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Existence of nontrivial solutions for perturbed p-Laplacian system involving critical nonlinearity and magnetic fields

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Abstract

Under the suitable assumptions, we establish the existence of nontrivial solutions for a perturbed p-Laplacian system in $\mathbb{R}^N$ with critical nonlinearity and magnetic fields by using the variational method.

MSC: 35B33; 35J60; 35J65

Keywords: p-Laplacian system; critical nonlinearity; magnetic fields; variational method

1 Introduction

In this paper, we consider a class of quasi-linear elliptic systems of the form

$$\begin{cases} -\varepsilon^p \Delta_{p,A} u + V(x)|u|^{p-2}u = H_s(|u|^p, |v|^p)|u|^{p-2}u + K(x)|u|^{p^*-2}u, & x \in \mathbb{R}^N, \\ -\varepsilon^p \Delta_{p,A} v + V(x)|v|^{p-2}v = H_t(|u|^p, |v|^p)|v|^{p-2}v + K(x)|v|^{p^*-2}v, & x \in \mathbb{R}^N, \end{cases} \tag{1.1}$$

where $\Delta_{p,A} u = \operatorname{div}(|\nabla u + iA(x)u|^{p-2}(\nabla u + iA(x)u))$, i is the imaginary unit, $A(x) : \mathbb{R}^N \to \mathbb{R}^N$ is real vector potential, $1 < p < N$, $V(x)$ is a non-negative potential, $p^* = Np/(N-p)$ denotes the Sobolev critical exponent for $N \geq 3$ and $K(x)$ is a bounded positive coefficient.

The scalar case corresponding to (1.1) has received considerable attention in recent years. For $p = 2$ and $A(x) \equiv 0$, the scalar case corresponding to (1.1) turns into

$$-\varepsilon^2 \Delta u + V(x)u = K(x)|u|^{2^*-2}u + f\big(x, |u|^2\big)u, \quad x \in \mathbb{R}^N. \tag{1.2}$$

The equation (1.2) arises in finding standing wave solutions of the nonlinear Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta \psi + W(x)\psi - g\big(x, |\psi|\big)\psi. \tag{1.3}$$

A standing wave solution of (1.3) is a solution of the form

$$\psi(x,t) = u(x)\exp^{\frac{-iEt}{\hbar}}.$$

Then $\psi(x,t)$ solves (1.3) if and only if $u(x)$ solves (1.2) with $V(x) = W(x) - E$ and $\varepsilon^2 = \frac{\hbar^2}{2m}$.

The equation (1.2) has been extensively investigated in the literature based on various assumptions of the potential $V(x)$ and the nonlinearity $f(x,u)$. See, for example, [1–15] and the references therein.

There are also many works dealing with the magnetic fields $A(x) \neq 0$ and $p = 2$ for the scalar case corresponding to (1.1). In [16], the authors firstly obtained the existence of standing waves for special classes of magnetic fields. For many results, we refer the reader to [17–22].

For general $p > 1$, most of the work, as we know, consider the scalar case which corresponds to (1.1) with $A(x) \equiv 0$. See [23–27] and the references therein. We especially mention [24] for the existence of positive solutions for a class of p-Laplacian equations. Gloss [24] studied the existence and asymptotic behavior of positive solutions for quasi-linear elliptic equations of the form

$$-\varepsilon^p \Delta_p u + V(x)|u|^{p-2}u = f(u), \quad x \in \mathbb{R}^N, \tag{1.4}$$

where f is a subcritical nonlinearity without some growth conditions such as the Ambrosetti-Rabinowitz condition. The problem (1.4) has also been studied in [28–32]. The main difficulty in treating this class of equation (1.4) is a possible lack of compactness due to the unboundedness of the domain.

However, to our best knowledge, it seems there is almost no work on the existence of non-trivial solutions to the problem (1.1) involving critical nonlinearity and magnetic fields. We mainly follow the idea of [7]. Observe that though the idea was used in other problems, the adaption of the procedure to the problem is not trivial at all. Because of the appearance of magnetic fields $A(x)$, we must deal with the problem for complex-valued functions and therefore we need more delicate estimates.

The outline of the paper is as follows. The forthcoming section is the main result and preliminary results including the appropriate space setting to work with. In Section 3, we study the behavior of $(PS)_c$ sequence. Section 4 gets that the functional associated to the problem possesses the mountain geometry structure, and the last section concludes the proof of the main result.

2 Main results and preliminaries

Firstly, we make the following assumptions on $V(x)$, $A(x)$, $H(s,t)$ and $K(x)$ throughout the paper:

(V_0) $V \in C(\mathbb{R}^N, \mathbb{R})$, $V(0) = \inf_{x\in\mathbb{R}^N} V(x) = 0$ and there exists $b > 0$ such that the set $\nu^b := \{x \in \mathbb{R}^N : V(x) < b\}$ has finite Lebesgue measure;

(A_0) $A \in C(\mathbb{R}^N, \mathbb{R}^N)$ and $A(0) = 0$;

(K_0) $K(x) \in C(\mathbb{R}^N, \mathbb{R}^+)$, $0 < \inf K \leq \sup K < \infty$;

(H_1) $H(s,t) \in C^1(\mathbb{R}^+ \times \mathbb{R}^+, \mathbb{R})$ and $H_s, H_t = o(|s| + |t|)$ as $|s| + |t| \to 0$;

(H_2) there exist $c_1 > 0$ and $p < \alpha < p^*$ such that

$$\big|H_s(s,t)\big|, \big|H_t(s,t)\big| \leq c_1\big(1 + |s|^{\frac{\alpha-p}{p}} + |t|^{\frac{\alpha-p}{p}}\big);$$

(H_3) there are $a_0 > 0$, $\theta \in (p, p^*)$ and $\alpha, \beta > p$ such that $H(s,t) \geq pa_0(|s|^{\frac{\alpha}{p}} + |t|^{\frac{\beta}{p}})$ and $0 < \frac{\theta}{p}H(s,t) \leq sH_s + tH_t$.

Under the above mentioned conditions, we get the following result.

Theorem 1 *Suppose that the assumptions* (V_0), (A_0), (K_0) *and* (H_1)-(H_3) *hold. Then for any* $\sigma > 0$, *there is* $\varepsilon_\sigma > 0$ *such that if* $\varepsilon < \varepsilon_\sigma$, *the problem* (1.1) *has at least one solution* $(u_\varepsilon, v_\varepsilon)$ *which satisfies*

$$\frac{\theta - p}{p\theta} \int_{\mathbb{R}^N} \left(\varepsilon^p \left(\left|\nabla |u_\varepsilon|\right|^p + \left|\nabla |v_\varepsilon|\right|^p\right) + V(x)|u_\varepsilon|^p + V(x)|v_\varepsilon|^p\right) \le \sigma \varepsilon^N.$$

Setting $\lambda = \varepsilon^{-p}$, the problem (1.1) is equivalent to the following problem:

$$\begin{cases} -\Delta_{p,A} u + \lambda V(x)|u|^{p-2}u = \lambda H_s(|u|^p, |v|^p)|u|^{p-2}u + \lambda K(x)|u|^{p^*-2}u, & x \in \mathbb{R}^N, \\ -\Delta_{p,A} v + \lambda V(x)|v|^{p-2}v = \lambda H_t(|u|^p, |v|^p)|v|^{p-2}v + \lambda K(x)|v|^{p^*-2}v, & x \in \mathbb{R}^N. \end{cases} \tag{2.1}$$

We are going to prove the following result.

Theorem 2 *Suppose that the assumptions* (V_0), (A_0), (K_0) *and* (H_1)-(H_3) *hold. Then for any* $\sigma > 0$, *there is* $\Lambda_\sigma > 0$ *such that if* $\lambda > \Lambda_\sigma$, *the problem* (2.1) *has at least one solution* (u_λ, v_λ) *which satisfies*

$$\frac{\theta - p}{p\theta} \int_{\mathbb{R}^N} \left(\left(\left|\nabla |u_\lambda|\right|^p + \left|\nabla |v_\lambda|\right|^p\right) + \lambda V(x)|u_\lambda|^p + \lambda V(x)|v_\lambda|^p\right) \le \sigma \lambda^{1-\frac{p}{N}}. \tag{2.2}$$

For convenience, we quote the following notations. Let $E_{\lambda,A}$ denote the Banach space

$$E_{\lambda,A} = \left\{ u \in W^{1,p}\left(\mathbb{R}^N\right) : \int_{\mathbb{R}^N} \lambda V(x)|u|^p < \infty \right\}, \quad \lambda > 0$$

equipped with the norm

$$\|u\|_{\lambda,A} = \left(\int_{\mathbb{R}^N} \left(\left|\nabla u + i\lambda^{\frac{1}{p}} A(x) u\right|^p + \lambda V(x)|u|^p \right) \right)^{\frac{1}{p}}.$$

Set $E = E_{\lambda,A} \times E_{\lambda,A}$ and $\|(u,v)\|_E^p = \|u\|_{\lambda,A}^p + \|v\|_{\lambda,A}^p$ for any $(u,v) \in E$.

Similar to the diamagnetic inequality [16], we have

$$\left|\nabla |u(x)|\right| \le \left| \nabla u \frac{\bar{u}}{|u|} \right| = \left| \mathrm{Re}\left(\nabla u + i\lambda^{\frac{1}{p}} A u\right) \frac{\bar{u}}{|u|} \right| \le \left|\nabla u + i\lambda^{\frac{1}{p}} A u\right| \tag{2.3}$$

(the bar denotes complex conjugation). This inequality shows that if $u \in E_{\lambda,A}$, then $|u| \in W^{1,p}(\mathbb{R}^N)$ and therefore $u \in L^q(\mathbb{R}^N)$ for any $q \in [p, p^*)$. That is to say, if $u_n \rightharpoonup u$ in $E_{\lambda,A}$, then $u_n \to u$ in $L^q_{\mathrm{loc}}(\mathbb{R}^N)$ for any $q \in [p, p^*)$ and $u_n \to u$ a.e. in $\mathbb{R}^N$.

The energy functional associated with (2.1) is defined by

$$\begin{aligned} I_\lambda(u,v) &= \frac{1}{p} \int_{\mathbb{R}^N} \left(\left|\nabla u + i\lambda^{\frac{1}{p}} A u\right|^p + \lambda V(x)|u|^p + \left|\nabla v + i\lambda^{\frac{1}{p}} A v\right|^p + \lambda V(x)|v|^p \right) \\ &\quad - \frac{\lambda}{p^*} \int_{\mathbb{R}^N} K(x)\left(|u|^{p^*} + |v|^{p^*}\right) - \frac{\lambda}{p} \int_{\mathbb{R}^N} H\left(|u|^p, |v|^p\right) \\ &= \frac{1}{p} \left\|(u,v)\right\|_E^p - \lambda \int_{\mathbb{R}^N} G(u,v), \end{aligned}$$

where $G(u,v) = \frac{1}{p^*} K(x)(|u|^{p^*} + |v|^{p^*}) + \frac{1}{p} H(|u|^p, |v|^p)$.

Under the assumptions of Theorem 2, standard arguments [33] show that $I_\lambda \in C^1(E_{\lambda,A}, \mathbb{R})$ and its critical points are weak solutions of the equation (2.1).

3 $(PS)_c$ condition

We call a sequence $\{(u_n, v_n)\} \subset E$ a $(PS)_c$ sequence if $I_\lambda(u_n, v_n) \to c$ and $I'_\lambda(u_n, v_n) \to 0$ strongly in E' (E' is the dual space of E). I_λ is said to satisfy the $(PS)_c$ condition if any $(PS)_c$ sequence contains a convergent subsequence.

The main result of Section 3 is the following compactness result.

Proposition 3.1 *Let the assumptions of Theorem* 2 *be satisfied. There exists a constant* $\alpha_0 \geq 0$ *independent of* λ *such that, for any* $(PS)_c$ *sequence* $\{(u_n, v_n)\} \subset E$ *for* I_λ *with* $(u_n, v_n) \rightharpoonup (u, v)$, *either* $(u_n, v_n) \to (u, v)$ *or* $c - I_\lambda(u, v) \geq \alpha_0 \lambda^{1-\frac{N}{p}}$.

As a consequence, we obtain the following result.

Proposition 3.2 *Assume that the assumptions of Proposition* 3.1 *hold,* $I_\lambda(u, v)$ *satisfies the* $(PS)_c$ *condition for all* $c \leq \alpha_0 \lambda^{1-\frac{N}{p}}$.

In order to prove Proposition 3.1, we need the following lemmas.

Lemma 3.1 *Let the assumptions of Theorem* 2 *be satisfied.* $\{(u_n, v_n)\}$ *is a* $(PS)_c$ *sequence of* I_λ. *Then* $c \geq 0$ *and* $\{(u_n, v_n)\}$ *is bounded in the space* E.

Proof One has

$$\begin{aligned}
&I_\lambda(u_n, v_n) - \frac{1}{\theta} I'_\lambda(u_n, v_n)(u_n, v_n) \\
&\quad = \left(\frac{1}{p} - \frac{1}{\theta}\right) \|(u_n, v_n)\|_E^p + \left(\frac{1}{\theta} - \frac{1}{p^*}\right) \lambda \int_{\mathbb{R}^N} K(x)\left(|u_n|^{p^*} + |v_n|^{p^*}\right) \\
&\qquad + \lambda \int_{\mathbb{R}^N} \left(\frac{1}{\theta}\left(|u_n|^p H_s\left(|u_n|^p, |v_n|^p\right) + |v_n|^p H_t\left(|u_n|^p, |v_n|^p\right)\right) - \frac{1}{p} H\left(|u_n|^p, |v_n|^p\right)\right) \\
&\quad \geq \left(\frac{1}{p} - \frac{1}{\theta}\right) \|(u_n, v_n)\|_E^p.
\end{aligned}$$

Together with $I_\lambda(u_n, v_n) \to c$ and $I'_\lambda(u_n, v_n) \to 0$ as $n \to \infty$, we have

$$\left(\frac{1}{p} - \frac{1}{\theta}\right) \|(u_n, v_n)\|_E^p \leq c + o(1) + \varepsilon_n \|(u_n, v_n)\|_E.$$

Then $\{(u_n, v_n)\}$ is bounded and $c \geq 0$. □

From Lemma 3.1, we may assume $(u_n, v_n) \rightharpoonup (u, v)$ in E and $(u_n, v_n) \to (u, v)$ in $L^q_{\mathrm{loc}}(\mathbb{R}^N) \times L^q_{\mathrm{loc}}(\mathbb{R}^N)$ for any $q \in [p, p^*)$ and $u_n \to u$, $v_n \to v$ a.e. in $\mathbb{R}^N$.

Lemma 3.2 *Let* $\gamma \in [p, p^*)$. *There is a subsequence* $\{(u_{n_j}, v_{n_j})\}$ *such that for any* $\varepsilon > 0$, *there is* $r_\varepsilon > 0$ *with* $r \geq r_\epsilon$

$$\limsup_{j \to \infty} \int_{B_j \setminus B_r} \left(|u_{n_j}|^\gamma + |v_{n_j}|^\gamma\right) \leq \varepsilon,$$

where $B_r := \{x \in \mathbb{R}^N : |x| \leq r\}$.

Proof The proof of Lemma 3.2 is similar to that of Lemma 3.2 of [27], so we omit it. □

Let $\eta \in C^\infty(\mathbb{R}^+)$ be a smooth function satisfying $0 \le \eta(t) \le 1$, $\eta(t) = 1$ if $t \le 1$ and $\eta(t) = 0$ if $t \ge 2$. Define $\widetilde{u}_j(x) = \eta(2|x|/j)u(x)$, $\widetilde{v}_j(x) = \eta(2|x|/j)v(x)$. Obviously, we have

$$\|u - \widetilde{u}_j\|_{E_{\lambda,A}} \to 0 \quad \text{and} \quad \|v - \widetilde{v}_j\|_{E_{\lambda,A}} \to 0 \quad \text{as } j \to \infty. \tag{3.1}$$

Lemma 3.3 *One has*

$$\begin{aligned}
&\lim_{j\to\infty} \operatorname{Re} \int_{\mathbb{R}^N} \big(H_s\big(|u_{n_j}|^p, |v_{n_j}|^p\big)|u_{n_j}|^{p-2}u_{n_j} \\
&\quad - H_s\big(|u_{n_j} - \widetilde{u}_j|^p, |v_{n_j} - \widetilde{v}_j|^p\big)|u_{n_j} - \widetilde{u}_j|^{p-2}(u_{n_j} - \widetilde{u}_j) - H_s\big(|\widetilde{u}_j|^p, |\widetilde{v}_j|^p\big)|\widetilde{u}_j|^{p-2}\widetilde{u}_j\big)\bar{\varphi} = 0
\end{aligned}$$

and

$$\begin{aligned}
&\lim_{j\to\infty} \operatorname{Re} \int_{\mathbb{R}^N} \big(H_t\big(|u_{n_j}|^p, |v_{n_j}|^p\big)|v_{n_j}|^{p-2}v_{n_j} \\
&\quad - H_t\big(|u_{n_j} - \widetilde{u}_j|^p, |v_{n_j} - \widetilde{v}_j|^p\big)|v_{n_j} - \widetilde{v}_j|^{p-2}(v_{n_j} - \widetilde{v}_j) - H_t\big(|\widetilde{u}_j|^p, |\widetilde{v}_j|^p\big)|\widetilde{v}_j|^{p-2}\widetilde{v}_j\big)\bar{\psi} = 0
\end{aligned}$$

uniformly in $(\varphi, \psi) \in E$ *with* $\|(\varphi, \psi)\|_E \le 1$.

Proof The local compactness of Sobolev embedding implies that for any $r \ge 0$, we have

$$\begin{aligned}
&\lim_{j\to\infty} \operatorname{Re} \int_{B_r} \big(H_s\big(|u_{n_j}|^p, |v_{n_j}|^p\big)|u_{n_j}|^{p-2}u_{n_j} \\
&\quad - H_s\big(|u_{n_j} - \widetilde{u}_j|^p, |v_{n_j} - \widetilde{v}_j|^p\big)|u_{n_j} - \widetilde{u}_j|^{p-2}(u_{n_j} - \widetilde{u}_j) - H_s\big(|\widetilde{u}_j|^p, |\widetilde{v}_j|^p\big)|\widetilde{u}_j|^{p-2}\widetilde{u}_j\big)\bar{\varphi} = 0
\end{aligned}$$

uniformly in $\|\varphi\|_{E_{\lambda,A}} \le 1$. For any $\varepsilon > 0$, there exists $r_\varepsilon > 0$ such that

$$\lim_{j\to\infty} \sup \int_{B_j \setminus B_r} |\widetilde{u}_j|^\gamma \le \int_{\mathbb{R}^N} |u|^\gamma \le \varepsilon$$

for all $r \ge r_\epsilon$. Together with the assumption (H_2) and the Hölder inequality, it follows from Lemma 3.2 that

$$\begin{aligned}
&\lim_{j\to\infty} \sup \operatorname{Re} \int_{\mathbb{R}^N} \big(H_s\big(|u_{n_j}|^p, |v_{n_j}|^p\big)|u_{n_j}|^{p-2}u_{n_j} \\
&\qquad - H_s\big(|u_{n_j} - \widetilde{u}_j|^p, |v_{n_j} - \widetilde{v}_j|^p\big)|u_{n_j} - \widetilde{u}_j|^{p-2}(u_{n_j} - \widetilde{u}_j) - H_s\big(|\widetilde{u}_j|^p, |\widetilde{v}_j|^p\big)|\widetilde{u}_j|^{p-2}\widetilde{u}_j\big)\bar{\varphi} \\
&= \lim_{j\to\infty} \sup \operatorname{Re} \int_{B_j \setminus B_r} \big(H_s\big(|u_{n_j}|^p, |v_{n_j}|^p\big)|u_{n_j}|^{p-2}u_{n_j} \\
&\qquad - H_s\big(|u_{n_j} - \widetilde{u}_j|^p, |v_{n_j} - \widetilde{v}_j|^p\big)|u_{n_j} - \widetilde{u}_j|^{p-2}(u_{n_j} - \widetilde{u}_j) - H_s\big(|\widetilde{u}_j|^p, |\widetilde{v}_j|^p\big)|\widetilde{u}_j|^{p-2}\widetilde{u}_j\big)\bar{\varphi} \\
&\le c_1 \lim_{j\to\infty} \sup \int_{B_j \setminus B_r} \big(|u_{n_j}|^{p-1} + |v_{n_j}|^{p-1} + |\widetilde{u}_j|^{p-1} + |\widetilde{v}_j|^{p-1}\big)|\bar{\varphi}| \\
&\qquad + c_2 \lim_{j\to\infty} \sup \int_{B_j \setminus B_r} \big(|u_{n_j}|^{\alpha-1} + |v_{n_j}|^{\alpha-1} + |\widetilde{u}_j|^{\alpha-1} + |\widetilde{v}_j|^{\alpha-1}\big)|\bar{\varphi}|
\end{aligned}$$

$$
\begin{aligned}
&\le c_1 \limsup_{j\to\infty}\big(\|u_{n_j}\|^{p-1}_{L^p(B_j\setminus B_r)} + \|v_{n_j}\|^{p-1}_{L^p(B_j\setminus B_r)} \\
&\quad + \|\widetilde{u}_j\|^{p-1}_{L^p(B_j\setminus B_r)} + \|\widetilde{v}_j\|^{p-1}_{L^p(B_j\setminus B_r)}\big)\|\bar{\varphi}\|_{L_p(B_j\setminus B_r)} \\
&\quad + c_2 \limsup_{j\to\infty}\big(\|u_{n_j}\|^{\alpha-1}_{L^\alpha(B_j\setminus B_r)} + \|v_{n_j}\|^{\alpha-1}_{L^\alpha(B_j\setminus B_r)} \\
&\quad + \|\widetilde{u}_j\|^{\alpha-1}_{L^\alpha(B_j\setminus B_r)} + \|\widetilde{v}_j\|^{\alpha-1}_{L^\alpha(B_j\setminus B_r)}\big)\|\bar{\varphi}\|_{L_\alpha(B_j\setminus B_r)} \\
&\le c_3\varepsilon^{\frac{p-1}{p}} + c_4\varepsilon^{\frac{\alpha-1}{\alpha}},
\end{aligned}
$$

where c_i $(i = 1, 2, 3, 4)$ are positive constants. Similarly, we can prove

$$
\begin{aligned}
&\lim_{j\to\infty}\operatorname{Re}\int_{\mathbb{R}^N}\big(H_t\big(|u_{n_j}|^p, |v_{n_j}|^p\big)|v_{n_j}|^{p-2}v_{n_j} \\
&\quad - H_t\big(|u_{n_j}-\widetilde{u}_j|^p, |v_{n_j}-\widetilde{v}_j|^p\big)|v_{n_j}-\widetilde{v}_j|^{p-2}(v_{n_j}-\widetilde{v}_j) - H_t\big(|\widetilde{u}_j|^p, |\widetilde{v}_j|^p\big)|\widetilde{v}_j|^{p-2}\widetilde{v}_j\big)\bar{\psi} = 0. \qquad \square
\end{aligned}
$$

Lemma 3.4 *Let* $\{(u_n, v_n)\}$ *and* $\{(\widetilde{u}_n, \widetilde{v}_n)\}$ *be as defined above. Then the following conclusions hold*:

$$
I_\lambda(u_n-\widetilde{u}_n, v_n-\widetilde{v}_n) \to c - I_\lambda(u, v)
$$

and

$$
I'_\lambda(u_n-\widetilde{u}_n, v_n-\widetilde{v}_n) \to 0 \quad \text{in } E' \text{ (the dual space of } E).
$$

Proof By using the similar arguments of [34, 35], we have

$$
\begin{aligned}
&I_\lambda(u_n-\widetilde{u}_n, v_n-\widetilde{v}_n) \\
&\quad = I_\lambda(u_n, v_n) - I_\lambda(\widetilde{u}_n, \widetilde{v}_n) \\
&\qquad + \frac{\lambda}{p^*}\int_{\mathbb{R}^N} K(x)\big(\big(|u_n|^{p^*} - |u_n-\widetilde{u}_n|^{p^*} - |\widetilde{u}_n|^{p^*}\big) + \big(|v_n|^{p^*} - |v_n-\widetilde{v}_n|^{p^*} - |\widetilde{v}_n|^{p^*}\big)\big) \\
&\qquad + \frac{\lambda}{p}\int_{\mathbb{R}^N}\big(H\big(|u_n|^p, |v_n|^p\big) - H\big(|u_n-\widetilde{u}_n|^p, |v_n-\widetilde{v}_n|^p\big) - H\big(|\widetilde{u}_n|^p, |\widetilde{v}_n|^p\big)\big) + o(1).
\end{aligned}
$$

By (3.1) and the similar idea of proving the Brézis-Lieb lemma [36], it is easy to get

$$
\lim_{n\to\infty}\int_{\mathbb{R}^N} K(x)\big(\big(|u_n|^{p^*} - |u_n-\widetilde{u}_n|^{p^*} - |\widetilde{u}_n|^{p^*}\big) + \big(|v_n|^{p^*} - |v_n-\widetilde{v}_n|^{p^*} - |\widetilde{v}_n|^{p^*}\big)\big) = 0
$$

and

$$
\lim_{n\to\infty}\int_{\mathbb{R}^N}\big(H\big(|u_n|^p, |v_n|^p\big) - H\big(|u_n-\widetilde{u}_n|^p, |v_n-\widetilde{v}_n|^p\big) - H\big(|\widetilde{u}_n|^p, |\widetilde{v}_n|^p\big)\big) = 0.
$$

Furthermore, using the fact $I_\lambda(u_n, v_n) \to c$ and $I_\lambda(\widetilde{u}_n, \widetilde{v}_n) \to I_\lambda(u, v)$, we obtain

$$
I_\lambda(u_n-\widetilde{u}_n, v_n-\widetilde{v}_n) \to c - I_\lambda(u, v).
$$

In order to prove $I'_\lambda(u_n-\widetilde{u}_n,v_n-\widetilde{v}_n)\to 0$ in E^{-1}, for any $(\varphi,\psi)\in E$, it follows that

$$\begin{aligned}
&I'_\lambda(u_n-\widetilde{u}_n,v_n-\widetilde{v}_n)(\varphi,\psi)\\
&\quad= I'_\lambda(u_n,v_n)(\varphi,\psi)-I'_\lambda(\widetilde{u}_n,\widetilde{v}_n)(\varphi,\psi)\\
&\qquad+\lambda\,\mathrm{Re}\int_{\mathbb{R}^N}K(x)\big(|u_n|^{p^*-2}u_n-|u_n-\widetilde{u}_n|^{p^*-2}(u_n-\widetilde{u}_n)-|\widetilde{u}_n|^{p^*-2}\widetilde{u}_n\big)\bar{\varphi}\\
&\qquad+\lambda\,\mathrm{Re}\int_{\mathbb{R}^N}K(x)\big(|v_n|^{p^*-2}v_n-|v_n-\widetilde{v}_n|^{p^*-2}(v_n-\widetilde{v}_n)-|\widetilde{v}_n|^{p^*-2}\widetilde{v}_n\big)\bar{\psi}\\
&\qquad+\lambda\,\mathrm{Re}\int_{\mathbb{R}^N}\big(H_s\big(|u_n|^p,|v_n|^p\big)|u_n|^{p-2}u_n\\
&\qquad-H_s\big(|u_n-\widetilde{u}_n|^p,|v_n-\widetilde{v}_n|^p\big)|u_n-\widetilde{u}_n|^{p-2}(u_n-\widetilde{u}_n)-H_s\big(|\widetilde{u}_n|^p,|\widetilde{v}_n|^p\big)|\widetilde{u}_n|^{p-2}\widetilde{u}_n\big)\bar{\varphi}\\
&\qquad+\lambda\,\mathrm{Re}\int_{\mathbb{R}^N}\big(H_t\big(|u_n|^p,|v_n|^p\big)|v_n|^{p-2}v_n\\
&\qquad-H_t\big(|u_n-\widetilde{u}_n|^p,|v_n-\widetilde{v}_n|^p\big)|v_n-\widetilde{v}_n|^{p-2}(v_n-\widetilde{v}_n)-H_t\big(|\widetilde{u}_n|^p,|\widetilde{v}_n|^p\big)|\widetilde{v}_n|^{p-2}\widetilde{v}_n\big)\bar{\psi}\\
&\qquad+o(1).
\end{aligned}$$

It is standard to check that

$$\lim_{n\to\infty}\int_{\mathbb{R}^N}K(x)\big(|u_n|^{p^*-2}u_n-|u_n-\widetilde{u}_n|^{p^*-2}(u_n-\widetilde{u}_n)-|\widetilde{u}_n|^{p^*-2}\widetilde{u}_n\big)\bar{\varphi}=0$$

and

$$\lim_{n\to\infty}\int_{\mathbb{R}^N}K(x)\big(|v_n|^{p^*-2}v_n-|v_n-\widetilde{v}_n|^{p^*-2}(v_n-\widetilde{v}_n)-|\widetilde{v}_n|^{p^*-2}\widetilde{v}_n\big)\bar{\psi}=0$$

uniformly in $(\varphi,\psi)\in E$ with $\|(\varphi,\psi)\|_E\le 1$. Together with Lemma 3.3, we have

$$I'_\lambda(u_n-\widetilde{u}_n,v_n-\widetilde{v}_n)\to 0\quad\text{in } E'.$$ □

Let $u_n^1=u_n-\widetilde{u}_n$, $v_n^1=v_n-\widetilde{v}_n$, then $u_n-u=u_n^1+(\widetilde{u}_n-u)$, $v_n-v=v_n^1+(\widetilde{v}_n-v)$. From (3.1), we get $(u_n,v_n)\to(u,v)$ in E if and only if $(u_n^1,v_n^1)\to(0,0)$ in E.

Observe that

$$\begin{aligned}
&I_\lambda\big(u_n^1,v_n^1\big)-\frac{1}{p}I'_\lambda\big(u_n^1,v_n^1\big)\big(u_n^1,v_n^1\big)\\
&\quad=\left(\frac{1}{p}-\frac{1}{p^*}\right)\lambda\int_{\mathbb{R}^N}K(x)\big(\big|u_n^1\big|^{p^*}+\big|v_n^1\big|^{p^*}\big)\\
&\qquad+\frac{\lambda}{p}\int_{\mathbb{R}^N}\big(\big|u_n^1\big|^pH_s\big(\big|u_n^1\big|^p,\big|v_n^1\big|^p\big)+\big|v_n^1\big|^pH_t\big(\big|u_n^1\big|^p,\big|v_n^1\big|^p\big)-H\big(\big|u_n^1\big|^p,\big|v_n^1\big|^p\big)\big)\\
&\quad\ge\frac{\lambda}{N}\int_{\mathbb{R}^N}K(x)\big(\big|u_n^1\big|^{p^*}+\big|v_n^1\big|^{p^*}\big)\\
&\quad\ge\frac{\lambda}{N}K_{\min}\int_{\mathbb{R}^N}\big(\big|u_n^1\big|^{p^*}+\big|v_n^1\big|^{p^*}\big),
\end{aligned}$$

where $K_{\min}=\inf_{x\in\mathbb{R}^N}K(x)>0$. Furthermore, we get

$$\big\|\big(u_n^1,v_n^1\big)\big\|_{p^*}^{p^*}\le\frac{N(c-I_\lambda(u,v))}{\lambda K_{\min}}+o(1).\tag{3.2}$$

Now, we consider the energy level of the functional I_λ below which the $(PS)_c$ condition holds.

Let $V_b(x) := \max\{V(x), b\}$, where b is a positive constant in the assumption (V_0). Since the set ν_b has finite measure, we get

$$\int_{\mathbb{R}^N} V(x)\big(|u_n^1|^p + |v_n^1|^p\big) = \int_{\mathbb{R}^N} V_b(x)\big(|u_n^1|^p + |v_n^1|^p\big) + o(1). \tag{3.3}$$

In connection with the assumptions (H_1)-(H_3) and the Young inequality, there exists $C_b > 0$ such that

$$\begin{aligned}&\int_{\mathbb{R}^N} \big(K(x)\big(|u|^{p^*} + |v|^{p^*}\big) + |u|^p H_s\big(|u|^p, |v|^p\big) + |v|^p H_t\big(|u|^p, |v|^p\big)\big)\\ &\quad \le b\big(\|u\|_p^p + \|v\|_p^p\big) + C_b\big(\|u\|_{p^*}^{p^*} + \|v\|_{p^*}^{p^*}\big).\end{aligned} \tag{3.4}$$

Let S be the best Sobolev constant of the immersion

$$S\|u\|_{p^*}^p \le \int_{\mathbb{R}^N} |\nabla u|^p \quad \text{for all } u \in W^{1,p}\big(\mathbb{R}^N\big).$$

Proof of Proposition 3.1 Assume that $(u_n, v_n) \rightharpoonup (u, v)$, then

$$\liminf_{n\to\infty} \big\|\big(u_n^1, v_n^1\big)\big\|_E > 0$$

and

$$c - I_\lambda(u, v) > 0.$$

By the Sobolev embedding inequality and the diamagnetic inequality, we get

$$\begin{aligned}&S\big(\|u_n^1\|_{p^*}^p + \|v_n^1\|_{p^*}^p\big)\\ &\quad \le \int_{\mathbb{R}^N} \big(|\nabla u_n^1|^p + |\nabla v_n^1|^p\big)\\ &\quad \le \int_{\mathbb{R}^N} \big(|\nabla u_n^1 + i\lambda^{\frac{1}{p}} A(x) u_n^1|^p + |\nabla v_n^1 + i\lambda^{\frac{1}{p}} A(x) v_n^1|^p\big)\\ &\quad = \int_{\mathbb{R}^N} \big(|\nabla u_n^1 + i\lambda^{\frac{1}{p}} A(x) u_n^1|^p + \lambda V(x)|u_n^1|^p + |\nabla v_n^1 + i\lambda^{\frac{1}{p}} A(x) v_n^1|^p + \lambda V(x)|v_n^1|^p\big)\\ &\qquad - \lambda \int_{\mathbb{R}^N} V(x)\big(|u_n^1|^p + |v_n^1|^p\big)\\ &\quad = \lambda \int_{\mathbb{R}^N} K(x)\big(|u_n^1|^{p^*} + |v_n^1|^{p^*}\big) + |u_n^1|^p H_s\big(|u_n^1|^p, |v_n^1|^p\big) + |v_n^1|^p H_t\big(|u_n^1|^p, |v_n^1|^p\big)\\ &\qquad - \lambda \int_{\mathbb{R}^N} V_b(x)\big(|u_n^1|^p + |v_n^1|^p\big) + o(1)\\ &\quad \le \lambda b\big(\|u_n^1\|_p^p + \|v_n^1\|_p^p\big) + \lambda C_b\big(\|u_n^1\|_{p^*}^{p^*} + \|v_n^1\|_{p^*}^{p^*}\big) - \lambda b\big(\|u_n^1\|_p^p + \|v_n^1\|_p^p\big) + o(1)\\ &\quad = \lambda C_b\big(\|u_n^1\|_{p^*}^{p^*} + \|v_n^1\|_{p^*}^{p^*}\big) + o(1).\end{aligned}$$

This, together with (3.2), gives

$$\begin{aligned} S &\le \lambda C_b\left(\left\|u_n^1\right\|_{p^*}^{p^*} + \left\|v_n^1\right\|_{p^*}^{p^*}\right)^{\frac{p^*-p}{p^*}} + o(1) \\ &\le \lambda C_b\left(\frac{N(c - I_\lambda(u,v))}{\lambda K_{\min}}\right)^{\frac{p}{N}} + o(1) \\ &= \lambda^{1-\frac{p}{N}} C_b\left(\frac{N}{K_{\min}}\right)^{\frac{p}{N}} \left(c - I_\lambda(u,v)\right)^{\frac{p}{N}} + o(1). \end{aligned}$$

Set $\alpha_0 = S^{\frac{N}{p}} C_b^{-\frac{N}{p}} N^{-1} K_{\min}$, then

$$\alpha_0 \lambda^{1-\frac{N}{p}} \le c - I_\lambda(u,v) + o(1).$$

This completes the proof of Proposition 3.1. □

Proof of Proposition 3.2 Since $c \le \alpha_0 \lambda^{1-\frac{N}{p}}$, we have

$$c - I_\lambda(u,v) \le \alpha_0 \lambda^{1-\frac{N}{p}} - I_\lambda(u,v).$$

In connection with $I_\lambda(u,v) \ge 0$ and Proposition 3.1, we complete this proof. □

4 The mountain-pass structure

In the following, we always consider $\lambda \ge 1$. We will prove that I_λ possesses the mountain-pass structure which has been carefully discussed in the works [37, 38].

Lemma 4.1 *Let the assumptions of Theorem* 2 *be satisfied. There exist* $\alpha_\lambda, \rho_\lambda > 0$ *such that*

$$I_\lambda(u,v) > 0 \quad \text{if } 0 < \left\|(u,v)\right\|_E < \rho_\lambda \quad \text{and} \quad I_\lambda(u,v) \ge \alpha_\lambda \quad \text{if } \left\|(u,v)\right\|_E = \rho_\lambda.$$

Proof By (3.4), for any $\delta > 0$, there is $C_\delta > 0$ such that

$$\int_{\mathbb{R}^N} G(u,v) \le \delta\left(\|u\|_p^p + \|v\|_p^p\right) + C_\delta\left(\|u\|_{p^*}^{p^*} + \|v\|_{p^*}^{p^*}\right).$$

Thus,

$$\begin{aligned} I_\lambda(u,v) &= \frac{1}{p}\left\|(u,v)\right\|_E^p - \lambda \int_{\mathbb{R}^N} G(u,v) \\ &\ge \frac{1}{p}\left\|(u,v)\right\|_E^p - \lambda\delta\left(\|u\|_p^p + \|v\|_p^p\right) - \lambda C_\delta\left(\|u\|_{p^*}^{p^*} + \|v\|_{p^*}^{p^*}\right). \end{aligned}$$

In connection with $\|u\|_p^p + \|v\|_p^p \le C_1 \|(u,v)\|_E^p$, we may choose $\delta \le (2p\lambda C_1)^{-1}$ such that

$$I_\lambda(u,v) \ge \frac{1}{2p}\left\|(u,v)\right\|_E^p - \lambda C_\delta\left(\|u\|_{p^*}^{p^*} + \|v\|_{p^*}^{p^*}\right).$$

The fact $p^* > p$ implies the desired conclusion. □

Lemma 4.2 *Under the assumptions of Lemma* 4.1, *for any finite dimensional subspace* $F \subset E$, *we have*

$$I_\lambda(u,v) \to -\infty \quad as\ (u,v) \in F, \qquad \|(u,v)\|_E \to \infty.$$

Proof Together with the fact $H(s,t) \geq pa_0(|s|^{\frac{\alpha}{p}} + |t|^{\frac{\beta}{p}})$, we have

$$I_\lambda(u,v) \leq \frac{1}{p}\|(u,v)\|_E^p - \lambda a_0\big(\|u\|_\alpha^\alpha + \|v\|_\beta^\beta\big) \quad \text{for all } (u,v) \in E.$$

Since all norms in a finite-dimensional space are equivalent and $\alpha, \beta > p$, we complete the proof. □

In the following, we will find special finite-dimensional subspaces by which we establish sufficiently small mini-max levels.

Define the functional

$$\Phi_\lambda(u,v) = \frac{1}{p}\|(u,v)\|_E^p - \lambda a_0 \int_{\mathbb{R}^N} \big(|u|^\alpha + |v|^\beta\big).$$

Obviously, it follows that $\Phi_\lambda \in C^1(E)$ and $I_\lambda(u,v) \leq \Phi_\lambda(u,v)$ for all $(u,v) \in E$.

Observe that

$$\inf\left\{\int_{\mathbb{R}^N} |\nabla\phi|^p : \phi \in C_0^\infty\big(\mathbb{R}^N,\mathbb{R}\big), \|\phi\|_{L^\alpha(\mathbb{R}^N)} = 1\right\} = 0$$

and

$$\inf\left\{\int_{\mathbb{R}^N} |\nabla\psi|^p : \psi \in C_0^\infty\big(\mathbb{R}^N,\mathbb{R}\big), \|\psi\|_{L^\beta(\mathbb{R}^N)} = 1\right\} = 0.$$

Then, for any $\delta > 0$, there are $\phi_\delta, \psi_\delta \in C_0^\infty(\mathbb{R}^N,\mathbb{R})$ with $\|\phi_\delta\|_{L^\alpha(\mathbb{R}^N)} = \|\psi_\delta\|_{L^\beta(\mathbb{R}^N)} = 1$ and $\operatorname{supp}\phi_\delta, \operatorname{supp}\psi_\delta \subset B_{r_\delta}(0)$ such that $\|\nabla\phi_\delta\|_p^p, \|\nabla\psi_\delta\|_p^p < \delta$.

Set $e_\lambda(x) = (\phi_\delta(\sqrt[p]{\lambda}x), \psi_\delta(\sqrt[p]{\lambda}x))$. Then $\operatorname{supp} e_\lambda \subset B_{\lambda^{-\frac{1}{p}} r_\delta}(0)$. For $t \geq 0$, we get

$$\begin{aligned}\Phi_\lambda(te_\lambda) &= \frac{t^p}{p}\|e_\lambda\|_E^p - a_0\lambda t^\alpha \int_{\mathbb{R}^N} \big|\phi_\delta\big(\sqrt[p]{\lambda}x\big)\big|^\alpha - a_0\lambda t^\beta \int_{\mathbb{R}^N} \big|\psi_\delta\big(\sqrt[p]{\lambda}x\big)\big|^\beta \\ &= \lambda^{1-\frac{N}{p}} J_\lambda(t\phi_\delta, t\psi_\delta),\end{aligned}$$

where

$$\begin{aligned}J_\lambda(u,v) = \frac{1}{p}\int_{\mathbb{R}^N} &\big(|\nabla u|^p + |\nabla v|^p + \big(A\big(\lambda^{-\frac{1}{p}}x\big) + V\big(\lambda^{-\frac{1}{p}}x\big)\big)\big(|u|^p + |v|^p\big)\big) \\ &- a_0 \int_{\mathbb{R}^N} \big(|u|^\alpha + |v|^\beta\big).\end{aligned}$$

By direct computation, we have

$$\max_{t\geq 0} J_\lambda(t\phi_\delta, t\psi_\delta) \leq \frac{\alpha - p}{p\alpha(\alpha a_0)^{\frac{p}{\alpha-p}}}\left\{\int_{\mathbb{R}^N} \big(|\nabla\phi_\delta|^p + \big(A\big(\lambda^{-\frac{1}{p}}x\big) + V\big(\lambda^{-\frac{1}{p}}x\big)\big)|\phi_\delta|^p\big)\right\}^{\frac{\alpha}{\alpha-p}}$$

$$+\frac{\beta-p}{p\beta(\beta a_0)^{\frac{p}{\beta-p}}}\left\{\int_{\mathbb{R}^N}\left(|\nabla\psi_\delta|^p+\left(A\left(\lambda^{-\frac{1}{p}}x\right)+V\left(\lambda^{-\frac{1}{p}}x\right)\right)|\psi_\delta|^p\right)\right\}^{\frac{\beta}{\beta-p}}.$$

Since $A(0)=0$, $V(0)=0$ and $\|\nabla\phi_\delta\|_p^p, \|\nabla\psi_\delta\|_p^p<\delta$, we know that there is $\Lambda_\delta>0$ such that for all $\lambda\geq\Lambda_\delta$, we have

$$\max_{t\geq0} I_\lambda(t\phi_\delta,t\psi_\delta)\leq\left(\frac{\alpha-p}{p\alpha(\alpha a_0)^{\frac{p}{\alpha-p}}}(5\delta)^{\frac{\alpha}{\alpha-p}}+\frac{\beta-p}{p\beta(\beta a_0)^{\frac{p}{\beta-p}}}(5\delta)^{\frac{\beta}{\beta-p}}\right)\lambda^{1-\frac{N}{p}}. \tag{4.1}$$

Lemma 4.3 *For any $\sigma>0$, there is $\Lambda_\sigma>0$ such that $\lambda\geq\Lambda_\sigma$, there is $\bar{w}_\lambda\in E$ with $\|\bar{w}_\lambda\|_E>\rho_\lambda$, $I_\lambda(\bar{w}_\lambda)\leq0$ and*

$$\max_{t\geq0} I_\lambda(t\bar{w}_\lambda)\leq\sigma\lambda^{1-\frac{N}{p}},$$

where ρ_λ is defined in Lemma 4.1.

Proof This proof is similar to that of Lemma 4.3 in [7], so we omit the details. □

5 Proof of Theorem 2

Proof By using Lemma 4.3, for any $\sigma>0$ with $0<\sigma<\alpha_0$, we choose $\Lambda_\sigma>0$ and define the mini-max level

$$c_\lambda=\inf_{\gamma\in\Gamma_\lambda}\max_{t\in[0,1]} I_\lambda\left(\gamma(t)\right)\leq\sigma\lambda^{1-\frac{N}{p}}\quad\text{for all }\lambda\geq\Lambda_\sigma,$$

where $\Gamma_\lambda=\{\gamma\in C([0,1],E):\gamma(0)=0,\gamma(1)=\bar{w}_\lambda\}$.

By Proposition 3.1, we know that I_λ satisfies the $(PS)_{c_\lambda}$ condition. Hence, by the mountain-pass theorem, there is $(u_\lambda,v_\lambda)\in E$ such that $I_\lambda(u_\lambda,v_\lambda)=c_\lambda$ and $I'_\lambda(u_\lambda,v_\lambda)=0$. This shows (u_λ,v_λ) is a weak solution of (2.1).

Moreover, note that $I_\lambda(u_\lambda,v_\lambda)\leq\sigma\lambda^{1-\frac{N}{p}}$ and $I'_\lambda(u_\lambda,v_\lambda)=0$. Then

$$\begin{aligned}
I_\lambda(u_\lambda,v_\lambda)&=I_\lambda(u_\lambda,v_\lambda)-\frac{1}{\theta}I'_\lambda(u_\lambda,v_\lambda)(u_\lambda,v_\lambda)\\
&=\left(\frac{1}{p}-\frac{1}{\theta}\right)\|(u_\lambda,v_\lambda)\|_E^p+\left(\frac{1}{\theta}-\frac{1}{p^*}\right)\lambda\int_{\mathbb{R}^N}K(x)\left(|u_\lambda|^{p^*}+|v_\lambda|^{p^*}\right)\\
&\quad+\lambda\int_{\mathbb{R}^N}\left(\frac{1}{\theta}\left(|u_\lambda|^pH_s\left(|u_\lambda|^p,|v_\lambda|^p\right)+|v_\lambda|^pH_t\left(|u_\lambda|^p,|v_\lambda|^p\right)\right)\right.\\
&\quad\left.-\frac{1}{p}H\left(|u_\lambda|^p,|v_\lambda|^p\right)\right)\\
&\geq\left(\frac{1}{p}-\frac{1}{\theta}\right)\|(u_\lambda,v_\lambda)\|_E^p.
\end{aligned}$$

Furthermore, together with the diamagnetic inequality, we prove that (u_λ,v_λ) satisfies the estimate (2.2). The proof is complete. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed equally in this article. All authors read and approved the final manuscript.

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