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Research Article

Free Will with Afterthoughts: A Quasichemical Model

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A model of free will is proposed, appealing to the similarity with simple, two-body chemical reactions where the energy curves for the reagents and for the products cross. The system at the crossing point has a freedom of choice to perform the reaction or not. The Landau-Zener formula, corresponding to the opportunity of meeting twice the crossing point, is interpreted as free will with an afterthought and generalized to the cases when a subject thinks about a choice n times. If the probability distribution p_n of afterthoughts is known, the probability of a final yes decision is given. The results are generalized to situations where a preference for or against a change exists or where the freedom is only partial, has to fight with conditioning factors, and possibly decreases with increasing instances of free choice.

1. Introduction

The problem of free will is very ancient, and even the modern literature on it is vast and highly controversial. The attempts of a number of intellectuals (usually nonphysicists) to justify human free will via quantum-mechanical indeterminacy often lack precision and are not really meaningful. Fortunately, however, some serious and profound discussions of the relation between quantum physics and at least some instances of free will do exist: a good example is an article by Peres [1]. An even more interesting development has led to the so-called *free will theorems* of Conway and Kochen [2, 3], showing that due to some subtle properties of quantum mechanics elementary particles (in particular, spin-1 particles) possess a sort of *"free will,"* in the sense that their response to an appropriate set of experiments is not a function of earlier properties of the universe. Equally interesting is the recent progress in neurological studies, involving in particular the famous experiment by Libet et al. [4] (showing that volition, as measured neurologically, takes place *earlier* than conscious determination of volition), as well as more recent developments refining it [5].

In the present paper the above difficult (although interesting) issues are not addressed, but a much simpler question is considered: how is a choice influenced by a number of afterthoughts (whatever the status of free will may be)? Although, as said above, naive attempts to understand free will via quantum-mechanical indeterminacy generally fail, it may be worthwhile, nevertheless, to explore (as it appears to be possible) a model actually possessing some of the desired properties. Human (and also animal) free will is actually a *freedom of choice*: from some generic situation which may evolve in two (or more, but two is sufficient for the present purposes) different directions, one is chosen, and, indeed, quantum chemistry affords many circumstances of this kind.

Let us consider, for example, the collision of a proton H^+ with a negative hydrogen ion H^-. For most distances the process takes place adiabatically, and nothing happens; at some specific distance, however, the curve giving the energy of the atomic state H-H crosses (the states considered are asymptotic, corresponding to well-defined charge distributions and atomic states of the colliding partners; the curves giving the true energy eigenvalues never cross) that of the ionic state H^--H^+. At this distance the nonadiabatic process yielding the mutual neutralization reaction

$$H^- + H^+ \longrightarrow 2H \qquad (1)$$

becomes possible. There is, so to speak, a choice between the ionic and the atomic state, with probabilities depending on the matrix elements of the interaction Hamiltonian and on the relative velocity of the colliding ions (the theory of such processes goes back to the 1930s, with the contributions of famous physicists such as Landau, Zener, and Stückelberg [6–8]; the problem of hydrogen mutual neutralization has recently been treated by much more advanced and exact methods, e.g., by Fussen and Kubach and by Stenrup et al. [9, 10], but only the Landau-Zener approach and its generalizations are relevant here; it was shown, on the other hand, that the results of the Landau-Zener approach are surprisingly good, when compared with present-time methods [9]). Such behaviour exhibits an impressive resemblance with the true choices performed by living beings.

Of course the latter are very different in the sense that they are not decided only by chance, but by the body structure and psyche of the performer and by the state in which the performer is (more complex factors determining actions, such as aims and plans, knowledge of the environment and of other individuals, etc., should also be considered, especially in the human case). But this is not strange from the physicochemical point of view: it is enough to suppose (as is clearly realistic) that the chemical reactions envisaged take place in the presence of a complex physical environment (we might in principle describe this by adding to the interaction Hamiltonian between atoms a further interaction Hamiltonian (as complex as is necessary) with the remainder of the system).

We propose thus here a model of free will, treated as a freedom of choice, appealing to the similarity with simple, two-body chemical reactions, where the energy curve corresponding to the initial state (i.e., to the reagent elements or compounds) crosses, at a well-defined distance, the energy curve corresponding to the final state (i.e., to the products). The system at this crossing has indeed a "freedom of choice" to perform, or not to perform, the reaction. There is here, moreover, a direct application to the problem of afterthoughts, since the molecules meet a decision point (a crossing) when they approach, but then they meet it again when they separate, where they necessarily have what may be termed an afterthought.

The crossing problem for the hydrogen neutralization reaction, as studied by Zener [7], is shown in Figure 1 (see also Appendix A).

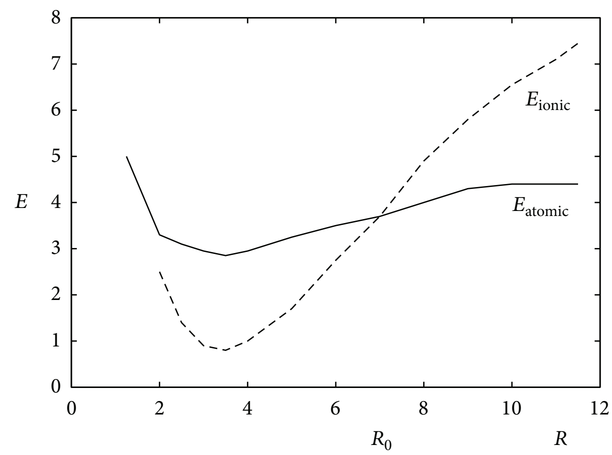

Figure 1: The crossing of energy curves for mutual neutralization of hydrogen, from Zener [7].

2. Free Will with Afterthoughts

A central result of Landau and Zener, giving the probability that the reaction takes place, is given by the little formula

$$P = 2s(1-s) = 2z(1-z), \tag{2}$$

where s is the probability that *no* transition occurs at the crossing point and $z = 1 - s$, correspondingly, is the probability that a transition does occur there (s can be computed in terms of matrix elements of the Hamiltonian (see Appendix A)). This is because the crossing must be met twice, first when the atoms approach and then when they part. We propose here the following interpretation: z is the probability to perform a change at each individual time of choice; P_n is the probability to perform it when thinking about it n times (or after a first thought and $n-1$ afterthoughts).

The Landau-Zener formula, quite generally, holds whenever a decision point must be met twice (actually, in the chemical case, z depends on the *impact parameter* of the collision, and P is obtained as an integral over it; but this is irrelevant in the present case). Calling $P = P_2$ the probability that a decision for a change is taken by someone thinking twice about the choice, the Landau-Zener formula (if the above interpretation is accepted) affords immediately interesting consequences: in particular, P_2 can never be larger than 1/2. In other words, for anybody thinking *twice* about a possible change, the probability of performing it can never be larger than the probability of leaving things as they stand. Thinking twice is thus a clearly conservative attitude.

Generalizing, let us call P_n the probability that a decision for a change is taken when one thinks n times about the choice. Then

$$P_n = (1-z)\,P_{n-1} + z\left(1 - P_{n-1}\right), \tag{3}$$

with the solution

$$P_n = \frac{1}{2}\left[1 - (1-2z)^n\right], \tag{4}$$

where $P_n = 1/2$ for $z = 1/2$; if n is even this is the maximum; if n is odd, on the contrary, it is only an inflection point and P_n tends to 1 when $z \to 1$. For $z < 1/2$, P_n increases with n, tending to 1/2 from below: that is, increasing the number of afterthoughts, the probability of performing a change increases, remaining, however, always less than the probability not to perform it; see Table 1 (according to Appendix A, the parameter u is taken to be $u = [-\ln(1 - z)]^{1/2}$).

Is it reasonable to assume that the probability to change status is always the same, z, at no matter which crossing? For z rather small (say, $z < 1/2$) it appears to be reasonable, under the condition that the subject is basically *indifferent* to the choice. The case where there is a bias, that is, a *preference* for one of the alternatives, is considered below (see (17)–(19)).

Let us consider now the case of $z > 1/2$ ($u > 0.83255$), where, so to speak, one changes one's mind every time. Also in

Table 1: Probability to make a change thinking about it n times, if the probability of making it the first time is 10% ($z = 0.1$, $u = 0.3246$).

n	P_n
1	0.1000
2	0.1800
3	0.2440
4	0.2952
5	0.3362
6	0.3689
7	0.3951
8	0.4161
9	0.4329
10	0.4463

Table 2: Probability to make a change thinking about it n times, if the probability of making it the first time is 90% ($z = 0.9$, $u = 1.5174$).

n	P_n
1	0.9000
2	0.1800
3	0.7560
4	0.2952
5	0.6638
6	0.3689
7	0.6049
8	0.4161
9	0.5671
10	0.4463

this case P_n tends to $1/2$ as n increases (the decisions in favour of a change or against it tend to become equiprobable), but now with an oscillating behaviour (see Table 2). If someone thinks over it an odd number of times the change is slightly more probable, if an even number of times the refusal of change. This simple "chemical" model of free will, however, in this case perhaps should be taken less seriously: not because chemistry fails, but because supposing that at each afterthought z remains exactly the same, always somewhat artificial, is more critical in this case.

Now let us ask: if the afterthoughts have a particular probability distribution p_n, which is the probability, in the end, to make a yes decision? In general the answer will be

$$\langle P\rangle = \sum_n p_n P_n. \tag{5}$$

It is immediate to calculate $\langle P\rangle$ if there is perfect randomness, that is, if p_n is the Poisson distribution

$$p_n = e^{-N}\frac{N^n}{n!}, \tag{6}$$

where N is the mean number of afterthoughts ($N = \nu T$ if T is the total time we have at our disposal and ν is the frequency of afterthoughts). In this case $\langle P\rangle$ is given by

$$\langle P\rangle = \frac{1}{2}\left(1 - e^{-2Nz}\right). \tag{7}$$

Psychophysicists, however, assert on the basis of a statistical theory (but have also demonstrated experimentally) that the distribution of times τ by which two competitive percepts alternate (typically in the interpretation of an ambiguous figure) is the gamma distribution [11–13]

$$p(\tau) = \frac{b^\sigma e^{-b\tau}\tau^{\sigma-1}}{\Gamma(\sigma)}, \tag{8}$$

where for $\sigma = 1$ the gamma distribution reduces to an exponential, corresponding to the Poisson distribution.

Let us note that for $\sigma > 1$ the gamma distribution has a smaller variance than the Poisson distribution and tends to certainty as $\sigma \to \infty$. In the latter case $\langle P\rangle$ is directly given by (4):

$$\langle P\rangle = \frac{1}{2}\left[1 - (1-2z)^N\right]. \tag{9}$$

Equation (9) may be compared to (7); see Appendix B. As N increases it appears that in all cases $\langle P\rangle$ tends to 0.5: thinking about a choice between A and B too many times, the probabilities to choose A or B become the same. But, if z is small, the probabilities given by (7) and (9) are similar; on the contrary, if z is large, (7) continues increasing with N, whereas (9) tends to 0.5 oscillating.

Let now the gamma distribution be considered in detail. Going back to (5), the problem is to determine p_n, the probability of making n attempts, if the distribution of times between an attempt and the next is $p(\tau)$. Let us take for the latter the gamma distribution. This corresponds [11] to the characteristic function

$$\frac{b^\sigma}{(b-ix)^\sigma}. \tag{10}$$

The characteristic function for n events obtains simply by taking the nth power; thus

$$\frac{b^{\sigma n}}{(b-ix)^{\sigma n}} \tag{11}$$

and the probability density $p_n(\tau)$ will again be a gamma distribution with σn in place of σ.

The probability p_n that exactly n events occur during the time T will be the difference between the integral from 0 to T of $p_n(\tau)$ and that of $p_{n+1}(\tau)$. For gamma distributions

$$p_n = \frac{1}{\Gamma(\sigma n)}\int_0^{bT} e^{-u}u^{\sigma n-1}du - \frac{1}{\Gamma[\sigma(n+1)]}\int_0^{bT} e^{-u}u^{\sigma(n+1)-1}du. \tag{12}$$

If σ is 1, the result

$$p_n = \frac{e^{-bT}b^nT^n}{n!}, \tag{13}$$

that is, the Poisson distribution, is immediately obtained. But for any value of σ (provided it is an integer) p_n can be simply obtained:

$$p_n = e^{-bT}\sum_{j=\sigma n}^{\sigma n+\sigma-1}\frac{(bT)^j}{j!}. \tag{14}$$

In particular, for $\sigma = 2$,

$$p_n = e^{-bT}\left[\frac{(bT)^{2n}}{(2n)!} + \frac{(bT)^{2n+1}}{(2n+1)!}\right]. \quad (15)$$

For $\sigma = 2$ the result for $\langle P\rangle$ (corresponding to (7) for $\sigma = 1$) is, if $z < 1/2$ and letting $Z = \sqrt{1-2z}$,

$$\langle P\rangle = \frac{1}{2} - \frac{1}{4Z}e^{-m}\left[(1+Z)\,e^{mZ} - (1-Z)\,e^{-mZ}\right]. \quad (16)$$

The latter expression can also be written using hyperbolic sine and cosine; if $z > 1/2$ instead, a similar expression holds with trigonometric sine and cosine. If, as above, N is interpreted as the *mean number of afterthoughts*, then the parameter m equals a good approximation $2N + 1/2$ (more exactly, $N = (1/2)m - (1/4)(1 - e^{-2m})$). See Appendix B for numerical values.

The present theory can be immediately extended to the case where a bias or preference (in favour of or against the change to a new state) exists. Indeed, if there is such a bias, two different probabilities of change, z_+ (forward) and z_- (backward), replace the single z considered up to here. Then (3) can be effectively replaced by

$$P_n = (1 - z_-)\,P_{n-1} + z_+\left(1 - P_{n-1}\right), \quad (17)$$

with the solution

$$P_n = \frac{z_+}{z_+ + z_-}\left[1 - \left(1 - z_+ - z_-\right)^n\right]. \quad (18)$$

But (18), letting $z = (z_+ + z_-)/2$, can be rewritten,

$$P_n = \frac{z_+}{2z}\left[1 - (1-2z)^n\right], \quad (19)$$

identical to (4), apart from a trivial coefficient z_+/z. Thus all results extend to the biased case, provided the latter coefficient is taken into account. Notice that, if there is a bias or preference, the upper bound to P_2 is no longer 1/2, but rather $z_+/(z_+ + z_-)$ (which is larger than 1/2 if $z_+ > z_-$).

3. Partially Free Will

Baumeister [14] has proposed the idea that the will is, in fact, only partially free and that (in agreement with Kant [15]) people have a capacity for free action but only use it sometimes (in the *Critique of Practical Reason*, Kant discusses at length the distinction between the instances of free choice, corresponding to the self-imposed moral law, and the instances of choice conditioned by a material purpose (which may simply be the search of happiness)). According to Baumeister, free will is expensive, and using it consumes a reserve and makes subsequent items of free will more difficult (the reserve, very materialistically, can be replenished by furnishing glucose to the brain, as an experiment performed by his group shows [16]). Let us see how such ideas can affect the probabilities of afterthoughts.

Assume for the nth afterthought a *freedom fraction* $\alpha_n < 1$. In the simplest situation, α_n will be a constant independent of n. In a more complex situation, α_n will decrease with increasing n (perhaps for increasing lack of glucose): we will assume $\alpha_n = \alpha^n$, with $\alpha < 1$ as an appropriate parameter. In the present section, for simplicity, we will not consider a bias, although this would not be difficult.

If α_n is the freedom fraction, the remainder $1 - \alpha_n$ will be conditioned (by surroundings, by orders, by advice, by fatigue, by immediate pleasure or pain or desire, etc.) and it will *discourage* the change (the probabilities in this case will be indicated as $P_{d,n}$) or *encourage* it (with probabilities $P_{e,n}$). Then (3) will be generalized to

$$\begin{aligned} P_{d,n} &= \alpha_n\left[(1-z)\,P_{d,n-1} + z\left(1 - P_{d,n-1}\right)\right], \\ P_{e,n} &= \alpha_n\left[(1-z)\,P_{e,n-1} + z\left(1 - P_{e,n-1}\right)\right] + 1 - \alpha_n. \end{aligned} \quad (20)$$

If α_n is a constant α, these equations can be immediately solved (the solutions are obvious generalizations of (4)):

$$\begin{aligned} P_{d,n} &= P_d\left[1 - \alpha^n(1-2z)^n\right], \\ P_{e,n} &= P_e\left[1 - \alpha^n(1-2z)^n\right], \end{aligned} \quad (21)$$

where

$$P_d = \frac{\alpha z}{1 - \alpha + 2\alpha z}, \qquad P_e = \frac{1 - \alpha + \alpha z}{1 - \alpha + 2\alpha z}. \quad (22)$$

As an example, we give here the discouraged ($P_{d,n}$) and encouraged ($P_{e,n}$) probabilities at the nth afterthought for the case $\alpha = 9/10$ (the will is nearly free), $z = 1/4$ (for comparison we also give P_n for totally free will, from (4)). It is clear that $P_{d,n} < P_n < P_{e,n}$; moreover, $P_d + P_e = 1$.

For the situation where $\alpha_n = \alpha^n$ decreases with increasing n, we give a numerical table, with the same parameters. Here

$$\begin{aligned} P_{d,n} &= z\sum_{i=0}^{n-1}\alpha^{(i+1)(2n-i)/2}(1-2z)^i, \\ P_{e,n} &= 1 - P_{d,n} - \alpha^{n(n+1)/2}(1-2z)^n. \end{aligned} \quad (23)$$

The situation is very different, as expected, and ultimately the conditioned contribution largely prevails (see Table 4).

The present results are represented in Figure 2.

4. Conclusions

The present paper shows how the phenomena of afterthoughts can simply be described mathematically, provided that a given probability z to decide for a change (or to come back from it) is taken to hold at each afterthought. The probabilities of decision are computed in particular in three cases:

(1) assuming perfectly free will and indifference (4);

(2) assuming perfectly free will, but a preference for or against the change considered (see (19));

(3) assuming partial conditioning, such that the change is encouraged or discouraged (Tables 3 and 4).

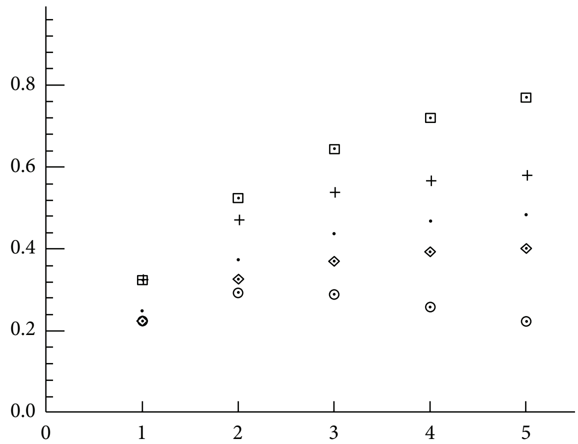

FIGURE 2: Probabilities of change for afterthoughts up to $n = 5$, for $z = 1/4$. Dots: $\alpha = 1$ (perfect free will). In the other cases $\alpha = 0.9$. Diamonds: $P_{d,n}$, $\alpha_n = \alpha$. Crosses: $P_{e,n}$, $\alpha_n = \alpha$. Circles: $P_{d.n}$, $\alpha_n = \alpha^n$. Squares: $P_{e,n}$, $\alpha_n = \alpha^n$.

TABLE 3: $\alpha_n = \alpha = 0.9$, $z = 0.25$.

n	$P_{d,n}$	$P_{e,n}$	P_n
1	0.2250	0.3250	0.2500
2	0.3262	0.4712	0.3750
3	0.3718	0.5371	0.4375
4	0.3923	0.5667	0.4687
5	0.4015	0.5800	0.4844
6	0.4057	0.5860	0.4922
7	0.4076	0.5887	0.4961
8	0.4084	0.5899	0.4980
9	0.4088	0.5905	0.4990
10	0.4090	0.5907	0.4995

TABLE 4: $\alpha_n = \alpha^n$, $\alpha = 0.9$, and $z = 0.25$.

n	$P_{d,n}$	$P_{e,n}$
1	0.2250	0.3250
2	0.2936	0.5241
3	0.2893	0.6443
4	0.2589	0.7193
5	0.2241	0.7695
6	0.1924	0.8059
7	0.1656	0.8340
8	0.1433	0.8567
9	0.1246	0.8754
10	0.1089	0.8911

I believe that these cases cover many interesting situations. The possible criticism, according to which the probability of change may vary at subsequent afterthoughts, is in fact partly included: for example, Table 4 describes the situation where the will becomes less and less free at subsequent afterthoughts, because of some sort of tiredness (freedom is expensive).

TABLE 5

u	z
0	0
0.5	0.2212
1	0.6321
1.5	0.8946
2	0.9817

Experiments to test the present formulae are somewhat difficult to devise (although some ideas to that end exist) and unfortunately have not been done yet (of course they would be very welcome). For example, to verify the treatment of the biased case, (19) depends on two parameters; hence all P_n's (in particular P_3) should be given in terms of P_1 and P_2. Letting $P_2 = P_1(1 + \omega)$, according to the present theory P_3 should be given by $P_3 = P_1(1 + \omega + \omega^2)$, and this in principle could be verified experimentally. Equally welcome would be a wider description of free will with afterthoughts, covering cases only marginally discussed here.

Appendices

A. Zener's Results and Their Generalization

Let us consider $s = 1 - z$ explicitly, following, for example, Geltman [17], who gives a detailed account of Zener's theory [7]. s has approximately a Gaussian form and, neglecting some nonorthogonality problems, is given (see formula (27.23) of Geltman's book) by

$$s = e^{-u^2}, \tag{A.1}$$

where u is defined as

$$u = \beta\left(\frac{2\pi}{\alpha v}\right)^{1/2}, \tag{A.2}$$

where, with reference to the matrix elements of the Hamiltonian operator H and using subscript i for the ionic state and a for the atomic state,

$$H_{aa} - H_{ii} = \hbar\left(R - R_0\right)\alpha, \qquad H_{ai} = \hbar\beta, \tag{A.3}$$

where R is the distance, R_0 in particular the crossing point distance, and v the relative velocity at crossing.

$z = 1 - s = 1 - e^{-u^2}$ trivially increases with u (see Table 5).

Formula (A.2) at first sight may seem strange: v is in the denominator, so that the transition happens more easily when the motion is slow (u and z decrease with increasing v). One would expect, on the contrary, that the process would be more strongly nonadiabatic when the collision is fast. The point is that Zener's treatment, although nonadiabatic, remains close to adiabatic conditions and the transition occurs more easily when more time is spent near the crossing point.

For really fast processes (A.2) will no longer hold and for large velocities (short duration times δt) z will increase again. Let us give an (altogether nonrigorous) plausibility argument for this, trying to generalize Zener's ideas. In fact, a short time

Table 6

N	$\langle P\rangle(N,1)$	$\langle P\rangle(N,2)$	$\langle P\rangle(N,\infty)$
		For $z = 0.1$	
1	0.0906	0.0936	0.1000
2	0.1648	0.1707	0.1800
3	0.2256	0.2334	0.2440
4	0.2753	0.2842	0.2952
5	0.3161	0.3252	0.3362
6	0.3494	0.3585	0.3689
7	0.3767	0.3854	0.3951
8	0.3991	0.4072	0.4161
9	0.4174	0.4249	0.4329
10	0.4323	0.4392	0.4463
		For $z = 0.2$	
1	0.1648	0.1748	0.2000
2	0.2753	0.2923	0.3200
3	0.3494	0.3677	0.3920
4	0.3991	0.4157	0.4352
5	0.4323	0.4463	0.4611
6	0.4546	0.4658	0.4767
7	0.4696	0.4782	0.4860
8	0.4796	0.4861	0.4916
9	0.4863	0.4911	0.4950
10	0.4908	0.4944	0.4970
		For $z = 0.3$	
1	0.2256	0.2450	0.3000
2	0.3494	0.3767	0.4200
3	0.4174	0.4408	0.4680
4	0.4546	0.4716	0.4872
5	0.4751	0.4864	0.4949
6	0.4863	0.4935	0.4980
7	0.4925	0.4969	0.4992
8	0.4959	0.4985	0.4997
9	0.4977	0.4993	0.4999
10	0.4988	0.4997	0.4999
		For $z = 0.4$	
1	0.2753	0.3052	0.4000
2	0.3991	0.4332	0.4800
3	0.4546	0.4778	0.4960
4	0.4796	0.4926	0.4992
5	0.4908	0.4976	0.4998
6	0.4959	0.4992	0.5000
7	0.4982	0.4997	0.5000
8	0.4992	0.4999	0.5000
9	0.4996	0.5000	0.5000
10	0.4998	0.5000	0.5000

Table 6: Continued.

N	$\langle P\rangle(N,1)$	$\langle P\rangle(N,2)$	$\langle P\rangle(N,\infty)$
	For $z = 0.5$		
1	0.3161	0.3564	0.5000
2	0.4323	0.4695	0.5000
3	0.4751	0.4944	0.5000
4	0.4908	0.4990	0.5000
5	0.4966	0.4998	0.5000
6	0.4988	0.5000	0.5000
7	0.4995	0.5000	0.5000
8	0.4998	0.5000	0.5000
9	0.4999	0.5000	0.5000
10	0.5000	0.5000	0.5000
	For $z = 0.6$		
1	0.3494	0.3995	0.6000
2	0.4546	0.4911	0.4800
3	0.4863	0.5003	0.5040
4	0.4959	0.5002	0.4992
5	0.4988	0.5000	0.5002
6	0.4996	0.5000	0.5000
7	0.4999	0.5000	0.5000
8	0.5000	0.5000	0.5000
9	0.5000	0.5000	0.5000
10	0.5000	0.5000	0.5000
	For $z = 0.7$		
1	0.3767	0.4355	0.7000
2	0.4696	0.5028	0.4200
3	0.4925	0.5014	0.5320
4	0.4982	0.5001	0.4872
5	0.4995	0.5000	0.5051
6	0.4999	0.5000	0.4980
7	0.5000	0.5000	0.5008
8	0.5000	0.5000	0.4997
9	0.5000	0.5000	0.5001
10	0.5000	0.5000	0.4999
	For $z = 0.8$		
1	0.3991	0.4652	0.8000
2	0.4796	0.5076	0.3200
3	0.4959	0.5007	0.6080
4	0.4992	0.4999	0.4352
5	0.4998	0.5000	0.5389
6	0.5000	0.5000	0.4767
7	0.5000	0.5000	0.5140
8	0.5000	0.5000	0.4916
9	0.5000	0.5000	0.5050
10	0.5000	0.5000	0.4970

Table 6: Continued.

N	$\langle P\rangle(N,1)$	$\langle P\rangle(N,2)$	$\langle P\rangle(N,\infty)$
		For $z = 0.9$	
1	0.4174	0.4892	0.9000
2	0.4863	0.5083	0.1800
3	0.4977	0.4997	0.7560
4	0.4996	0.4999	0.2952
5	0.4999	0.5000	0.6638
6	0.5000	0.5000	0.3689
7	0.5000	0.5000	0.6049
8	0.5000	0.5000	0.4161
9	0.5000	0.5000	0.5671
10	0.5000	0.5000	0.4463

δt involves an indeterminacy $\hbar/\delta t$. This amounts to a possible transition if it is larger than, say, $|H_{aa}-H_{ii}| = \hbar(R-R_0)\alpha$, that is, taking for $R-R_0$ a typical length L (which will presumably be of the order of the range of the interaction potential), if $\hbar/\delta t \approx \hbar v/L > \hbar L\alpha$ and hence if

$$v > \alpha L^2. \quad \text{(A.4)}$$

Conversely, no transition of this kind will occur in the opposite case, which may be represented by a factor in s of the form $e^{-v/(\alpha L^2)}$, that is, by a generalization of u:

$$u^2 = u_1^2 + u_2^2, \qquad u_1^2 = \frac{2\pi\beta^2}{\alpha v}, \qquad u_2^2 = \frac{v}{\alpha L^2}, \quad \text{(A.5)}$$

where of course u_1 coincides with (A.2) but the additional term involving u_2 yields a strong contribution to the transition probability for fast collisions; (A.1) still applies but s has a maximum and z a minimum for a velocity $v = \sqrt{2\pi}L\beta$: for higher velocities u and z increase again. The point is that for really fast collisions β becomes unimportant, the transition being determined no longer by the nondiagonal element H_{ai} of the Hamiltonian, but solely by the indeterminacy $\hbar/\delta t$, hence by the diagonal elements H_{ii} and H_{aa} crossing at R_0.

B. Mean Probability $\langle P\rangle(N,\sigma)$ for Different Values of z

Table 6 gives $\langle P\rangle$, the second column for $\sigma = 1$ (Poisson), the third column for the particular case of the gamma distribution with $\sigma = 2$, and the fourth column for $\sigma \to \infty$ (i.e., (9); for $z = 0.1$ this column coincides with Table 1, for $z = 0.9$ with Table 2).

$\langle P\rangle$ is the probability of a yes decision in time T if the frequency of afterthoughts is v and $N = vT$. It is seen that, at least for $z < 1/2$, the values of $\langle P\rangle$ for $\sigma = 2$ are intermediate, as expected, between Poisson statistics (13) and certainty, represented by (9).

C. Stückelberg Oscillations

It is interesting to explore a genuine quantum effect: the *Stückelberg oscillations* [6, 8]. If the Landau-Zener model is really taken seriously, the *phase* that each crossing (in our case, each afterthought) produces must be taken into account; such phases accumulate over the successive afterthoughts. These effects are under active study at present (of course in physics, not in psychology) because they are the basis of a new and interesting interferometry, amply discussed in a review article by Shevchenko et al. [18]. The resulting probability distributions are very different from (4), and moreover they differ completely according to whether the interference is constructive or destructive.

Table 7: $z = 0.1$.

n	$\widetilde{P}_n$	P_n	10%
1	0.0967	0.1000	0.0997
2	0.3494	0.1800	0.1969
3	0.6604	0.2440	0.2856
4	0.9093	0.2952	0.3566
5	0.9999	0.3362	0.4026
6	0.8971	0.3689	0.4217
7	0.6407	0.3951	0.4197
8	0.3298	0.4161	0.4075
9	0.0848	0.4329	0.3981
10	0.0004	0.4463	0.4017

I will consider the particularly interesting case of constructive interference, described, in a good approximation, by the article of Shevchenko et al. [18, equation (43)]. Indicating such probabilities of constructive interference by $\widetilde{P}_n$ (Shevchenko et al. write P_{1Z} in place of z and $P_+(t)$ in place of $\widetilde{P}_n$),

$$\widetilde{P}_n = \sin^2\left(n\sqrt{z}\right). \quad \text{(C.1)}$$

In Table 7 I show, for $z = 0.1$, the probabilities given by (C.1), to be compared with those (very different and, for n small but larger than 1, much less) given by (4).

Obviously, while P_n for $z < 1/2$ gradually increases and tends to 1/2 (and for $z > 1/2$ oscillates), $\widetilde{P}_n$ always oscillates, with a period $\pi/\sqrt{z}$ definitely longer than the period (=2) of the "classical" oscillation that P_n for $z > 1/2$ shows.

It is not easy to say whether these *Stückelberg oscillations* have a bearing on the problem of free will. One could imagine that some internal, spontaneous oscillation of the soul does exist, and that, if the afterthoughts happen to be in phase with such oscillation, something like a constructive interference may occur. The latter can never be so strong as the $\widetilde{P}_n$ indicates, but a partial constructive interference contribution could be there. In the last column of Table 7 I give the result that would be obtained if the amount of such contribution was 10%. Comparing this column with the column giving P_n, the main effect appears to be that, for a few afterthoughts, the probabilities approach 1/2 faster than in the absence of interference (but for a larger number of afterthoughts the opposite may happen).

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