * & * & * & * & \widetilde{R}_i \end{bmatrix} < 0, \tag{19}$$

where

$$\begin{aligned} \overline{S}_i^T(X) &= \left[\sqrt{\overline{\lambda}_{i1}} X_i^T \ \cdots \ \sqrt{\overline{\lambda}_{i(i-1)}} X_i^T \ \ \sqrt{\overline{\lambda}_{i(i+1)}} X_i^T \ \cdots \ \sqrt{\overline{\lambda}_{iN}} X_i^T\right] \\ \widetilde{S}_i^T(X) &= \left[\sqrt{\overline{\lambda}_{i1}} X_i^T E^T \ \cdots \ \sqrt{\overline{\lambda}_{i(i-1)}} X_i^T E^T \ \ \sqrt{\overline{\lambda}_{i(i+1)}} X_i^T E^T \ \cdots \ \sqrt{\overline{\lambda}_{iN}} X_i^T E^T\right] \\ \overline{R}_i &= -\operatorname{diag}\{4T_{i1}, \ldots, 4T_{i(i-1)}, 4T_{i(i+1)}, \ldots, 4T_{iN}\} \\ \widetilde{R}_i &= -\operatorname{diag}\left\{(X_1)^* - T_{i1}, \ldots, (X_{i-1})^* - T_{i(i-1)}, (X_{i+1})^* - T_{i(i+1)}, \ldots, (X_N)^* - T_{iN}\right\} \end{aligned} \tag{20}$$

hold for all $i \in S$.

Proof. From (3) and (4), we have

$$\begin{aligned} \sum_{j=1}^{N} \lambda_{ij} E^T P_j &= \sum_{j=1, j\ne i}^{N} \lambda_{ij} E^T P_j - \sum_{j=1, j\ne i}^{N} \lambda_{ij} E^T P_i \\ &\le \sum_{j=1, j\ne i}^{N} \overline{\lambda}_{ij} E^T P_j - \sum_{j=1, j\ne i}^{N} \underline{\lambda}_{ij} E^T P_i. \end{aligned} \tag{21}$$

Then, by (8) and (21), we conclude that

$$\begin{bmatrix} \left(A_i^T P_i\right)^\star + \sum_{j=1,j\neq i}^{N} \overline{\lambda}_{ij} E^T P_j - \sum_{j=1,j\neq i}^{N} \underline{\lambda}_{ij} E^T P_i & P_i^T F_i & C_i^T \\ * & -\gamma^2 I & D_i^T \\ * & * & -I \end{bmatrix} < 0. \tag{22}$$

Since $[(1/2)T_{ij}^{-(1/2)} - E^T P_j T_{ij}^{(1/2)}][(1/2)T_{ij}^{-(1/2)} - E^T P_j T_{ij}^{(1/2)}]^T \geq 0$ and $T_{ij} > 0$, $\forall i, j \in \mathbb{S}$ and $i \neq j$, we have

$$E^T P_j \leq \frac{1}{4} T_{ij}^{-1} + E^T P_j T_{ij} P_j^T E, \tag{23}$$

$$-P_i^{-T} T_{ij}^{-1} P_i^{-1} \leq T_{ij} - P_i^{-1} - P_i^{-T}. \tag{24}$$

Via (22) and (23), we have (25) implying (22); that is,

$$\begin{bmatrix} \overline{\Omega}_i & P_i^T F_i & C_i^T \\ * & -\gamma^2 I & D_i^T \\ * & * & -I \end{bmatrix} < 0, \tag{25}$$

where

$$\overline{\Omega}_i = \left(A_i^T P_i\right)^\star - \sum_{j=1,j\neq i}^{N} \underline{\lambda}_{ij} E^T P_i + \sum_{j=1,j\neq i}^{N} \frac{1}{4} \overline{\lambda}_{ij} T_{ij}^{-1} + \sum_{j=1,j\neq i}^{N} \overline{\lambda}_{ij} E^T P_j T_{ij} P_j^T E. \tag{26}$$

Let $X_i = P_i^{-1}$; then pre- and postmultiply (25) by $\mathrm{diag}\{X_i^T, I, I\}$ and its transpose. Taking into account (24) and via Schur complement, we obtain that (19) and (21) imply (8). This completes the proof. □

Remark 11. Since to the term $E^T P_j$ gives some problems, [25] presented a method to deal with problem; that is, $E^T P_j \leq (1/4)\epsilon_j^{-1} I + \epsilon_j E^T P_j P_j^T E$. In this paper, we overcome it by (23); let $T_{ij}^{-1} = \epsilon_j^{-1}$; we have $E^T P_j \leq (1/4)\epsilon_j^{-1} I + \epsilon_j E^T P_j P_j^T E$. Therefore, we have $E^T P_j \leq (1/4)T_{ij}^{-1} + E^T P_j T_{ij} P_j^T E \leq (1/4)\epsilon_j^{-1} I + \epsilon_j E^T P_j P_j^T E$.

Theorem 12. *Given a scalar $\gamma > 0$, there exists a state feedback controller in the form of* (7) *such that the resulting closed-loop system is stochastically admissible with* (6)*, if there exist matrices X_i, $T_{ij} > 0$, and Y_i such that*

$$X_i^T E^T = E X_i \geq 0, \tag{27}$$

$$\begin{bmatrix} \Omega_i & F_i & X_i^T C_i^T & \overline{S}_i^T(X) & \widetilde{S}_i^T(X) \\ * & -\gamma^2 I & D_i^T & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & \overline{R}_i & 0 \\ * & * & * & * & \widetilde{R}_i \end{bmatrix} < 0, \tag{28}$$

where

$$\begin{aligned} \Omega_i &= \left(A_i X_i + B_i Y_i\right)^\star - \sum_{j=1,j\neq i}^{N} \underline{\lambda}_{ij} X_i^T E^T \\ \widetilde{S}_i^T(X) &= \left[\ \sqrt{\overline{\lambda}_{i1}} X_i^T \ \cdots \ \sqrt{\overline{\lambda}_{i(i-1)}} X_i^T \quad \sqrt{\overline{\lambda}_{i(i+1)}} X_i^T \ \cdots \ \sqrt{\overline{\lambda}_{iN}} X_i^T\ \right] \\ \overline{S}_i^T(X) &= \left[\ \sqrt{\overline{\lambda}_{i1}} X_i^T E^T \ \cdots \ \sqrt{\overline{\lambda}_{i(i-1)}} X_i^T E^T \quad \sqrt{\overline{\lambda}_{i(i+1)}} X_i^T E^T \ \cdots \ \sqrt{\overline{\lambda}_{iN}} X_i^T E^T\ \right] \\ \overline{R}_i &= -\mathrm{diag}\left\{4T_{i1}, \ldots, 4T_{i(i-1)}, 4T_{i(i+1)} \ldots, 4T_{iN}\right\} \\ \widetilde{R}_i &= -\mathrm{diag}\left\{(X_1)^\star - T_{i1}, \ldots, (X_{i-1})^\star - T_{i(i-1)}, (X_{i+1})^\star - T_{i(i+1)}, \ldots, (X_N)^\star - T_{iN}\right\} \end{aligned} \tag{29}$$

hold for all $i \in \mathbb{S}$ with constraint (4)*. The controller gain K_i can be constructed as*

$$K_i = Y_i X_i^{-1}. \tag{30}$$

Proof. Substituting A_i with $A_i + B_i K_i$, then by similar method in Theorem 10 with (28), we can obtain Theorem 12. Thus, it is omitted here. □

Remark 13. It is seen that such conditions have equation constraint (28), which are not strict LMIs. In addition, the desired controller (7) is mode dependent, which requires its system mode to be available online. This makes the scope of application limited. In order to deal with this practical condition, mode-independent control is usually used. Based on the methods in [28, 29], such problems can be solved easily.

3.2. Discrete-Time Markovian Jump Linear Systems

Theorem 14. *Given a scalar $\gamma > 0$, the unforced system* (9) *with constraint* (12) *is stochastically admissible with H_∞ performance if there exist matrices P_i, G_i, and $T_{ij} > 0$ such that*

$$E^T P_i E \geq 0, \tag{31}$$

$$\begin{bmatrix} \Phi_{i11} & \Phi_{i12} & 0 & 0 & G_i^T C_i^T & \widehat{G}_i^T & 0 \\ * & \Phi_{i22} & \widehat{G}_i^T & 0 & 0 & 0 & 0 \\ * & * & -2I + \widehat{T}_{ij} & \widehat{T}_{ij} F_i & 0 & 0 & \widetilde{S}_{i1}(P) \\ * & * & * & -\gamma^2 I + F_i^T \widehat{T}_{ij} F_i & D_i^T & 0 & \widetilde{S}_{i2}(P) \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -\underline{\pi}_i E^T P_i E - 2I & 0 \\ * & * & * & * & * & * & \widetilde{R}_i \end{bmatrix} < 0, \tag{32}$$

where

$$\begin{gathered} \Phi_{i11} = \left(A_i G_i - G_i\right)^{\star}, \qquad \Phi_{i12} = G_i^T A_i^T - G_i, \\ \Phi_{i22} = -2G_i - 2G_i^T \\ \widehat{T}_{ij} = \frac{1}{4}\sum_{j=1}^{N} \overline{\pi}_{ij} T_{ij}, \qquad \widehat{G}_i = G_i + I, \\ \underline{\pi}_i = \sum_{j=1}^{N} \underline{\pi}_{ij}, \qquad \widetilde{R}_i = -\operatorname{diag}\{T_{i1}, \ldots, T_{iN}\} \\ \widetilde{S}_{i1}^T(P) = \left[\sqrt{\overline{\pi}_{i1}} P_1 \ \cdots \ \sqrt{\overline{\pi}_{iN}} P_N\right], \\ \widetilde{S}_{i2}^T(P) = \left[\sqrt{\overline{\pi}_{i1}} F_i^T P_1 \ \cdots \ \sqrt{\overline{\pi}_{iN}} F_i^T P_N\right] \end{gathered} \tag{33}$$

hold for all $i \in S$.

Proof. From (16) and (17), similar to [23], we have

$$(18) = Z_1^T Z_2^T Z_3^T \left(\Psi_i + \Pi_i^T \widehat{P}_i \Pi_i\right) Z_3 Z_2 Z_1, \tag{34}$$

where

$$Z_1 = \begin{bmatrix} G_i^{-1} & 0 & 0 \\ G_i^{-1} A_i & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \qquad Z_2 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & G_i & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix},$$

$$Z_3 = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \\ G_i & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \Pi_i^T = \begin{bmatrix} 0 \\ 0 \\ I \\ F_i^T \\ 0 \\ 0 \end{bmatrix},$$

$$\Psi_i = \begin{bmatrix} \Phi_{i11} & \Phi_{i12} & 0 & 0 & G_i^T C_i^T & \widehat{G}_i^T \\ * & \Phi_{i22} & \widehat{G}_i^T & 0 & 0 & 0 \\ * & * & -2I & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & D_i^T & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -E^T P_i E - 2I \end{bmatrix}. \tag{35}$$

Since $\sum_{j=1}^{N} \pi_{ij} P_j \leq \sum_{j=1}^{N} \pi_{ij}(\frac{1}{4} T_{ij} + P_j T_{ij}^{-1} P_j)$ with $T_{ij} > 0$, $\forall i, j \in \mathbb{S}$ and (12), we have

$$\Pi_i^T \widehat{P}_i \Pi_i \leq \Pi_i^T \left(\sum_{j=1}^{N} \overline{\pi}_{ij} \left(\frac{1}{4} T_{ij} + P_j T_{ij}^{-1} P_j\right)\right) \Pi_i. \tag{36}$$

From (12), we obtain that

$$\underline{\pi}_i E^T P_i E = \sum_{j=1}^{N} \underline{\pi}_{ij} E^T P_i E \leq \sum_{j=1}^{N} \pi_{ij} E^T P_i E. \tag{37}$$

Taking into account (34), (36), and (37), we have

$$\Psi_i + \Pi_i^T \widehat{P}_i \Pi_i \leq \overline{\Psi}_i + \Pi_i^T \left(\sum_{j=1}^{N} \overline{\pi}_{ij} P_j T_{ij}^{-1} P_j\right) \Pi_i < 0, \tag{38}$$

where

$$\overline{\Psi}_i = \begin{bmatrix} \Phi_{i11} & \Phi_{i12} & 0 & 0 & G_i^T C_i^T & \widehat{G}_i^T \\ * & \Phi_{i22} & \widehat{G}_i^T & 0 & 0 & 0 \\ * & * & -2I + \widehat{T}_{ij} & \widehat{T}_{ij} F_i & 0 & 0 \\ * & * & * & -\gamma^2 I + F_i^T \widehat{T}_{ij} F_i & D_i^T & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\underline{\pi}_i E^T P_i E - 2I \end{bmatrix}. \tag{39}$$

Then, by the Schur complement, we see that (32) is equivalent to (38). This completes the proof. □

Remark 15. It should be remarked that when $E = I$ in (9), [24] gave important criteria under the constraint (12) via $\sum_{j=1}^N \pi_{ij} P_j \le \sum_{j=1}^N \overline{\pi}_{ij} P_j$ and $\sum_{j=1}^N \underline{\pi}_{ij} P_i \le \sum_{j=1}^N \pi_{ij} P_i$, where P_j must be positive-definite matrix which is not applicable in SMJSs. In SMJS (9), P_i is only required to be nonsingular matrix. In the above proof, we remove this constraint via (36).

In particular, if $E = I$, then $\widehat{P}_i \le \sum_{j=1}^N \overline{\pi}_{ij}((1/4)T_{ij} + P_j T_{ij}^{-1} P_j)$. Let $T_{ij} = 2P_j$; we have $(1/4)T_{ij} + P_j T_{ij}^{-1} P_j = P_j$; thus, we have $\widehat{P}_i \le \sum_{j=1}^N \overline{\pi}_{ij}((1/4)T_{ij} + P_j T_{ij}^{-1} P_j) \le \sum_{j=1}^N \overline{\pi}_{ij} P_j$.

Theorem 16. *Given a scalar $\gamma > 0$, there exists a state feedback controller in the form of* (15) *such that the resulting closed-loop system* (9) *is stochastically admissible with* (14), *if there exist matrices G_i, P_i, Y_i, and $T_{ij} > 0$ such that*

$$E^T P_i E \ge 0 \tag{40}$$

$$\begin{bmatrix} \Phi_{i11} & \Phi_{i12} & 0 & 0 & G_i^T C_i^T & \widehat{G}_i^T & 0 \\ * & \Phi_{i22} & \widehat{G}_i^T & 0 & 0 & 0 & 0 \\ * & * & -2I + \widehat{T}_{ij} & \widehat{T}_{ij} F_i & 0 & 0 & \widetilde{S}_{i1}(P) \\ * & * & * & -\gamma^2 I + F_i^T \widehat{T}_{ij} F_i & D_i^T & 0 & \widetilde{S}_{i2}(P) \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -\underline{\pi}_i E^T P_i E - 2I & 0 \\ * & * & * & * & * & * & \widetilde{R}_i \end{bmatrix} < 0, \tag{41}$$

where

$$\widehat{\Phi}_{i11} = \left(A_i G_i + B_i Y_i - G_i\right)^\star, \qquad \widehat{\Phi}_{i12} = G_i^T A_i^T + Y_i^T B_i^T - G_i \tag{42}$$

hold for all $i \in \mathbb{S}$ with constraint (12). *The controller gain K_i can be constructed as*

$$K_i = Y_i G_i^{-1}. \tag{43}$$

4. Numerical Examples

In this section, numerical examples are given to demonstrate the effectiveness of proposed theory.

Example 1. Consider the following continuous-time SMJS as (1) described as

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0.1 & -0.2 & 0 \\ 0.5 & -1 & -0.5 \\ 0.1 & 0 & 0.4 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.1 & -1 & 0 \\ -0.3 & -1 & 0.4 \\ 0 & 0.2 & 0.1 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} 0.6 & 0 & 0.4 \\ -0.4 & 0 & 0.7 \\ -0.3 & 0.1 & -0.4 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \qquad B_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ -0.1 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0.1 \\ -0.2 \\ 0.1 \end{bmatrix}, \qquad F_3 = \begin{bmatrix} -0.1 \\ 0.2 \\ 0.1 \end{bmatrix},$$

$$C_1 = [1 \;\; 0 \;\; 0], \qquad C_2 = [0 \;\; 1 \;\; 1], \qquad C_3 = [1 \;\; -1 \;\; 1],$$
$$D_1 = 0.2, \qquad D_2 = 0.1, \qquad D_3 = -0.3. \tag{44}$$

Assume the bound transition probability matrices are

$$\underline{\Lambda} = \begin{bmatrix} -2.5 & 1.3 & 1.2 \\ 0.8 & -1.9 & 1.1 \\ 0.8 & 0.7 & -1.5 \end{bmatrix}, \qquad \overline{\Lambda} = \begin{bmatrix} -2.9 & 1.5 & 1.4 \\ 1 & -2.2 & 1.2 \\ 1 & 0.9 & -1.9 \end{bmatrix}. \tag{45}$$

By the method in [25], we find that it is infeasible for this system no matter γ takes any values. However, for $\gamma = 0.8$, via Theorem 12, we obtain the controller gain

$$K_1 = [-6.3315 \;\; 10.0669 \;\; 96.5153],$$
$$K_2 = [2.9966 \;\; -9.0875 \;\; -59.5261], \tag{46}$$
$$K_3 = [24.7500 \;\; -22.4333 \;\; -17.1457].$$

Example 2. Consider the following discrete-time SMJS as (9) described as

$$E = \begin{bmatrix} 0.5 & 0.6 \\ 1 & 1.2 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0.1 & 0.8 \\ 0.15 & 1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1.1 & 0.6 \\ 0.5 & -1 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} 0.1 & 1 \\ 0.5 & -0.9 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad B_3 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \tag{47}$$
$$F_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}, \qquad F_3 = \begin{bmatrix} -0.1 \\ 0.2 \end{bmatrix},$$
$$C_1 = [1 \;\; 0], \qquad C_2 = [0 \;\; 1], \qquad C_3 = [1 \;\; -1],$$
$$D_1 = D_2 = 0.1, \qquad D_3 = -0.1.$$

Assume the bound transition probability matrices are

$$\underline{\Lambda} = \begin{bmatrix} 0.2 & 0.4 & 0 \\ 0.3 & 0 & 0.3 \\ 0.1 & 0.4 & 0.1 \end{bmatrix}, \qquad \overline{\Lambda} = \begin{bmatrix} 0.6 & 0.8 & 0 \\ 0.7 & 0 & 0.5 \\ 0.3 & 0.7 & 0.3 \end{bmatrix}. \tag{48}$$

Let $\gamma = 1$; then via Theorem 16, we obtain the controller gain

$$K_1 = [12.3040 \;\; 9.9997]$$
$$K_2 = [-13.0975 \;\; -4.5714] \tag{49}$$
$$K_3 = [10.6808 \;\; -7.6430].$$

Example 3. In order to do some comparison, consider the following discrete-time normal state-space MJS which can be obtained by (9) with $E = I$ and is described as

$$A_1 = \begin{bmatrix} 0.8 & -0.1 \\ 0.2 & 0.5 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0.6 & 0 \\ -0.1 & 0.2 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.6 & 1 \\ 0 & 0.7 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$
$$B_3 = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \qquad C_1 = [-0.2 \;\; 0], \qquad C_2 = [0.6 \;\; 0],$$
$$C_3 = [0.3 \;\; 0] \qquad D_1 = -0.4, \qquad D_2 = 0.2, \qquad D_3 = 0.3. \tag{50}$$

Assume the bound transition probability matrices are

$$\underline{\Lambda} = \begin{bmatrix} 0.2 & 0.4 & 0 \\ 0.3 & 0 & 0.3 \\ 0.1 & 0.4 & 0.1 \end{bmatrix}, \qquad \overline{\Lambda} = \begin{bmatrix} 0.6 & 0.8 & 0 \\ 0.7 & 0 & 0.5 \\ 0.3 & 0.7 & 0.3 \end{bmatrix}. \tag{51}$$

By the method in [24], we have the minimum $\gamma^* = 2.45$, while via Theorem 16, we obtain $\gamma^* = 0.43$. By this example, it is seen that our method is less conservative.

5. Conclusions

In this paper, the problems of H_∞ control for both continuous-time and discrete-time SMJSs with bounded transition probabilities have been investigated. Less conservative sufficient criteria for SMJSs to be stochastically admissible with H_∞ performance are given by the LMI approach. Based on the obtained conditions, a kind of state feedback controller, such that not only stochastic admissibility but also a prescribed H_∞ performance level is guaranteed, can be computed. Finally, numerical examples are provided to illustrate the advantage and effectiveness of the presented results in this paper.

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