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IEA-Task 31 WAKEBENCH: Towards a protocol for wind farm flow model evaluation. Part 2: Wind farm wake models

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Abstract. Researchers within the International Energy Agency (IEA) Task 31: Wakebench have created a framework for the evaluation of wind farm flow models operating at the microscale level. The framework consists of a model evaluation protocol integrated with a web-based portal for model benchmarking (www.windbench.net). This paper provides an overview of the building-block validation approach applied to wind farm wake models, including best practices for the benchmarking and data processing procedures for validation datasets from wind farm SCADA and meteorological databases. A hierarchy of test cases has been proposed for wake model evaluation, from similarity theory of the axisymmetric wake and idealized infinite wind farm, to single-wake wind tunnel (UMN-EPFL) and field experiments (Sexbierum), to wind farm arrays in offshore (Horns Rev, Lillgrund) and complex terrain conditions (San Gregorio). A summary of results from the axisymmetric wake, Sexbierum, Horns Rev and Lillgrund benchmarks are used to discuss the state-of-the-art of wake model validation and highlight the most relevant issues for future development.

1. Introduction

The IEA Task 31 Wakebench was initiated in 2011 to establish an international research collaboration in the field of wind farm flow modeling. The objective of the project is to establish a framework for verification, validation and uncertainty quantification (VV&UQ) that will first be used to produce best-practice guidelines for flow-over-terrain and wind farm wake models. The scientific scope of the project is mainly addressing microscale atmospheric boundary layer (ABL) and wind farm wake (far-wake) models suitable for wind resource assessment and wind farm design applications. The framework consists of a model evaluation protocol integrated with a web-based portal for model benchmarking (www.windbench.net) [1], which contains a repository of test cases, an inventory of models and a set of online tools for reviewing, discussion and reporting of benchmark results. In the future, the scientific scope will be extended to neighboring research communities, notably mesoscale and near-wake models, and will define a basis for uncertainty quantification of the model-chain.

The building-block validation approach, as implemented in Wakebench, is useful for analyzing highly complex wind farm systems, consisting of individual turbines with unique locations and

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environmental conditions, by subdividing it in subsystems and unit problems to form a hierarchy of benchmarks with systematically increasing levels of complexity [2][3][4]. This approach allows isolation of individual or combined elements of the model-chain and evaluation of the potential impact of each element on the full system performance. The process typically involves analyzing idealized conditions using theoretical approaches, parametric testing in controlled environments (e.g. wind tunnels) and field testing of scaled or full-scale prototypes in research conditions and, at the most complex level, operational units in industrial conditions. The higher levels of physical complexity are often associated with lower levels of data quality and resolution because of physical and economic limitations. Whereas the lower levels of complexity oversimplify the physical processes of full scale reality. In practice an overlapping integrated set of benchmarks assures a balanced and useful approach for model VV&UQ. An essential aspect of the evaluation process is the definition of pertinent metrics to assess the performance of models on variables of interest for the target application [5].

This paper provides an overview of the building-block validation approach applied to wind farm wake models, including best practices for the benchmarking from meteorological and wind farm SCADA databases. A summary of the results from the first benchmarks executed under IEA Task 31 is provided in order to set the basis for future studies.

2. Wake Models

Wake models are intrinsically embedded in ABL models that deal with the background flow field physics. Hence, the model evaluation procedure should begin with a systematic validation of ABL models over a range of terrain conditions from offshore to flat and eventually hilly and mountainous terrain, which is the focus of the companion paper [6]. Turbine wake models differ in physical basis and complexity that is often bound by the computational resources of the end user [7][8][9]. In this paper, we shall focus on microscale wind farm models that produce mean wake characteristics, since this approximation is used in the wind farm design process where power estimates are calculated for long time periods. The initial benchmarks will therefore not focus on the dynamics of the flow within the wind farm.

Outside of the benchmark group, model and user names are kept anonymous. During this initial stage the focus is on identifying consistency among groups of models instead of evaluating details of individual models. This task is left to the user, who benefits from the benchmarking activities by having access to detailed information about the simulations of the group. With this philosophy in mind, the following model categories have been identified:

- **Engineering model (ENG)**: fast wake model for layout optimization based on similarity theory of the actuator disk, analytical functions and semi-empirical relations [10][11][12][13]
- **Reynolds Averaged Navier-Stokes model (RANS):** CFD models using rotor models (actuator disk, actuator line, full rotor, etc.) including both axial and tangential forces in the rotor, and turbulence based on eddy-viscosity approximations[14][15][16][17][18][19][20]
- **Large Eddy Simulation model (LES)**: unsteady CFD using rotor models and an eddy-resolving turbulence computational approach [21][22][23][24][25][26]

The ABL models in which the wake models are embedded are used to simulate neutral background atmospheric conditions with flat or no terrain for the first set of benchmarks presented here.

3. Model Evaluation Procedure

The credibility of a model is built upon two essential principles: verification and validation, defined by the AIAA (1998) guide as [2]:

- **Verification** is the process of determining that the model implementation accurately represents the developer's conceptual description of the model and the solution of the model.
- **Validation** is the process of determining the degree to which the model is an accurate representation of the real world from the perspective of the intended uses of the model.

Accuracy is evaluated differently in verification and validation:

- In verification activities accuracy is measured with respect to benchmark solutions of simplified model problems
- In validation activities accuracy is measured with respect to experimental observations

In this paper, only results from validation are presented, while verification will be considered in future publications.

Accuracy is measured on a selected set of variables within a range of application, relevant for the intended use of the model. Typically, in wind assessment studies, the main focus shall be on mean velocity (*U*) and turbulent kinetic energy (*tke*) as they are directly related to the annual energy production (*AEP*) and turbulence intensity (*I*), target parameters for wind turbine siting. Concerning wake models, the most relevant parameter is the wake efficiency η_{wake}:

$$\eta_{wake} = \frac{P_{wake}}{P_{free}} \quad (1)$$

where P_{wake} and P_{free} are the power production of a turbine in waked and undisturbed conditions respectively. Other relevant variables to diagnose wake model outputs are the velocity/momentum or power deficit, added turbulence, dissipation distance and wake width.

Comparison between simulated (*sim*) and observed (*obs*) data is visualized with conventional 2D profile plots and quantified with statistical metrics. The selection of good metrics is essential to compare results from different benchmarks. Among the wide variety of statistical tools, we shall use the normalized mean absolute error *NMAE*:

$$NMAE(\hat{\delta}_{ijk}) = \frac{1}{\overline{\overline{\delta_{ijk}^{obs}}} \cdot N_s} \sum_{s=1}^{N_s} \left| \hat{\delta}_{ijk,s}^{obs} - \hat{\delta}_{ijk,s}^{sim} \right|; \quad \hat{\delta}_{ijk} = \frac{\delta_{ijk}}{\delta_0} \quad (2)$$

where $\hat{\delta}_{ijk}$ is any variable of interest made dimensionless by dividing by a reference value δ_0 that typically corresponds to the inflow conditions, prescribed in the benchmark, or a reference location in the site of interest, such as the top level of a reference mast or an undisturbed turbine. The *ijk* subscript refers to a certain class of wind conditions (flow case) which is defined from observations in terms of ensemble or bin averages for a range of velocity, turbulence intensity, wind direction and stability. The summation is done for N_s sensor locations typically related to a spatial profile of measurements.
Generating validation data from operational wind farms deserves some attention since this operation requires a rather exhaustive filtering for quality control [27].

4. Test Cases and Benchmarks

4.1. Axisymmetric Wake: Similarity Theory

The building-block approach for wake models starts by investigating fundamental physical properties of the steady, turbulent, axisymmetric wake. Under these idealized conditions, similarity theory [28] shows wake width δ^* growth scaling with $x^{1/3}$ and the center-line velocity deficit U_0 decay scaling with $x^{-2/3}$, where *x* is the downstream distance from the rotor disk. The benchmark objective is to verify if wake models satisfy this background theory concerning these two variables of interest. Additionally, the momentum deficit should be conserved. Figure 1 shows the behaviour for the wake width and wake deficit as a line, where the exact curve uses the parameters from [29]. In these plots, the momentum thickness, θ, freestream velocity, U_∞, and virtual origin, x_0, have been used to non-dimensionalize the data.

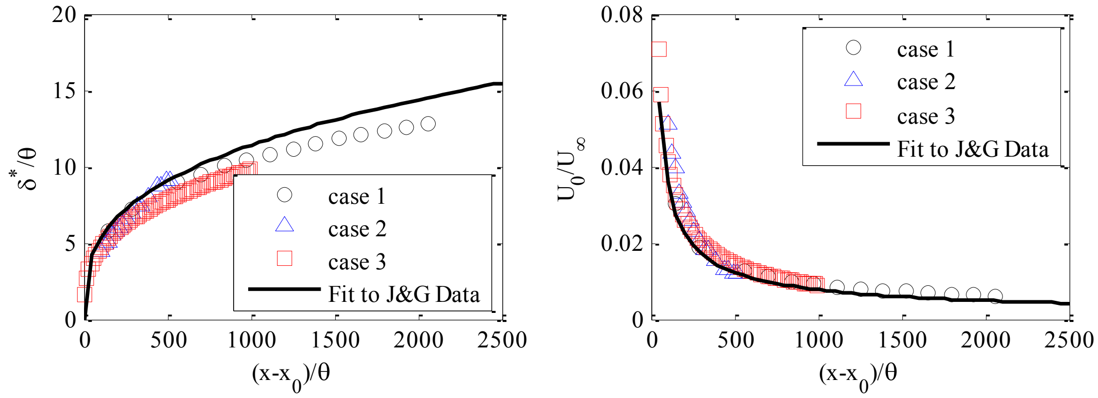


Figure 1: Dimensionless wake width (left) and momentum deficit (right) as a function of downstream dimensionless distance from the wake-generating disk

Three cases have been simulated using different models. As shown in Figure 1, the results agree fairly well with the experimental result, particularly the centerline velocity deficit.

$$\frac{\delta^*(x)}{\theta} = a\left[\frac{x-x_0}{\theta}\right]^{1/3} \quad (3)$$

$$\frac{U_0(x)}{U_\infty} = b\left[\frac{x-x_0}{\theta}\right]^{-2/3} \quad (4)$$

Due to the different means of producing the wake, similarity theory suggests that the constants a and b in the equations above can be slightly different for different tests or simulations. As a result, each of the datasets was curve fit to determine its own constants, which are given in Table 1. In addition, the NMAE was determined for each for δ^*/θ and U_0/U_∞ data set using the fit parameters determined for that case. As can be seen in Figure 1, the simulations perform relatively well when estimating the wake deficit, but they are less accurate when predicting the wake growth as accurately at large downstream distances. For comparison, the data of Johansson and George [29] and their fit is given for reference. Notable is the performance of the prediction of wake deficit for case 1 (a RANS model) out to a non-dimensional axial distance of 2000. It is interesting to point out that the parameters a and b vary significantly, but this is not entirely surprising considering that all the wakes were simulated differently. Work is still ongoing to determine the discrepancy between the experimental constants and those for some of the simulations. For the current set of cases, it appears that the case 1 computational approach has the best agreement with data. However, the inclusion of more downstream simulation results may cause other two cases to perform better.

Table 1: Summary of simulation cases, the constants they produced for the wake values, and the error between the calculated and fit results. Fits were limited to 50<(x-x0)/θ<1000 in order to compare all three approaches over the same range of data.

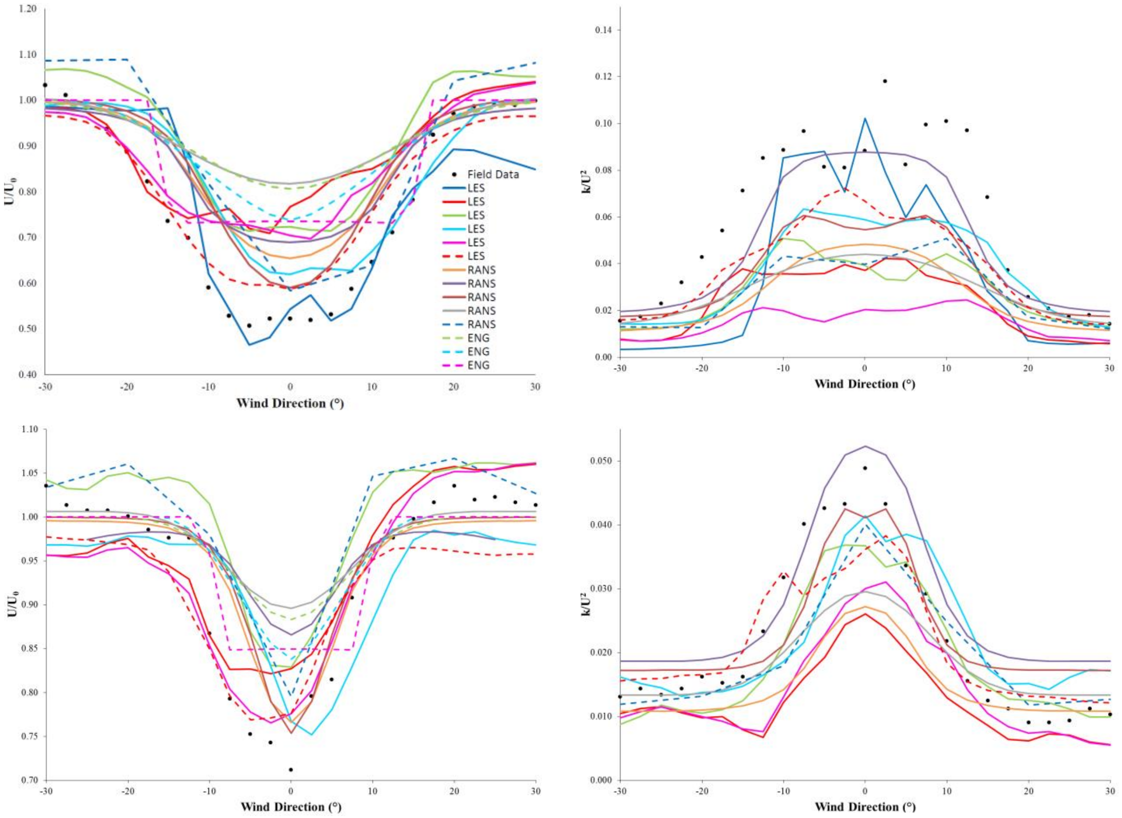
Case	Description	a	b	NMAE δ^*/θ (%)	NMAE U_0/U_∞ (%)
1	RANS	1.09	0.84	1.5	2.8
2	Unsteady RANS	1.15	0.76	8.1	21.8
3	Rotationally Symmetric BL Eqns	1.07	0.98	7.6	8.6
	Johannson & George [29]	1.14	0.77	1.0	3.0

4.2. Sexbierum: Single-Wake Conditions

An experimental campaign was carried out in 1992 at the Dutch Experimental Wind Farm at Sexbierum, in the northern part of The Netherlands about 4 km from the shore. The wind farm is in flat, homogeneous terrain characterized by grassland. It consists of 18 HOLEC turbines with rated powers of 310 kW, rotor diameters of 30 m and hub heights of 35 m. Making use of mobile masts, two measurement campaigns where conducted to measure the wind conditions in the wake of the first [30] and the second [31] turbine in a line of turbines. Two benchmarks are configured accordingly, with the results for the single-turbine case presented here.

The experimental setup for the single-wake case consists of four met masts situated at -2.8*D*, 2.5*D*, 5.5*D* and 8*D* (*D* is the rotor diameter), corresponding to freestream, near-wake, intermediate-wake and far-wake conditions. Prescribed input conditions consist of an incoming hub-height velocity of 10 m/s, turbulence intensity of 10%, roughness length of 0.047 m, turbine power (C_p) and thrust coefficient (C_t) curves and neutral atmospheric conditions (assumed because validation data are long-term averages).

The variables of interest are the dimensionless velocity (U/U_0) and turbulent kinetic energy (tke/tke_0), where the reference values refer to the inflow conditions at hub-height. Figure 2 shows the transverse velocity and *tke* profiles at hub-height and downstream distances corresponding to the near-wake, intermediate-wake and far-wake. The dotted points are the observational data whereas the lines are simulated results for 14 different simulation tools. From these plots, it is obvious that most simulation tools reproduce the general trends in the data, but it is difficult to tell which models may be the most accurate.



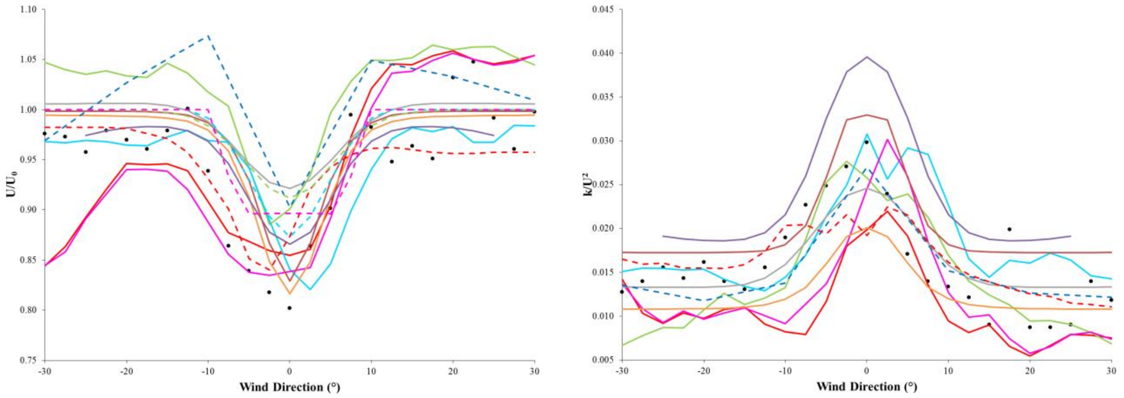


Figure 2: Transverse velocity (left) and *tke* (right) profiles at hub-height and downstream distances 2.5*D* (top), 5.5*D* (middle) and 8*D* (bottom). The variables are made dimensionless by dividing by the upstream freestream conditions at hub-height.

Table 2: Summary of normalized averaged error for Sexbierum benchmarks of transverse profiles of mean velocity and turbulence kinetic energy. Results are from numerous models grouped into three categories described above.

	NMAE of U/U_0 [%]			*NMAE* of tke/tke_0 [%]		
	2.5D	5.5D	8.0D	2.5D	5.5D	8.0D
ENG	20.0	5.0	4.3	-	-	-
RANS	16.9	4.4	3.3	35.8	30.9	32.9
LES	15.0	5.7	5.6	50.1	29.3	27.6

Table 2 encapsulates the normalized mean average error for the three downstream distances for models within the three different categories of fidelity. Notice in both Figure 2 and Table 2 that all models are worst at predicting the properties of the near wake, which is expected for most of these models do not fully resolve turbine blades. Another interesting conclusion to be drawn is that higher fidelity models in the far wake do not always produce the lowest error metric. There are many possible explanations for this, including that the observations are averaged over long periods of time and different atmospheric stabilities, for which it is difficult to simulate the entire range of inflow conditions. More detailed observations over shorter periods of time should provide more useful validation results.

4.3. Horns Rev and Lillgrund: Wake Efficiency as a Function of Wind Direction, Sector Width and Inflow Turbulence

The Horns Rev offshore wind farm, with shared ownership by Vattenfall AB (60%) and DONG Energy AS (40%), has a rated capacity of 160 MW comprised of 80 Vestas V80-2MW wind turbines, with 70 m hub-height, arranged in a regular array of 8 by 10 turbines, and a spacing of 560 m (7*D*) in both directions (Figure 3, left).

The Lillgrund wind farm, operated by Vattenfall, is located at Öresund, the body of water between Malmö, Sweden and Copenhagen, Denmark. The farm consists of 48 Siemens SWT-2.3-93 wind turbines, each producing a rated power of 2.3 MW with a rotor diameter of 93 m and a hub height of 65 m. The turbines are arranged in a dense array with separation of 3.3 rotor diameters (D) within a row and 4.3 D between rows (Figure 3, right). A reference mast measures the western wind without disturbance from the wind farm.

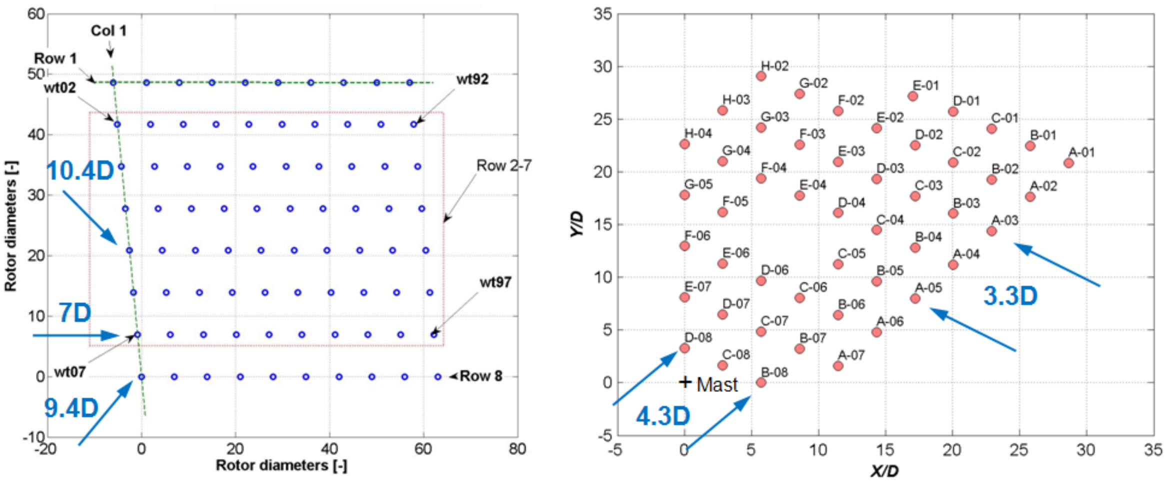


Figure 3: Layout of Horns Rev (left) and Lillgrund (right) with spacing distance along major rows denoted.

In cooperation with the EERA-DTOC European project [33], a series of benchmarks have been defined for both these wind farms by comparing wake efficiency versus wind direction, sector width, and incoming turbulence intensity.

Due to a lack of meteorological measurement at Horns Rev, the inflow conditions have been derived from the wind turbine SCADA data. The ambient wind speed is derived from the power values combined with the official power curve for a undisturbed wind turbines. The wind direction is derived from the undisturbed wind turbine yaw position [27]. The wake deficit is determined as the ratio between the mean power of each turbine and the undisturbed wind turbine.

Considering the 270º wind direction in the Horns Rev wind farm, where the turbines are aligned at 7D spacing, the sector width is varied between 5º and 30º to produce different bin-averaged profiles of wake efficiency down the rows of wind turbines (Figure 4). The wind direction and wake efficiency are derived from the undisturbed reference wind turbine *wt07* for neutral conditions. Meteorological mast data are used to filter for stability (|L| > 500 m, where L is the surface-layer Obukhov length) and define an incoming wind speed of 8±0.5 m/s and turbulence intensity of 7% at hub-height. In this figure, the yellow boxed line represents the observations while all other lines are simulation results.

As discussed by [33] and [34], the ensemble averaging process integrates a large amount of spatio-temporal variability in the wind conditions, most notably the wind direction. Hence, it is important to mimic this variability when post-processing simulations, which are often run with a fixed wind direction, to compare simulated and measured profiles in the same way. Even so, as with the Sexbierum benchmarks, it is again difficult to discern which model is most accurate in Figure 4.

The impact of the wind direction on park efficiencies for the whole wind farm is highlighted in Figure 5. Considering a narrow sector width, the wake efficiency is largely underestimated when the wind direction is aligned with the main turbine rows (full-wake conditions). Introducing a Gaussian weighted-average binning in some of the simulations, to represent the wind direction variability, corrects this effect substantially and produces reasonably accurate results for the whole wind farm.

The development of the wind farm flow depends on the degree of mixing between the wind farm canopy and the freestream above: as the freestream turbulence increases the wake efficiency increases due to a better exchange of momentum. Figure 6 shows the maximum wake deficit for 7D and 3.3D spacing as a function of ambient turbulence intensity. Here the slope of the line for power loss as a function of turbulence intensity is reflected by all models, but the absolute accuracy varies greatly. This is particularly true of the Lillgrund benchmark where turbine operate in the near wake and models are known to be inaccurate.

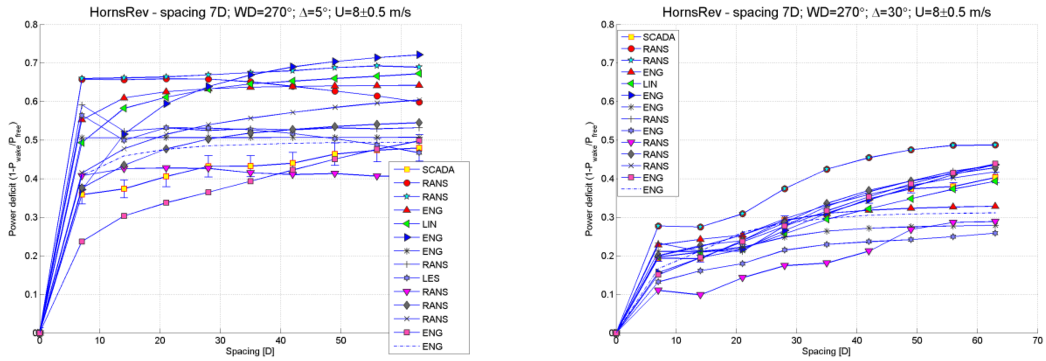


Figure 4: Wake efficiency down the lines of wind turbines in Horns Rev at 270º±2.5º (left, narrow bin) and 270º±15º (right, large bin).

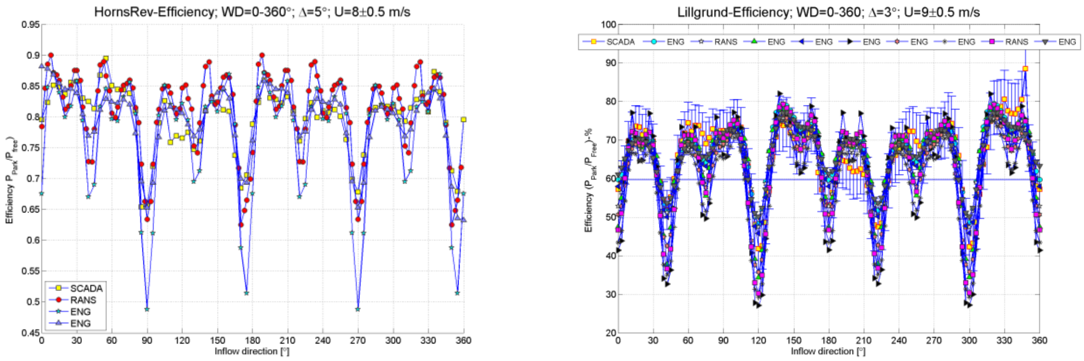


Figure 5: Total wake efficiency per wind direction for Horns Rev (left) and Lillgrund (right) representing similar inflow conditions.

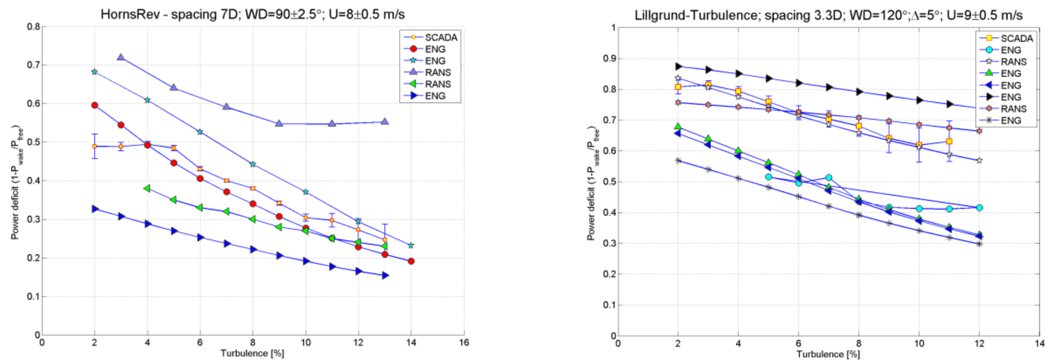


Figure 6: Maximum wake deficit for 7D spacing at Horns Rev (left) and for 3.3 D spacing at Lillgrund (right) representing similar inflow conditions.

Table 3 quantifies the normalized average error for each of these individual benchmarks and a number of simulation tools applied to the Horns Rev wind farm. Simulations from LES models were only available for the narrow wind sector benchmark as producing results for the other benchmarks was deemed too computationally expensive. Some interesting trends from these results are that the LES models do have lower error than the others in terms of wake efficiency. However, the RANS models have larger errors than engineering models for all benchmarks. This is in part due to the fact that most engineering models were tuned to these datasets before the comparison studies. And it may also reflect the coarseness of the observational data that consists of an average of numerous atmospheric and wind farm operating conditions. More precise measurements from these wind farms will be required to gain a better understanding of model performance and potential for improvement.

Table 3: Summary of normalized averaged error for Horns Rev benchmarks of wake efficiency. Results are from numerous models grouped into three categories described above.

	NMAE of η_{wake} [%]			
	ΔWD = 5º	ΔWD = 30º	Turbulence	Wind Direction
ENG	29.5	13.2	24.5	3.1
RANS	30.4	19.4	39.3	4.0
LES	20.6	-	-	-

5. Conclusions

The IEA Task 31; Wakebench is an international effort for verification and validation of wind farm modelling tools. Several benchmarks have been executed by task participants ranging from theoretical to full scale wind farms. In general, trends in the benchmark observations are reproduced by the simulation results, but accuracy is highly variable. In the comparison of accuracy between levels of model fidelity, there is no obvious winner, as sometimes LES or RANS models have lower normalized average error than lower fidelity models, but often their error is higher. This does not mean further high fidelity model development is not important for future work, but rather indicates more detailed insight into individual models and the experimental observations is required to better inform differences in model accuracy. For example, more precise and delineated observations of wind farms under many different operating and atmospheric conditions will provide better validation data than those averaged over long periods of time. Such insight will lead to informed best practices as to model usage for wake modelling and wind farm design, help quantify uncertainty bounds for different modelling tools and also help determine the most useful quantities for model validation that will guide future field measurement campaigns.

Wakebench will continue to provide more detailed and useful validation studies and best practices through further detailed analysis of existing benchmarks and the addition of new benchmarks. Examples of upcoming benchmarks include: a theoretical infinite wind farm, detailed lidar single and double wake measurements for a scale turbine, wind tunnel testing with varying atmospheric stability, additional offshore wind farms and wind farms in complex terrain and varying vegetation.

6. Aknowledgements

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