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Abstract

This paper describes the application of adaptive neuro-fuzzy inference system (ANFIS) methodology for the backcalculation of airport flexible pavement layer moduli. The proposed ANFIS-based backcalculation approach employs a hybrid learning procedure to construct a non-linear input-output mapping based on qualitative aspects of human knowledge and pavement engineering experience incorporated in the form of fuzzy if-then rules as well as synthetically generated Finite Element (FE) based pavement modeling solutions in the form of input-output data pairs. The developed neuro-fuzzy backcalculation methodology was evaluated using hypothetical data as well as extensive non-destructive field deflection data acquired from a state-of-the-art full-scale airport pavement test facility. It was shown that the ANFIS based backcalculation approach inherits the fundamental capability of a fuzzy model to especially deal with nonrandom uncertainties, vagueness, and imprecision associated with non-linear inverse analysis of transient pavement surface deflection measurements.

Keywords

Adaptive neuro-fuzzy inference system, airport flexible pavements, backcalculation

Disciplines

Civil and Environmental Engineering | Geotechnical Engineering

Comments

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Adaptive Neuro-Fuzzy Inference System-Based Backcalculation Approach to Airport Pavement Structural Analysis

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ABSTRACT: This paper describes the application of adaptive neuro-fuzzy inference system (ANFIS) methodology for the backcalculation of airport flexible pavement layer moduli. The proposed ANFIS-based backcalculation approach employs a hybrid learning procedure to construct a non-linear input-output mapping based on qualitative aspects of human knowledge and pavement engineering experience incorporated in the form of fuzzy if-then rules as well as synthetically generated Finite Element (FE) based pavement modeling solutions in the form of input-output data pairs. The developed neuro-fuzzy backcalculation methodology was evaluated using hypothetical data as well as extensive non-destructive field deflection data acquired from a state-of-the-art full-scale airport pavement test facility. It was shown that the ANFIS based backcalculation approach inherits the fundamental capability of a fuzzy model to especially deal with nonrandom uncertainties, vagueness, and imprecision associated with non-linear inverse analysis of transient pavement surface deflection measurements.

INTRODUCTION

Among the various pavements Non-Destructive Testing (NDT) techniques currently used, the Falling Weight Deflectometer (FWD) is the most common. A Heavy Weight Deflectometer (HWD), which is basically a FWD equipment capable of higher load magnitudes, is used for non-destructive evaluation of airport pavements. The FWD equipment measures pavement surface deflections from an applied dynamic load that simulates a moving wheel (FAA 2004). During FWD testing, typically, a 9,000-lb load is applied to the pavement surface by the intermediary of a circular plate (with a diameter of 12 in.) and the generated duration of the half-sine pulse is typically 30 ms. It corresponds to the loading time produced by a truck moving at 40 to 50 mph. The

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resulting pavement surface deflections are measured with six geophones at offsets of 0 (D0), 12 in. (D12), 24 in. (D24), 36 in. (D36), 48 in. (D48), and 60 in. (D60) intervals from the center of the load.

Inverse analysis (more commonly referred to as backcalculation) is used to determine the Young's modulus of pavement layers based on observed dynamic response (deflection) of the pavement surface to an impulse load (the falling weight). Backcalculation of pavement layer properties is a very useful pavement design tool to evaluate the structural condition of in-service pavements and to characterize the layer properties as inputs into available numerical or analytical programs. In the backcalculation process, a numerical optimization method is employed so that the measured deflection basin agrees with the deflections given by a numerical model. The optimization process is an iterative method which modifies the elastic modulus of the pavement layers until a better adjustment is produced. Moreover, the optimization process can be carried out by employing an algorithm of parameter identification like non-linear least-squares algorithm, research in a database, or soft computing methods such as Neural Networks (NNs), Genetic Algorithms (GAs), etc. Especially, in recent years, NNs have been shown to be capable of predicting the pavement layer moduli using the Falling Weight Deflectometer (FWD) field deflection measurements (Meier and Rix, 1995, Ceylan et al. 2007).

The objective of this paper is to investigate the feasibility of using Adaptive-Network-based Fuzzy Inference System (ANFIS) for inverse analysis of multi-layered airport flexible pavement structures based on FWD data. The approach is hybridized by employing a Finite Element (FE) structural model for computing responses of pavement structure with known characteristics of pavement materials subjected to FWD loading. The FE model captures the non-linear, stress-dependent behavior of geo-materials used in the underlying unbound pavement layers resulting in realistic materials characterization and modeling of responses.

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ADAPATIVE NEURO-FUZZY INFERENCE SYSTEM (ANFIS) METHODOLOGY

ANFIS Theory

One of the most important and promising research fields in recent years has been *Nature-Inspired Heuristics*, an area utilizing some analogies with natural or social systems for deriving non-deterministic heuristic methods to obtain better results in combinatorial optimization problems. Fuzzy logic approach (FLA) is one such heuristic method (Zadeh 1965).

In contrast to classical set theory, where membership of the elements are assessed in binary terms (an element either belongs to or does not belong to the set), fuzzy sets are sets whose elements have degrees of membership. The fuzzy set theory permits the gradual assessment of the membership of elements in a set with the aid of a membership function valued in the real unit interval [0, 1].

Fuzzy inference systems (FIS) are powerful tools for the simulation of nonlinear behaviors utilizing fuzzy logic and linguistic fuzzy rules. In the literature, there are several inference techniques developed for fuzzy rule-based systems, such as Mamdani and Sugeno (Jang et al. 1997). In the Mamdani fuzzy inference methodology, inputs and outputs are represented by fuzzy relational equations in canonical rule-based form. In Sugeno FIS, output of the fuzzy rule is characterized by a crisp function and it was developed to generate fuzzy rules from a given input-output data set. Neuro-fuzzy systems are multi-layer feed forward adaptive networks that realize the basic elements and functions of traditional fuzzy logic systems. Since it has been shown that fuzzy logic systems are universal approximators, neuro-fuzzy control systems, which are isomorphic to traditional fuzzy logic control systems in terms of their functions, are also universal approximators. ANFIS is an extension of the Sugeno fuzzy model.

The Sugeno model allows the fuzzy systems to learn the parameters using adaptive backpropagation learning algorithm. In general, ANFIS is much more complicated than FIS. A FIS can be considered to be a parameterized non-linear map or a crisp function in a consequence called f , namely:

$$f(x) = \frac{\sum_{l=1}^m y^l \left(\prod_{i=1}^n \mu_{A_i^l}(x_i) \right)}{\sum_{l=1}^m \left(\prod_{i=1}^n \mu_{A_i^l}(x_i) \right)} \quad (1)$$

where y^l is a part of output if Mamdani reasoning is applied or a constant if Sugeno reasoning is applied (Jang et al. 1997). The membership function $\mu_{A_i^l}(x_i)$ corresponds to the input $x = [x_1, \dots, x_n]$ of the rule l and m is the number of fuzzy rules. For the i th input predictor variable, x_i is the real data (for example, the measured FWD

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deflection) in one point from the set of observed values. The output values, $f(x)$ are the estimated values (for example, the backcalculated pavement layer modulus) of simulation function within the range of input set (Abolpour et al. 2007). The center of gravity method is used for defuzzification. This can be further written as:

$$f(x) = \sum_{l=1}^m w_l b_l(x) \quad (2)$$

where $w_i = y^l$ and

$$b_l(x) = \frac{\prod_{i=1}^n \mu_{A_i^l}(x_i)}{\sum_{l=1}^m \left(\prod_{i=1}^n \mu_{A_i^l}(x_i) \right)} \quad (3)$$

If F_S is a set of continuous estimated value functions on domain D , then f can approximate F_S to any desired accuracy. Let F_S be a bounded function on $[a, b]$ and $D = \{x^1, \dots, x^h\}$, a set of points in $[a, b]$. Then there exists the least squares polynomial of degree $\leq r$ between F_S and Q^h , which minimizes the following expression:

$$\sum_{j=1}^h \left| F_S(x^j) - Q(x^j) \right|^2 \quad (4)$$

Overall polynomial's degree is equal to or less than r . Where Q^h is real data of output values over h th point of input set (for each input predictor variable $i = 1, 2, \dots, n$ and for each point of real world data $j = 1, 2, \dots, h$).

In the Mamdani type of fuzzy system, the real data of the output values can be classified into classes such that the length of each class is equal to $[a, b]$. But in the Sugeno type, the length of $[a, b]$ is only determined over input data set (D), and f can be approximately equal to F_S ; hence, F_S is the output values of simulation model. In the interest of space, the derivation of equations for development and evaluation of rule base are not presented in this paper, but can be found in Jang et al. (1997). 'Learning' process in ANFIS methodology, namely adaptation of membership functions to emulate the training data, is commonly performed by two techniques: backpropagation and hybrid learning algorithms. The hybrid optimization method is a combination of Least Squares Error (LSE) and Backpropagation (BP) descent method. In hybrid learning algorithm, consequent parameters are identified in forward computation by LSE algorithm, and premise parameters are adjusted in backward computation using backpropagation algorithm.

Development of ANFIS-Based Backcalculation Approach

Recently, researchers have attempted to employ FIS and ANFIS methodologies to model the pavement deflection behavior under dynamic loading (Saltan et al. 2007)

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and backcalculate the mechanical properties of flexible pavements based on elastic layered analysis (Goktepe et al. 2004), respectively. This study focused on developing a FE-based ANFIS methodology for backcalculation of airport flexible pavement non-linear moduli based on HWD data.

As a first step towards employing ANFIS methodology in learning the inverse mapping between known input (pavement layer thickness, moduli, and Poisson's ratio) and output patterns in a supervised manner, synthetic training and testing databases were generated using a 2-D axisymmetric pavement finite-element software, ILLI-PAVE (Raad and Figueroa, 1980). The Hot-Mix Asphalt (HMA) surface layer was characterized as a linear elastic material. Stress-dependent elastic models along with Mohr-Coulomb failure criteria were applied for the base and subgrade layers. The *stress-hardening* K - θ model was used for the base layer ($E_R = K\theta^n$; where E_R is resilient modulus (psi), θ is bulk stress (psi) and K and n are statistical parameters). The fine-grained low-strength subgrade was modeled using the *bi-linear* model for characterizing the resilient modulus (Thompson and Elliot, 1985).

The pavement structure considered in this study is a conventional granular base flexible pavement section built over a medium-strength subgrade (referred to as MFC) at the U.S. National Airport Pavement Test Facility (NAPTF). The MFC test section consists of 5-in. of HMA surface layer, 8-in. of granular base, and 12-in. of granular subbase over medium-strength subgrade (CL-CH ASTM Unified Soil Classification with CBR of 8).

To illustrate the feasibility of ANFIS-based backcalculation approach for airport pavement analysis, a synthetic database was generated using ILLI-PAVE simulating the effect of 36-kip HWD loading on the MFC pavement structure with the as-constructed layer thicknesses. A total of 5,000 datasets were generated by randomly varying the layer moduli parameters over typical ranges. It is noted that the proposed ANFIS-based approach could be easily generalized for a variety of pavement structures with different layer thicknesses by developing a more comprehensive synthetic database.

In developing the ANFIS-based backcalculation approach, input parameters (six HWD surface deflections at 12-in. radial offsets starting with the deflection at the center of the HWD loading plate) were partitioned using *subtractive clustering* technique and Gaussian membership functions were used. First order Sugeno FIS with linear output function was selected as the inference system. ANFIS structure was completed by the selection of *hybrid learning* algorithm and a batch learning scheme was used. In this learning algorithm, the BP algorithm is applied to the learning of premise parameters, while LSE algorithm is applied to the learning of consequent parameters. In the rulebase, fuzzy variables were connected with T-norm (fuzzy AND) operators and rules were associated using max-min decomposition technique. The output part of each rule uses a linear defuzzifier formula; the total output of ANFIS is the weighting average of the output of each rule. The ANFIS backcalculation approach was implemented in MATLAB[®] using the in-built toolbox. The ANFIS methodology predictions for E_{AC} and E_{Ri} are shown in figure 1 at the end of 200 epochs for 1,000

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independent testing patterns along with the Root Mean Squared Errors (RMSEs).

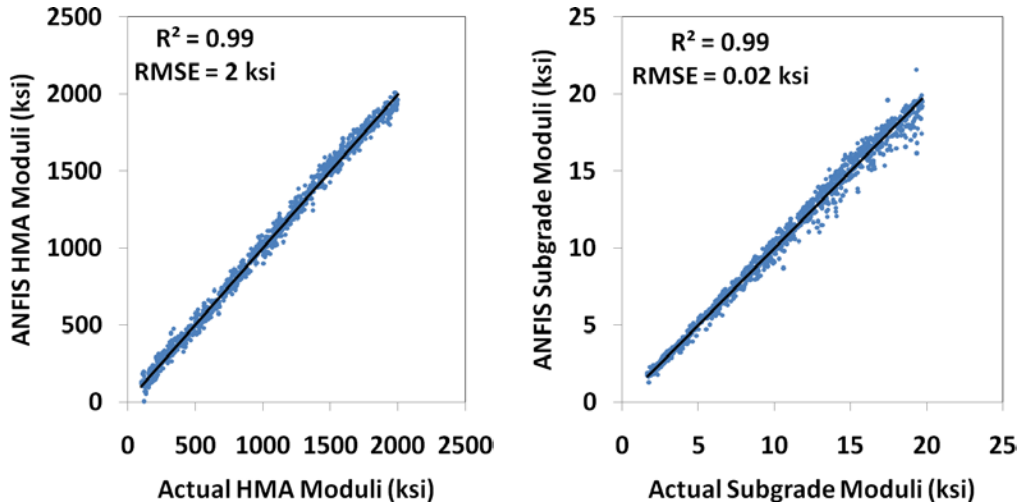


FIG. 1. ANFIS backcalculation model prediction accuracy.

EVALUATION OF ANFIS APPROACH USING AIRFIELD PAVEMENT DATA

The developed ANFIS-based backcalculation methodology was evaluated using actual field data acquired at the NAPTF for the MFC test section. It was constructed to generate full-scale test data needed to develop pavement design procedures for the new generation of large civil transport aircraft, including the Boeing 777 (B777) and Boeing 747 (B747). During the first series of tests, two gear configurations, a six-wheel dual-tridem landing gear (B777) in one lane and a four-wheel dual-tandem landing gear (B747) in the other lane were tested simultaneously (Gopalakrishnan and Thompson, 2006). HWD tests were conducted at regular time intervals as trafficking continued to monitor the effect of repeated traffic loading on the test pavements. For HWD testing, the Federal Aviation Administration (FAA) HWD equipment configured with a 12-in. loading plate and a 27~30 ms pulse width was used. The HWD tests were conducted on the untrafficked centerline (C/L), B777 traffic lane and B747 traffic lane at approximately 10-ft intervals along the length of the test section. All test data referenced in this paper are available for download on the FAA Airport Pavement Technology website (<http://www.airporttech.tc.faa.gov/naptf/>).

To study the loss of stiffness in NAPTF MFC pavement section resulting from trafficking, the HMA and non-linear subgrade layer moduli values were backcalculated from the 36-kip HWD data acquired at the NAPTF using the ANFIS backcalculation models developed in this study. By studying the changes in HMA and subgrade moduli values in the traffic lanes relative to those from the untrafficked C/L, the degree of structural deterioration induced by B777 and B747 trafficking on MFC

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test pavement can be assessed. The variations in ANFIS predicted HMA moduli values with the number of traffic load repetitions (N) are displayed in figure 2 for the MFC test section. The ANFIS backcalculated moduli values have successfully captured the relative severity effects of B777 and B747 traffic gear loading and are consistent with results from previous studies (Gopalakrishnan and Thompson, 2004).

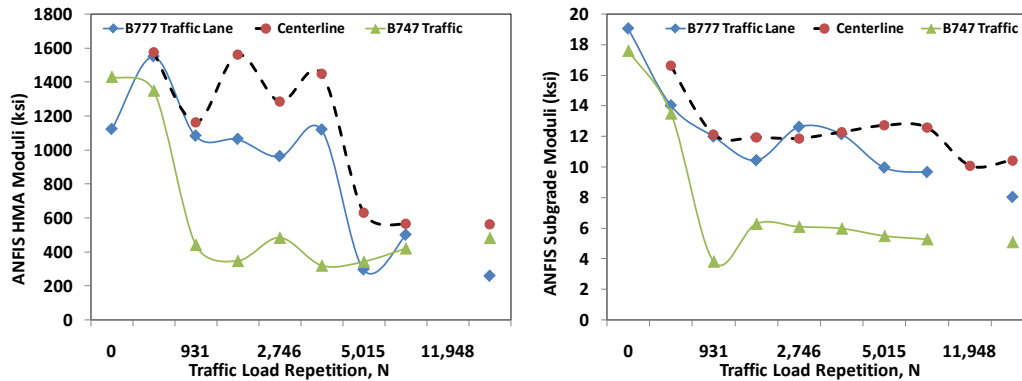


FIG. 2. ANFIS predicted backcalculated moduli vs NAPTF traffic load repetitions.

The ANN predicted results were then compared with those obtained using a conventional modulus backcalculation program, BAKFAA (previously known as FAABACKCAL) which assumes the pavement materials to be linear elastic. The BAKFAA was developed under the sponsorship of the FAA Airport Technology Branch and is based on the Layered Elastic Analysis program (LEAF) layered elastic computation program (Hayhoe, 2002). In this program, the pavement layer moduli are adjusted to minimize the root mean square (RMS) of the differences between FWD/HWD sensor measurements and the LEAF-computed deflection basin for a specified pavement structure. A standard multidimensional simplex optimization routine is then used to adjust the moduli values (McQueen et al. 2001). The comparison between the ANFIS predicted moduli values and BAKFAA backcalculated moduli values are presented for MFC untrafficked centerline in figure 3. The prediction trends are consistent although the magnitudes may vary which could be attributed to the pavement structural model used in response computations. BAKFAA uses the FAA LEAF multi-layered elastic analysis program whereas ANFIS utilizes synthetic database generated by ILLI-PAVE FE based program which models the pavement as a 2D axisymmetric solid of revolution and employs nonlinear stress-dependent models and failure criteria for granular materials and fine-grained soils.

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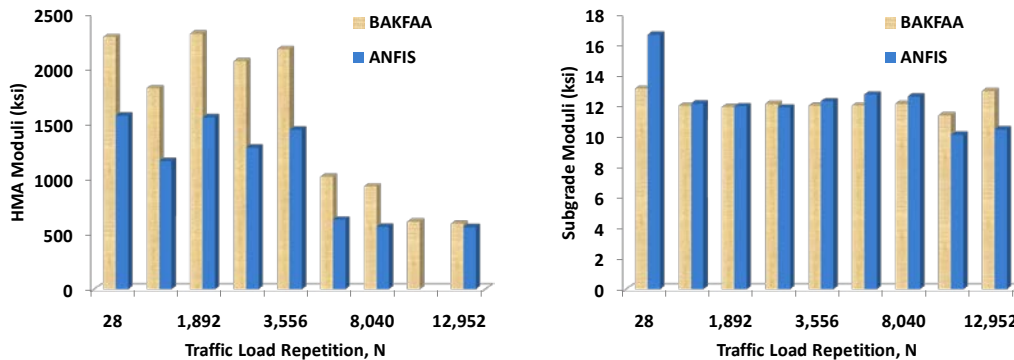


FIG. 3. Comparison of ANFIS predictions with conventional backcalculation results for untrafficked centerline.

CONCLUSIONS

An Adaptive Neuro-Fuzzy Inference System (ANFIS) based methodology was developed for the backcalculation of airport flexible pavement layer moduli. The approach was hybridized by employing a FE structural model for computing responses of pavement structure with known characteristics of pavement materials subjected to FWD loading. The FE model captures the non-linear, stress-dependent behavior of geo-materials used in the underlying unbound pavement layers resulting in realistic materials characterization and modeling of responses. The ANFIS methodology was successful in capturing the effect of simulated aircraft gear trafficking on backcalculated non-linear moduli. A significant advantage of ANFIS methodology over conventional backcalculation techniques is that it can embed qualitative aspects of human knowledge and pavement engineering experience in the form of fuzzy if-then rules and can thus deal with nonrandom uncertainties, vagueness, and imprecision associated with non-linear inverse analysis of transient pavement surface deflection measurements.

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