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Investigation of modified AD/RANS models for wind turbine wake predictions in large wind farm

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Abstract. Average power losses due to multiple wind turbine wakes in the large offshore wind farm is studied in this paper using properly modified $k-\omega$ SST turbulence models. The numerical simulations are carried out by the actuator disc methodology implemented in the flow solver EllipSys3D. In these simulations, the influence of different inflow conditions such as wind direction sectors are considered and discussed. Comparisons with measurements in terms of wake speed ratio and the corresponding power outputs show that the modified turbulence models had significant improvements; especially the SST-Csust model reflects the best ability in predicting the wake defect. The investigations of various inflow angles reveal that the agreement between predicted and measured data is improved for the wider sector case than the narrow case because of the wind direction uncertainty.

1. Introduction

For the limited area of wind farm, wind turbines are often installed as closely as possible to each other because of economical constraints and land constraints. In this case, power losses due to wake interaction from the surrounding wind turbines and higher loads at turbines which experiencing wakes are inevitable. Therefore, in order to properly assess the wind farm power efficiency, the characteristic of wind turbine wake should be accurately investigated at first.

Different approaches, from analytical and engineering models to computational fluid dynamics (CFD) of full rotor simulation using Reynolds Averaged Navier-Stokes (RANS) model or large eddy simulation (LES), have been proposed and developed to predict wind turbines induced wakes. Among the numerical methods, RANS model becomes widely used in wind energy projects because of the computational efficiency [1, 2], and meanwhile better accuracy than the existing engineering models (such as Jensen model [3] or Ainslie model [4]). However, problems exist [5] when it comes to the application of conventional RANS models in predicting the total energy output of large wind farms. Several studies [1, 2, and 6] indicated that the predicted power output has a large discrepancy when comparing with the field measurement. This disagreement is due to the fact that RANS model over-predicts power losses in a narrow wind sector of less than 10 degrees, this is even more noticeable in the full wake conditions where the simulated power outputs are under-predicted by up to 50% [5].



Most of the previous studies were carried out using turbulence model (e.g. steady state two-equation turbulence models such as k- ϵ and k- ω) coupled with actuator disc/line model for turbine rotor parameterization. The discrepancies between numerical predictions and experimental data reflect the need for improvement of both turbulence modeling and turbine parameterization. In which, a big part of these improvements is related with turbulence model modification. The processes of these works usually include two parts: investigating the performance of classical turbulence models firstly and then correcting these models according to their individual property, theoretical basis or experimental measurements. In 2008, Kasmin and Masson [7] proposed a modification for the dissipation term in the ϵ -equation representing the transmission of generated turbulence energy from large scale to small scale. Later, Prospathopoulos et al. [8] summarized four different revised models and the applications were made on single wind turbine as well as wind farm with five wind turbines in a row. In order to account for the anisotropy instinct of the atmospheric turbulence, a modified Reynolds Stress Model (RSM) was proposed by Makridis [9] and then was assessed in the study of wind turbine wake interactions in a complex wind farm. From the comparisons with field measurements it can be concluded that these newly developed models are much reliable.

In the previous work [10], three suitably modified turbulence closures which are based on k- ω SST turbulence model are assessed for predicting the wake of a stand-alone wind turbine Nibe-B. Comparisons with available field measurements show that these corrected models indeed give good agreements in terms of the predicted wind speed and turbulence intensity in the single wake flow. This is the initial work on turbulence model modifications by the present authors.

In order to ensure the accuracy and generality of these modified turbulence models, further validations are highly needed. In the present paper, numerical tests on the performance of these corrected models are carried out employing the EllipSys3D solver coupled with the actuator disc method. The simulation of a single turbine Vestas-V80 is carried out at first to test the precision of the developed models. Afterwards, the corrected models are applied to multi-wake simulations in a large offshore wind farm. The influences of different inflow conditions, such as wind directions and wind speed intervals, on the wake deficit and associated power output are investigated. Predictions are compared with measurements for the wind direction of 221° at various sector widths ($\pm 1^\circ$, $\pm 5^\circ$). The preliminary results reveal that the standard SST model highly over-predict wake losses while the developed models all show varying degrees of improvement, especially the SST-Csust model, provide satisfactory predictions for power losses in the large wind farm simulation. This proves that the newly proposed SST-Csust model is more accurate in both single and multi-wake predictions.

2. Numerical methodology

2.1. Standard k- ω SST model

Previous investigations [11] show that the k- ω SST turbulence model is acceptable in the wind engineering research field because of the superior performance in dealing with the relatively complex flow problems. According to this, the SST model [12] with the following form is employed in our work for the purposed numerical simulations.

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho U_j k)}{\partial x_j} = \tau_{ij} \frac{\partial U_i}{\partial x_j} - \beta^* \rho k \omega + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_j} \right] \quad (1)$$

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial(\rho U_j \omega)}{\partial x_j} = \frac{\gamma}{\nu_t} \tau_{ij} \frac{\partial U_i}{\partial x_j} - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_\omega \mu_t) \frac{\partial \omega}{\partial x_j} \right] + 2(1 - F_1) \frac{\rho \sigma_{\omega 2}}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \quad (2)$$

2.2. Modified turbulence models

2.2.1. Turbulence decay problem. Before introducing the modified turbulence models, a physical phenomenon which forms big part of the theoretical foundation of our performed modifications, needed to be explained.

When the wind shear is not taken into account for the atmospheric flow simulations, the flow field which positioned in the upstream of the wind turbine can be regarded as the free-stream flow area. Spalart et al. [13] pointed out that the decay of turbulence happens when employing RANS turbulence models predicting the free-stream flow. This means that the turbulence quantities specified at the inlet boundary suffer a quite rapid decay as the flow moves downstream, leading to highly decreased turbulence level at rotor plane. This unphysical decay can be theoretically explained as follows.

In the free-stream flow, the flow variables are consistently distributed, there is no mean velocity gradient to generate new turbulence and the variation of turbulence parameter also does not exist, therefore the production term and the diffusion term can be considered negligible. At this point, the standard equations (as in equation (1) and (2)) are reduced to a very simple form with only the destruction terms appear on the right side. Through solving the simplified equations, the decay of turbulence quantities can be expressed using the following solutions:

$$k = k_{inlet} \left(1 + \frac{\omega_{inlet}\beta x}{U} \right)^{-\frac{\beta^*}{\beta}} \quad (3)$$

$$\omega = \omega_{inlet} \left(1 + \frac{\omega_{inlet}\beta x}{U} \right)^{-1} \quad (4)$$

where the subscript 'inlet' represents the parameters specified at inflow boundary, x is the stream-wise distance, U is the local velocity, β and β^* are SST model constants with the value of 0.0828 and 0.09, respectively.

2.2.2. SST-sust model. In order to balance the unphysical decay of turbulence in the free-stream flow, the SST-sust model was proposed in reference [13], with the basic idea of adding sustain terms into the transport equations as the source terms. The new added terms have the function of accurately cancel out the destruction terms in free-stream flow, with the following expression:

$$S_k = \beta^* \rho k_{amb} \omega_{amb} \quad (5)$$

$$S_\omega = \beta \rho \omega_{amb}^2 \quad (6)$$

where the subscript 'amb' represents the ambient values. It should be noted that only when the 'inlet' value and the 'amb' value are set equal to the desired values the decay of turbulence can be fully eliminated.

2.2.3. SST-const model. According to the wind tunnel experiment and related theory about the neutral atmospheric flow, Prospathopoulos et al. [14] pointed out that the set of coefficients of k - ω turbulence model should be corrected into the following values:

$$\beta^* = 0.033; \quad \beta_1 = 0.025; \quad \gamma_1 = 0.3706 \quad (7)$$

2.2.4. SST-Csust model. The performed numerical experiments showed that the revised SST-sust model and SST-const model are still not enough to accurately predict the wake effect. This is mainly due to the fact that the turbulence intensity is under-predicted by these models. In order to remedy this issue, the SST-Csust model was proposed in reference [10]. It is the combination of SST-sust model and SST-const model, with the letter C represents the coefficients of the turbulence model is further corrected based on the SST-sust model.

Figure 1 shows the numerically predicted decay of turbulence kinetic energy as a function of the stream-wise distances in the free-stream flow and theoretical results from equation (3) are also included. From the red line in this figure, it can be seen that the initial value decays drastically as the

distance from the inlet increases. Most importantly, the agreement between results of standard SST model and the theoretical solutions approves the validity and usability of our flow solver EllipSys3D.

When it comes to the revised turbulence model, as illustrated in figure1, the models with additional sustain terms (such as SST-sust model and SST-Csust model) consistently show the ability of eliminating the decay of turbulence kinetic energy as expected (note that these two lines are overlapped and have a constant value of 1.0), while the SST-const model slightly controlled the decay to some degree.

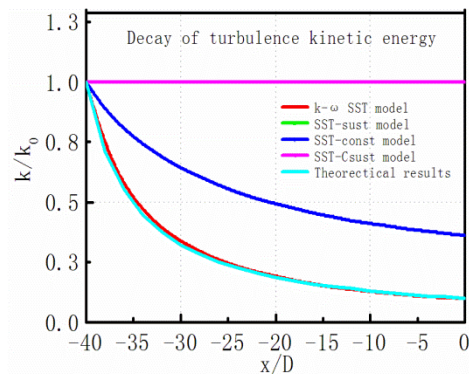


Figure 1. Normalized turbulence kinetic energy in the free-stream flow.

2.3. Actuator disc modelling

The actuator disc technique developed by Mikkelsen [15], representing an acceptable approximation of the wind turbine rotor without leaving the main essence of the involved physics, has been widely used in the wind turbine wake investigations, especially for the multi-wake predictions in large wind farms. This model combines a three-dimensional in-house developed flow solver EllipSys3D [15] with a technique in which the rotor is represented by a permeable disc of equivalent area where the body forces are distributed on. The body forces acting on the disc are calculated through the local angle of attack and a look-up table of airfoil coefficients.

3. Wind farm layout

To increase the understanding of wake effects in large wind farms a number of projects have been carried out with the purpose of investigating and quantifying the wake characteristic. Among all of these, Horns Rev offshore wind farm [2] which located about 12-17 km off the Danish west coast has become extensively studied since plenty of meteorological and SCADA data are available for comparisons. Besides that, this wind farm is similar to a number of exiting wind farms: it has a rectangular arrangement and the distance between each turbine is typical (there is a rule of thumb which used to design a wind farm layout project mentions that the suitable separation distance between two adjacent wind turbines is 7~15 rotor diameters when parallel to the predominant direction of wind [16]).

As shown in figure 2, 80 Vestas-V80 2MW wind turbines which have a feature of rotor diameter $D=80\text{m}$, hub height $H=70\text{m}$ are positioned in a matrix shape of 10 rows by 8 columns with the same turbine spacing $7D$. From the layout of wind turbines, it can be seen that the separation distance between surround wind turbines is highly relied on the incoming wind direction. For instance, the separation distance is $9.4D$ when the wind direction changes into 221° (as shown in figure 2). This situation is chose as the numerical test case to evaluate the performance of modified turbulence models at Horns Rev wind farm. In this case, two different direction intervals $[-1^\circ, +1^\circ]$ and $[-5^\circ, +5^\circ]$ are discussed. The relative numerical set-up will be discussed in detail in the following part.

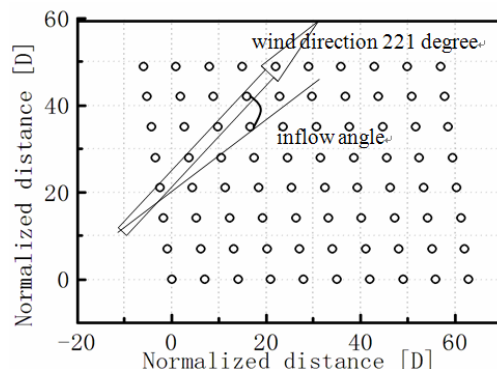


Figure 2. Layout of Horns Rev offshore wind farm.

4. Results and discussions

Standard SST turbulence model and its modifications, coupled with actuator disc method are employed to simulate the multiple wake interactions and to predict the associated power losses in the Horns Rev wind farm. Here we focus on the wind direction 221° with two sectors $\pm 1^\circ$ and $\pm 5^\circ$. For both of them, the available measurements are collected under near-neutral stability condition for the wind speed of 8m/s and the averaged turbulence intensity of 7% at hub height level. In the following conducted simulations, the incoming wind flow condition is assumed as uniform inflow, where the values of velocity and turbulence intensity are consistent in all the computational free-stream area. The turbulence intensity of 7% is numerically represented by boundary conditions specified at inflow with initial values of k and ω . Almost the same turbulence level is achieved at rotor plane using the modified RANS model.

4.1. Single wake test

Before the predictions on multi-wake interactions are performed, the simulation of a single wind turbine wake is carried out at first to test the validity and generality of the corrected models. Figure 3 shows the comparison of measured and simulated wake speed ratio in the cross-wind direction at downstream positions $x/D=9.4$. This distance is exactly the wind turbine separation distance when the incoming wind direction is 221° .

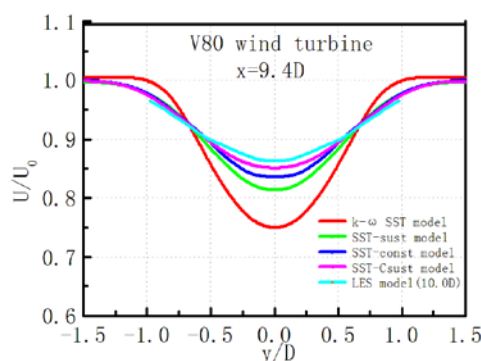


Figure 3. Wind speed ratio in the cross-wind direction at down-stream positions of 9.4D.

In this figure, the numerical results that obtained using the standard $k-\omega$ SST model and its corrections are compared with the LES model results that given by Wu [17]. As depicted in figure 3, all of the corrected models show improvement to some extent as compared with the standard SST model. Among of them, the predicted velocity deficit by SST-Csust model shows acceptable agreement with LES results and the observed little discrepancy is caused by the fact that the LES results are obtained

further downstream at the position of 10.0D. We can also find that both of the SST-const model and the SST-sust model under-estimate the wake speed with the former behaves better than the latter. While the original SST model obviously under-predict the wake speed, with the simulated minimum speed ratio in the wake center is nearly 15% less than the corresponding results of LES model.

4.2. Multi-wake tests (sector $221^\circ \pm 1^\circ$)

The flow direction at 221 degrees is a row consists of 5 turbines; figure 4 shows the simulated wind speed of the wake center-line at each turbine for this direction, where the numerical results are obtained using standard k- ω SST model and its corrections. In this figure, the wind speed is normalized to free-stream velocity which was derived from the measured electrical power output using the power curve of the Vestas-V80 wind turbine. The error bar included in the plot show the standard deviation of the measured 10-min averages and thus represents the scatter of the measurements. Similarly to this, the predicted power productions are presented in figure 5.

About the effect of the developed turbulence models, figures 4 and 5 reveal that the SST-Csust model gives the best agreement with the measurements, and then followed by SST-const model while SST-sust model comes the third, which is quite similar to their performances on single wake simulation (see figure 3). When compared with the results of standard SST turbulence model, it can be seen that the SST-Csust model has improved by 11.7% and 15.6% for the predictions of wake speed ratio and power production, respectively.

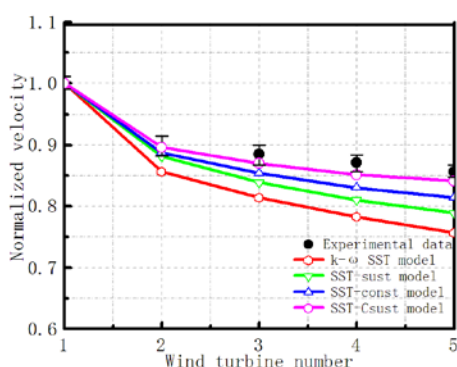


Figure 4. Comparisons of the measured and simulated (the inflow angle is 0°) normalized wake velocity deficit for the wind direction sector $221^\circ \pm 1^\circ$.

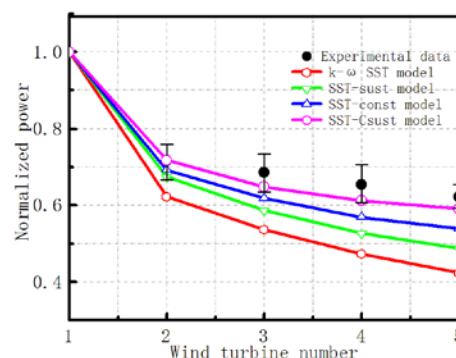


Figure 5. Comparisons of the measured and simulated (the inflow angle is 0°) normalized power production for the wind direction sector $221^\circ \pm 1^\circ$.

Although the SST-Csust model performs better than the others, as illustrated in figures 4 and 5, the small discrepancy between numerical and measured data still exists, especially for the last three wind turbines. This is mainly due to the fact that the numerical results in figures 4 and 5 are obtained under the situation that the inflow angle is 0° , while the measured data are collected for a narrow sector $\pm 1^\circ$. When the wake sector is 1° , the downstream wind turbine would be $0.164D$ apart from the wake center of the upstream one. This means less wake deficit would be generated than the full wake situation, in which the incoming wind direction is parallel to the turbine rows (which is the numerical case with inflow angle 0°). In order to fairly validate the behaviours of corrected turbulence models, the simulations corresponding to the narrow wake sector of $\pm 1^\circ$ are carried out. Figure 6 shows the predicted power losses using SST-Csust for two different inflow angles: 0° and 1° . Besides, the experimental data are also included for comparison.

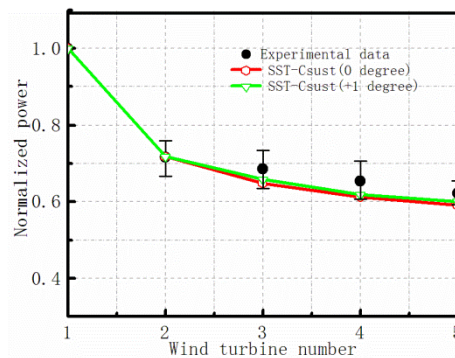


Figure 6. Simulated normalized power using SST-sust model for two kinds of inflow angle: 0° and 1° .

From figure 6 we find that the predicted power is increased a little bit for the case with 1° inflow angle than the full wake condition, even though the amount of growth is very limited. In general, it can be seen that the numerical models consistently under-estimate the wake velocity and related energy output for a narrow wind direction bin of $\pm 1^\circ$. Similar results were also reported in previous studies [18, 19]. In reference [19] the authors pointed out that the discrepancy between numerical predictions and the experimental data is not caused by the physical essence of the models, but rather by a wind direction uncertainty in the measured data. For a narrowest wind direction sectors $\pm 1^\circ$, there are only a limited number of available measurements for data analysis, this mainly attributes to the uncertainty of the dataset.

4.3. Multi-wake tests (sector $221^\circ \pm 5^\circ$)

Figures 7 and 8 present the comparisons of simulated wake velocity as well as power production with the measurements for each turbine when the wind direction sector is $221^\circ \pm 5^\circ$. In these two figures, the numerical results are obtained at the situation with an inflow angle of 2.5° , which is the intermediate value of the direction interval $[-5^\circ, +5^\circ]$. Unlike this, the measured data are obtained for the wind sector $\pm 5^\circ$.

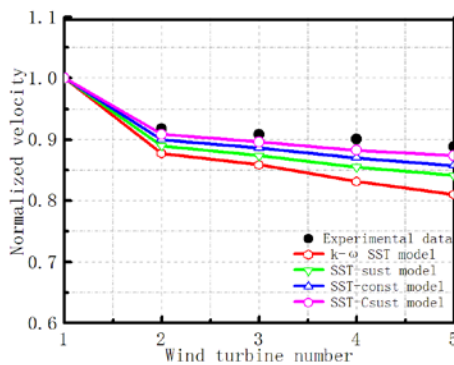


Figure 7. Comparisons of the measured and simulated (the inflow angle is 2.5°) normalized wake velocity deficit for the wind direction sector $221^\circ \pm 5^\circ$.

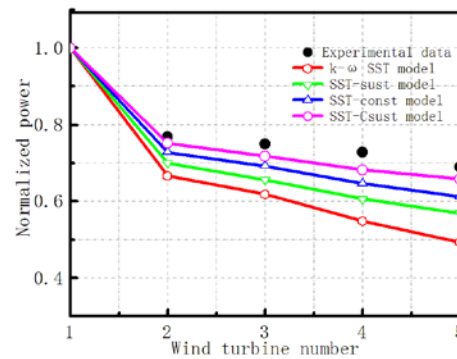


Figure 8. Comparisons of the measured and simulated (the inflow angle is 2.5°) normalized power production for the wind direction sector $221^\circ \pm 5^\circ$.

The improvement of the modified turbulence model is clearly shown in figures 7 and 8, where the predicted wake velocity has been increased by 10.7% as compared with the standard SST model. Besides, it should be noted that the differences between the results predicted by three modified turbulence models are not as obvious as that of the case with 0° inflow angle (as shown in figures 4

and 5). That is because for the two inflow angles 0 degree and 2.5 degrees, the downstream wind turbines are located along the wake center and 0.41D apart from the wake center of the upstream one, respectively. As illustrated in figure 3, there is a big gap between the predicted wind speed ratios at the position of wake center ($y/D=0.0$) while this gap becomes narrow at the position $y/D=0.41$ (here y means the distance to the wake center in the cross wind direction). This may explain why the numerical results are close to each other when the inflow angle is not 0 degree.

In order to fairly compare the numerical results with the measurements, simulations using SST-Csust model for inflow angle of 5 degree are performed and the results are presented in figure 9. In addition to that, the predictions corresponding to the inflow angle of 0° and 2.5° are also included in this figure. Here we average the simulated power results obtained from the three cases and finally get the mean value as shown in figure 9.

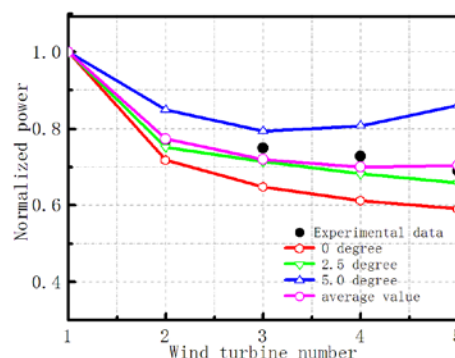


Figure 9. Simulated normalized power using SST-sust model for different kinds of inflow angle: 0° and 2.5° and 5° .

It should be noted that the mean value of simulated power production for the wider sector $221^\circ \pm 5^\circ$ is more close to the related measurements than the results of the narrow sector ($\pm 1^\circ$) case. This is mainly due to the fact that when the sector is increased the uncertainty of wind direction becomes less significant and less data are filter in the wrong intervals. From this it can be concluded that the relatively low accuracy of the modified models for the case with narrow sector $221^\circ \pm 1^\circ$ is mainly caused by the uncertainty of the dataset, while the acceptable accuracy of the modified models that obtained for the case with wider sector $221^\circ \pm 5^\circ$ is more representative.

5. Conclusions

Previous research performed on a single wake prediction has proved the significant improvement of each modified models, especially the proposed SST-Csust model. The main purpose of this work is to further evaluate the performance of the developed turbulence models in simulating the multi-wake interactions in a large offshore wind farm. The numerical simulations are carried out based on the actuator disc model implemented in the flow solver EllipSys3D.

In Horns Rev wind farm, the separation distance between upstream and downstream wind turbines is highly relied on the incoming wind direction. Here we choose a row consists of 5 wind turbines with the separation distance of 9.4D at the wind direction of 221 degree. In this case, the effect of different inflow angles on the predicted wake speed ratio and the corresponding power productions are discussed. The simulation results show that:

- 1) The predicted velocity and power using SST-Csust model are in good agreement with the measured data from Horns Rev wind farm. While the left two models over-predict the wake deficit and power losses.

- 2) For narrow sector of 1 degree, because of the wind direction uncertainty, the performance of SST-Csust model is not as good as that for the relatively wider sector of 5 degree.
- 3) For a row of wind turbines, that second one experiences the most serious wake effect, where the power decreased nearly 40% as compared with upstream turbine. Continuous decrease is still observed when the flow moving downstream through the whole wind farm but the decrease speed has been obviously slow down.

Through the comparisons with measured data and the previous work [17], the versatility of the developed SST-Csust model has been assessed in the single and multiple wake simulations performed in this work. Despite this, more future research is still needed to further validate the correct model for flow cases at Horns Rev wind farm and other data available wind farms.

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