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Research Article

Design of Wideband Multifunction Antenna Array Based on Multiple Interleaved Subarrays

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A new Modified Iterative Fourier Technique (MIFT) is proposed for the design of interleaved linear antenna arrays which operate at different frequencies with no grating lobes, low-sidelobe levels, and wide bandwidths. In view of the Fourier transform mapping between the element excitations and array factor of uniform linear antenna array, the spectrum of the array factor is first acquired with FFT and its energy distributions are investigated thoroughly. The relationship between the carrier frequency and the element excitation is obtained by the density-weighting theory. In the following steps, the element excitations of interleaved subarrays are carefully selected in an alternate manner, which ensures that similar patterns can be achieved for interleaved subarrays. The Peak Sidelobe Levels (PSLs) of the interleaved subarrays are further reduced by the iterative Fourier transform algorithm. Numerical simulation results show that favorable design of the interleaved linear antenna arrays with different carrier frequencies can be obtained by the proposed method with favorable pattern similarity, low PSL, and wide bandwidths.

1. Introduction

Wideband multifunction antenna arrays are given a distinctive attention, in particular for applications involving radar tracking, biomedical imaging, wireless communications, location and remote sensing, and so forth. A popular solution for assuring the necessary bandwidth of the multifunction array is to use radiators with a very large bandwidth [1]. However, when wideband radiators are used, it is difficult to assure a good isolation between different functions supported by the array due to the mutual coupling. Another way to design wideband antenna array is to interleave several subarrays on a shared aperture [2, 3]. However, this approach needs to ensure that the interleaved subarrays are designed with no grating lobes, similar patterns, and low Peak Sidelobe Levels (PSLs).

In order to address these challenges, several deterministic methods have been proposed in the literature. Multiple interleaved subarrays are obtained by means of the difference sets (DS) [4–6]. The patterns of these interleaved subarrays exhibited no major grating lobes and low PSLs. Three approaches to interleave two subarrays on the same aperture are presented in [7]. The available aperture is efficiently used by these interleaved subarrays. With the help of a finite-by-infinite-array approach, the element pattern of wideband arrays is analyzed in [8]. Compared with the infinite-array approach, the finite-by-infinite-array approach becomes useful for arrays whose side dimension is larger than 1.5 to 2 wavelengths. In [9], a timescale model is proposed as a discrete characterization of wideband time-varying systems. Reference [10] demonstrates that the bandwidth of antenna array can be extended by incorporating a simple perturbation scheme into the basic array generation process. However, because all the subarrays interleaved by difference sets operate at the same frequency, just a limited extension of the bandwidth can be obtained. Interleaving multiple subarrays with different carrier frequencies is a powerful and versatile way to design wideband antenna array. Meanwhile, a problem that the grating lobes of the subarrays which operate at high frequency are hard to be avoided must be solved in the design of multi-interleaved subarrays. To the best of our knowledge, efficient design methods for the interleaved linear antenna array with multiple subarrays and variable carrier frequencies have not been reported in the literature.

To address these challenging problems effectively, a Modified Iterative Fourier Technique (MIFT) for interleaving linear antenna array with different carrier frequencies is proposed in this paper. The Iterative Fourier Technique (IFT), which was earlier presented by Carroll for synthesizing array patterns [11, 12], had been further developed by Keizer in recent years for the design of thinned antenna array [13]. As a version of the alternating projection method, the IFT derives the element excitations from the array factor using successive direct and inverse fast Fourier transform. Array thinning can be accomplished by forcing element excitations with higher amplitude values to be equal to one and others with smaller amplitude to be zero in every iteration cycle. In this study, according to the density-weighting theory, the spectrum energy distribution can be selected in an equal-proportion with different carrier frequencies for the interleaved subarrays, which ensures that similar patterns can be obtained. Furthermore, the IFT is utilized to reduce the PSLs of the interleaved subarrays. After several iterations, the interleaved subarrays which operate at variable frequencies can be obtained by MIFT method with low PSLs and favorable pattern similarity. Due to the fact that the subarrays operate at different frequencies, the bandwidth is expanded substantially, which ensures that the wideband linear antenna array can be achieved. Due to nonuniform arrangement, the mutual couplings between the interleaved subarrays were reduced considerably [7] and thus the mutual coupling effects are not considered in this paper.

The rest of the paper is organized as follows. In order to convey the technical approach in a clear manner, the density-weighting theory was briefly described and the description of the MIFT algorithm was presented in Section 2. In Section 3, the processes of the proposed method are given in detail. Numerical simulation results are described and discussed in Section 4. Finally, some conclusions are drawn in Section 5.

2. Description of the MIFT

In this section, the density-weighting theory is first briefly reviewed. Density-weighting method developed by Skolnik is a statistic technique for the design of an equally weighted thinned antenna array (i.e., the excitations are equal to 1 or 0) [14, 15]. Considering a linear array with N elements, which are arranged along a periodic grid at distance half wavelength apart, the array factor, AF, can be written as follows:

$$\mathrm{AF}(\theta) = \sum_{n=0}^{N-1} A_n e^{jnkd(\sin\theta - \sin\theta_0)}, \quad (1)$$

where $k = 2\pi/\lambda$, λ denotes the wavelength, and θ is the azimuth angle. θ_0 is the direction of main beam and A_n is the source distribution of full antenna array. Because the antenna array is a density-weighting array, the array factor can be equivalently rewritten as follows:

$$F(\theta) = \sum_{n=0}^{N-1} I_n e^{jnkd\sin\theta}, \quad (2)$$

where I_n is equal to “1” or “0.” “1” indicates that the element is “turned ON,” and “0” means that the element is “turned OFF.” In accordance with the density-weighting theory, the turning on probability of the array elements ($P(I_{mn} = 1)$) depends on the ratio of element excitation amplitude to the maximum element excitation [14]:

$$P(I_n = 1) = \upsilon \cdot \frac{A(n)}{A_{\max}}, \quad (3)$$

where $A_{\max}$ indicates the maximum excitation of the uniform antennas array. υ is a constant value. Equation (3) shows that the turning on probability of the elements is increased with its excitations.

Since the interleaved subarrays operate at different frequency, the interelement spaces and subarray aperture sizes are different. Without loss of generality, considering a shared aperture linear antenna array consisting of T interleaved subarrays at T different carrier frequencies, $f_1, f_2, \ldots, f_T$, the relationships of frequencies are depicted as

$$f_1 = a_1 f_2 = \cdots = a_{T-1} f_T, \quad (4)$$

where $a_1, a_2, \ldots, a_{T-1}$ are a set of constant values. If the array aperture size is measured by the wavelength λ_0, the performance of our method will depend on the carrier frequency, f_0. According to (5), the array factors for the interleaved subarrays which operate at T different frequencies can be depicted as in (6). Consider

$$\lambda_1 = \frac{1}{a_1}\lambda_2 = \cdots = \frac{1}{a_{T-1}}\lambda_T, \quad (5)$$

$$\begin{aligned} F_1(\theta) &= \sum_{n_1=0}^{N_1-1} A_{n_1} e^{jn_1\pi d\sin\theta}, \\ F_2(\theta) &= \sum_{n_2=0}^{N_2-1} A_{n_2} e^{jn_2\pi d\sin\theta}, \\ &\vdots \\ F_T(\theta) &= \sum_{n_T=0}^{N_T-1} A_{n_T} e^{jn_T\pi d\sin\theta}, \end{aligned} \quad (6)$$

where $\{A_{ni}\}$ denotes the element excitations of ith subarray, N_i denotes the number of elements of ith subarray, and $F_i(\theta)$ indicates the array factor of ith subarray. Because the array factor is related to the element excitations through a discrete inverse Fourier transform, a discrete direct Fourier transform applied to array factor over the period λ/d will map the element excitations, A_n. The calculation of the array factor is carried out with K-point inverse FFT. Therefore, (6) can be equivalently rewritten as

$$\begin{aligned} F_1(\theta) &= \mathrm{IFFT}(A_{n1}\theta), \\ F_2(\theta) &= \mathrm{IFFT}(a_1 A_{n2}\theta), \\ &\vdots \\ F_T(\theta) &= \mathrm{IFFT}(a_{T-1} A_{nT}\theta), \end{aligned} \quad (7)$$

where IFFT indicates inverse fast Fourier transform. If the array factors of the interleaved subarrays are taken as probabilistic events, the expected value of the array factor can be defined as

$$E(F(\theta)) = \sum_{n=0}^{N-1} P_n A_n e^{jnkd \sin\theta}, \quad (8)$$

where P_n indicates the turning on probability value of nth array element and is described in (3). To achieve the optimal design of interleaved linear antenna array, several performance indexes should be taken into consideration, such as similar overall power patterns and a low Peak Sidelobe Level [7]. The target for the design of interleaved linear antenna array is finding a set of optimal positions of the array elements to reduce the PSL of the subarray simultaneously

$$\eta = \frac{\text{number of subarray elements in the aperture}}{\text{total number of elements in the aperture}} \times 100\%. \quad (9)$$

In the traditional iterative Fourier transform algorithm [13, 14] FFT mapping between the element excitations and array factor AF is utilized in the synthesis of low PSL antenna array. Besides, the iterative Fourier transform can also be devoted to the design of thinned planar antenna array [14]. In every iteration cycle of [14], $N \times \eta$ (i.e., η denotes thinned ratio of the subarray which can be defined as (9)) element excitations with higher amplitude are forced to be 1 and those with smaller amplitude to be 0. After several iterations, the array thinning is accomplished by retaining the elements with excitation one and deleting the elements with excitation zero.

In this study, by changing the selection of element excitations, a new interleaving method is proposed for the design of interleaved linear antenna array with different carrier frequencies. To achieve an efficient design of the interleaved linear antenna array, the expected value of the array factor AF for each interleaved subarray should be kept the same as much as possible, which ensures that the performance-similarity of the interleaved subarrays can be obtained. In every iteration of our proposed method, the PSLs of the interleaved subarrays are decreased by an adaptation of the sidelobe region of AF to the sidelobe level threshold (SLT). The SLT plays a key role in obtaining a low-sidelobe interleaved array by MIFT method, and a high or low value of SLT can largely raise PSLs of interleaved subarrays. To get an interleaved array with the minimum PSL, the MIFT should be performed with a suitable SLT. However, the SLT value that fits the method best is difficult to find because the considerable computational burden is required by repeatedly adjusting the value of SLT. According to many synthesis trials, a good value of the SLT suitable for the MIFT can be confined in a small interval P_0:

$$P_0 = [S_0 - 12, S_0 - 15\,\text{dB}], \quad (10)$$

where S_0 corresponds to the PSL of a same-size periodic antenna array. According to (10), when S_0 is obtained, interleaved arrays are usually achieved by using the MIFT method to interleave a same-size array with the SLT value varying slightly within the interval P_0. Furthermore, in the realization of our proposed method, the element excitations in the allowable aperture are sorted by its amplitude. If T is the number of subarrays, $\mathbf{A}_f$ denotes the sorted excitation vector, and then the element excitation coefficients $\mathbf{A}_n$ of interleaved subarrays are carefully selected in an alternate manner. The excitations of the first subarray element are treated as the updated input for the next iteration; after several iterations, interleaved subarrays which operate at different frequencies can be obtained with low PSLs.

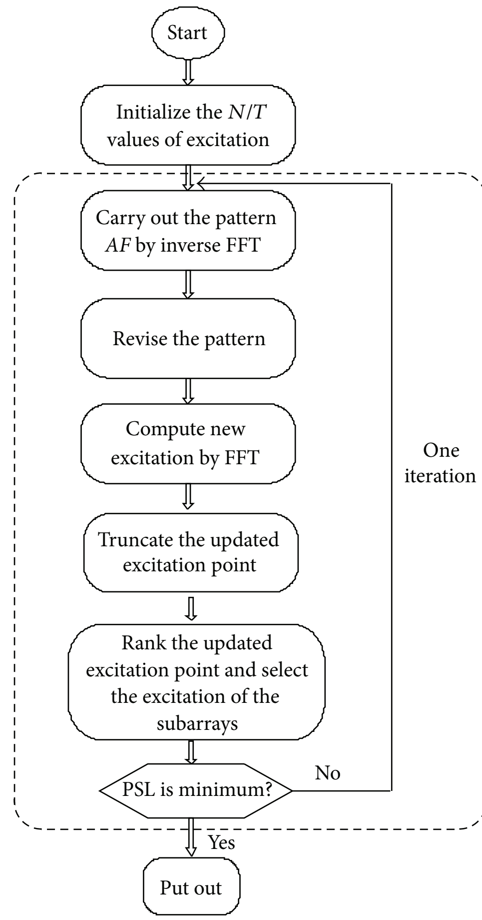

Figure 1: Flowchart of the MIFT algorithm.

3. The Proceeds of the MIFT Algorithm

The flowchart of the MIFT algorithm for the design of interleaved linear antenna array with different carrier frequencies is shown in Figure 1. At first we make the values of the initial element excitations $\{A\}$ be equal to 1 with a sparse ratio as $1/T$ (i.e., T denotes the number of subarrays) and the rest of the element excitations be equal to 0. To ensure that the narrow main-beam width is obtained, the initial and end point of the excitation sequence should be 1. The synthesis procedure starts with the calculation of the array factor AF through a K-point inverse FFT of initial element excitations, where the value of K is larger than the numbers of the linear array elements N (i.e., $K > N$). It is followed by an adaptation of the sidelobe region of the array factor to the SLT. The values of sidelobe levels for array factor that exceed SLT are corrected, and the other samples of array factor remain unchanged. After this revision, a K-point FFT is performed on the updated AF to get a new set of excitation coefficients. Because the dominating energy of frequency spectrum for the pattern is focused on the front part of the sampling points, from those K excitations, only the N samples which belong to the linear antenna array are retained. Sort the truncated excitations to get a new excitation vector $\mathbf{A}_f$, and then the element excitations of subarrays are selected in an alternate manner (i.e., the element excitations of ith subarray should be selected as $A_f((i-1)T+1)$, $A_f((i-1)T+1)$,$\ldots$, $A_f(iT)$, $A_f((i-1)T+1+T^2)$, $A_f((i-1)T+2+T^2), \ldots, A_f(iT+T^2)$, $A_f((i-1)T+1+2T^2), \ldots, A_f(N+1+iT-T^2-T), \ldots, A_f(N+iT-T^2))$. The excitations of the first subarray element are treated as the updated input for the next iteration, after which a new updated array factor would be calculated. Repeat the process until the Peak Sidelobe Levels of the first subarray do not change or the iterative times are out of requirements. In the above procedures, due to the fact that the elements which are turned on in one thinned subarray are not allowed to be used by the other subarrays and the mapping between excitation point and the element position is a one-to-one mapping, the elements belonging to different subarrays are prevented from overlapping.

To convey the proposed method in a clear manner, suppose that $T = 3$ and the schematic diagram of the element excitations selecting process for the interleaved subarrays can be presented in Figure 2. To sum up, the procedure of the MIFT for the design of interleaved linear antenna array can be described as follows.

(1) Set the values of the initial element excitations as 1 with a probability of $1/T$ and the rest of the element excitations as 0. To ensure that the narrow main-beam width is obtained, the initial and end point of the excitation sequence should be 1.

(2) Compute AF from element excitation vector using a K-point inverse FFT, with $K > N$.

(3) Adjust the values of AF to adapt to the constraint of the SLT. In more detail, only the samples of AF that exceed the SLTs are corrected, and the rest of the samples of AF are left unchanged.

(4) Calculate the updated element excitation $\mathbf{A}_n$ for the adapted AF using a K-point FFT.

(5) Truncate $\mathbf{A}_n$ from the K samples to N samples by making all the samples outside the array aperture be equal to 0.

(6) Sort the truncated excitation to get a new excitation vector $\mathbf{A}_f$, and then the element excitations of ith subarray should be selected as $A_f((i-1)T+1)$, $A_f((i-1)T+1), \ldots, A_f(iT)$, $A_f((i-1)T+1+T^2)$, $A_f((i-1)T+2+T^2), \ldots, A_f(iT+T^2)$, $A_f((i-1)T+1+2T^2), \ldots$.

(7) Take the excitations of the first subarray to be the updated input for the next iteration.

(8) Repeat Steps (2)–(7) until the prescribed sidelobe requirements for AF are unchanged or the allowed number of iterations is reached.

(9) Set the element excitations of all the interleaved subarrays equal to "1" and output the results.

4. Numerical Analysis

This section is aimed at assessing the performance of multiple interleaved subarrays with different carrier frequencies based on MIFT method. An interleaved linear antenna array consisting of 100 elements spaced half wavelength apart is concerned. In the iterations, the value of the sidelobe level threshold is set as -20 dB, and the 1024-point direct and inverse FFT are utilized. Without loss of generality, the MIFT method is exploited to interleave four subarrays (i.e., $T = 4$) on this linear array aperture and the carrier frequencies of the four interleaved subarrays are set as f_0, $0.75f_0$, $0.5f_0$, and $0.25f_0$, respectively. The following equation shows the simulation results of array aperture architecture of the interleaved subarrays:

$$
\begin{array}{ll}
\text{Subarray 1} & \begin{array}{l} 1,0,0,0,0,0,0,0,1,0,1,1,0,0,0,0,0,0,0,0,0,0,0,1,0,0,1,1,0,0,1,0,1,1,0,0,1,0,0,1,0,0,1,0,0,1,0,0,0, \\ 0,0,0,1,0,0,0,0,0,1,1,0,0,0,1,0,0,0,0,0,1,0,0,0,1,1,0,0,0,1,1,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0 \end{array} \\
\text{Subarray 2} & \begin{array}{l} 0,0,0,0,0,1,1,1,0,1,0,0,0,0,0,0,1,1,0,0,1,1,1,0,0,0,1,0,0,1,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0, \\ 0,0,1,0,0,0,0,1,1,0,0,1,1,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,1,1,0,1,0,0,0,0,1,0,0,0,1,0,0,0 \end{array} \\
\text{Subarray 3} & \begin{array}{l} 0,0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,1,0,0,0,1, \\ 1,1,0,0,0,1,1,0,0,0,0,0,0,1,0,0,1,0,0,1,0,0,1,0,0,0,0,1,1,0,0,1,0,0,0,0,0,0,0,1,1,1,0,0,0,0,0,1,1,0 \end{array} \\
\text{Subarray 4} & \begin{array}{l} 0,1,1,0,1,0,0,0,0,0,0,0,0,1,1,1,0,0,1,0,0,0,0,1,0,0,0,0,0,0,0,0,1,0,0,1,0,0,1,1,0,0,0,0,1,0,0,0,1,0, \\ 0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,1,0,0,1,0,0,0,0,1,0,0,1,0,0,0,0,0,0,1,0,0,1,0,0,0,0,0,0,1,1,1,0,0,0,1, \end{array}
\end{array}
\quad (11)
$$

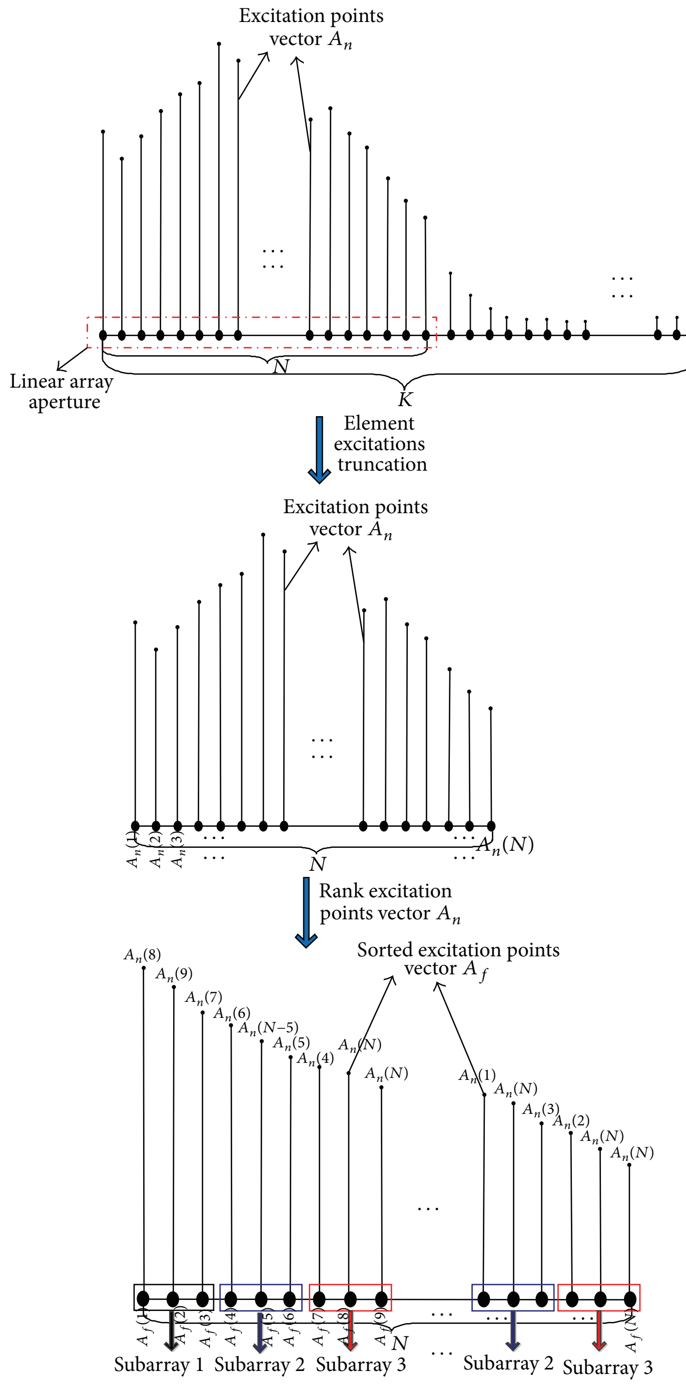

Figure 2: The schematic diagram of the element excitations selecting process for the interleaved subarrays ($T = 3$).

where "1" indicates that the element of the subarray is turned on and "0" indicates that the element of the subarray is turned off. It can be observed that the four interleaved subarrays occupy the same aperture without elements overlapping. Define an aperture occupancy ratio as

$$\zeta = \frac{L_{\text{subarray}i}}{L_{\text{array}}} \times 100\%, \tag{12}$$

where $L_{\text{subarray}i}$ is the aperture size of ith subarray and L_{array} is the aperture size of the periodic linear antenna array. It can be observed that the aperture occupancy ratio ζ of the four interleaved subarrays is 89%, 93%, 96%, and 99%, respectively, which indicates that the array aperture is used efficiently. The normalized power patterns of the four interleaved subarrays are shown in Figure 3. It can be seen that the sidelobe levels of the interleaved subarrays are almost the same. Because of

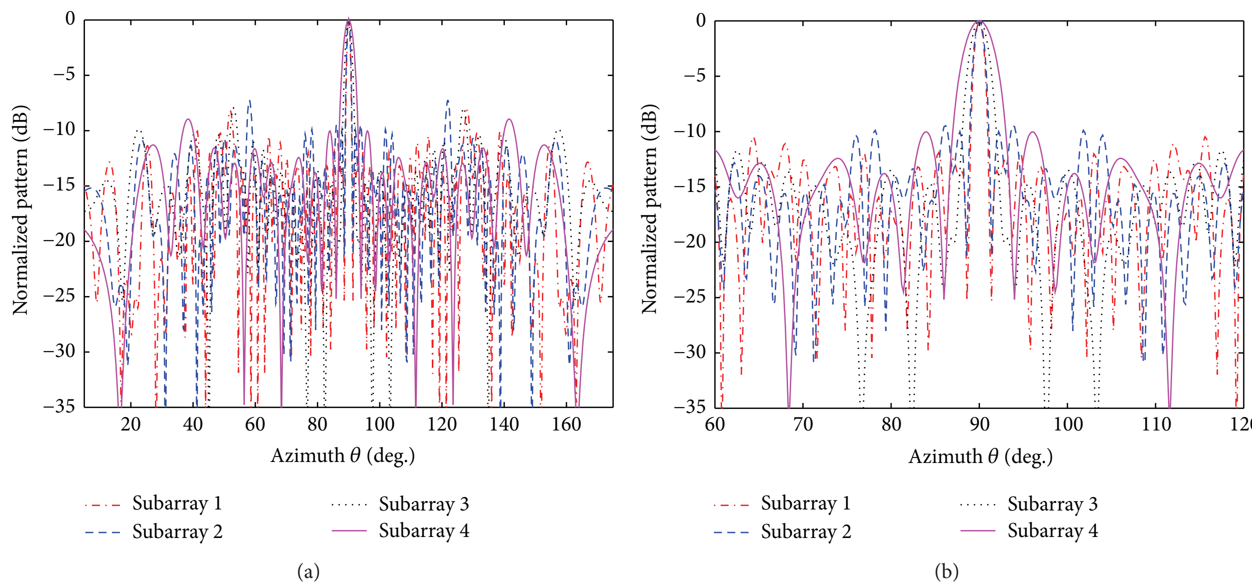

Figure 3: (a) Normalized power pattern for the subarrays. (b) Magnification of the pattern for the interleaved subarrays.

Table 1: Performance parameters for the interleaved subarrays.

Type	Aperture occupancy ratio	Carrier frequency	Main-beam width	PSL
Subarray 1	89%	f	2.4°	−7.29 dB
Subarray 2	93%	$0.75f$	2.8°	−8.17 dB
Subarray 3	96%	$0.5f$	5.6°	−7.96 dB
Subarray 4	99%	$0.25f$	8°	−8.96 dB

the different carrier frequencies, which makes the aperture size of each interleaved subarrays different, the main-beam widths of the interleaved subarrays are variable. For smaller carrier frequencies, the meaning aperture size is smaller, which makes the main-beam width wider than the subarrays with higher carrier frequencies. The performance parameters for the subarrays are shown in Table 1, in which aperture occupancy ratio, carrier frequencies, PSLs, and main-beam widths are listed, respectively, in detail. It can be observed that the main-beam widths of subarrays are 2.4°, 2.8°, 5.6°, and 8°, respectively. The values of PSLs for the interleaved subarrays are −7.29 dB, −8.17 dB, −7.96 dB, and −8.96 dB. The minimum value of difference between the PSL for the subarrays is 0.21 dB. It can be concluded from Table 1 that the subarrays which are interleaved by MIFT algorithm exhibit favorable pattern similarity, low PSLs, and no grating lobes. These results demonstrate the effectiveness of the proposed approach for interleaving subarrays.

The next simulation is to assess the performance of the bandwidth and main-beam width when the carrier frequency of interleaved subarrays is increased. The curve of the relationship between the carrier frequency and main-beam lobe width is shown in Figure 4. It can be observed that the main-beam lobe widths of the interleaved subarrays narrow down with the increased carrier frequencies. The curve of relationship between the carrier frequency and the sidelobe levels of interleaved subarrays is presented in Figure 5. It can be seen that the PSLs of the interleaved subarrays are enlarged with the increased carrier frequencies. All the subarrays have almost the same pattern-broadband widths. The broadband widths of the interleaved subarrays with different carrier frequencies can be stacked, which makes the bandwidth of the shared aperture antenna array be extended substantially. These simulation results prove that the wideband multifunction antenna array based on multiple interleaved subarrays with different carrier frequencies can be designed by MIFT algorithm.

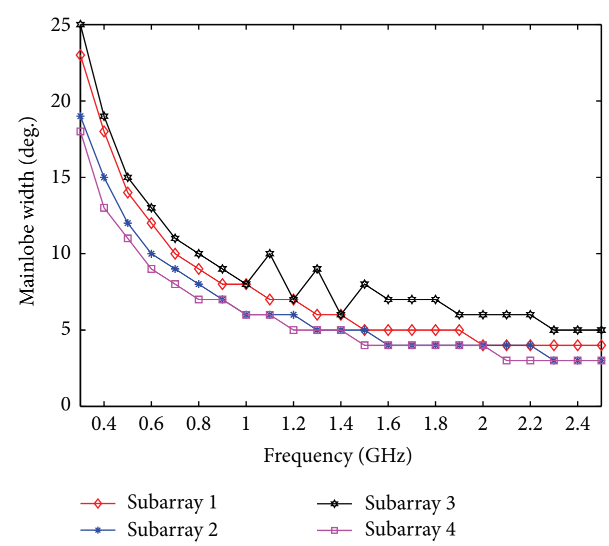

Figure 4: The curve of the relation between the carrier frequency and main-beam width.

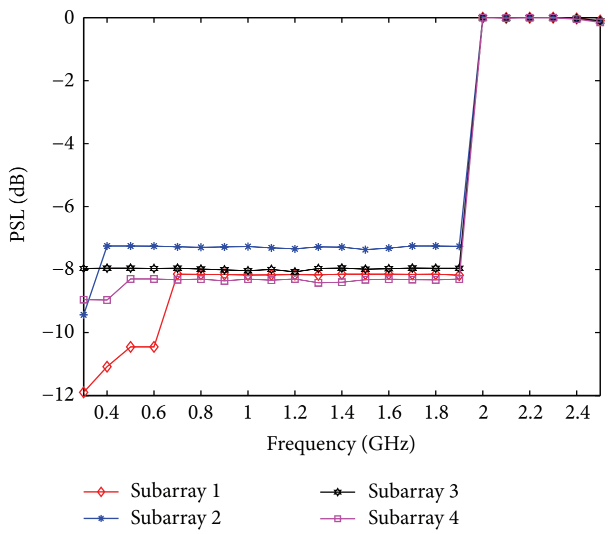

FIGURE 5: The curve of the relation between the carrier frequency and PSL.

5. Conclusion and Discussion

In this paper a new method is demonstrated to interleave multiple subarrays which share a common aperture with different carrier frequencies. Through MIFT algorithm, low-sidelobe levels and similar radiation patterns for multi-interleaved subarrays can be achieved and the available aperture space is efficiently utilized. Some numerical simulations are presented to assess the performance of the MIFT method for the design of interleaved antenna array with different carrier frequencies. The performance of the bandwidth and main-beam width with the increased carrier frequencies is also evaluated. The broadband widths of the interleaved subarrays with different carrier frequencies is stacked and the wideband multifunction antenna array based on multiple interleaved subarrays with different carrier frequencies can be designed by MIFT algorithm. Future efforts will be devoted to extending the proposed method to two-dimensional planar antenna array in the presence of mutual coupling effect.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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