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Harvesting analysis of a discrete competitive system

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Abstract

In this paper, we discuss a discrete competitive system based on density dependence to obtain a set of sufficient conditions for the existence and asymptotic stability of the equilibrium of systems. By obtaining the optimal harvest strategy of systems through the extreme value method and the discrete Pontryagin maximum principle, we provide a theoretical direction for the actual production.

Keywords: discrete; competitive systems; equilibrium; global stability; fishing companies; profit

1 Introduction

Stability and permanence of a biological system have been studied by several authors [1–4]. The problem of fractional differential equation was also studied [5–13]. However, the rational development and management of the biological resources were directly related to sustainable development. In recent years, continuous system capture has received many scholars' attention [14–22]. Similarly, the optimal control theory is a good method [23–27]. In fact, as we know, fish distribution is inhomogeneous and it is not possible to capture successively. Therefore, it is more reasonable to consider the discrete system's capture. Not only it will keep the biological balance but it will also save time and produce more economic revenue for fishermen. Due to the peculiarity of the discrete system, it is difficult to study its stability and capture, and there are few related studies. Therefore, in this paper, we consider the following discrete two species competitive system and discuss the system's stability and capturing strategy,

$$\begin{cases} \Delta x_n = x_n(a_1 - b_1 x_n - c_1 y_n) - E_1 x_n = P_1(x_n, y_n), \\ \Delta y_n = y_n(a_2 - b_2 x_n - c_2 y_n) - E_2 y_n = P_2(x_n, y_n). \end{cases} \tag{1.1}$$

Here a_1 and a_2 ($a_1 > 0, a_2 > 0$) denote the intrinsic growth rate of two species x_n and y_n (or life factor). b_1 and c_2 ($b_1, c_2 > 0$) denote the density-dependent entry. Generally speaking, two populations are both caught by fishermen. It has practical significance to take the capture effect into consideration in order to reap the maximum economic benefits. Let E_1, E_2 ($E_1, E_2 > 0$) be the capture intensity of the two populations (that is, fishing effort multiplies the capture coefficients) ($E_1 + E_2 = E$), and let the capture per unit time be proportional to the stock and population, $a_1 > E_1$ and $a_2 > E_2$. Under this assumption, we can get the following competitive capture systems.

The rest of the paper is arranged as follows. We discuss a set of sufficient conditions for the stability of system (1.1) equilibrium based on density dependence in Section 2. It is discussed that system (1.1) stable equilibrium in the optimal acquisition strategy through the extreme value method, by using structuring discrete Hamiltonian function and discrete Pontryagin maximum principle, it is to obtain optimal harvest policy by three equilibriums in Section 3.

2 Equilibrium and stability

2.1 Equilibrium

By calculating, we can get that system (1.1) has the following equilibriums: $O(0,0)$, $P_0(\frac{a_1-E_1}{b_1},0)$, $\overline{P}(0,\frac{a_2-E_2}{c_2})$, $P(x^*,y^*)$, where

$$x^* = \frac{(a_1-E_1)c_2-(a_2-E_2)c_1}{b_1c_2-b_2c_1}, \qquad y^* = \frac{(a_2-E_2)b_1-(a_1-E_1)b_2}{b_1c_2-b_2c_1}.$$

Theorem 2.1 *O, P_0 and $\overline{P}$ are non-negative equilibrium points; $P(x^*,y^*)$ is a positive equilibrium if and only if*

$$\frac{b_1}{b_2} > \frac{a_1-E_1}{a_2-E_2} > \frac{c_1}{c_2}. \tag{2.1}$$

2.2 Stability of the positive equilibrium

For any initial value (x_0,y_0), let $\{(x_n,y_n)\}$ be the solution sequence of system (1.1).

Theorem 2.2 *Under the conditions of Theorem* 2.1 *and further assumption that system* (1.1) *satisfies the following conditions*:

(1) $(1+a_1-E_1)^2 \le 4b_1x^*$,

(2) $(1+a_2-E_2)^2 \le 4c_2y^*$,

the positive equilibrium $P(x^*,y^*)$ is locally asymptotically stable in the region $D_1 = \{(x,y)|0<x\le x^*, 0<y\le y^*\}$, which is called the attraction domain of the positive equilibrium point $P(x^*,y^*)$ in system (1.1).

Proof Let $(x_0,y_0)\in D_1$, considering the function:

$$u = b_1x^2-(1+a_1-E_1)x+x^*,$$

from condition (1), we have $\Delta_1 = (1+a_1-E_1)^2-4b_1x^* \le 0$ and $b_1>0$, hence $u\ge 0$.

When $x>0$, $y>0$, we have

$$b_1x_n^2+c_1x_ny_n-(1+a_1-E_1)x_n+x^*>0. \tag{2.2}$$

For $(x_0,y_0)\in D_1$, according to (1.1) and (2.2), we get

$$x_1 = (1+a_1-E_1)x_0-b_1x_0^2-c_1x_0y_0 < x^*.$$

Because D_1 is in the region surrounded by

$$a_1 - E_1 - b_1x_n - c_1y_n = 0, \qquad a_2 - E_2 - b_2x_n - c_2y_n = 0, \qquad x = 0, \qquad y = 0,$$

it follows that $\Delta x_0 = x_0(a_1 - E_1 - b_1x_0 - c_1y_0) > 0$. That is, $x_0 < x_1$, then $0 < x_0 < x_1 < x^*$.

Similarly, consider the following function:

$$v = c_2y^2 - (1 + a_2 - E_2)y + y^*.$$

From condition (2) we have

$$\Delta_2 = (1 + a_2 - E_2)^2 - 4c_2y^* \leq 0$$

and $c_2 > 0$, so $v \geq 0$. When $x > 0$, $y > 0$, we have

$$b_2x_ny_n + c_2y_n^2 - (1 + a_2 - E_2)y_n + y^* > 0. \tag{2.3}$$

For $(x_0, y_0) \in D_1$, from (1.1) and (2.3) we get

$$y_1 = (1 + a_2 - E_2)y_0 - b_2x_0y_0 - c_2y_0^2 < y^*.$$

Because D_1 is in the region surrounded by

$$a_1 - E_1 - b_1x_n - c_1y_n = 0, \qquad a_2 - E_2 - b_2x_n - c_2y_n = 0,$$
$$y_n = 0, \qquad x = 0, \qquad y = 0,$$

it follows that

$$\Delta y_n = y_n(a_2 - E_2 - b_2x_n - c_2y_n) > 0.$$

That is,

$$y_1 > y_0,$$

thus

$$0 < y_0 < y_1 < y^*, \quad (x_1, y_1) \in D_1.$$

By the recursive method, the solutions $(x_n, y_n) \in D_1$ of system (1.1) satisfy the conditions of theorem and $0 < x_n < x_{n+1} < x^*$, $0 < y_n < y_{n+1} < y^*$ $(n = 1, 2, \ldots)$.

According to the monotone bounded theorem $\lim_{n\to\infty} x_n = M$, $\lim_{n\to\infty} y_n = N$.

Let $n \to \infty$. In (1.1), $\{x_n\}$, $\{y_n\}$ are monotonically increasing sequences and the positive equilibrium point of system (1.1) is unique, we get $M = x^*$, $N = y^*$. So the sequence of $\{x_n\}$, $\{y_n\}$ converges to the positive equilibrium P. □

Theorem 2.3 *Under the conditions of Theorem* 2.1 *and further assumption that system* (1.1) *satisfies the following conditions*:

(1) $(c_2 + a_1c_2 - E_1c_2 - c_1a_2 + c_1E_2)^2 > 4b_1c_2^2x^*$;

(2) $c_2 + a_1c_2 + c_1E_2 \le E_1c_2 + c_1a_2 + 2b_1c_2^2x^*$;

(3) $c_2 + c_1E_2 \ge a_1c_2 + c_2E_1 + c_1a_2$;

(4) $(b_1 + b_1a_2 - b_1E_2 + a_1b_2 - E_1b_2)^2 > 4c_2b_1^2y^*$;

(5) $b_1 + b_1a_2 + E_1b_2 \le a_1b_2 + b_1E_2 + 2b_1c_2^2x^*$;

(6) $b_1a_2 + a_1b_2 + b_1E_2 \le b_1 + b_2E_1$,

$P(x^*, y^*)$ *in system* (1.1) *is locally asymptotically stable in the region*

$$D_2 = \left\{(x,y)\middle| x^* < x \le \frac{a_1 - E_1}{b_1}, y^* < y \le \frac{a_2 - E_2}{c_2}\right\},$$

which is the attraction domain of $P(x^*, y^*)$.

Proof Let $(x_0, y_0) \in D_2$, since D_2 is included in the region on the top of the two straight lines $a_1 - E_1 - b_1x_n - c_1y_n = 0$, $a_2 - E_2 - b_2x_n - c_2y_n = 0$ and $\Delta x_0 = x_0(a_1 - E_1 - b_1x_0 - c_1y_0) < 0$, that is, $x_1 < x_0$.

Consider the following function:

$$u = b_1t^2 - \left(1 + a_1 - E_1 - \frac{c_1a_2 - c_1E_2}{c_2}\right)t + x^*.$$

From condition (1) we get $\Delta_1 = \frac{1}{c_2^2}(c_2 + a_1c_2 - E_1c_2 - c_1a_2 + c_1E_2)^2 - 4b_1x^* > 0$. This function has two real zero points:

$$t_{12} = \frac{c_2 + a_1c_2 - E_1c_2 - c_1a_2 + c_1E_2 \pm \sqrt{\Delta_1c_2^2}}{2b_1c_2}.$$

From condition (2) we get

$$c_2 + a_1c_2 - E_1c_2 - c_1a_2 + c_1E_2 - 2b_1c_2^2x^* \le 0 \le \sqrt{(c_2 + a_1c_2 - c_1a_2 - E_1c_2)^2 - 4b_1c_2^2x^*},$$

hence $t_1 \le x^*$.

From condition (3) we get

$$t_2 = \frac{c_2 + a_1c_2 - c_2E_1 - c_1a_2 + c_1E_2 + \sqrt{\Delta_1c_2^2}}{2b_1c_2} > \frac{a_1 - E_1}{b_1},$$

$t_1 < x^* < \frac{a_1 - E_1}{b_1} < t_2$. And when $t_1 < t < t_2$, $u < 0$, so for $x^* < t < \frac{a_1 - E_1}{b_1}$,

$$b_1t^2 + \left(\frac{c_1a_2 - c_1E_2}{c_2} - 1 - a_1 + E_1\right)t + x^* < 0.$$

For $(x, y) \in D_2$,

$$b_1x^2 + c_1xy - (1 + a_1 - E_1)x + x^* < b_1x^2 + \frac{c_1(a_2 - E_2)}{c_2}x - (1 + a_1 - E_1)x + x^* < 0,$$

then $x_1 = (1 + a_1 - E_1)x_0 - b_1x_0^2 - c_1x_0y_0 > x^*$, hence $x^* < x_1 < x_0$.

Consider the auxiliary functions

$$v = c_2s^2 - \left(1 + a_2 - E_2 - \frac{a_1b_2 - E_1b_2}{b_1}\right)s + y^*.$$

From conditions (4), (5) and (6), $y^* < y_1 < y_0$ can also be proved. From the recursive method available, the solution $(x_n, y_n) \in D_2$ of system (1.1) satisfies the conditions of the theorem, and $x^* < x_n < x_{n+1}$, $y^* < y_n < y_{n+1}$ $(n = 1, 2, \ldots)$. By the same method used in Theorem 2.2, it can be proved that the solution sequence of system (1.1) converges to the positive equilibrium point P. □

Based on the actual situation, population $x_n > 0$, $y_n > 0$, then we have the following.

Theorem 2.4 *If Theorem* 2.2 *is satisfied, and system* (1.1) *satisfies the following conditions*:

$$2a_2 - E_2 < \frac{c_1b_1 + b_2c_2}{b_1c_2 - b_2c_1}, \tag{2.4}$$

then system (1.1) *is globally asymptotically stable*.

Proof Define a Lyapunov function, $V_n(x_n, y_n) = b_2x_n + c_1y_n$, then

$$\begin{aligned}
\Delta V_n &= b_2\Delta x_n + c_1\Delta y_n \\
&= b_2x_n(a_1 - E_1 - b_1x_n - c_1y_n) + c_1y_n(a_2 - E_2 - b_2x_n - c_2y_n) \\
&= b_2x_n(a_1 - E_1) - b_1b_2x_n^2 - c_1b_2x_ny_n + c_1y_n(a_2 - E_2) - b_2c_1x_ny_n - c_1c_2y_n^2 \\
&\leq b_2x_n(a_1 - E_1) - b_1b_2x_n^2 + c_1y_n(a_2 - E_2) - c_1c_2y_n^2 \\
&= -b_1b_2\left(x_n - \frac{a_1 - E_1}{2b_1}\right)^2 - c_1c_2\left(y_n - \frac{a_2 - E_2}{2c_2}\right)^2 \\
&\quad + \frac{b_2(a_1 - E_1)^2}{4b_1} + \frac{c_1(a_2 - E_2)^2}{4c_2}.
\end{aligned}$$

From conditions (1) and (2) of Theorem 2.2 we get

$$\begin{aligned}
&\frac{(a_1 - E_1)^2}{4b_1} \leq x^* - \frac{1}{4b_1} - \frac{a_1 - E_1}{2b_1}, \qquad \frac{(a_2 - E_2)^2}{4c_2} \leq y^* - \frac{1}{4c_2} - \frac{(a_2 - E_2)}{2c_2}, \\
&\Delta V_n \leq -b_1b_2\left(x_n - \frac{a_1 - E_1}{2b_1}\right)^2 - c_1c_2\left(y_n - \frac{a_2 - E_2}{2c_2}\right)^2 + b_2x^* - \frac{b_2}{4b_1} - \frac{b_2(a_1 - E_1)}{2b_1} \\
&\qquad + c_1y^* - \frac{c_1}{4c_2} - \frac{c_1(a_2 - E_2)}{2c_2} \\
&\quad = -b_1b_2\left(x_n - \frac{a_1 - E_1}{2b_1}\right)^2 - c_1c_2\left(y_n - \frac{a_2 - E_2}{2c_2}\right)^2 + (a_2 - E_2) - \frac{b_2}{4b_1} - \frac{b_2(a_1 - E_1)}{2b_1}
\end{aligned}$$

$$\begin{aligned}
&- \frac{c_1}{4c_2} - \frac{c_1(a_2 - E_2)}{2c_2} \\
&= -b_1 b_2 \left(x_n - \frac{a_1 - E_1}{2b_1}\right)^2 - c_1 c_2 \left(y_n - \frac{a_2 - E_2}{2c_2}\right)^2 \\
&- \frac{(b_2 c_2 + b_1 c_1) + 2(a_2 - E_2)(b_2 c_1 - b_1 c_2)}{4b_1 c_2}.
\end{aligned}$$

From $2(a_2 - E_2) < \frac{c_1 b_1 + b_2 c_2}{b_1 c_2 - b_2 c_1}$, $\Delta V_n < 0$, then system (1.1) is globally asymptotically stable. □

3 The optimal economic benefit

As we know, both the fishermen and the fishing companies must consider the cost-effectiveness when catching all kinds of fish in terms of the sale price and the capture cost. Suppose that the largest capture intensity is E_m, then $0 < E_1 + E_2 = E \leq E_m$, the cost is C and the price of the two kinds of group are p_1, p_2. The economic profit is $L = p_1 E_1 x + p_2 E_2 y - CE$.

For the positive equilibrium point $P(x^*, y^*)$, the economic benefits (profits) are

$$\begin{aligned}
L &= p_1 E_1 x^* + p_2 E_2 y^* - CE \\
&= p_1 E_1 \frac{(a_1 - E_1)c_2 - (a_2 - E_2)c_1}{b_1 c_2 - b_2 c_1} + p_2 E_2 \frac{(a_2 - E_2)b_1 - (a_1 - E_1)b_2}{b_1 c_2 - b_2 c_1} - CE \\
&= A(E_1 + B)^2 + D,
\end{aligned}$$

where

$$\begin{aligned}
A &= -\frac{p_1(c_1 + c_2) - p_2(b_1 + b_2)}{b_1 c_2 - b_2 c_1}, \\
B &= -\frac{p_1(Ec_1 + a_1 c_2 - a_2 c_1) + p_2(Eb_2 + 2Eb_1 + a_1 b_2 - a_2 b_1)}{2p_1(c_1 + c_2) - 2p_2(b_1 + b_2)}, \\
D &= \frac{p_2 E(a_2 b_1 - a_1 b_2 - Eb_1)}{b_1 c_2 - b_2 c_1} \\
&\quad - \frac{[p_1(Ec_1 + a_1 c_2 - a_2 c_1) + p_2(Eb_2 + 2Eb_1 + a_1 b_2 - a_2 b_1)]^2}{4(b_1 c_2 - b_2 c_1)[p_1(c_1 + c_2) - p_2(b_1 + b_2)]} - CE.
\end{aligned}$$

Due to the limitation of capture ability $0 < E_1 + E_2 \leq E_m$, from the knowledge of calculus, $A < 0$ (that is, $\frac{p_1}{p_2} > \frac{b_1 + b_2}{c_1 + c_2}$), so L has a maximum value.

If $Ec_1 + a_1 c_2 > a_2 c_1$, $2Eb_1 + Eb_2 + a_1 b_2 > a_2 b_1$, then, when

$$E_1 = -B = \frac{p_1(Ec_1 + a_1 c_2 - a_2 c_1) + p_2(Eb_2 + 2Eb_1 + a_1 b_2 - a_2 b_1)}{2p_1(c_1 + c_2) + 2p_2(b_1 + b_2)} > 0,$$

L reaches the maximum:

$$\begin{aligned}
L_{\max} = D &= \frac{p_2 E(a_2 b_1 - a_1 b_2 - Eb_1)}{b_1 c_2 - b_2 c_1} \\
&\quad - \frac{[p_1(Ec_1 + a_1 c_2 - a_2 c_1) + p_2(Eb_2 + 2Eb_1 + a_1 b_2 - a_2 b_1)]^2}{4(b_1 c_2 - b_2 c_1)[p_1(c_1 + c_2) - p_2(b_1 + b_2)]} - CE.
\end{aligned}$$

For the non-negative equilibrium point $\overline{P}(0, \frac{a_2 - E_2}{c_2})$ $(0 < E_2 = E \leq E_m)$, we obtain the optimal harvest strategy of the non-negative equilibrium point by using the discrete

Pontryagin maximum principle and the optimal control theory. To obtain the optimal capture, seeking to capture the best efforts of degrees E_2^*, the goal of functions are given:

$$\overline{L} = \sum_{n=1}^{\infty} \alpha^{n-1}(p_2 y_n - C)E_2.$$

According to the discrete maximum principle, seeking optimal control E_2, the following Hamilton function is introduced:

$$\overline{H}_n = \alpha^{n-1}(p_2 y_n - C)E_2 + \lambda_n(a_2 + b_2 x_n - c_2 y_n - E_2)y_n, \tag{3.1}$$

where $\alpha = \frac{1}{1+i}$, i is the instantaneous discount rate for periods, λ_n are variables, E_2 gets maximum value H_n, which is accompanied by the following equations:

$$\Delta\lambda_n = \lambda_n - \lambda_{n-1} = -\frac{\partial \overline{H}_n}{\partial y_n} = -\alpha^{n-1}p_2 E_2 + c_2 y_n \lambda_n, \tag{3.2}$$

$$\Delta^2\lambda_n = \lambda_n - 2\lambda_{n-1} + \lambda_{n-2} = \alpha^{n-2}p_2 E_2(1-\alpha) + c_2 y_n(\lambda_n - \lambda_{n-1}), \tag{3.3}$$

that is,

$$(1 - c_2 y_n)\lambda_n + (c_2 y_n - 2)\lambda_{n-1} + \lambda_{n-2} = \alpha^{n-2}p_2 E_2(1-\alpha).$$

Substituting n into $n-2$ type, we have

$$(1 - c_2 y_n)\lambda_{n+2} + (c_2 y_n - 2)\lambda_{n+1} + \lambda_n = \alpha^n p_2 E_2(1-\alpha), \tag{3.4}$$

$$\Delta = (c_2 y_n - 2)^2 - 4(1 - c_2 y_n) = (c_2 y_n)^2 > 0. \tag{3.5}$$

If $c_2 y_n < 1$, we have a solution

$$\lambda_n = -\frac{p_2 E_2 \alpha^n}{\alpha - 1 + \alpha c_2 y_n}. \tag{3.6}$$

By $\frac{\partial H}{\partial E} = 0$, we have

$$\lambda_n = \alpha^{n-1}(p_2 y_n - C)/y_n \tag{3.7}$$

because

$$E_2 = a_2 - c_2 y_n. \tag{3.8}$$

By (3.6), (3.7) and (3.8), we have

$$y_\alpha = \frac{C(1-\alpha)}{p_2(1-\alpha) + \alpha C c_2 - a_2 p_2 \alpha}. \tag{3.9}$$

From (3.9), we have $y^* = y_\alpha$ as the optimal equilibrium solution. So, seeking to capture the best efforts of degrees

$$E_2^* = a_2 - \frac{c_2 C(1-\alpha)}{p_2(1-\alpha) + \alpha C c_2 - a_2 p_2 \alpha},$$

this is the optimal equilibrium program. Then the economic profit of captured populations is completely controlled by the discount rates α, C, p_2.

Similarly, consider the non-negative equilibrium point $P_0(\frac{a_1-E_1}{b_1},0)$ $(0 < E_1 = E \le E_m)$. If $b_1 x_n < 1$, we have a solution

$$x_\alpha = \frac{C(1-\alpha)}{p_1(1-\alpha) + \alpha C b_1 - a_1 p_1 \alpha}. \tag{3.10}$$

From (3.10), we have $y^* = y_\alpha$ as the optimal equilibrium solution. So, seeking to capture the best efforts of degrees

$$E_1^* = a_1 - \frac{b_1 C(1-\alpha)}{p_1(1-\alpha) + \alpha C b_1 - a_1 p_2 \alpha},$$

this is the optimal equilibrium program. Then the economic profit of captured populations is completely controlled by the discount rates α, C, p_1.

4 Conclusion

This paper qualitatively analyzes a competitive system in situations that are density constrained. We have discussed the stability of equilibrium point in different regions, improved methods of proof in reference. Using the extreme value method to analyze the stable positive equilibrium point is the most optimal way to capture it. By using the Pontryagin maximum principle, through introduces the Hamilton function obtains of the non-negative equilibrium point most superior capture strategy.

Competing interests
The author declares that they have no competing interests.

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