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CHARACTERIZATION AND OPTIMAL DESIGN **OF NONLINEAR MEMS RESONATORS** S. Dou¹, B.S. Strachan^{2,3}, S.W. Shaw², J.S. Jensen⁴

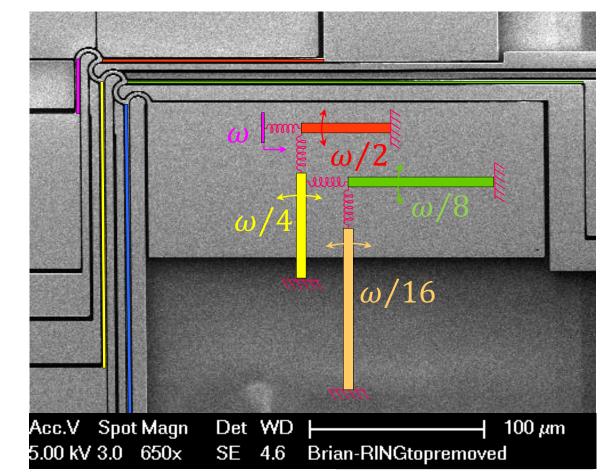


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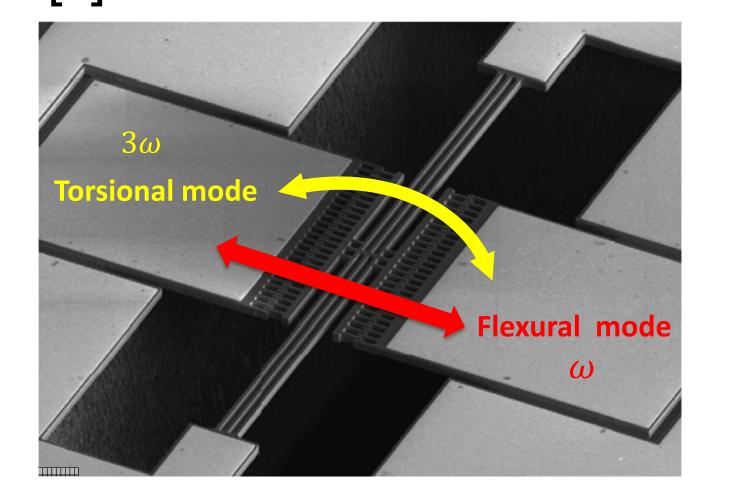
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I. Motivation

Optimization of energy transfer in a frequency divider cascade [1]

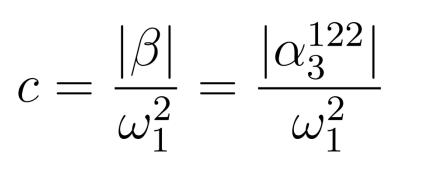


Minimization of phase noise of a coupled mode oscillator [2]



IV. T-Bar Divider: Design Optimization

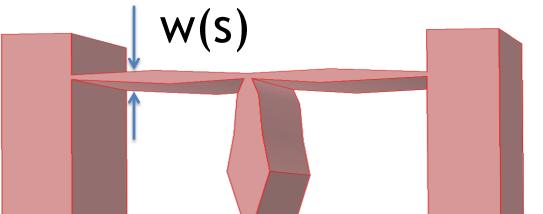
Objective Function: Maximize the normalized coupling coefficient:



Design variables w(s)

Constraints:

- 2:1 Frequency Ratio
- Min/Max Beam Width
- Volume constraint



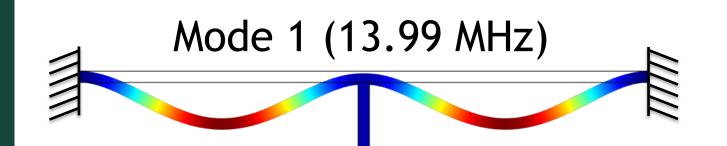
(left) Courtesy of K. Turner, UCSB; (right) Courtesy of D. Lopez, ANL

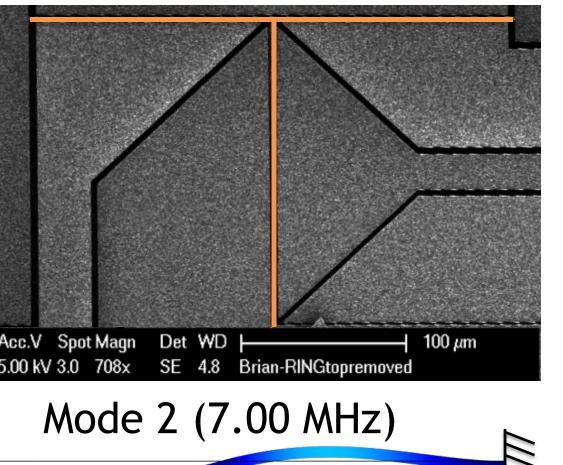
II. Example: T-Bar Frequency Divider

Generic two mode model for divide-by-two device. Note that frequency ratio must be 2:1 and that division requires energy transfer via nonlinear effects, captured by the coupling coefficient β . Approach allows determination of nonlinear coefficients γ_n and β from finite element model.

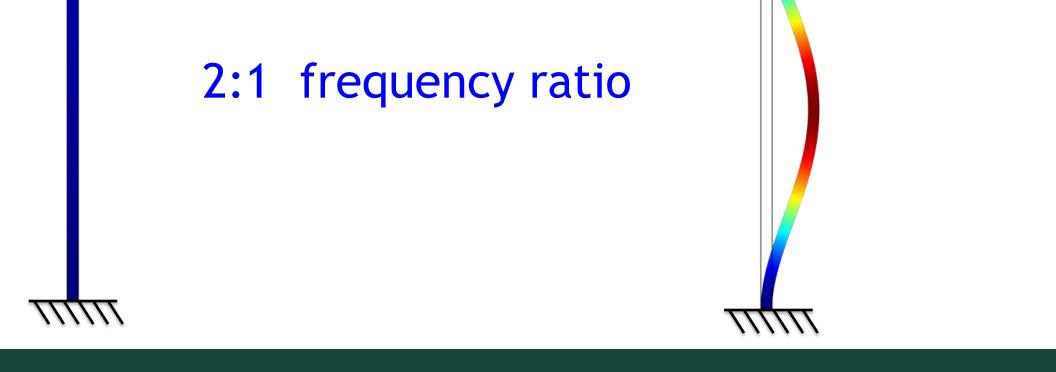
$\ddot{x}_1 + \omega_1^2 x_1 + \gamma_1 x_1^3 + \beta x_2^2 = F_1(t)$ $\ddot{x}_2 + (\omega_2^2 + 2\beta x_1)x_2 + \gamma_2 x_2^3 = F_2(t)$

(right) SEM two-mode T-bar cascade; orange lines show structure; courtesy of K.Turner, UCSB (below) two coupled modes simulated in COMSOL





the beam width					
n=1 (initial design) n=25 itera		tions	n=200 (final design)		
Effects of optimization 1. Frequency ration but frequencies vary 2. Vertical beam remuniform, thinned; 3. Horizontal beam h	aintained nains almost	Objective function $\begin{pmatrix} C_1 \\ C_1 \end{pmatrix}$			
thickness, thickened;					-
4. Modal coupling coefficient $3^{12}_{0}_{50}_{50}_{50}_{100}_{150}_{150}_{20}_{100}_{150}_{20}_{20}_{100}_{100}_{150}_{100}$					200
Mode 1 (18.05 MHz			e 2 (9.05 M		

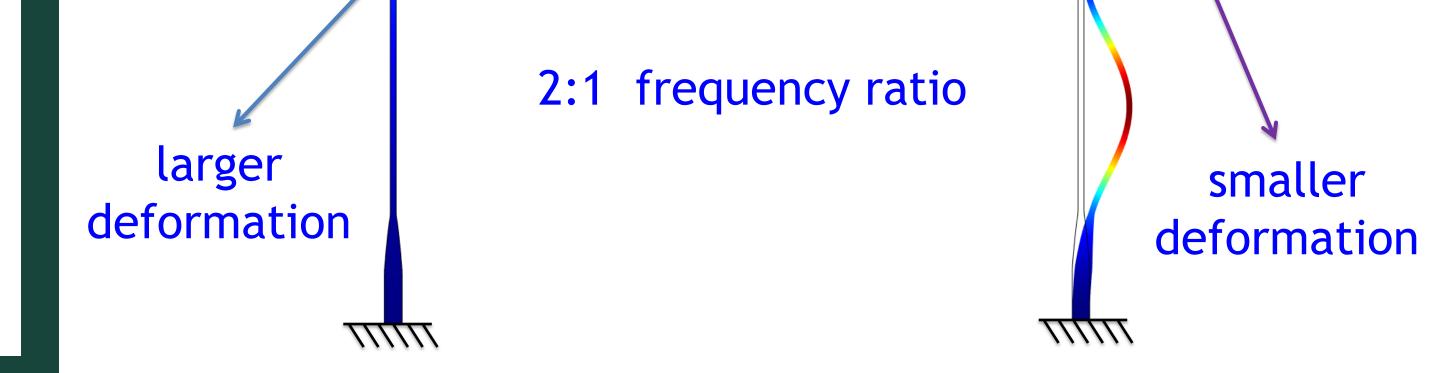


III. Optimization Procedure

- Develop crude design based on nonlinear phenomena and create a finite element model.
- Solve the finite element model eigenproblem. Use N 2. modal eigenvectors to obtain the nonlinear coefficients of the potential of the reduced order model [3] :

$\sum \alpha_3^{i,j,k} x_i x_j x_k + \sum \alpha_4^{i,j,k,l} x_i x_j x_k x_l$ $i,j \ge i,k \ge j$ $i,j \ge i,k \ge j,l \ge k$

- 3. Define the objective function and constraints. These can be based on:
 - nonlinear modal frequency responses
 - reduced order model coefficients
 - a function of the model coefficients



Benefits:

- Characterization of nonlinear resonators from FE model
- \succ Systematic optimization tools for nonlinear resonators



V. Acknowledgements

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VI. References

[1] B.S. Strachan, K.R. Qalandar, B. Gibson, M. Sharma, P. Polunin, S.W. Shaw, K.L. Turner "A Passive Micro-mechanical Frequency Divider", Technical Digest, Solid-State Sensor, Actuator and Microsystems Workshop, Hilton Head Island, SC, June 2014

[2] D. Czaplewski, B.S. Strachan, S.W. Shaw, M.I. Dykman, D. Lopez, "Phase Noise Reduction in an

4. Update design variables based on shape optimization

scheme and repeat steps 2-4 until convergence.

Oscillator Through Coupling to an Internal Resonance Mode", Technical Digest, Solid-State Sensor, Actuator and Microsystems Workshop, Hilton Head Island, SC, June 2014 [3] C. Touze, M. Vidrascu, D. Chapelle, "Direct Finite Element Computation of Non-Linear Modal Coupling Coefficients for Reduced-Order Shell Models", Computational Mechanics, pp1-14, 2014