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Research Article

Robust H_∞ Control for Singular Time-Delay Systems via Parameterized Lyapunov Functional Approach

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A new version of delay-dependent bounded real lemma for singular systems with state delay is established by parameterized Lyapunov-Krasovskii functional approach. In order to avoid generating nonconvex problem formulations in control design, a strategy that introduces slack matrices and decouples the system matrices from the Lyapunov-Krasovskii parameter matrices is used. Examples are provided to demonstrate that the results in this paper are less conservative than the existing corresponding ones in the literature.

1. Introduction

During the last decade, a considerable amount of attention has been paid to stability and control of singular systems with delay, since they can better describe and analyze physical systems than the state-space time-delay ones [1, 2]. It should be pointed out that the control problems for singular systems are much more complicated than that for state-space systems, because singular systems usually have three types of modes, namely, finite dynamic modes, impulse modes, and nondynamic modes, whereas the latter two do not appear in state-space systems [1]. Recently, the problem of H_∞ control for singular systems with delay has been investigated by many researchers and various approaches have been adopted [3–15]. There are two performance indexes which are used to judge the conservatism of the derived conditions. One is the H_∞ performance index while the other is admissible upper bound of the delay. For a given time-delay, the smaller the H_∞ performance index, the better the conditions; for a prescribed H_∞ performance level, the larger the admissible upper bound of the time-delay, the less conservative the conditions.

It is well-known that Lyapunov-Krasovskii theorems are basic theories for the study of all types of time-delay system [3–22]. The choice of appropriate Lyapunov-Krasovskii functional (LKF) is crucial for obtaining stability criteria and bounded real lemmas (BRLs) and, as a result, for obtaining solutions to various control problems. In [3–5], the simple form LKFs $V(t, x_t)$ containing definition integral terms were used to obtain the delay-independent bounded real lemma for singular systems with state delay. Generally speaking, delay-dependent conditions are less conservative than the delay-independent ones, especially when the size of delay is small. To obtain delay-dependent conditions, many efforts have been made in the literature [6–14]. Improved LKFs $V(t, x_t)$ containing double integral terms were given in [6–13]. Along the system trajectory, there will be definition integral term in $\dot{V}(t, x_t)$, which causes an unwieldy question: how to deal with it to get less conservative conditions. Some strategies are usually adopted to solve the problem such as different classes of model transformations and over-bounding cross terms [6–8]; free weighting matrix approach [9–12]; delay fractioning technique [13, 14]; integral inequalities approach [15]. However, there are no obvious ways to obtain less conservative results even if one is willing to commit more computational effort to the problem and to find a more tighter bounding for cross terms. This is the serious limitation for these criteria. To overcome the limitation, one has to find some more general LKFs to handle the stability problem for singular systems.

On the other hand, LMI stability conditions via complete quadratic LKF and discretization were introduced by Gu in [17] for time delay system and appeared to be very efficient, leading in some examples to results close to analytical ones. The discretized LKF method has been extended to singular time-delay systems in [16]. An improved bounded real lemma (BRL) is presented by discretization LKF method which greatly lowers the conservatism, but the initial parameter is needed to introduce controller design. To further improve the conclusions, the H_∞ control for singular time-delay systems is proceeded with studies by parameterized LKF approach.

The contribution of this paper lies in three aspects. First, a singular-type complete quadratic Lyapunov-Krasovskii functional is introduced in which the quadratic weighting matrices are defined using matrix polynomial functions. This class of LKF $V(t, x_t)$ covers those considered in [3–15] as special cases, where the quadratic weighting matrices are defined by constant matrices. Second, a delay-dependent BRL is presented to ensure the system to be regular, impulse free, and stable with prescribed H_∞ performance level, which greatly lower the conservatism. Third, the free weighting matrix approach is employed to decouple the system matrices from the weighting matrices of LKF, which urge that the robust H_∞ control problem is solved and an explicit expression of the desired state-feedback control law is also derived.

Notation. Throughout this note, the superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space with vector norm $\|\cdot\|$, $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices, and the notation $P > 0$ means that P is symmetric and positive definite. The symmetric elements of the symmetric matrix will be denoted by $*$. $\mathscr{L}_2^n[0,\infty]$ is the space of square integrable functions $f : [0,\infty] \to \mathbb{R}^n$ with the norm $\|f\|_{L_2} = [\int_0^\infty \|f(t)\|^2 dt]^{1/2}$.

2. Preliminaries and Problem Formulation

Consider the following linear singular system with state delay and parameter uncertainties described by

$$\Sigma_1 : \begin{cases} E\dot{x}(t) = (A_0 + \Delta A_0)\,x(t) + (A_1 + \Delta A_1)\,x(t-r) \\ \qquad\qquad + (B + \Delta B)\,u(t) + D\omega(t), \\ z(t) = (C_0 + \Delta C_0)\,x(t) + (C_1 + \Delta C_1)\,x(t-r) \\ \qquad\qquad + (H + \Delta H)\,u(t), \\ x(t) = \phi(t), \quad t \in [-r, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the control input vector; $\omega(t) \in \mathbb{R}^p$ is the disturbance input which belongs to $\mathscr{L}_2[0,\infty)$, and $z(t) \in \mathbb{R}^l$ is the controlled output. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular. We will assume that $\operatorname{rank}(E) = q \le n$. The matrices A_0, A_1, B, C_0, C_1, D, and H are known real constant matrices with appropriate dimensions. $r > 0$ is a constant time delay. $\phi(t)$ is a compatible vector valued initial function. ΔA_0, ΔA_1, ΔB, ΔC_0, ΔC_1, and ΔH are time-invariant matrices representing norm-bounded parameter uncertainties and are assumed to be of the form

$$\begin{bmatrix} \Delta A_0 & \Delta A_1 & \Delta B \\ \Delta C_0 & \Delta C_1 & \Delta H \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} F \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix}, \tag{2}$$

where M_1, M_2, N_1, N_2, and N_3 are constant matrices and F is an unknown real matrix satisfying

$$F^T F \le I. \tag{3}$$

The parameter uncertainties ΔA_0, ΔA_1, ΔB, ΔC_0, ΔC_1, and ΔH are said to be admissible if both (2) and (3) hold.

The nominal singular time-delay system of Σ_1 with $u(t) = 0$ can be written as

$$\Sigma_0 : \begin{cases} E\dot{x}(t) = A_0 x(t) + A_1 x(t-r) + D\omega(t), \\ z(t) = C_0 x(t) + C_1 x(t-r), \\ x(t) = \phi(t), \quad t \in [-r, 0]. \end{cases} \tag{4}$$

Throughout the paper, we will adopt the following definition.

Definition 1 (see [1]). (1) The pair (E, A_0) is said to be regular if $\det(sE - A_0)$ is not identically zero.

(2) The pair (E, A_0) is said to be impulse free if $\deg(\det(sE - A_0)) = \operatorname{rank} E$.

Lemma 2 (see [3]). *Suppose the pair (E, A_0) is regular and impulsive free, then the solution to unforced systems Σ_0 exists and is impulse free and unique on $[0,\infty)$.*

Remark 3. The regularity of pair (E, A_0) guarantees the existence and uniqueness of solution for system Σ_0; impulse terms are generally not expected to appear since strong impulse behavior may stop the system from work or even destroy it.

In view of this, we introduce the following definition for singular time-delay system Σ_0.

Definition 4 (see [3]). (1) The singular time-delay system Σ_0 is said to be regular and impulse free, if the pair (E, A_0) is regular and impulse free.

(2) The singular time-delay system Σ_0 is said to be stable if, for any $\varepsilon > 0$, there exists a scalar $\delta(\varepsilon) > 0$, such that for any compatible initial function $\phi(t)$ satisfies $\sup_{-r\le t\le 0}\|\phi(t)\| \le \delta(\varepsilon)$, and the solutions $x(t)$ of system Σ_0 satisfy $\|x(t)\| \le \varepsilon$ for $t \ge 0$. Furthermore, $x(t) \to 0$ as $t \to \infty$.

Definition 5. The uncertain singular time-delay system Σ_1 is said to be robustly stable if the unforced system of Σ_1 is regular, impulse free, and stable for all admissible uncertainties.

Definition 6. For a given scalar $\gamma > 0$, the uncertain singular time-delay system Σ_1 with $u(t) = 0$ is said to be robustly stable with H_∞ performance γ, if it is robustly stable and under zero initial condition $\|z(t)\|_2 < \gamma\|\omega(t)\|_2$ is satisfied for any nonzero $\omega(t) \in \mathscr{L}_2^p[0,\infty)$ and all admissible uncertainties.

The aim of this paper is for prescribed scalars $r > 0$ and $\gamma > 0$ to develop a state feedback controller

$$u(t) = K_0 x(t) + K_1 x(t-r) \tag{5}$$

such that closed-loop system is robustly stable with H_∞ performance γ, where K_0 and K_1 are matrices to be determined.

We conclude this section by presenting the following lemma, which is extensively used in uncertain system research.

Lemma 7 (see [23]). *For appropriate dimensional matrices* Γ, Ξ *and symmetric matrix* Ω, *all the* $F(t)$ *satisfied* $F^T(t)F(t) \leq I$,

$$\Omega + \Gamma F(t)\Xi + \Xi^T F^T(t)\Gamma^T < 0 \tag{6}$$

if and only if there exists a constant $\varepsilon > 0$ *such that*

$$\Omega + \varepsilon\Gamma\Gamma^T + \varepsilon^{-1}\Xi^T\Xi < 0. \tag{7}$$

3. Main Results

First of all, we present delay-dependent result that assures the nominal unforced singular system Σ_0 to be regular, impulse free, and stable with H_∞ performance γ, which will play a key role in solving the H_∞ control problem.

3.1. A New Version of Delay-Dependent Bounded Real Lemma. We introduce a singular-type complete quadratic Lyapunov-Krasovskii functional

$$\begin{aligned} V(t,x_t) &= x^T(t)E^T PEx(t) \\ &+ 2x^T(t)E^T\int_{-r}^{0} Q(\xi)x(t+\xi)\,d\xi \\ &+ \int_{-r}^{0}\int_{-r}^{0} x^T(t+\xi)R(\xi,\eta)x(t+\eta)\,d\xi d\eta \\ &+ \int_{-r}^{0} x^T(t+\xi)S(\xi)x(t+\xi)\,d\xi, \end{aligned} \tag{8}$$

where $Q(\xi), S(\xi) = S^T(\xi)$ and $R(\xi,\eta) = R^T(\eta,\xi)$ are matrix polynomial functions in the form of

$$\begin{aligned} Q(\xi) &= \mathcal{Q}W(\xi), \\ S(\xi) &= S + (r+\xi)W^T(\xi)\mathcal{T}W(\xi), \\ R(\xi,\eta) &= W^T(\xi)\mathcal{R}W(\eta), \end{aligned} \tag{9}$$

where

$$\begin{aligned} W^T(\xi) &= \left(I_n \;\; \xi I_n \;\; \cdots \;\; \xi^{N-1}I_n\right), \\ \mathcal{Q} &= [Q_1 \;\; Q_2 \;\; \cdots \;\; Q_N], \\ \mathcal{T} &= \begin{bmatrix} T_{11} & T_{12} & \cdots & T_{1N} \\ * & T_{22} & \cdots & T_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & T_{NN} \end{bmatrix}, \\ \mathcal{R} &= \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ * & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & R_{NN} \end{bmatrix}. \end{aligned} \tag{10}$$

Noting that the differentiation of the partitioned matrix $\dot{W}(\xi) = \mathcal{D}W(\xi)$, where

$$\mathcal{D} = D \otimes I_n, \qquad D = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & N-1 & 0 \end{bmatrix}. \tag{11}$$

Based on it, the differentiation of the functions $Q(\xi)$, $S(\xi)$, and $R(\xi,\eta)$ is given by

$$\begin{aligned} \dot{Q}(\xi) &= \mathcal{Q}\mathcal{D}W(\xi), \\ \dot{S}(\xi) &= W^T(\xi)\left[\mathcal{T} + (r+\xi)\left(\mathcal{D}^T\mathcal{T} + \mathcal{T}\mathcal{D}\right)\right]W(\xi), \\ \frac{\partial R(\xi,\eta)}{\partial\xi} &= W^T(\xi)\mathcal{D}^T\mathcal{R}W(\eta), \\ \frac{\partial R(\xi,\eta)}{\partial\eta} &= W^T(\xi)\mathcal{R}\mathcal{D}W(\eta). \end{aligned} \tag{12}$$

A new version of bounded real lemma is obtained via the singular-type complete quadratic Lyapunov-Krasovskii functional (8) with polynomial parameter (9).

Theorem 8. *For a given scalar* $\gamma > 0$, *the system* Σ_0 *is regular, impulse free, and stable with* H_∞ *performance* γ; *suppose that there exist symmetric positive definite matrices* $P \in \mathbb{R}^{n\times n}$, $S \in \mathbb{R}^{n\times n}$, $\mathcal{T} \in \mathbb{R}^{nN\times nN}$, *and* $\mathcal{R} \in \mathbb{R}^{nN\times nN}$ *and matrices* $\mathcal{Q} \in \mathbb{R}^{n\times nN}$, $U \in \mathbb{R}^{(n-q)\times n}$, $P_2 \in \mathbb{R}^{n\times n}$, $P_3 \in \mathbb{R}^{n\times nN}$, *and* $P_4 \in \mathbb{R}^{n\times p}$ *such that*

$$\Phi = \begin{bmatrix} E^T PE & E^T\mathcal{Q} \\ * & \mathcal{R} \end{bmatrix} \geq 0, \tag{13}$$

$$\Pi = \mathcal{T} + r\left(\mathcal{D}^T\mathcal{T} + \mathcal{T}\mathcal{D}\right) + \mathcal{T}\mathfrak{D} + \mathfrak{D}\mathcal{T} > 0, \tag{14}$$

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & A_0^T P_2 & \Psi_{14} & A_0^T P_4 + P_1^T D & C_0^T \\ * & -S & A_1^T P_2 & \Psi_{24} & A_1^T P_4 & C_1^T \\ * & * & -P_2 - P_2^T & \mathcal{Q} - P_3 & P_2^T D - P_4 & 0 \\ * & * & * & \Psi_{44} & P_3^T D & 0 \\ * & * & * & * & \Psi_{55} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{15}$$

hold, where $V \in \mathbb{R}^{n\times(n-q)}$ is any matrix with full column rank and satisfies $E^TV = 0$ and

$$\Psi_{11} = P_1^T A_0 + A_0^T P_1 + E^T \mathcal{Q} W(0) + W(0)^T \mathcal{Q}^T E + S + rW(0)^T \mathcal{T} W(0),$$
$$\Psi_{12} = P_1^T A_1 - E^T \mathcal{Q} W(-r),$$
$$\Psi_{14} = A_0^T P_3 - E^T \mathcal{Q}\mathcal{D} + W(0)^T \mathcal{R},$$
$$\Psi_{24} = A_1^T P_3 - W^T(-r)\mathcal{R},$$
$$\Psi_{44} = -\frac{\Pi}{r} - \mathcal{D}^T\mathcal{R} - \mathcal{R}\mathcal{D},$$
$$\Psi_{55} = P_4^T D + D^T P_4 - \gamma^2 I,$$
$$\mathfrak{D} = \operatorname{diag}\{0,1,\dots,N-1\}\otimes I_n, \qquad P_1 = (PE + VU). \tag{16}$$

Proof. Since $\operatorname{rank}(E) = q \le n$, there exist nonsingular matrices $\overline{G}$ and $\overline{H}$ such that $\overline{E} = \overline{G}E\overline{H} = \left[\begin{smallmatrix} I_q & 0 \\ 0 & 0\end{smallmatrix}\right]$. Noting this and $E^TV = 0$, $\operatorname{rank}(V) = n - q$, we can show that V can be parameterized as $V = \overline{G}^T \left[\begin{smallmatrix} 0 \\ \overline{V}\end{smallmatrix}\right]$, where $\overline{V} \in \mathbb{R}^{(n-q)\times(n-q)}$ is any nonsingular matrix. Accordingly, we define the following transformations:

$$\begin{aligned}
\overline{P} &= \overline{G}^{-T} P \overline{G}^{-1} = \begin{bmatrix} \overline{P}_{11} & \overline{P}_{12} \\ \overline{P}_{12}^T & \overline{P}_{22}\end{bmatrix},\\
\overline{A}_0 &= \overline{G} A_0 \overline{H} = \begin{bmatrix} \overline{A}_{01} & \overline{A}_{02} \\ \overline{A}_{03} & \overline{A}_{04}\end{bmatrix},\\
\overline{S} &= \overline{H}^T S \overline{H} = \begin{bmatrix} \overline{S}_{11} & \overline{S}_{12} \\ \overline{S}_{12}^T & \overline{S}_{22}\end{bmatrix},\\
\overline{T}_{11} &= \overline{H}^T T_{11} \overline{H} = \begin{bmatrix} \overline{T}_{111} & \overline{T}_{112} \\ \overline{T}_{113} & \overline{T}_{114}\end{bmatrix},\\
\overline{Q}_1 &= \overline{G}^{-T} Q_1 \overline{H} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} \\ \overline{Q}_{13} & \overline{Q}_{14}\end{bmatrix},\\
\overline{U} &= U\overline{H} = \begin{bmatrix} \overline{U}_1 & \overline{U}_2 \end{bmatrix}.
\end{aligned} \tag{17}$$

Then, pre- and postmultiplying Ψ_{11} by $\overline{H}^T$ and $\overline{H}$ get

$$\begin{bmatrix} \star & \star \\ \star & \overline{A}_{04}^T \overline{V}\overline{U}_2 + \overline{U}_2^T \overline{V}^T \overline{A}_{04} + \overline{S}_{22} + r\overline{T}_{114}\end{bmatrix} < 0, \tag{18}$$

which obviously implies that $\overline{A}_{04}$ is nonsingular. Then we can deduce system Σ_0 is regular and impulse free. Here, the terms denoted $\star$ are irrelevant to the results of the above discussion, so the real expression of these variables is omitted.

Next, we will show the nominal singular system Σ_0 is stable and has the H_∞ performance γ. Consider the functional (8) with the functions $Q(\xi)$, $R(\xi,\eta)$, and $S(\xi)$ as in (9), and define the vector $\psi(t) = \int_{-r}^{0} W(\xi)x(t+\xi)d\xi$; then

$$\begin{aligned}
V(t,x_t) &= x^T(t)E^TPEx(t) + 2x^T(t)E^T\mathcal{Q}\psi(t) + \int_{-r}^{0} x^T(t+\xi)Sx(t+\xi)d\xi \\
&\quad + \psi^T(t)\mathcal{R}\psi(t) + \int_{-r}^{0}(r+\xi)x^T(t+\xi)W^T(\xi)\mathcal{T}W(\xi)x(t+\xi)d\xi.
\end{aligned} \tag{19}$$

Denote $\xi^T(t) = \begin{pmatrix} x^T(t) & \psi^T(t)\end{pmatrix}$; the functional (8) satisfies

$$\begin{aligned}
V(t,x_t) &\ge \xi^T(t)\Phi\xi(t) + \int_{-r}^{0} x^T(t+\xi)Sx(t+\xi)d\xi \\
&\quad + \int_{-r}^{0}(r+\xi)x^T(t+\xi)W^T(\xi)\mathcal{T}W(\xi)x(t+\xi)d\xi.
\end{aligned} \tag{20}$$

If $\mathcal{T} > 0$, $S > 0$, and $\Phi \ge 0$, then $V(t,x_t)$ is positive definite. The derivation of $V(t,x_t)$ along system Σ_0 gives

$$\begin{aligned}
&\dot{V}(t,x_t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \\
&= x^T(t)\left[E^T\mathcal{Q}W(0) + W^T(0)\mathcal{Q}^TE + S + rW^T(0)\mathcal{T}W(0) + C_0^TC_0\right]x(t) \\
&\quad + 2x^T(t)\left[-E^T\mathcal{Q}W(-r) + C_0^TC_1\right]x(t-r) \\
&\quad + 2x^T(t)\left[E^TP + U^TV^T\right]E\dot{x}(t) \\
&\quad + 2x^T(t)\left[W^T(0)\mathcal{R} - E^T\mathcal{Q}D\right]\psi(t) \\
&\quad + x^T(t-r)\left[-S + C_1^TC_1\right]x(t-r) \\
&\quad - 2x^T(t-r)W^T(-r)\mathcal{R}\psi(t) + 2\dot{x}^T(t)E^T\mathcal{Q}\psi(t) \\
&\quad - \psi^T(t)\left(\mathcal{D}^T\mathcal{R} + \mathcal{R}\mathcal{D}\right)\psi(t) \\
&\quad - \int_{-r}^{0} x^T(t+\xi)W^T(\xi)\Pi W(\xi)x(t+\xi)d\xi \\
&\quad - \gamma^2\omega^T(t)\omega(t).
\end{aligned} \tag{21}$$

As $\Pi > 0$, Jensen's inequality ensures that

$$-\int_{-r}^{0} x^T(t+\xi)W^T(\xi)\Pi W(\xi)x(t+\xi)d\xi \le -\frac{1}{r}\psi^T(t)\Pi\psi(t). \tag{22}$$

On the other hand, for any appropriate dimensional matrices P_2, P_3, and P_4, the following equation is true:

$$\begin{aligned} &2\left[x^T(t)P_1^T + \dot{x}^T(t)E^TP_2^T + \psi^T(t)P_3^T + \omega^T(t)P_4^T\right] \\ &\quad\times\left[A_0x(t) + A_1x(t-r) - E\dot{x}(t) + D\omega(t)\right] = 0. \end{aligned} \tag{23}$$

Hence

$$\dot{V}(t,x_t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \le \zeta^T(t)\Psi\zeta(t), \tag{24}$$

where $\zeta^T(t) = \left(x^T(t)\ \ x^T(t-r)\ \ \dot{x}^T(t)E^T\ \ \psi^T(t)\ \ \omega^T(t)\right)$. Under the zero initial condition, it can be shown that, for any nonzero $\omega(t) \in \mathscr{L}_2[0,\infty)$, the following index is

$$\begin{aligned} J_{z\omega} &= \int_0^\infty \left(z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\right)dt \\ &\le \int_0^\infty \left[z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(t,x_t)\right]dt \\ &\le \int_0^\infty \zeta^T(t)\Omega\zeta(t)\,dt, \end{aligned} \tag{25}$$

where

$$\Omega = \begin{bmatrix} \Psi_{11} + C_0^TC_0 & \Psi_{12} + C_0^TC_1 & A_0^TP_2 & \Psi_{14} & A_0^TP_4 + P_1^TD \\ * & -S + C_1^TC_1 & A_1^TP_2 & \Psi_{24} & A_1^TP_4 \\ * & * & -P_2 - P_2^T & \mathscr{Q} - P_3 & P_2^TD - P_4 \\ * & * & * & \Psi_{44} & P_3^TD \\ * & * & * & * & \Psi_{55} \end{bmatrix}. \tag{26}$$

Thus, $\Psi < 0$ implies that $J_{z\omega} < 0$; that is, $\|z(t)\|_2 < \gamma\|\omega(t)\|_2$ for any nonzero $\omega(t) \in \mathscr{L}_2[0,\infty)$. Moreover, similar to [16], the feasibility of LMIs (13), (14), and

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & A_0^TP_2 & \Psi_{14} \\ * & -S & A_1^TP_2 & \Psi_{24} \\ * & * & -P_2 - P_2^T & \mathscr{Q} - P_3 \\ * & * & * & \Psi_{44} \end{bmatrix} < 0 \tag{27}$$

implies $\dot{V}(t,x_t) < 0$; then the system Σ_0 with $\omega(t) = 0$ is stable. This completes the proof. □

Remark 9. In Theorem 8, matrices P_i $(i = 1,2,3,4)$ are introduced to decouple the system matrices A_0, A_1 from the LKF weighting matrices P and Q_i, $(i = 1,2,\ldots,N)$, which provides convenience for controller design.

Remark 10. When E is nonsingular, the singular system Σ_0 reduces to a state-space system. In this case, if we choose $V = 0$ in the proof of Theorem 8, we can derive a new version of bounded real lemma for state-space system.

Based on Theorem 8, we obtain the following result on H_∞ performance analysis for the uncertain singular system Σ_1 with $u(t) = 0$.

Theorem 11. *For a given scalar $\gamma > 0$, the system Σ_1 is regular, impulse free, and stable with H_∞ performance γ; suppose that there exist symmetric positive definite matrices $P \in \mathbb{R}^{n\times n}$, $S \in \mathbb{R}^{n\times n}$, $\mathscr{T} \in \mathbb{R}^{nN\times nN}$, and $\mathscr{R} \in \mathbb{R}^{nN\times nN}$ and matrices $\mathscr{Q} \in \mathbb{R}^{n\times nN}$, $U \in \mathbb{R}^{(n-q)\times n}$, $P_2 \in \mathbb{R}^{n\times n}$, $P_3 \in \mathbb{R}^{n\times nN}$, and $P_4 \in \mathbb{R}^{n\times p}$ and scalar $\varepsilon > 0$ such that (13), (14), and*

$$\begin{bmatrix} \widetilde{\Psi}_{11} & \widetilde{\Psi}_{12} & A_0^TP_2 & \Psi_{14} & A_0^TP_4 + P_1^TD & C_0^T & P_1^TM_1 \\ * & \widetilde{\Psi}_{22} & A_1^TP_2 & \Psi_{24} & A_1^TP_4 & C_1^T & 0 \\ * & * & -P_2 - P_2^T & \mathscr{Q} - P_3 & P_2^TD - P_4 & 0 & P_2^TM_1 \\ * & * & * & \Psi_{44} & P_3^TD & 0 & P_3^TM_1 \\ * & * & * & * & \Psi_{55} & 0 & P_4^TM_1 \\ * & * & * & * & * & -I & M_2 \\ * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{28}$$

hold, where $\widetilde{\Psi}_{11} = \Psi_{11} + \varepsilon N_1^TN_1$, $\widetilde{\Psi}_{12} = \Psi_{12} + \varepsilon N_1^TN_2$, and $\widetilde{\Psi}_{22} = -S + \varepsilon N_2^TN_2$.

Proof. Suppose that there exist symmetric positive definite matrices $P \in \mathbb{R}^{n\times n}$, $S \in \mathbb{R}^{n\times n}$, $\mathscr{T} \in \mathbb{R}^{nN\times nN}$, and $\mathscr{R} \in \mathbb{R}^{nN\times nN}$ and matrices $\mathscr{Q} \in \mathbb{R}^{n\times nN}$, $U \in \mathbb{R}^{(n-q)\times n}$, $P_2 \in \mathbb{R}^{n\times n}$, $P_3 \in \mathbb{R}^{n\times nN}$, and $P_4 \in \mathbb{R}^{n\times p}$ and scalar ε such that (13), (14), and (28) hold. Then, by Schur complement, it follows from (28) that

$$\Psi + \varepsilon\Gamma^T\Gamma + \frac{1}{\varepsilon}\Xi\Xi^T < 0, \tag{29}$$

where

$$\begin{aligned} \Xi^T &= \left[M_1^TP_1\ \ 0\ \ M_1^TP_2\ \ M_1^TP_3\ \ M_1^TP_4\ \ M_2^T\right], \\ \Gamma &= \left[N_1\ \ N_2\ \ 0\ \ 0\ \ 0\ \ 0\right]. \end{aligned} \tag{30}$$

By Lemma 7, $\Psi + \Xi F\Gamma + (\Xi F\Gamma)^T < 0$, which is in the form of Ψ by replacing A_0, A_1, C_0, and C_1 with $A_0 + M_1FN_1$, $A_1 + M_1FN_2$, $C_0 + M_2FN_1$, and $C_1 + M_2FN_2$, respectively. Thus, by Theorem 8, we have that the uncertain singular system Σ_1 with $u(t) = 0$ is robustly stable with H_∞ performance γ. □

3.2. Robust H_∞ Controller Design. In this subsection, we will apply the bounded real lemma obtained above to design the state feedback controller (5) such that the resultant closed-loop system is regular, impulse free, and stable with H_∞ performance γ.

The nominal closed-loop singular system of Σ_0 can be written as

$$\Sigma_{0c}: \begin{cases} E\dot{x}(t) = \left(A_0 + BK_0\right)x(t) + \left(A_1 + BK_1\right)x(t-r) + D\omega(t), \\ z(t) = \left(C_0 + HK_0\right)x(t) + \left(C_1 + HK_1\right)x(t-r), \\ x(t) = \phi(t), \quad t \in [-r,0]. \end{cases} \tag{31}$$

Theorem 12. *For prescribed scalar $\gamma > 0$, the closed-loop singular system Σ_{0c} is regular, impulse free, and stable with*

H_∞ performance γ; suppose that there exist symmetric positive definite matrices $\overline{S} \in \mathbb{R}^{n\times n}$, $\overline{\mathcal{T}} \in \mathbb{R}^{nN\times nN}$, and $\overline{\mathcal{R}} \in \mathbb{R}^{nN\times nN}$ and matrices $\overline{P} \in \mathbb{R}^{n\times n}$, $\overline{\mathcal{Q}} \in \mathbb{R}^{n\times nN}$, and $L_i \in \mathbb{R}^{m\times n}$, $i = 0, 1$ and scalars $\delta_j > 0$, $j = 1, 2$ such that

$$\overline{\Phi} = \begin{bmatrix} E\overline{P}E^T & E\overline{\mathcal{Q}} \\ * & \overline{\mathcal{R}} \end{bmatrix} \geq 0, \quad (32)$$

$$\overline{\Pi} = \overline{\mathcal{T}} + r\left(\mathcal{D}^T\overline{\mathcal{T}} + \overline{\mathcal{T}}\mathcal{D}\right) + \overline{\mathcal{T}}\mathfrak{D} + \mathfrak{D}\overline{\mathcal{T}} > 0, \quad (33)$$

$$\overline{\Psi} = \begin{bmatrix} \overline{\Psi}_{11} & \overline{\Psi}_{12} & \overline{\Psi}_{13} & \overline{\Psi}_{14} & D & \overline{\Psi}_{16} \\ * & -\overline{S} & \overline{\Psi}_{23} & \overline{\Psi}_{24} & 0 & \overline{\Psi}_{26} \\ * & * & \overline{\Psi}_{33} & \overline{\Psi}_{34} & \delta_1 D & 0 \\ * & * & * & \overline{\Psi}_{44} & \overline{\Psi}_{45} & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (34)$$

hold. Then a suitable state-feedback control law is given by

$$u(t) = L_0\overline{P}^{-1}x(t) + L_1\overline{P}^{-1}x(t-r), \quad (35)$$

where

$$\begin{aligned}
\overline{\Psi}_{11} &= A_0\overline{P} + BL_0 + \left(A_0\overline{P} + BL_0\right)^T \\
&\quad + E\overline{\mathcal{Q}}W(0) + W(0)^T\overline{\mathcal{Q}}^T E^T \\
&\quad + \overline{S} + r\overline{T}_{11}, \\
\overline{\Psi}_{12} &= A_1\overline{P} + BL_1 - E\overline{\mathcal{Q}}W(-r), \\
\overline{\Psi}_{13} &= \delta_1\left(\overline{P}^T A_0^T + L_0^T B^T\right), \\
\overline{\Psi}_{14} &= \left[\delta_2\left(\overline{P}^T A_0^T + L_0^T B^T\right) \ \cdots \ \delta_2\left(\overline{P}^T A_0^T + L_0^T B^T\right)\right] \\
&\quad - E\overline{\mathcal{Q}}\mathcal{D} + W(0)^T\overline{\mathcal{R}}, \\
\overline{\Psi}_{16} &= \overline{P}^T C_0^T + L_0^T H^T, \\
\overline{\Psi}_{26} &= \overline{P}^T C_1^T + L_1^T H^T, \\
\overline{\Psi}_{23} &= \delta_1\left(\overline{P}^T A_1^T + L_1^T B^T\right), \\
\overline{\Psi}_{24} &= \left[\delta_2\left(\overline{P}^T A_1^T + L_1^T B^T\right) \ \cdots \ \delta_2\left(\overline{P}^T A_1^T + L_1^T B^T\right)\right] \\
&\quad - W^T(-r)\overline{\mathcal{R}}, \\
\overline{\Psi}_{33} &= -\delta_1\left(\overline{P} + \overline{P}^T\right), \\
\overline{\Psi}_{34} &= \overline{\mathcal{Q}} - \left[\delta_2\overline{P}^T \ \cdots \ \delta_2\overline{P}^T\right], \\
\overline{\Psi}_{44} &= -\frac{\overline{\Pi}}{r} - \mathcal{D}^T\overline{\mathcal{R}} - \overline{\mathcal{R}}\mathcal{D}, \\
\overline{\Psi}_{45}^T &= \delta_2\left[D^T \ \cdots \ D^T\right].
\end{aligned} \quad (36)$$

Proof. According to Theorem 8, the closed-loop singular system Σ_{0c} is regular, impulse free, and stable with H_∞ performance γ, if there exist symmetric positive definite matrices $P \in \mathbb{R}^{n\times n}$, $S \in \mathbb{R}^{n\times n}$, and $\mathcal{T} \in R^{nN\times nN}$ and matrices $\mathcal{Q} \in R^{n\times nN}$, $\mathcal{R} \in R^{nN\times nN}$, $Q \in \mathbb{R}^{(n-q)\times n}$, $P_2 \in \mathbb{R}^{n\times n}$, $P_3 \in \mathbb{R}^{n\times nN}$, and $P_4 \in \mathbb{R}^{n\times p}$ such that (13), (14), and $\widehat{\Psi} < 0$ hold, where $\widehat{\Psi}$ is in the form of Ψ with A_0, A_1, C_0, C_1 replaced by $A_0 + BK_0$, $A_1 + BK_1$, $C_0 + HK_0$, and $C_1 + HK_1$. Let $P_2 = \delta_1 P_1$, $P_3 = \delta_2\left[P_1 \ \cdots \ P_1\right]$, $P_4 = [0]_{n\times p}$, and $P_1^{-1} = \overline{P}$. Pre- and postmultiply Φ by $\operatorname{diag}\{\overline{P}^T, \mathcal{P}^T\}$ and its transpose; pre- and postmultiply Π by $\mathcal{P}^T$ and its transpose; pre- and postmultiply $\widehat{\Psi}$ by $\operatorname{diag}\{\overline{P}^T, \overline{P}^T, \overline{P}^T, \mathcal{P}^T, I, I\}$ and its transpose. We introduce the following:

$$\begin{gathered}
\overline{\mathcal{Q}} = \overline{P}\mathcal{Q}\mathcal{P}, \\
\overline{\mathcal{R}} = \mathcal{P}^T\mathcal{R}\mathcal{P}, \\
\overline{S} = \overline{P}^T S\overline{P}, \\
\overline{\mathcal{T}} = \mathcal{P}^T\mathcal{T}\mathcal{P}, \\
\mathcal{P} = \operatorname{diag}\left\{\overline{P}, \overline{P}, \ldots, \overline{P}\right\} \in R^{nN\times nN}, \\
L_0 = K_0\overline{P}, \qquad L_1 = K_1\overline{P}.
\end{gathered} \quad (37)$$

After some manipulation, we can obtain (32), (33), and (34), and the desired controller gains are given by $K_0 = L_0\overline{P}^{-1}$, $K_1 = L_1\overline{P}^{-1}$. □

Remark 13. In this case, we can assume that the matrix P_1 is nonsingular. If this is not the case, we can choose some $\theta \in (0, 1)$ such that $\widetilde{P}_1 = P_1 + \theta\widehat{P}$ is nonsingular and satisfies (32) and (34), in which $\widehat{P}$ is any nonsingular matrix satisfying $E^T\widehat{P} = \widehat{P}^T E$.

Remark 14. In the case where $x(t-r)$ is available for feedback, the use of K_0 and K_1 can lead to a significative reduction on the values of γ. On the other hand, when $x(t-r)$ is not available for feedback, a memoryless control law is required; it is enough to set $L_1 = 0$ in LMI (34).

Now we are in a position to present the result on the problem of delay-dependent robust H_∞ control for the uncertain singular system Σ_1. The closed-loop singular system of Σ_1 can be written as

$$\Sigma_{1c}: \begin{cases} E\dot{x}(t) = \left[\left(A_0 + \Delta A_0\right) + (B + \Delta B)K_0\right]x(t) \\ \qquad + \left[\left(A_1 + \Delta A_1\right) + (B + \Delta B)K_1\right] \\ \qquad \times x(t-r) + D\omega(t), \\ z(t) = \left[\left(C_0 + \Delta C_0\right) + (H + \Delta H)K_0\right]x(t) \\ \qquad + \left[\left(C_1 + \Delta C_1\right) + (H + \Delta H)K_1\right] \\ \qquad \times x(t-r), \\ x(t) = \phi(t), \quad t \in [-r, 0]. \end{cases} \quad (38)$$

Theorem 15. *For prescribed scalar $\gamma > 0$, the closed-loop singular system Σ_{0c} is regular, impulse free, and stable with H_∞ performance γ; suppose that there exist symmetric positive definite matrices $\overline{S} \in \mathbb{R}^{n\times n}$, $\overline{\mathcal{T}} \in R^{nN\times nN}$, $\overline{\mathcal{R}} \in R^{nN\times nN}$ and* *matrices* $\overline{P} \in \mathbb{R}^{n\times n}$, $\overline{Q} \in R^{n\times nN}$, $L_i \in \mathbb{R}^{m\times n}$, $i = 0, 1$ *and scalars* $\delta_j > 0$, $(j = 1, 2)$, $\varepsilon > 0$ *such that* (32), (33) *and*

$$\widehat{\Psi} = \begin{bmatrix} \widehat{\Psi}_{11} & \overline{\Psi}_{12} & \widehat{\Psi}_{13} & \widehat{\Psi}_{14} & D & \widehat{\Psi}_{16} & \widehat{\Psi}_{17} \\ * & -\overline{S} & \overline{\Psi}_{23} & \overline{\Psi}_{24} & 0 & \overline{\Psi}_{26} & \widehat{\Psi}_{27} \\ * & * & \widehat{\Psi}_{33} & \widehat{\Psi}_{34} & \delta_1 D & \varepsilon\delta_1 M_1 M_2^T & 0 \\ * & * & * & \widehat{\Psi}_{44} & \overline{\Psi}_{45} & \widehat{\Psi}_{46} & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & -I + \varepsilon M_2 M_2^T & 0 \\ * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{39}$$

hold. Then a suitable state-feedback control law is given by

$$u(t) = L_0 \overline{P}^{-1} x(t) + L_1 \overline{P}^{-1} x(t - r), \tag{40}$$

where

$$\begin{aligned} &\widehat{\Psi}_{11} = \overline{\Psi}_{11} + \varepsilon M_1 M_1^T, \qquad \widehat{\Psi}_{13} = \overline{\Psi}_{13} + \varepsilon\delta_1 M_1 M_1^T, \\ &\widehat{\Psi}_{14} = \overline{\Psi}_{14} + \varepsilon\delta_2 M_1 \begin{bmatrix} M_1^T & \cdots & M_1^T \end{bmatrix}, \\ &\widehat{\Psi}_{16} = \overline{\Psi}_{16} + \varepsilon M_1 M_2^T, \\ &\widehat{\Psi}_{33} = \overline{\Psi}_{33} + \varepsilon\delta_1^2 M_1 M_1^T, \\ &\widehat{\Psi}_{34} = \overline{\Psi}_{34} + \varepsilon\delta_1\delta_2 M_1 \begin{bmatrix} M_1^T & \cdots & M_1^T \end{bmatrix}, \\ &\widehat{\Psi}_{44} = \overline{\Psi}_{44} + \varepsilon\delta_2^2 \begin{bmatrix} M_1 M_1^T & \cdots & M_1 M_1^T \\ \vdots & \ddots & \vdots \\ M_1 M_1^T & \cdots & M_1 M_1^T \end{bmatrix}, \\ &\widehat{\Psi}_{46} = \varepsilon\delta_2 M_1 \begin{bmatrix} M_2^T \\ \vdots \\ M_2^T \end{bmatrix}, \\ &\widehat{\Psi}_{17} = \overline{P}^T N_1^T + L_0^T N_3^T, \qquad \widehat{\Psi}_{27} = \overline{P}^T N_2^T + L_1^T N_3^T. \end{aligned} \tag{41}$$

The proof can be carried out by resorting to Theorem 12 and following a similar line as in the proof of Theorem 11 and is thus omitted.

4. Examples

Example 1. Consider a singular time-delay system in the form of Σ_0 with

$$\begin{gathered} E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A_0 = \begin{bmatrix} -0.3012 & 0.1257 \\ 0.2351 & -2.5652 \end{bmatrix}, \\ A_1 = \begin{bmatrix} -0.5124 & 0.9648 \\ 0.1023 & 0.8197 \end{bmatrix}, \\ D = \begin{bmatrix} 0.2102 \\ -0.8152 \end{bmatrix}, \qquad C_0 = \begin{bmatrix} 1.2321 & 0.3185 \end{bmatrix}, \\ C_1 = \begin{bmatrix} 0.8765 & 0.8231 \end{bmatrix}. \end{gathered} \tag{42}$$

Table 1: Comparisons of maximum allowed delay r^* for Example 1.

γ	2	2.5	3	3.5
r^* by [6]	1.8116	2.0029	2.1465	2.2587
r^* by [11]	2.2761	2.6131	2.8739	3.0855
r^* by [16]	3.0710	3.4810	3.8030	4.0640
r^* by Theorem 8	3.1303	3.5666	3.9113	4.1860

Table 2: Comparisons of maximum allowed delay r^* for Example 2.

γ	4.0	4.5	5.0	5.5	6.0	6.5
r^* by [10]	0.9425	0.9635	0.9801	0.9938	1.0053	1.0151
r^* by Theorem 11	1.0383	1.0497	1.0589	1.0666	1.0730	1.0784

Table 3: Comparisons of minimum allowed γ for Example 2.

r	0.35	0.40	0.45	0.50	0.55	0.60
γ by [10]	1.4160	1.4676	1.5251	1.600	1.6652	1.7546
γ by Theorem 11	1.0392	1.0488	1.0583	1.0677	1.0817	1.0954

For given $\gamma > 0$, we can calculate the maximum allowed delay r^* satisfying the LMIs in Theorem 8. To show the low conservativeness of the result, we compare ours with the criteria of [6, 11, 16] in Table 1. It is clear that the characterization of bounded realness in this paper is an improvement over the previous ones.

Example 2. Consider the uncertain singular time-delay system Σ_1 with $u(t) = 0$, where

$$\begin{gathered} E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A_0 = \begin{bmatrix} 0.5 & 0 \\ 0 & -1 \end{bmatrix}, \\ A_1 = \begin{bmatrix} -1 & 1 \\ 0 & -0.5 \end{bmatrix}, \\ D = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix}, \qquad C_0 = \begin{bmatrix} 0.5 & 1 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0 & 0 \end{bmatrix}, \\ M_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad N_1 = N_2 = \begin{bmatrix} 0.025 & 0 \\ 0 & 0.025 \end{bmatrix}. \end{gathered} \tag{43}$$

Tables 2 and 3 give the comparison results on the maximum allowed delay r^* for given γ and the minimum allowed γ for given $r > 0$, respectively. It is clear that the results of Theorem 11 are significantly better than those in [10].

Example 3. We consider the problem of H_∞ control for the singular time-delay system Σ_0 with the following parameters:

$$\begin{gathered} E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\ A_1 = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix}, \\ D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad C_0 = \begin{bmatrix} 1 & 0.2 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0 & 0 \end{bmatrix}, \qquad H = 0.1. \end{gathered} \tag{44}$$

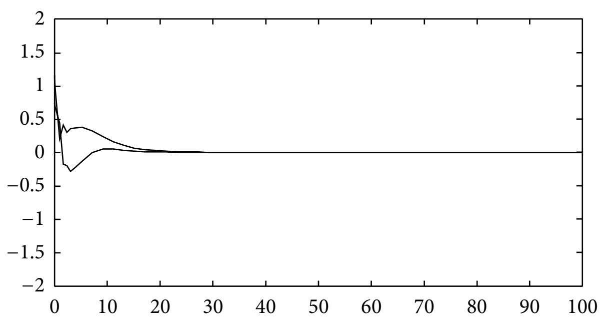

FIGURE 1: The state response of the closed-loop system in Example 3.

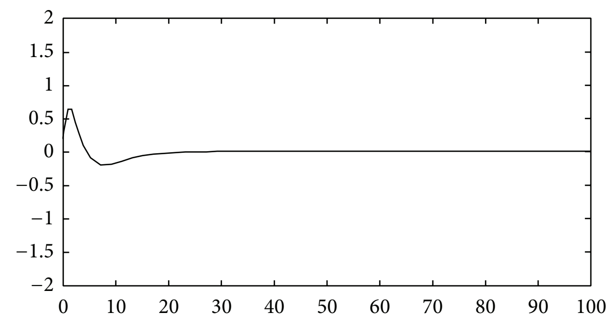

FIGURE 2: The controlled output of the closed-loop system in Example 3.

TABLE 4: Comparisons of minimum H_∞ performance index γ^* for Example 3.

Methods	[6]	[11]	[12]	[15]	[16]	Theorem 12
γ^*	21	15.0268	9.9514	9.6754	6.61	4.8610

For a given delay $r = 1.2$, Table 4 provides the comparison results on the minimum H_∞ performance index for given delay via the methods in [6, 11, 12, 15, 16] and Theorem 12 in this paper, which shows that Theorem 12 in this paper can lower the H_∞ performance index. The state feedback controller achieving the minimum H_∞ performance level can be obtained as

$$u(t) = \begin{bmatrix} -0.3369 & -0.4316 \end{bmatrix} x(t) + \begin{bmatrix} -0.8625 & 1.1541 \end{bmatrix} x(t-r). \tag{45}$$

The state response of the closed-loop system is shown in Figure 1. We can see that the state responses are converging. The controlled output of the closed-loop system is shown in Figure 2.

5. Conclusions

The problem of delay-dependent robust H_∞ control for singular time-delay system with admissible uncertainties has been investigated by parameterized LKF method. A new version of bounded real lemma is presented to ensure the system to be regular, impulse free, and robustly stable with H_∞ performance condition. The slack matrices are suitably introduced to decouple the systems matrices from the LKF parameter, so that the H_∞ controller law can be derived directly. Three examples are given to illustrate the effectiveness of our method and the improvement over some existing ones. As a future research direction, it would be of interest to apply the parameterized LKF method in passivity and H_∞ filtering for neutral systems and singularly perturbed systems [19–22].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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