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Research Article

Average Consensus in Multiagent Systems with the Problem of Packet Losses When Using the Second-Order Neighbors' Information

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This paper mainly investigates the average consensus of multiagent systems with the problem of packet losses when both the first-order neighbors' information and the second-order neighbors' information are used. The problem is formulated under the sampled-data framework by discretizing the first-order agent dynamics with a zero-order hold. The communication graph is undirected and the loss of data across each communication link occurs at certain probability, which is governed by a Bernoulli process. It is found that the distributed average consensus speeds up by using the second-order neighbors' information when packets are lost. Numerical examples are given to demonstrate the effectiveness of the proposed methods.

1. Introduction

In recent years, there has been an increasing research in coordination control of multiagent systems. Information consensus has attracted more and more attentions from many engineering application fields, such as formation control, flocking, artificial intelligence, and automatic control [1–4]. A critical problem in distributed control is to develop distributed protocols under which agents can reach an agreement on a common decision.

An excellent protocol can reduce cost, increase efficiency, and can optimize performance. Convergence rate is an important index to evaluate the performance of consensus. There has been much research interest in dealing with this issue. In [5], the authors pointed out that the second smallest eigenvalue of its Laplacian matrix was a measure of speed of solving consensus problems. From [6], we know that the convergence speeds up by finding the optimal weight associated with each communication link, where the global structure of the network must be known beforehand. Reference [7] accelerated the convergence rate by using the polynomial filtering algorithms. In [8], the authors presented randomized gossip algorithm on an arbitrary connected network and showed its performance precisely in the terms of the second largest eigenvalue of an appropriate stochastic matrix. The above literatures all tried to seek a suitable topology communication to achieve a fast convergence. However, in practice, it is more useful to design a protocol to obtain a better convergence performance under a given topology. In order to get a better convergence speed without changing the topology and edge weights, the authors in [9] proposed a protocol in an unchanged topology network that each node got its state value updated by using the information of multihop communication and showed that the protocol increased the convergence speed effectively for the first time. Then, in [10], the authors discussed that the node in the network topology updated its current state value not only from its immediate neighbors but also from its second-order neighbors for both the discrete-time case and the continuous-time case. Further, the authors in [11] extended the systems to second-order case and made comparisons between the convergence rate of second-order neighbor protocol and the general protocol. What is more, the delay margins of general protocol and second-order neighbor protocol were derived.

It is noted that the literatures mentioned above mainly focus on consensus problem for agents under first-order dynamics with time delay. In reality, the agents exchange data over fading communication channels instead of ideal ones. In fact, in many practical applications, this data exchange between sensors is done by wireless communication, which has a possibility of packets lost. Thereby, the packet losses should be taken into consideration. Many related works have been reported. Reference [12] dealt with consensus with random delay and data losses. Reference [13] compared the memory and memoryless consensus protocols in the presence of uniform packet losses. In [14, 15], the authors discussed the average consensus in first-order agents and analyzed the convergence speed under data losses. Furthermore, [16] showed that packet dropouts can be treated as an absence of a communication link over time. In addition, [17–19] studied stochastic consensus subject to a random process.

Inspired by the above references, we consider multiagent systems with the problem of packet losses based on the second-order neighbors' information. We construct a group of agents, which can communicate with their second-order neighbors and each communication link has a probability of failure. We assume that all channels are independent and subject to a distributed random process. Thereby, they have the same probability of data loss. Each agent is equipped with a sampler and a zero-order hold, which are synchronized in time. Then, by converting the system to the equivalent error dynamics, stochastic stability of the error dynamic system is studied. Here, a Lyapunov function is constructed and a sufficient condition is established to guarantee the average consensus in the form of linear matrix inequality (LMI). We are curious about whether the protocol based on the second-order neighbors' information can accelerate the convergence speed with the problem of packet losses. Then, a simulation comparison of the convergence rate between the protocol based on the second-order neighbors and the one in general linear is shown. Comparison of the convergence speed between different probabilities of packet losses is also simulated.

The rest of this paper is organized as follows. Section 2 provides some preliminaries on graph theory and gives the designed protocol. Section 3 analyses the average consensus and gives a sufficient condition. Section 4 includes some numerical examples, which demonstrate the effectiveness of the proposed approach. Finally, Section 5 offers the concluding remarks.

Notations. The set of real numbers is denoted by $\mathbb{R}$. For any matrix $Q \in \mathbb{R}^{n\times n}$, $\operatorname{sym}(Q) = Q + Q^T$. The index set $\Lambda_n = \{1, 2, \ldots, n\}$ is a group of consecutive integers from 1 to n. The vector $1_n = [1, 1, \ldots, 1]^T \in \mathbb{R}^n$ has all of its elements equal to 1. The mathematical expectation is denoted by $E\{\cdot\}$ and $P\{\cdot\}$ is the probability operator.

2. Problem Formation

2.1. Preliminaries on Graph Theory. In this paper, the interaction among n agents is modeled by an undirected graph $G = \{\nu, \varepsilon, A\}$, where $\nu = \{\nu_1, \nu_2, \ldots, \nu_n\}$ is the node set. The edge set $\varepsilon \subseteq \nu \times \nu$ contains ordered pairs of nodes. The neighbor set of agent i is denoted by N_i, which includes agents from which agent i receives information. The adjacency matrix $A = [w_{ij}] \in \mathbb{R}^{n\times n}$ is a nonnegative matrix, where $w_{ij} > 0$ if and only if $(\nu_j, \nu_i) \in \varepsilon$; otherwise, $w_{ij} = 0$. We assume that there is no self-loop, so $w_{ii} = 0$. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ is defined as

$$
\begin{aligned}
l_{ij} &= -w_{ij}, \quad \text{if } i \neq j; \\
l_{ij} &= \sum_{k\in N_i} w_{ik}.
\end{aligned}
\tag{1}
$$

From the above definitions, we know some facts: A and L determine each other uniquely, and L has nonnegative eigenvalues. Moreover, L has at least one zero eigenvalue with the associated eigenvector 1_n^T ($L1_n^T = 0$); that is, $\operatorname{span}\{1_n^T\} \subseteq \operatorname{null}\{L\}$, where $\operatorname{null}\{L\}$ is the null space of L. For the undirected graph, we further have $L = L^T$, $1_n^T L = 0$. From [20], it is known that $\operatorname{span}\{1_n^T\} = \operatorname{null}\{L\}$ if and only if the undirected graph G is connected.

2.2. Protocols Based on Second-Order Neighbor with Packet Losses. Consider the following first-order dynamics:

$$\dot{x}_i = u_i, \quad i \in \Lambda_n, \tag{2}$$

where $x_i, u_i \in \mathbb{R}$ are the state and the input of agent i, respectively. With sampling period T and a zero-order hold, the agent dynamics is discretized as

$$x_i(k+1) = x_i(k) + Tu_i(k), \quad i \in \Lambda_n. \tag{3}$$

Considering the protocol based on second-order neighbors' information, if there is no communication constraint taken into account, the following control protocol can be used:

$$
\begin{aligned}
u_i(k) = -r_c \sum_{j\in N_i} w_{ij} \Bigg[& \left(x_i(k) - x_j(k)\right) \\
& + \sum_{h\in N_j} w_{jh}\left(x_i(k) - x_h(k)\right) \Bigg],
\end{aligned}
\tag{4}
$$

where the control gain r_c is to be designed.

Next, we consider the packet losses among agents. The following control protocol is designed:

$$u_i(k) = -r_c \left\{ \sum_{j\in N_i} \gamma_{ij}(k) w_{ij} \left[\left(x_i(k) - x_j(k)\right) + \sum_{h\in N_j} \gamma_{jh}(k) w_{jh} \times \left(x_i(k) - x_h(k)\right) \right] \right\}, \tag{5}$$

where $\gamma_{ij}(k) = 1$, if there is no packet loss between agents i and j; $\gamma_{ij}(k) = 0$, otherwise.

Furthermore, we assume that the occurrence of packet loss is governed by a Bernoulli process with uniform probability p satisfying $0 < p < 1$; that is,

$$P\left\{\gamma_{ij}(k) = 1\right\} = p, \quad P\left\{\gamma_{ij}(k) = 0\right\} = 1 - p, \quad \forall i \neq j. \tag{6}$$

As a result, we have $E\{\gamma_{ij}(k)\} = p$.

Assumption 1. The undirected topology is coupled; that is, for any pair of agents i and j, the communication channels between them exist or vanish simultaneously.

Assumption 1 ensures that the communication topology is always symmetric, so the average of agents' states can be retained during dynamic evolution.

We define two sets of matrices $L_1(k) = [l_{1ij}(k)] \in \mathbb{R}^{n\times n}$ and $L_2(k) = [l_{2ij}(k)] \in \mathbb{R}^{n\times n}$ as follows:

$$\begin{aligned} l_{1ij}(k) &= -\gamma_{ij}(k) w_{ij}, \\ l_{2ij}(k) &= -\sum_{\substack{h\in N_i \\ j\in N_h}} \gamma_{ih}(k) w_{ih} \gamma_{hj}(k) w_{hj}, \quad i \neq j, \\ l_{1ii}(k) &= \sum_{j\in N_i} \gamma_{ij}(k) w_{ij}, \\ l_{2ii}(k) &= \sum_{h\in N_i} \gamma_{ih}(k) w_{ih} \sum_{l\in N_h} \gamma_{hl}(k) w_{hl}. \end{aligned} \tag{7}$$

Denote the vectors $x(k)$ and $u(k)$ by

$$\begin{aligned} x(k) &= \left[x_1(k), x_2(k), \ldots, x_n(k)\right]^T, \\ u(k) &= \left[u_1(k), u_2(k), \ldots, u_n(k)\right]^T. \end{aligned} \tag{8}$$

Then, the control protocol can be rewritten as

$$u(k) = -r_c L(k) x(k), \tag{9}$$

where $L(k) = L_1(k) + L_2(k)$.

So, the system dynamics can be written as

$$x(k+1) = x(k) - r_c T L(k) x(k) = \left[I - r_c T L(k)\right] x(k). \tag{10}$$

By taking the mathematical expectation of $L_1(k)$ and $L_2(k)$, we have $E\{L_1(k)\} = p \times L^{(1)}$, $E\{L_2(k)\} = p^2 \times L^{(2)}$, where $L^{(1)}$ is the nominal Laplacian matrix of full weights where there is no packet loss and $L^{(2)}$ is the nominal Laplacian matrix of the system which is only based on the second-order neighbors' information with full weights and without packet loss.

Assumption 2. The nominal communication topologies $G^{(1)}$ associated with $L^{(1)}$ and $G^{(2)}$ associated with $L^{(2)}$ are all connected.

The above assumption is necessary for consensus because if the undirected graph is not connected, then it does not have a spanning tree. From [21, Lemma 1] and [22, Theorem 5], we know that there exist two nonempty, disjoint groups of agents that have no communication with each other at any time. In this case, consensus cannot be reached.

3. Consensus Analysis

The average states of the agents

$$\alpha = \text{Ave}(x(k)) = \frac{1}{n}\sum_{i=1}^{n} x_i(k) = \frac{1}{n} 1_n^T x(k) \tag{11}$$

are invariant. We say the *average consensus* problem is solved, if

$$\lim_{k\to+\infty} x_i(k) = \alpha, \quad i = 1, \ldots, n. \tag{12}$$

Each agent state can be presented by the form

$$x(k) = \alpha 1_n + \delta(k), \tag{13}$$

where the variable $\delta(k) = [\delta_1(k), \delta_2(k), \ldots, \delta_n(k)]^T$ satisfies

$$1_n^T \delta = 0. \tag{14}$$

The following error dynamics are obtained:

$$\delta(k+1) = \delta(k) - r_c T L(k) \delta(k) = \left[I - r_c T L(k)\right] \delta(k). \tag{15}$$

Obviously, the stability of (15) is equivalent to the consensus in (2). Then, we introduce the following lemma, which plays an important role in the stability analysis of (15).

Lemma 3 (see [16]). *For an undirected graph, given the Laplacian matrix $L_1(k)$, $L_2(k)$, and a symmetric matrix*

$Q = Q^T$, $E\{(L_1(k)+L_2(k))\ Q(L_1(k)+L_2(k))\}$ can be calculated as follows:

$$\begin{aligned}
&E\left\{\left(L_1(k)+L_2(k)\right)\ Q\left(L_1(k)+L_2(k)\right)\right\}\\
&\quad = E\left\{\mathrm{I}\begin{bmatrix} L_1(k) & 0\\ 0 & L_2(k)\end{bmatrix}\mathrm{I}^T Q \mathrm{I}\begin{bmatrix} L_1(k) & 0\\ 0 & L_2(k)\end{bmatrix}\mathrm{I}^T\right\}\\
&\quad = \mathrm{I}E\left\{\begin{bmatrix} L_1(k) & 0\\ 0 & L_2(k)\end{bmatrix}\Gamma\begin{bmatrix} L_1(k) & 0\\ 0 & L_2(k)\end{bmatrix}\right\}\mathrm{I}^T\\
&\quad = \mathrm{I}\left\{\hat{P}\left(L^{(0)}\right)\hat{P}\left(L^{(0)}\right)L^{(0)}\Gamma L^{(0)}\right.\\
&\qquad \left. +\hat{P}\left(L^{(0)}\right)\left(I-\hat{P}\left(L^{(0)}\right)\right)\Xi(\Gamma)\right\}\mathrm{I}^T,
\end{aligned}\tag{16}$$

where $L^{(0)} = \left[\begin{smallmatrix} L^{(1)} & 0\\ 0 & L^{(2)}\end{smallmatrix}\right]$, $\mathrm{I}^T = \left[\begin{smallmatrix} I\\ I\end{smallmatrix}\right]_{2n*n}$, $\hat{P}\left(L^{(0)}\right) = \left[\begin{smallmatrix} p & 0\\ 0 & p^2\end{smallmatrix}\right]$, $\Gamma = \left[\begin{smallmatrix} Q & Q\\ Q & Q\end{smallmatrix}\right]$, and $\Xi(\Gamma)$ is a function of Q, defined as

$$\begin{aligned}
&\Xi(\Gamma)\\
&= \sum_{m=1}^{n}\sum_{q=1}^{n}\begin{bmatrix} E_{1(m,q)}^T & 0\\ 0 & E_{2(m,q)}^T\end{bmatrix}\Gamma\begin{bmatrix} E_{1(m,q)} & 0\\ 0 & E_{2(m,q)}\end{bmatrix}\\
&\qquad \times\begin{bmatrix} a_{1(m,q)}^2 & 0\\ 0 & a_{2(m,q)}^2\end{bmatrix}\\
&\quad + \sum_{m=1}^{n}\sum_{q=1}^{n}\begin{bmatrix} E_{1(m,m)}^T & 0\\ 0 & E_{2(m,m)}^T\end{bmatrix}\Gamma\begin{bmatrix} E_{1(m,m)} & 0\\ 0 & E_{2(m,m)}\end{bmatrix}\\
&\qquad \times\begin{bmatrix} a_{1(m,q)}^2 & 0\\ 0 & a_{2(m,q)}^2\end{bmatrix}\\
&\quad - \sum_{m=1}^{n}\sum_{q=1}^{n}\operatorname{sym}\left(\begin{bmatrix} E_{1(m,q)}^T & 0\\ 0 & E_{2(m,q)}^T\end{bmatrix}\Gamma\begin{bmatrix} E_{1(m,m)} & 0\\ 0 & E_{2(m,m)}\end{bmatrix}\right)\\
&\qquad \times\begin{bmatrix} a_{1(m,q)}^2 & 0\\ 0 & a_{2(m,q)}^2\end{bmatrix}\\
&\quad + \sum_{j=1}^{n}\sum_{m=j+1}^{n}\operatorname{sym}\left(2\begin{bmatrix} E_{1(j,j)}^T & 0\\ 0 & E_{2(j,j)}^T\end{bmatrix}\Gamma\begin{bmatrix} E_{1(m,m)} & 0\\ 0 & E_{2(m,m)}\end{bmatrix}\right.\\
&\qquad -\begin{bmatrix} E_{1(j,j)}^T & 0\\ 0 & E_{2(j,j)}^T\end{bmatrix}\Gamma\begin{bmatrix} E_{1(m,j)} & 0\\ 0 & E_{2(m,j)}\end{bmatrix}\\
&\qquad -\begin{bmatrix} E_{1(j,m)}^T & 0\\ 0 & E_{2(j,m)}^T\end{bmatrix}\\
&\qquad \left.\times\Gamma\begin{bmatrix} E_{1(m,m)} & 0\\ 0 & E_{2(m,m)}\end{bmatrix}\right)\\
&\qquad \times\begin{bmatrix} a_{1(m,q)}^2 & 0\\ 0 & a_{2(m,q)}^2\end{bmatrix}.
\end{aligned}\tag{17}$$

The following theorem gives a sufficient condition on the average consensus of the system (2).

Theorem 4. *Given the scalar r_c, the average consensus of the system (2) is achieved if there exists a matrix $Q > 0$, such that the following LMI holds:*

$$\begin{aligned}
&-\left[L^{(1)}Q+QL^{(1)}+p\left(L^{(2)}Q+QL^{(2)}\right)\right]\\
&\quad + r_cT\mathrm{I}\left\{\hat{P}\left(L^{(0)}\right)\hat{P}\left(L^{(0)}\right)L^{(0)}\Gamma L^{(0)}\right.\\
&\qquad \left.+\hat{P}\left(L^{(0)}\right)\left(I-\hat{P}\left(L^{(0)}\right)\right)\Xi(\Gamma)\right\}\mathrm{I}^T < 0.
\end{aligned}\tag{18}$$

Proof. Construct the candidate Lyapunov function as $V(k) = \delta^T(k)Q\delta(k)$. We have

$$\begin{aligned}
&E\{\Delta V(k)\}\\
&\quad = E\{V(k+1)-V(k)\}\\
&\quad = E\left\{\delta^T(k+1)Q\delta(k+1)-\delta^T(k)Q\delta(k)\right\}\\
&\quad = E\left\{\delta^T(k)\left[I-r_cTL(k)\right]Q\left[I-r_cTL(k)\right]\delta(k)\right.\\
&\qquad \left.-\delta^T(k)Q\delta(k)\right\}\\
&\quad = E\left\{\delta^T(k)\left[-r_cTL(k)Q-r_cTQL(k)\right.\right.\\
&\qquad \left.\left.+r_c^2TL(k)QL(k)\right]\delta(k)\right\}\\
&\quad = -r_cT\delta^T(k)\left(pL^{(1)}+p^2L^{(2)}\right)Q\delta(k)\\
&\qquad -r_cT\delta^T(k)Q\left(pL^{(1)}+p^2L^{(2)}\right)\delta(k)\\
&\qquad +r_c^2T^2\delta^T(k)E\{L(k)QL(k)\}\delta(k)\\
&\quad = -r_cTp\delta^T(k)\left[L^{(1)}Q+QL^{(1)}\right.\\
&\qquad \left.+p\left(L^{(2)}Q+QL^{(2)}\right)\right]\delta(k)\\
&\qquad +r_c^2T^2\delta^T(k)\mathrm{I}\left\{\hat{P}\left(L^{(0)}\right)\hat{P}\left(L^{(0)}\right)L^{(0)}\Gamma L^{(0)}\right.\\
&\qquad \left.+\hat{P}\left(L^{(0)}\right)\left(I-\hat{P}\left(L^{(0)}\right)\right)\Xi(\Gamma)\right\}\\
&\qquad \times\mathrm{I}^T\delta(k).
\end{aligned}\tag{19}$$

Thus, from the Lyapunov stability theory, we know that if $E\{\Delta V(k)\}$ is negative, then (15) is asymptotically stable. Thereby, the states of all agents will converge to their average state; that is, the average consensus of the system (2) is achieved. □

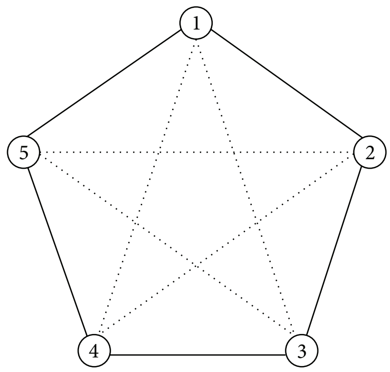


Figure 1: Nominal communication topology and topology based on the second-order neighbors' information.

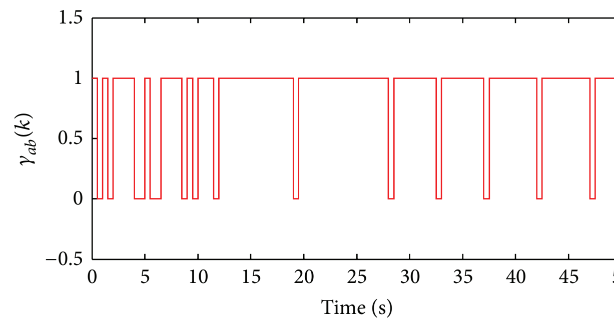


Figure 2: Time history of the Bernoulli variable $\gamma_{ab}(k)$.

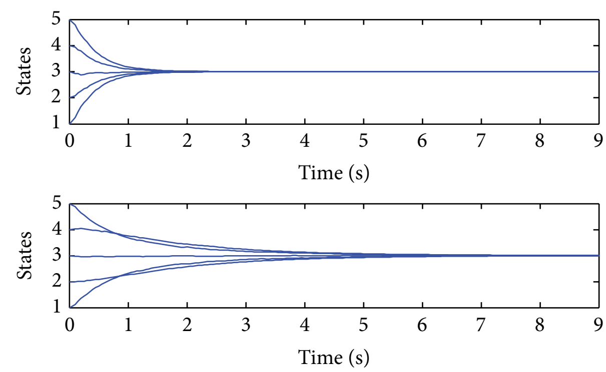
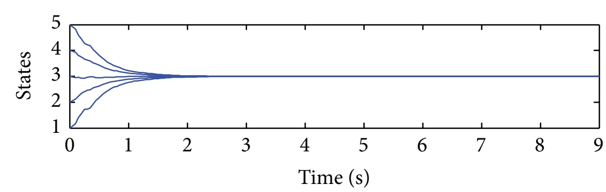
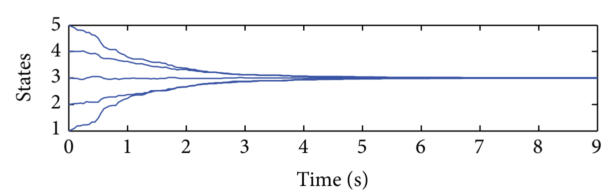


Figure 3: Nominal communication and the topology based on second-order neighbors' information with $P = 0.9$.



(a) $P = 0.9$



(b) $P = 0.5$

Figure 4: Comparison of the convergence speed based on second-order neighbors' information with different package loss probability.

4. Simulations and Analyses

To illustrate the average consensus of the system (2) under the condition of packet losses and fast convergence rate of the protocol based on second-order neighbors' information, a numerical example is provided. The nominal interaction topology $G^{(1)}$ and topology only based on second-order neighbors' information $G^{(2)}$ among five agents are shown in Figure 1.

The weights are set to unity for simplicity here. We set the corresponding Laplacian matrices $L^{(1)}$ and $L^{(2)}$ as follows:

$$
\begin{gathered}
L^{(1)} = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}, \\
L^{(2)} = \begin{bmatrix} 2 & 0 & -1 & -1 & 0 \\ 0 & 2 & 0 & -1 & -1 \\ -1 & 0 & 2 & 0 & -1 \\ -1 & -1 & 0 & 2 & 0 \\ 0 & -1 & -1 & 0 & 2 \end{bmatrix}.
\end{gathered} \tag{20}
$$

We choose the sampling period as $T = 0.05$ sec, control gain as $r_c = 0.5$, and the probability of successfully receiving information as $P = 0.9$. The initial condition is set to be $x(0) = [1, 2, 3, 4, 5]^T$, and it will be shown that the agents' states finally converge to the average value $\alpha = \text{Ave}(x(0)) = (1+2+3+4+5)/5 = 3$. Then, by solving the LMI in Theorem 4, the result shows that it is feasible. Thus, consensus will be achieved. The time history of the Bernoulli variable $\gamma_{ab}(k)$ is shown in Figure 2. Figure 3 compares the convergence speed of the nominal communication and the topology based on second-order neighbors' information with $P = 0.9$, from which we can see that the protocol we designed is more effective. Figure 4 compares the convergence speed based on second-order neighbors' information with $P = 0.9$ and $P = 0.5$, from which we can see the influence of packet losses.

5. Conclusions

In this paper, we have investigated the average consensus in multiagent systems with the problem of packet losses when second-order neighbors' information was used. The convergence rates of general protocol and second-order neighbor protocol with packet losses have been compared and it is concluded that second-order neighbor protocol speeds up the consensus rate. What is more, we can see the influence of packet losses. Future work will extend the agent dynamics to second-order or higher-order dynamics with data loss and time-varying delay.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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