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Dou, Suguang; Strachan, B.S.; Shaw, S.W.; Jensen, Jakob Søndergaard

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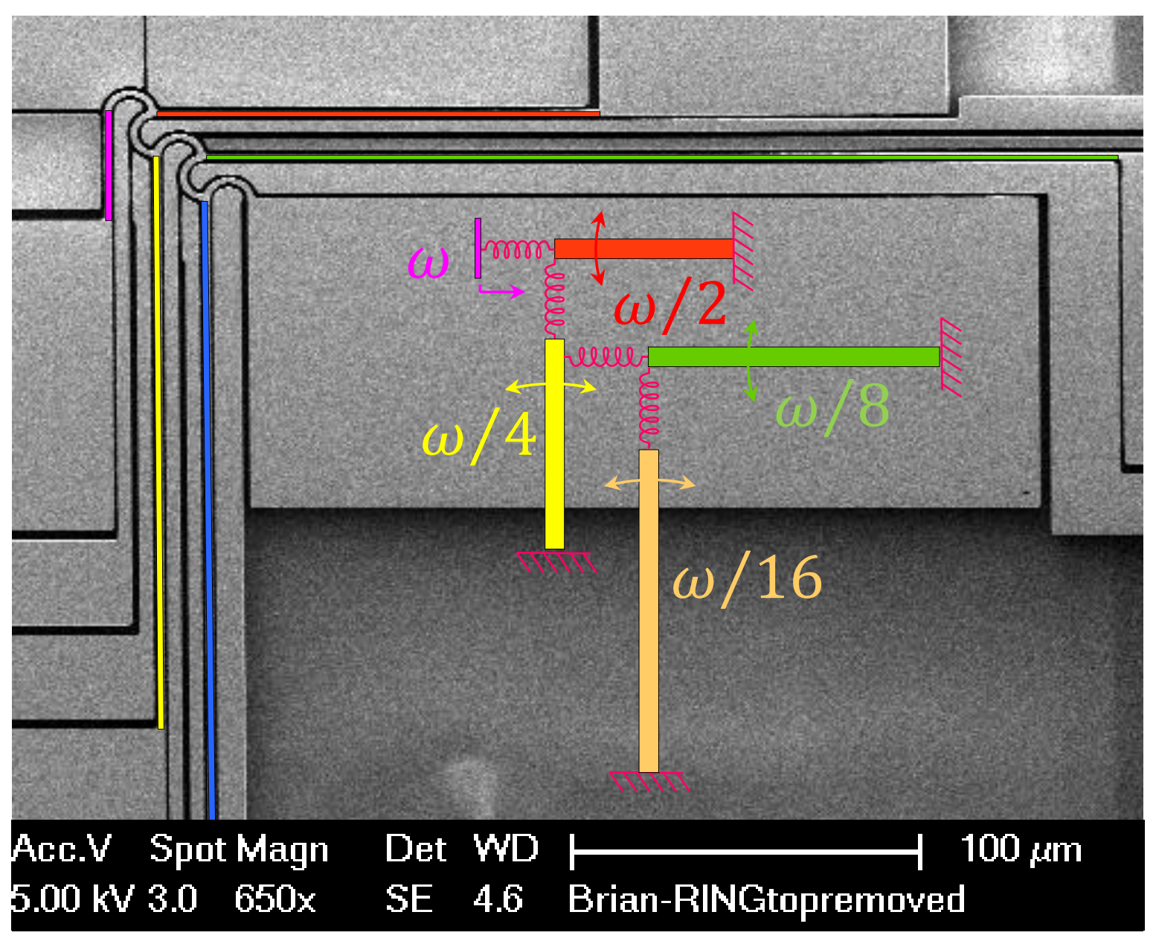
CHARACTERIZATION AND OPTIMAL DESIGN OF NONLINEAR MEMS RESONATORS

S. Dou[1], B.S. Strachan[2,3], S.W. Shaw[2], J.S. Jensen[4]

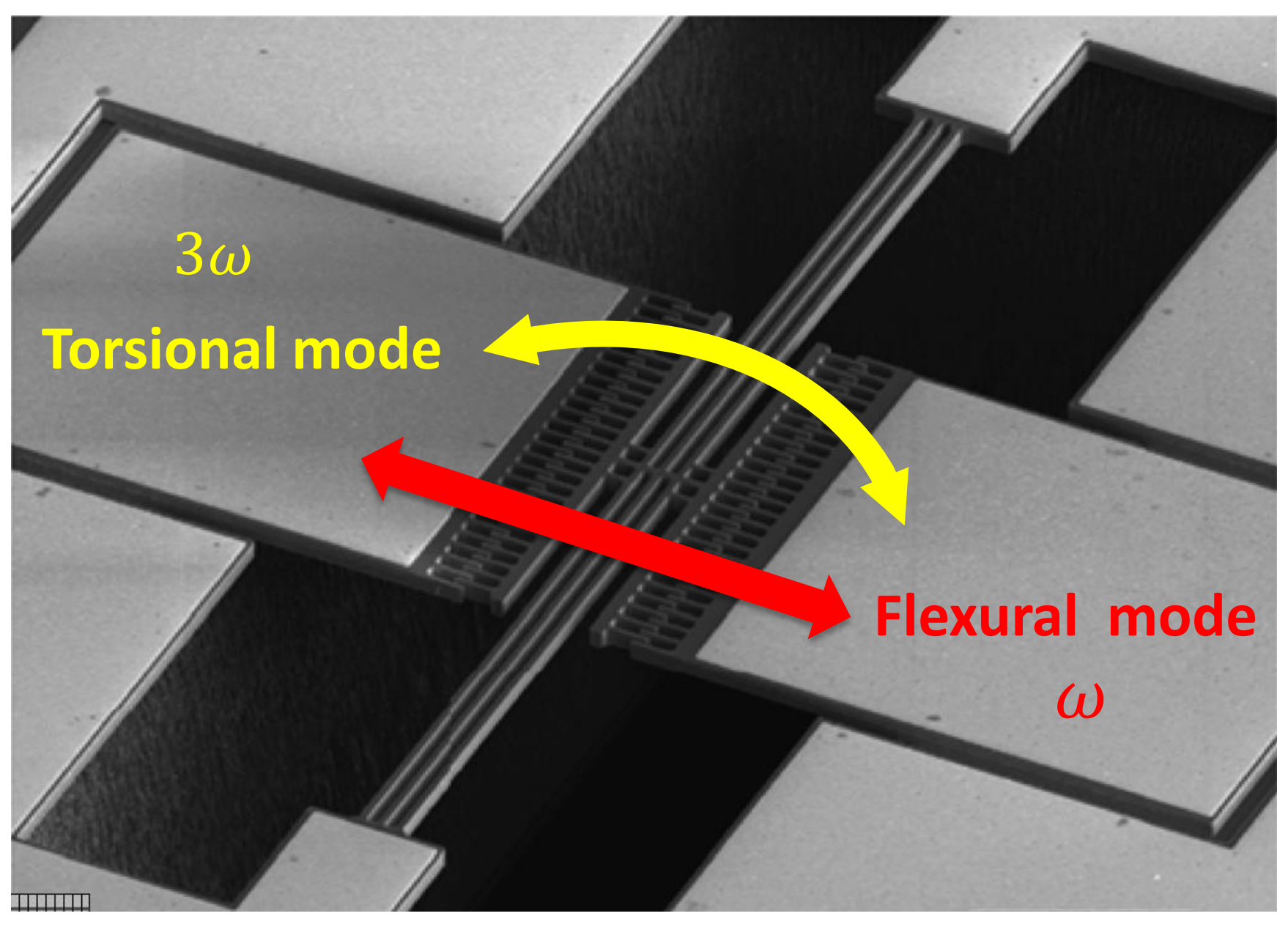
[1]Dep. of Mechanical and [4]Dep. of Electrical Engineering, Technical University of Denmark, Denmark
[2]Dep. of Mechanical and [3]Dep. of Electrical Engineering, Michigan State University, USA

I. Motivation

Optimization of energy transfer in a frequency divider cascade [1]



Minimization of phase noise of a coupled mode oscillator [2]



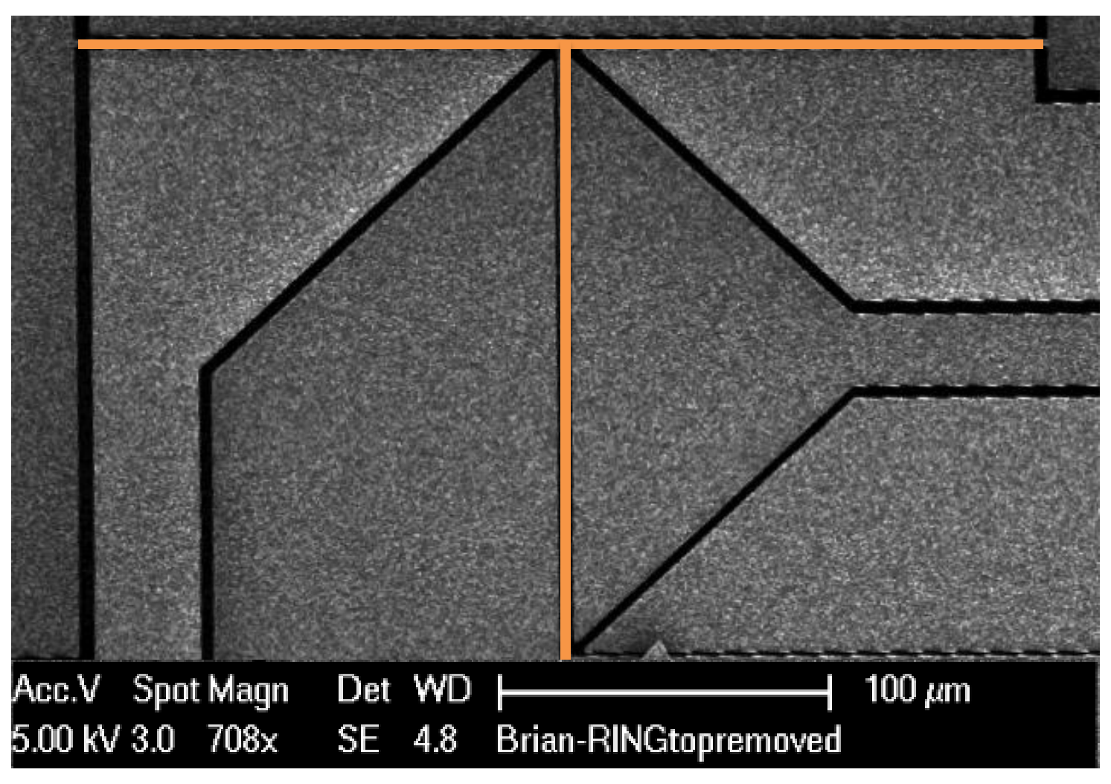
(left) Courtesy of K. Turner, UCSB; (right) Courtesy of D. Lopez, ANL

II. Example: T-Bar Frequency Divider

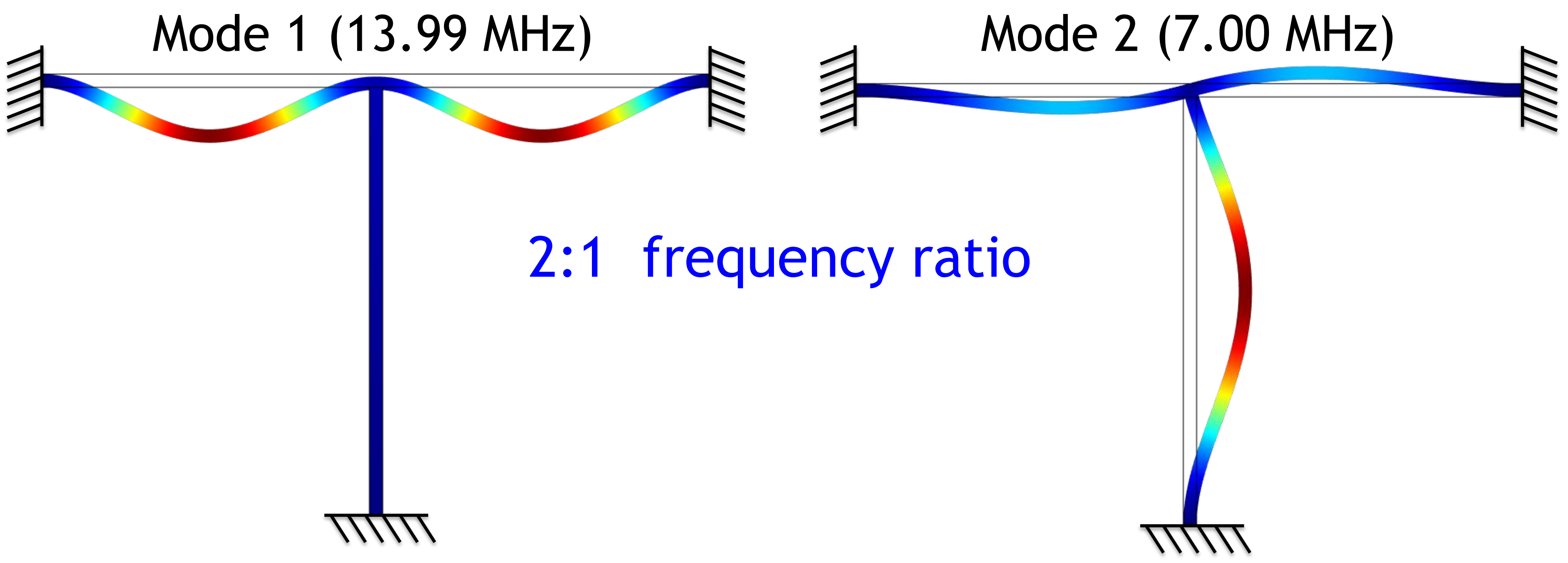
Generic two mode model for divide-by-two device. Note that frequency ratio must be 2:1 and that division requires energy transfer via nonlinear effects, captured by the coupling coefficient β. Approach allows determination of nonlinear coefficients γ_n and β from finite element model.

$$\ddot{x}_1 + \omega_1^2 x_1 + \gamma_1 x_1^3 + \beta x_2^2 = F_1(t)$$

$$\ddot{x}_2 + (\omega_2^2 + 2\beta x_1) x_2 + \gamma_2 x_2^3 = F_2(t)$$



(right) SEM two-mode T-bar cascade; orange lines show structure; courtesy of K.Turner, UCSB
(below) two coupled modes simulated in COMSOL



III. Optimization Procedure

1. Develop crude design based on nonlinear phenomena and create a finite element model.
2. Solve the finite element model eigenproblem. Use N modal eigenvectors to obtain the nonlinear coefficients of the potential of the reduced order model [3] :

$$\sum_{i,j\geq i,k\geq j}^{N} \alpha_3^{i,j,k} x_i x_j x_k + \sum_{i,j\geq i,k\geq j,l\geq k}^{N} \alpha_4^{i,j,k,l} x_i x_j x_k x_l$$

3. Define the objective function and constraints. These can be based on:
    - nonlinear modal frequency responses
    - reduced order model coefficients
    - a function of the model coefficients
4. Update design variables based on shape optimization scheme and repeat steps 2-4 until convergence.

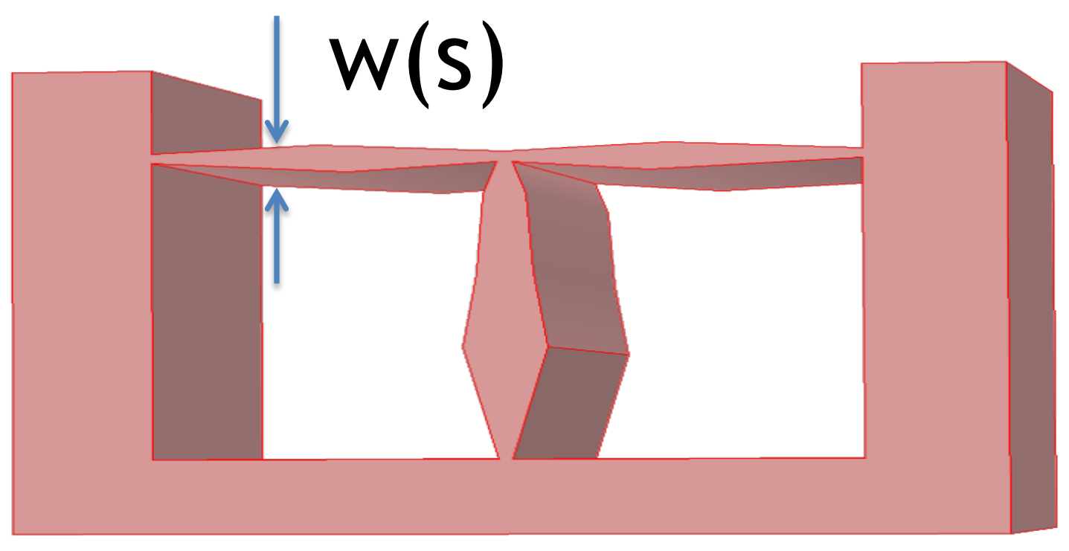
IV. T-Bar Divider: Design Optimization

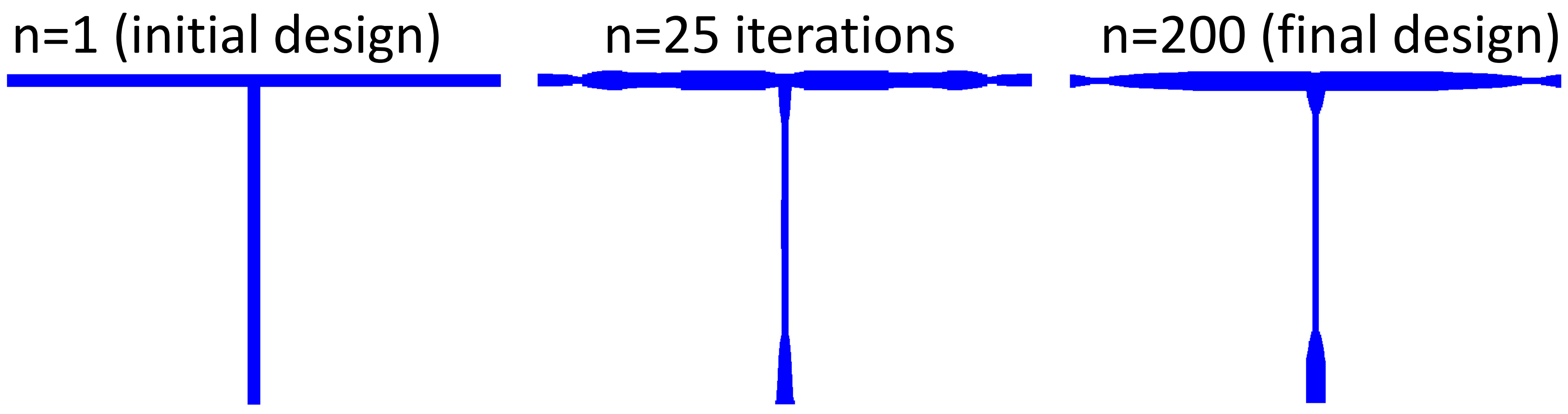
Objective Function:
Maximize the normalized coupling coefficient:

$$c = \frac{|\beta|}{\omega_1^2} = \frac{|\alpha_3^{122}|}{\omega_1^2}$$

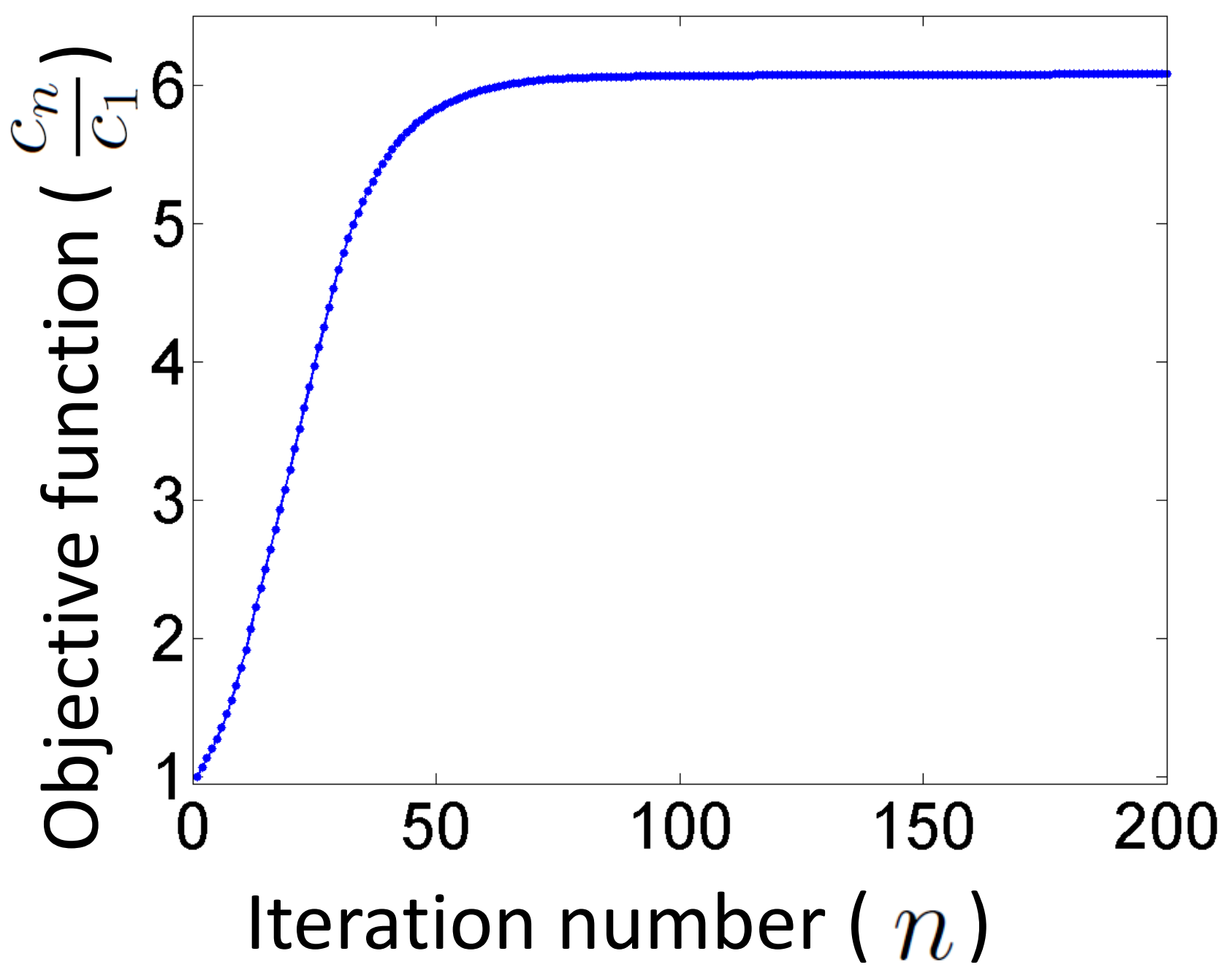
Design variables: w(s), the beam width

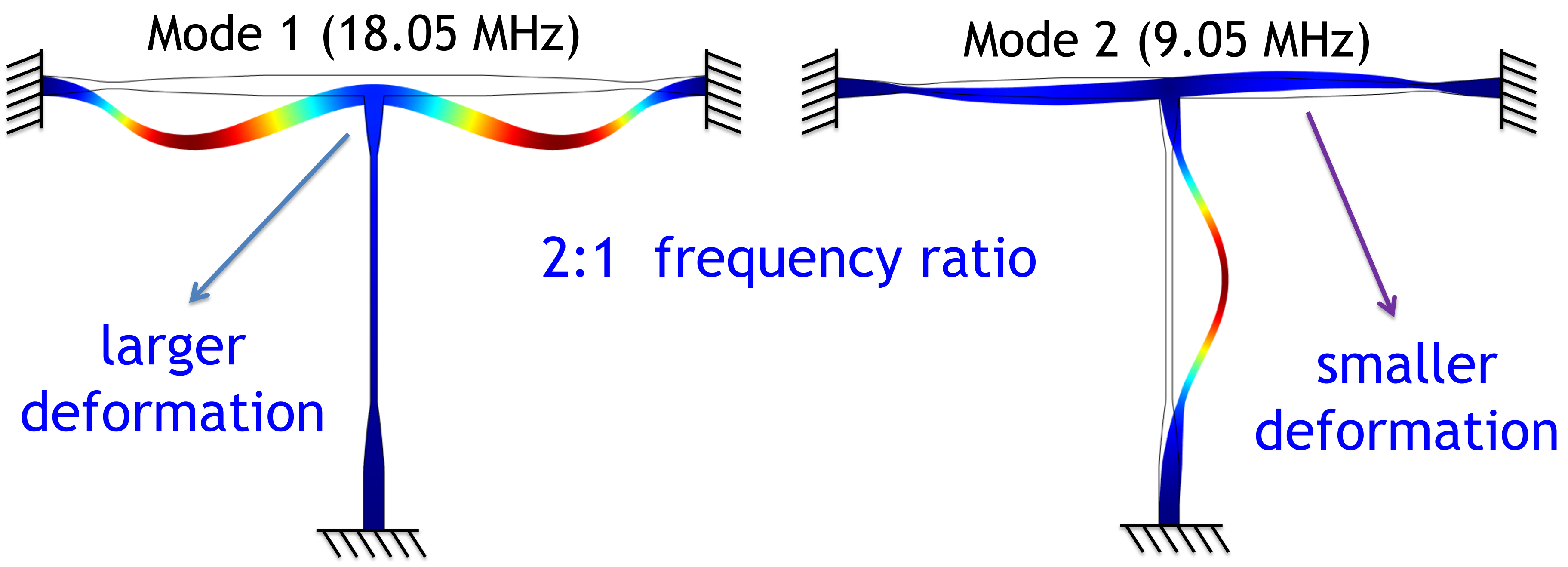
Constraints:
- 2:1 Frequency Ratio
- Min/Max Beam Width
- Volume constraint





Effects of optimization:
1. Frequency ratio maintained but frequencies vary.
2. Vertical beam remains almost uniform, thinned;
3. Horizontal beam has variable thickness, thickened;
4. **Modal coupling coefficient is increased by a factor of 6.**





Benefits:
- ➢ Characterization of nonlinear resonators from FE model
- ➢ Systematic optimization tools for nonlinear resonators

V. Acknowledgements

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VI. References

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