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Common coupled fixed point theorems for weakly compatible mappings in fuzzy metric spaces

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Abstract

In this paper, we prove a common fixed point theorem for weakly compatible mappings under ϕ-contractive conditions in fuzzy metric spaces. We also give an example to illustrate the theorem. The result is a genuine generalization of the corresponding result of Hu (Fixed Point Theory Appl. 2011:363716, 2011, doi:10.1155/2011/363716). We also indicate a minor mistake in Hu (Fixed Point Theory Appl. 2011:363716, 2011, doi:10.1155/2011/363716).

1 Introduction

In 1965, Zadeh [1] introduced the concept of fuzzy sets. Then many authors gave the important contribution to development of the theory of fuzzy sets and applications. George and Veeramani [2, 3] gave the concept of a fuzzy metric space and defined a Hausdorff topology on this fuzzy metric space, which have very important applications in quantum particle physics, particularly, in connection with both string and E-infinity theory.

Bhaskar and Lakshmikantham [4], Lakshmikantham and Ćirić [5] discussed the mixed monotone mappings and gave some coupled fixed point theorems, which can be used to discuss the existence and uniqueness of solution for a periodic boundary value problem. Sedghi *et al.* [6] gave a coupled fixed point theorem for contractions in fuzzy metric spaces, and Jin-xuan Fang [7] gave some common fixed point theorems for compatible and weakly compatible ϕ-contractions mappings in Menger probabilistic metric spaces. Xin-Qi Hu [8] proved a common fixed point theorem for mappings under φ-contractive conditions in fuzzy metric spaces. Many authors [9–26] proved fixed point theorems in (intuitionistic) fuzzy metric spaces or probabilistic metric spaces.

In this paper, we give a new coupled fixed point theorem under weaker conditions than in [8] and give an example, which shows that the result is a genuine generalization of the corresponding result in [8].

2 Preliminaries

First, we give some definitions.

Definition 2.1 (see [2]) A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if $*$ satisfies the following conditions:

(1) $*$ is commutative and associative,

(2) $*$ is continuous,
(3) $a * 1 = a$ for all $a \in [0,1]$,
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 2.2 (see [27]) Let $\sup_{0<t<1} \Delta(t,t) = 1$. A t-norm Δ is said to be of H-type if the family of functions $\{\Delta^m(t)\}_{m=1}^{\infty}$ is equicontinuous at $t = 1$, where

$$\Delta^1(t) = t\Delta t, \qquad \Delta^{m+1}(t) = t\Delta\bigl(\Delta^m(t)\bigr), \quad m = 1, 2, \ldots, t \in [0,1]. \tag{2.1}$$

The t-norm $\Delta_M = \min$ is an example of t-norm of H-type, but there are some other t-norms Δ of H-type [27].

Obviously, Δ is a t-norm of H-type if and only if for any $\lambda \in (0,1)$, there exists $\delta(\lambda) \in (0,1)$ such that $\Delta^m(t) > 1 - \lambda$ for all $m \in \mathbb{N}$, when $t > 1 - \delta$.

Definition 2.3 (see [2]) A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, +\infty)$ satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$,
(FM-1) $M(x, y, t) > 0$,
(FM-2) $M(x, y, t) = 1$ if and only if $x = y$,
(FM-3) $M(x, y, t) = M(y, x, t)$,
(FM-4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(FM-5) $M(x, y, \cdot) : (0, \infty) \to [0,1]$ is continuous.

We shall consider a fuzzy metric space $(X, M, *)$, which satisfies the following condition:

$$\lim_{t \to +\infty} M(x, y, t) = 1, \quad \forall x, y \in X. \tag{2.2}$$

Let $(X, M, *)$ be a fuzzy metric space. For $t > 0$, the open ball $B(x, r, t)$ with a center $x \in X$ and a radius $0 < r < 1$ is defined by

$$B(x, r, t) = \bigl\{y \in X : M(x, y, t) > 1 - r\bigr\}. \tag{2.3}$$

A subset $A \subset X$ is called open if for each $x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Let τ denote the family of all open subsets of X. Then τ is called the topology on X, induced by the fuzzy metric M. This topology is Hausdorff and first countable.

Example 2.4 Let (X, d) be a metric space. Define t-norm $a * b = ab$ or $a * b = \min\{a, b\}$ and for all $x, y \in X$ and $t > 0$, $M(x, y, t) = \frac{t}{t + d(x,y)}$. Then $(X, M, *)$ is a fuzzy metric space.

Definition 2.5 (see [2]) Let $(X, M, *)$ be a fuzzy metric space. Then
(1) a sequence $\{x_n\}$ in X is said to be convergent to x (denoted by $\lim_{n\to\infty} x_n = x$) if

$$\lim_{n \to \infty} M(x_n, x, t) = 1$$

for all $t > 0$.

(2) A sequence $\{x_n\}$ in X is said to be a Cauchy sequence if for any $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$, such that

$$M(x_n, x_m, t) > 1 - \varepsilon$$

for all $t > 0$ and $n, m \geq n_0$.

(3) A fuzzy metric space $(X, M, *)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Remark 2.6 (see [9]) Let $(X, M, *)$ be a fuzzy metric space. Then

(1) for all $x, y \in X$, $M(x, y, \cdot)$ is non-decreasing;

(2) if $x_n \to x$, $y_n \to y$, $t_n \to t$, then

$$\lim_{n \to \infty} M(x_n, y_n, t_n) = M(x, y, t);$$

(3) if $M(x, y, t) > 1 - r$ for x, y in X, $t > 0$, $0 < r < 1$, then we can find a t_0, $0 < t_0 < t$ such that $M(x, y, t_0) > 1 - r$;

(4) for any $r_1 > r_2$, we can find a r_3 such that $r_1 * r_3 \geq r_2$, and for any r_4, we can find a r_5 such that $r_5 * r_5 \geq r_4$ $(r_1, r_2, r_3, r_4, r_5 \in (0, 1))$.

Define $\Phi = \{\phi : R^+ \to R^+\}$, where $R^+ = [0, +\infty)$ and each $\phi \in \Phi$ satisfies the following conditions:

(ϕ-1) ϕ is non-decreasing,

(ϕ-2) ϕ is upper semi-continuous from the right,

(ϕ-3) $\sum_{n=0}^{\infty} \phi^n(t) < +\infty$ for all $t > 0$, where $\phi^{n+1}(t) = \phi(\phi^n(t))$, $n \in \mathbb{N}$.

It is easy to prove that if $\phi \in \Phi$, then $\phi(t) < t$ for all $t > 0$.

Lemma 2.7 (see [7]) *Let $(X, M, *)$ be a fuzzy metric space, where $*$ is a continuous t-norm of H-type. If there exists $\phi \in \Phi$ such that*

$$M\bigl(x, y, \phi(t)\bigr) \geq M(x, y, t) \tag{2.4}$$

for all $t > 0$, then $x = y$.

Definition 2.8 (see [4]) An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F : X \times X \to X$ if

$$F(x, y) = x, \qquad F(y, x) = y. \tag{2.5}$$

Definition 2.9 (see [5]) An element $(x, y) \in X \times X$ is called a coupled coincidence point of the mappings $F : X \times X \to X$ and $g : X \to X$ if

$$F(x, y) = g(x), \qquad F(y, x) = g(y). \tag{2.6}$$

Definition 2.10 (see [5]) An element $(x, y) \in X \times X$ is called a common coupled fixed point of the mappings $F : X \times X \to X$ and $g : X \to X$ if

$$x = F(x, y) = g(x), \qquad y = F(y, x) = g(y). \tag{2.7}$$

Definition 2.11 (see [5]) An element $x \in X$ is called a common fixed point of the mappings $F : X \times X \to X$ and $g : X \to X$ if

$$x = g(x) = F(x, x). \tag{2.8}$$

Definition 2.12 (see [8]) The mappings $F : X \times X \to X$ and $g : X \to X$ are said to be compatible if

$$\lim_{n\to\infty} M\big(gF(x_n, y_n), F\big(g(x_n), g(y_n)\big), t\big) = 1 \tag{2.9}$$

and

$$\lim_{n\to\infty} M\big(gF(y_n, x_n), F\big(g(y_n), g(x_n)\big), t\big) = 1 \tag{2.10}$$

for all $t > 0$ whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X, such that

$$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x, \qquad \lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y, \tag{2.11}$$

for all $x, y \in X$ are satisfied.

Definition 2.13 (see [20]) The mappings $F : X \times X \to X$ and $g : X \to X$ are called weakly compatible mappings if $F(x, y) = g(x)$, $F(y, x) = g(y)$ implies that $gF(x, y) = F(gx, gy)$, $gF(y, x) = F(gy, gx)$ for all $x, y \in X$.

Remark 2.14 It is easy to prove that if F and g are compatible, then they are weakly compatible, but the converse need not be true. See the example in the next section.

3 Main results

For simplicity, denote

$$\big[M(x, y, t)\big]^n = \underbrace{M(x, y, t) * M(x, y, t) * \cdots * M(x, y, t)}_{n}$$

for all $n \in \mathbb{N}$.

Xin-Qi Hu [8] proved the following result.

Theorem 3.1 (see [8]) *Let $(X, M, *)$ be a complete FM-space, where $*$ is a continuous t-norm of H-type satisfying* (2.2). *Let $F : X \times X \to X$ and $g : X \to X$ be two mappings, and there exists $\phi \in \Phi$ such that*

$$M\big(F(x, y), F(u, v), \phi(t)\big) \ge M\big(g(x), g(u), t\big) * M\big(g(y), g(v), t\big) \tag{3.1}$$

for all $x, y, u, v \in X$, $t > 0$.

Suppose that $F(X \times X) \subseteq g(X)$, g is continuous, F and g are compatible. Then there exist $x, y \in X$ such that $x = g(x) = F(x, x)$; that is, F and g have a unique common fixed point in X.

Now we give our main result.

Theorem 3.2 *Let $(X, M, *)$ be a FM-space, where $*$ is a continuous t-norm of H-type satisfying* (2.2). *Let $F : X \times X \to X$ and $g : X \to X$ be two weakly compatible mappings, and there exists $\phi \in \Phi$ satisfying* (3.1).

Suppose that $F(X \times X) \subseteq g(X)$ and $F(X \times X)$ or $g(X)$ is complete. Then F and g have a unique common fixed point in X.

Proof Let $x_0, y_0 \in X$ be two arbitrary points in X. Since $F(X \times X) \subseteq g(X)$, we can choose $x_1, y_1 \in X$ such that $g(x_1) = F(x_0, y_0)$ and $g(y_1) = F(y_0, x_0)$. Continuing this process, we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$g(x_{n+1}) = F(x_n, y_n), \qquad g(y_{n+1}) = F(y_n, x_n), \quad \text{for all } n \geq 0. \tag{3.2}$$

The proof is divided into 4 steps.

Step 1: We shall prove that $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences.

Since $*$ is a t-norm of H-type, for any $\lambda > 0$, there exists an $\mu > 0$ such that

$$\underbrace{(1-\mu) * (1-\mu) * \cdots * (1-\mu)}_{k} \geq 1 - \lambda$$

for all $k \in \mathbb{N}$.

Since $M(x, y, \cdot)$ is continuous and $\lim_{t \to +\infty} M(x, y, t) = 1$ for all $x, y \in X$, there exists $t_0 > 0$ such that

$$M(gx_0, gx_1, t_0) \geq 1 - \mu, \qquad M(gy_0, gy_1, t_0) \geq 1 - \mu. \tag{3.3}$$

On the other hand, since $\phi \in \Phi$, by condition (ϕ-3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any $t > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$t > \sum_{k=n_0}^{\infty} \phi^k(t_0). \tag{3.4}$$

From condition (3.1), we have

$$\begin{aligned}
M\bigl(gx_1, gx_2, \phi(t_0)\bigr) &= M\bigl(F(x_0, y_0), F(x_1, y_1), \phi(t_0)\bigr) \\
&\geq M(gx_0, gx_1, t_0) * M(gy_0, gy_1, t_0), \\
M\bigl(gy_1, gy_2, \phi(t_0)\bigr) &= M\bigl(F(y_0, x_0), F(y_1, x_1), \phi(t_0)\bigr) \\
&\geq M(gy_0, gy_1, t_0) * M(gx_0, gx_1, t_0).
\end{aligned}$$

Similarly, we have

$$\begin{aligned} M\bigl(gx_2, gx_3, \phi^2(t_0)\bigr) &= M\bigl(F(x_1, y_1), F(x_2, y_2), \phi^2(t_0)\bigr) \\ &\geq M\bigl(gx_1, gx_2, \phi(t_0)\bigr) * M\bigl(gy_1, gy_2, \phi(t_0)\bigr) \\ &\geq \bigl[M(gx_0, gx_1, t_0)\bigr]^2 * \bigl[M(gy_0, gy_1, t_0)\bigr]^2, \\ M\bigl(gy_2, gy_3, \phi^2(t_0)\bigr) &= M\bigl(F(y_1, x_1), F(y_2, x_2), \phi^2(t_0)\bigr) \\ &\geq \bigl[M(gy_0, gy_1, t_0)\bigr]^2 * \bigl[M(gx_0, gx_1, t_0)\bigr]^2. \end{aligned}$$

From the inequalities above and by induction, it is easy to prove that

$$\begin{aligned} M\bigl(gx_n, gx_{n+1}, \phi^n(t_0)\bigr) &\geq \bigl[M(gx_0, gx_1, t_0)\bigr]^{2^{n-1}} * \bigl[M(gy_0, gy_1, t_0)\bigr]^{2^{n-1}}, \\ M\bigl(gy_n, gy_{n+1}, \phi^n(t_0)\bigr) &\geq \bigl[M(gy_0, gy_1, t_0)\bigr]^{2^{n-1}} * \bigl[M(gx_0, gx_1, t_0)\bigr]^{2^{n-1}}. \end{aligned}$$

So, from (3.3) and (3.4), for $m > n \geq n_0$, we have

$$\begin{aligned} M(gx_n, gx_m, t) &\geq M\left(gx_n, gx_m, \sum_{k=n_0}^{\infty} \phi^k(t_0)\right) \\ &\geq M\left(gx_n, gx_m, \sum_{k=n}^{m-1} \phi^k(t_0)\right) \\ &\geq M\bigl(gx_n, gx_{n+1}, \phi^n(t_0)\bigr) * M\bigl(gx_{n+1}, gx_{n+2}, \phi^{n+1}(t_0)\bigr) * \cdots \\ &\quad * M\bigl(gx_{m-1}, gx_m, \phi^{m-1}(t_0)\bigr) \\ &\geq \bigl[M(gy_0, gy_1, t_0)\bigr]^{2^{n-1}} * \bigl[M(gx_0, gx_1, t_0)\bigr]^{2^{n-1}} * \bigl[M(gy_0, gy_1, t_0)\bigr]^{2^n} \\ &\quad * \bigl[M(gx_0, gx_1, t_0)\bigr]^{2^n} * \cdots * \bigl[M(gy_0, gy_1, t_0)\bigr]^{2^{m-2}} \\ &\quad * \bigl[M(gx_0, gx_1, t_0)\bigr]^{2^{m-2}} \\ &= \bigl[M(gy_0, gy_1, t_0)\bigr]^{2^{m-1}-2^{n-1}} * \bigl[M(gx_0, gx_1, t_0)\bigr]^{2^{m-1}-2^{n-1}} \\ &\geq \underbrace{(1-\mu) * (1-\mu) * \cdots * (1-\mu)}_{2^m - 2^n} \geq 1 - \lambda, \end{aligned}$$

which implies that

$$M(gx_n, gx_m, t) > 1 - \lambda \tag{3.5}$$

for all $m, n \in \mathbb{N}$ with $m > n \geq n_0$ and $t > 0$. So $\{g(x_n)\}$ is a Cauchy sequence.

Similarly, we can prove that $\{g(y_n)\}$ is also a Cauchy sequence.

Step 2: Now, we prove that g and F have a coupled coincidence point.

Without loss of generality, we can assume that $g(X)$ is complete, then there exist $x, y \in g(X)$, and exist $a, b \in X$ such that

$$\begin{aligned} \lim_{n\to\infty} g(x_n) &= \lim_{n\to\infty} F(x_n, y_n) = g(a) = x, \\ \lim_{n\to\infty} g(y_n) &= \lim_{n\to\infty} F(y_n, x_n) = g(b) = y. \end{aligned} \tag{3.6}$$

From (3.1), we get

$$M\big(F(x_n, y_n), F(a, b), \phi(t)\big) \geq M\big(gx_n, g(a), t\big) * M\big(gy_n, g(b), t\big).$$

Since M is continuous, taking limit as $n \to \infty$, we have

$$M\big(g(a), F(a, b), \phi(t)\big) = 1,$$

which implies that $F(a, b) = g(a) = x$.

Similarly, we can show that $F(b, a) = g(b) = y$.

Since F and g are weakly compatible, we get that $gF(a, b) = F(g(a), g(b))$ and $gF(b, a) = F(g(b), g(a))$, which implies that $g(x) = F(x, y)$ and $g(y) = F(y, x)$.

Step 3: We prove that $g(x) = y$ and $g(y) = x$.

Since $*$ is a t-norm of H-type, for any $\lambda > 0$, there exists an $\mu > 0$ such that

$$\underbrace{(1-\mu) * (1-\mu) * \cdots * (1-\mu)}_{k} \geq 1 - \lambda$$

for all $k \in \mathbb{N}$.

Since $M(x, y, \cdot)$ is continuous and $\lim_{t\to+\infty} M(x, y, t) = 1$ for all $x, y \in X$, there exists $t_0 > 0$ such that $M(gx, y, t_0) \geq 1 - \mu$ and $M(gy, x, t_0) \geq 1 - \mu$.

On the other hand, since $\phi \in \Phi$, by condition (ϕ-3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Thus, for any $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $t > \sum_{k=n_0}^{\infty} \phi^k(t_0)$. Since

$$\begin{aligned} M\big(gx, gy_{n+1}, \phi(t_0)\big) &= M\big(F(x, y), F(y_n, x_n), \phi(t_0)\big) \\ &\geq M(gx, gy_n, t_0) * M(gy, gx_n, t_0), \end{aligned}$$

letting $n \to \infty$, we get

$$M\big(gx, y, \phi(t_0)\big) \geq M(gx, y, t_0) * M(gy, x, t_0). \tag{3.7}$$

Similarly, we can get

$$M\big(gy, x, \phi(t_0)\big) \geq M(gx, y, t_0) * M(gy, x, t_0). \tag{3.8}$$

From (3.7) and (3.8), we have

$$M\big(gx, y, \phi(t_0)\big) * M\big(gy, x, \phi(t_0)\big) \geq \big[M(gx, y, t_0)\big]^2 * \big[M(gy, x, t_0)\big]^2.$$

From this inequality, we can get

$$\begin{aligned} M\big(gx, y, \phi^n(t_0)\big) * M\big(gy, x, \phi^n(t_0)\big) &\geq \big[M\big(gx, y, \phi^{n-1}(t_0)\big)\big]^2 * \big[M\big(gy, x, \phi^{n-1}(t_0)\big)\big]^2 \\ &\geq \big[M(gx, y, t_0)\big]^{2^n} * \big[M(gy, x, t_0)\big]^{2^n} \end{aligned}$$

for all $n \in \mathbb{N}$. Since $t > \sum_{k=n_0}^{\infty} \phi^k(t_0)$, then, we have

$$\begin{aligned} M(gx,y,t) * M(gy,x,t) &\geq M\left(gx,y,\sum_{k=n_0}^{\infty}\phi^k(t_0)\right) * M\left(gy,x,\sum_{k=n_0}^{\infty}\phi^k(t_0)\right) \\ &\geq M\big(gx,y,\phi^{n_0}(t_0)\big) * M\big(gy,x,\phi^{n_0}(t_0)\big) \\ &\geq \big[M(gx,y,t_0)\big]^{2^{n_0}} * \big[M(gy,x,t_0)\big]^{2^{n_0}} \\ &\geq \underbrace{(1-\mu)*(1-\mu)*\cdots*(1-\mu)}_{2^{2n_0}} \geq 1-\lambda. \end{aligned}$$

Therefore, for any $\lambda > 0$, we have

$$M(gx,y,t) * M(gy,x,t) \geq 1-\lambda \tag{3.9}$$

for all $t > 0$. Hence conclude that $gx = y$ and $gy = x$.

Step 4: Now, we prove that $x = y$.

Since $*$ is a t-norm of H-type, for any $\lambda > 0$, there exists an $\mu > 0$ such that

$$\underbrace{(1-\mu)*(1-\mu)*\cdots*(1-\mu)}_{k} \geq 1-\lambda$$

for all $k \in \mathbb{N}$.

Since $M(x,y,\cdot)$ is continuous and $\lim_{t\to+\infty} M(x,y,t) = 1$, there exists $t_0 > 0$ such that $M(x,y,t_0) \geq 1-\mu$.

On the other hand, since $\phi \in \Phi$, by condition (ϕ-3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then, for any $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $t > \sum_{k=n_0}^{\infty} \phi^k(t_0)$.

From (3.1), we have

$$\begin{aligned} M\big(gx_{n+1},gy_{n+1},\phi(t_0)\big) &= M\big(F(x_n,y_n),F(y_n,x_n),\phi(t_0)\big) \\ &\geq M(gx_n,gy_n,t_0) * M(gy_n,gx_n,t_0). \end{aligned}$$

Letting $n \to \infty$ yields

$$M\big(x,y,\phi(t_0)\big) \geq M(x,y,t_0) * M(y,x,t_0).$$

Thus, we have

$$\begin{aligned} M(x,y,t) &\geq M\left(x,y,\sum_{k=n_0}^{\infty}\phi^k(t_0)\right) \\ &\geq M\big(x,y,\phi^{n_0}(t_0)\big) \\ &\geq \big[M(x,y,t_0)\big]^{2^{n_0-1}} * \big[M(y,x,t_0)\big]^{2^{n_0-1}} \\ &\geq \underbrace{(1-\mu)*(1-\mu)*\cdots*(1-\mu)}_{2^{2n_0-2}} \geq 1-\lambda, \end{aligned}$$

which implies that $x = y$.

Thus, we proved that F and g have a common fixed point in X.

The uniqueness of the fixed point can be easily proved in the same way as above. This completes the proof of Theorem 3.2. □

Taking $g = I$ (the identity mapping) in Theorem 3.2, we get the following consequence.

Corollary 3.3 *Let $(X, M, *)$ be a FM-space, where $*$ is a continuous t-norm of H-type satisfying* (2.2). *Let $F : X \times X \to X$, and there exists $\phi \in \Phi$ such that*

$$M\big(F(x,y), F(u,v), \phi(t)\big) \geq M(x,u,t) * M(y,v,t) \tag{3.10}$$

for all $x, y, u, v \in X$, $t > 0$. $F(X)$ is complete.

Then there exist $x \in X$ such that $x = F(x,x)$; that is, F admits a unique fixed point in X.

Remark 3.4 Comparing Theorem 3.2 with Theorem 3.1 in [8], we can see that Theorem 3.2 is a genuine generalization of Theorem 3.2.

(1) We only need the completeness of $g(X)$ or $F(X \times X)$.

(2) The continuity of g is relaxed.

(3) The concept of compatible has been replaced by weakly compatible.

Remark 3.5 The Example 3 in [8] is wrong, since the t-norm $a * b = ab$ is not the t-norm of H-type.

Next, we give an example to support Theorem 3.2.

Example 3.6 Let $X = \{0, 1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\}$, $* = \min$, $M(x,y,t) = \frac{t}{|x-y|+t}$, for all $x, y \in X$, $t > 0$. Then $(X, M, *)$ is a fuzzy metric space.

Let $\phi(t) = \frac{t}{2}$. Let $g : X \to X$ and $F : X \times X \to X$ be defined as

$$g(x) = \begin{cases} 0, & x = 0, \\ 1, & x = \frac{1}{2n+1}, \\ \frac{1}{2n+1}, & x = \frac{1}{2n}, \end{cases} \qquad F(x,y) = \begin{cases} \frac{1}{(2n+1)^4}, & (x,y) = (\frac{1}{2n}, \frac{1}{2n}), \\ 0, & \text{others}. \end{cases}$$

Let $x_n = y_n = \frac{1}{2n}$. We have $gx_n = \frac{1}{2n+1} \to 0$, $F(x_n, y_n) = \frac{1}{(2n+1)^4} \to 0$, but

$$M\big(F(gx_n, gy_n), gF(x_n, y_n), t\big) = M(0, 1, t) \nrightarrow 0,$$

so g and F are not compatible. From $F(x,y) = g(x)$, $F(y,x) = g(y)$, we can get $(x,y) = (0,0)$, and we have $gF(0,0) = F(g0, g0)$, which implies that F and g are weakly compatible.

The following result is easy to verify

$$\frac{t}{X+t} \geq \min\left\{\frac{t}{Y+t}, \frac{t}{Z+t}\right\} \quad \Leftrightarrow \quad X \leq \max\{Y, Z\}, \quad \forall X, Y, Z \geq 0, t > 0.$$

By the definition of M and ϕ and the result above, we can get that inequality (3.1)

$$M\big(F(x,y), F(u,v), \phi(t)\big) \geq M\big(g(x), g(u), t\big) * M\big(g(y), g(v), t\big)$$

is equivalent to the following

$$2\left|F(x,y)-F(u,v)\right| \le \max\left\{\left|g(x)-g(u)\right|,\left|g(y)-g(v)\right|\right\}. \tag{3.11}$$

Now, we verify inequality (3.11). Let $A=\{\frac{1}{2n}, n\in\mathbb{N}\}$, $B=X-A$. By the symmetry and without loss of generality, (x,y), (u,v) have 6 possibilities.

Case 1: $(x,y)\in B\times B$, $(u,v)\in B\times B$. It is obvious that (3.11) holds.

Case 2: $(x,y)\in B\times B$, $(u,v)\in B\times A$. It is obvious that (3.11) holds.

Case 3: $(x,y)\in B\times B$, $(u,v)\in A\times A$. If $u\neq v$, (3.11) holds. If $u=v$, let $u=v=\frac{1}{2n}$, then

$$2\left|F(x,y)-F(u,v)\right| = \frac{2}{(2n+1)^4}, \qquad \max\left\{\left|g(x)-g(u)\right|,\left|g(y)-g(v)\right|\right\} = \frac{2n}{2n+1},$$

which implies that (3.11) holds.

Case 4: $(x,y)\in B\times A$, $(u,v)\in B\times A$. It is obvious that (3.11) holds.

Case 5: $(x,y)\in B\times A$, $(u,v)\in A\times A$. If $u\neq v$, (3.11) holds. If $u=v$, let $x\in B$, $y=\frac{1}{2j}$, $u=v=\frac{1}{2n}$, then

$$2\left|F(x,y)-F(u,v)\right| = \frac{2}{(2n+1)^4},$$
$$\max\left\{\left|g(x)-g(u)\right|,\left|g(y)-g(v)\right|\right\} = \max\left\{\frac{1}{2n+1},\left|\frac{1}{2j+1}-\frac{1}{2n+1}\right|\right\},$$

or

$$\max\left\{\left|g(x)-g(u)\right|,\left|g(y)-g(v)\right|\right\} = \max\left\{\frac{2n}{2n+1},\left|\frac{1}{2j+1}-\frac{1}{2n+1}\right|\right\},$$

(3.11) holds.

Case 6: $(x,y)\in A\times A$, $(u,v)\in A\times A$.

If $x\neq y$, $u\neq v$, (3.11) holds.

If $x\neq y$, $u=v$, let $x=\frac{1}{2i}$, $y=\frac{1}{2j}$, $i\neq j$, $u=v=\frac{1}{2n}$. Then

$$2\left|F(x,y)-F(u,v)\right| = \frac{2}{(2n+1)^4},$$
$$\max\left\{\left|g(x)-g(u)\right|,\left|g(y)-g(v)\right|\right\} = \max\left\{\left|\frac{1}{2i+1}-\frac{1}{2n+1}\right|,\left|\frac{1}{2j+1}-\frac{1}{2n+1}\right|\right\},$$

(3.11) holds.

If $x=y$, $u=v$, let $x=y=\frac{1}{2i}$, $u=v=\frac{1}{2n}$. Then

$$2\left|F(x,y)-F(u,v)\right| = 2\left|\frac{1}{(2i+1)^4}-\frac{1}{(2n+1)^4}\right|,$$
$$\max\left\{\left|g(x)-g(u)\right|,\left|g(y)-g(v)\right|\right\} = \left|\frac{1}{2i+1}-\frac{1}{2n+1}\right|,$$

(3.11) holds.

Then all the conditions in Theorem 3.2 are satisfied, and 0 is the unique common fixed point of g and F.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors read and approved the final manuscript.

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