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PARAMETRIC AMPLIFICATION/MIXING USING THE VARACTOR DIODE

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This paper deals with the large-signal analysis of parametric amplification/mixing using reverse biased p-n junction varactor diodes. Expressions are obtained for the current components of the parametric amplification/mixing. The special case of relatively small input amplitudes is considered and the results are compared with previously published results.

INTRODUCTION

Parametric amplifiers/mixers are built around devices whose reactances are varied in such a way that amplification/mixing results. Of particular interest here is the variable capacitor diode, or varactor, which is widely used for parametric amplification/mixing. Amplification/mixing, in this case, is obtained by varying the capacitive reactance of the varactor electronically at a frequency higher than the frequency of the signal being amplified.

It is well known that the dynamic junction capacitance of a reverse-biased p-n junction can be expressed by [1]

$$C = C_b(V_o + v_R)^{-1/2} \tag{1}$$

where V_o is the magnitude of the contact potential barrier, v_R is the reverse-bias voltage, $C_b = S\sqrt{qN_d\,\varepsilon/2}$, S is the cross-sectional area of the diode, q is the electronic charge, N_d is the donor density in the n-type semiconductor, and ε is the permitivity of the depletion layer in the junction of semiconductor. If the reverse bias voltage v_R is formed of the combination of a dc bias-voltage modulated by a local oscillator, or a pump oscillator, this voltage can be represented as:

$$v_R = V_{Ro} + V_p \sin(\omega_p t + \phi_p) \tag{2}$$

where V_{Ro} is the dc reverse-bias voltage and V_p is the amplitude of a sinusoidal voltage with frequency ω_p and phase angle ϕ_p, is applied across the capacitance, then combining (1) and (2) we get

$$C = C_o\left(1 + \frac{V_p}{V_o + V_{Ro}} \sin(\omega_p t + \phi_p)\right)^{-1/2} \tag{3}$$

where $C_o = C_b/(V_o + V_{Ro})^{1/2}$ is the junction capacitance without sinusoidal input voltage. From (3), it is essential to keep $\frac{V_p}{V_o + V_{Ro}} \leq 1$so that the capacitance C will be real [1].

The microwave voltage to be amplified/mixed is applied across the capacitance and can be expressed by

$$v_s(t) = V_s \sin(\omega_s t + \phi_s) \tag{4}$$

where V_s is the amplitude, ω_s is the frequency, and ϕ_s is the phase angle of the incident microwave voltage.

Now, the capacitor current can be expressed as

$$i_C = \frac{d}{dt}(Cv_s) = v_s \frac{dC}{dt} + C\frac{dv_s}{dt} \tag{5}$$

Combining (1)–(5), the capacitor current can be expressed as

$$i_c = C_o \frac{V_s \omega_s \cos(\omega_s t + \phi_s)}{\left(1 + \frac{V_p}{V_o + V_{Ro}} \sin(\omega_p t + \phi_p)\right)^{1/2}} - \frac{1}{2} \frac{C_o V_s V_p \omega_p \sin(\omega_p t + \phi_p) \cos(\omega_s t + \phi_s)}{(V_o + V_{Ro})\left(1 + \frac{V_p}{V_o + V_{Ro}} \sin(\omega_p t + \phi_p)\right)^{3/2}} \tag{6}$$

Equation (6) can be rewritten in the form

$$y = \frac{\cos \Omega_s}{(1 + x \sin \Omega_p)^{1/2}} - \frac{1}{2} \frac{\omega_p}{\omega_s} \frac{x \sin \Omega_s \cos \Omega_p}{(1 + x \sin \Omega_p)^{3/2}} \tag{7}$$

where $y = \frac{i_c}{\omega_s C_o V_s}$ is the normalized capacitance current, $x = \frac{V_p}{V_o + V_{Ro}}$ is the normalized input voltage, $\Omega_s = \omega_s t + \phi_s$, and $\Omega_p = \omega_p t + \phi_p$. It is obvious that (7) is nonlinear. Therefore, the capacitor current will contain harmonics, sums and differences of the input frequencies. However, (7) in its present form cannot be used for predicting the amplitudes of these current components. Therefore, Ishii [1] specialized his interest in the small signal conditions, under which the nonlinear terms of (7) can be expanded in a Taylor series. By truncating these series after the second terms and assuming that the input voltage amplitudes are relatively small, Ishii [1] obtained expressions for the current components at the signal frequency, ω_s, at the idler(difference) frequency, $(\omega_p - \omega_s)$, and at the sum frequency, $\omega_p + \omega_s$. The analysis of Ishii [1], cannot, therefore, predict the amplitudes of these current components under large signal conditions.

It is the major intention of this paper to present simple approximations for the nonlinear terms of (7). These approximations, which are valid over the full useful range of input voltages, are intended to provide simple analytical expressions for the amplitudes of the components of the capacitor current under large signal conditions. Such analytical expressions are important for evaluating the large signal performance of the varactor diode when used for parametric amplification/mixing.

PROPOSED APPROXIMATIONS

The development of the proposed approximations has proceeded along empirical lines by comparing the nonlinear terms

$$\Theta(z) = (1 + z)^{-1/2} \tag{8}$$

shown in Fig. 1(a) for $|z| \leq 0.9$, and

$$\Psi(z) = (1 + z)^{-3/2} \tag{9}$$

shown in Fig. 1(b) for $|z| \leq 0.9$, with the truncated Fourier-series

$$\Theta(z) \cong \gamma_o + \sum_{k=1}^{K} \left(\gamma_k \cos\left(\frac{2k\pi}{T} z \right) + \eta_k \sin\left(\frac{2k\pi}{T} z \right) \right) \tag{10}$$

and

$$\Psi(z) \cong \delta_o + \sum_{k=1}^{K} \left(\delta_k \cos\left(\frac{2k\pi}{T} z \right) + \zeta_k \sin\left(\frac{2k\pi}{T} z \right) \right) \tag{11}$$

respectively.

The parameters γ_o, γ_k, η_k, δ_o, δ_k, ζ_k, and T are fitting parameters selected to provide the best fit between the nonlinear terms of (8) and (9) and equations (10) and (11), respectively. In general, these parameters can be obtained using standard curve-fitting techniques. Alternatively, by removing the offset, at $z = 0$ in the curves of Fig. 1, and then mirror imaging, the resulting curves can be made periodic as shown in Fig. 2. Now if we choose a number of data points, join them end to end using straight line segments, and denoting the slope of each segment by α_m and β_m, respectively, as shown in Fig. 2(a),(b), it is easy, following the procedure described by Kreyszig [2], to show that the coefficients γ_o, γ_k, η_k, δ_o, δ_k, and ζ_k can be expressed by [3]

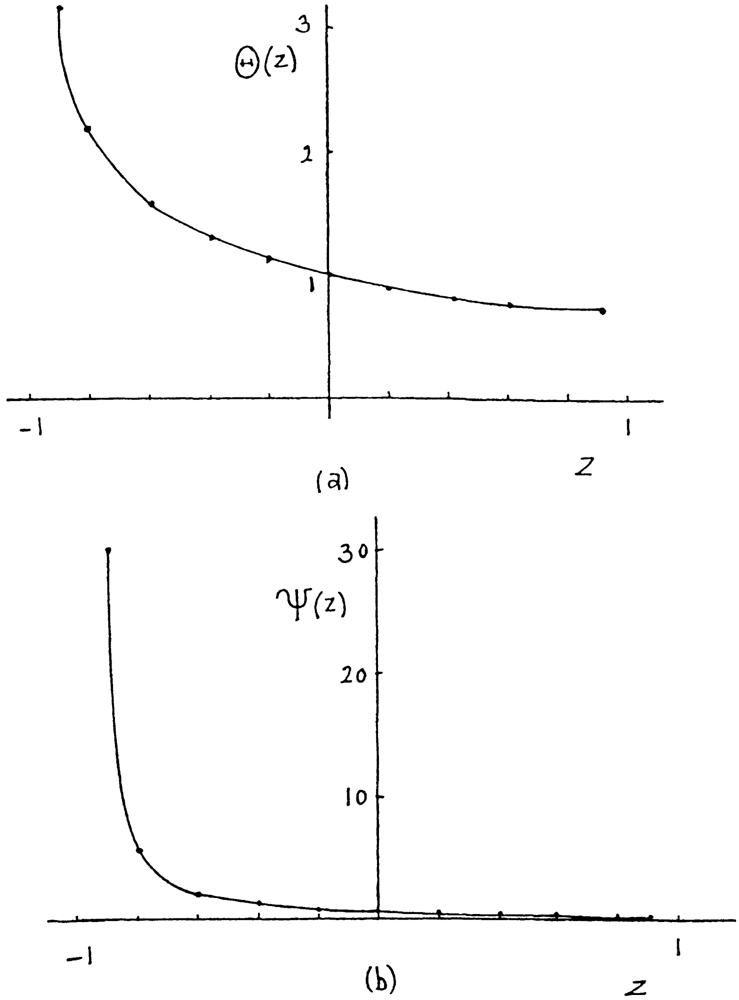

FIGURE 1 The functions $\Theta(z)$ and $\Psi(z)$ of eqns. (8) and (9).

$$\gamma_o = 1 + \frac{1}{T}\left(\frac{1}{2}z_2\Theta_2 + \frac{1}{2}(z_M - z_{M-1})\Theta_{M-1} + \sum_{m=2}^{M-2}\left((z_{m+1} - z_m)\Theta_{m+1} - \frac{1}{2}\right.\right.$$
$$\left.\left.(z_{m+1} - z_m)\left(\Theta_{m+1} - \Theta_m\right)\right)\right) \tag{12}$$

$$\eta_k = \frac{-T}{2(k\pi)^2}\left(\sum_{m=1}^{M-2}(\alpha_{m+1} - \alpha_m)\sin\left(\frac{2k\pi}{T}z_{m+1}\right)\right) \tag{13}$$

$$\gamma_k = \frac{-T}{2(k\pi)^2}\left(\alpha_1 - \alpha_{M-1} + \sum_{m=1}^{M-2}(\alpha_{m+1} - \alpha_m)\cos\left(\frac{2k\pi}{T}z_{m+1}\right)\right) \tag{14}$$

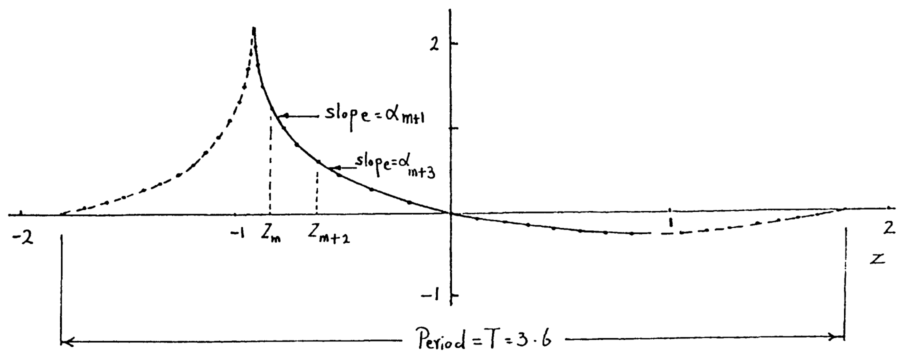

FIGURE 2 (a) The function $\Theta(z)$ of Fig. 1(a) after removing the offset at $Z = 0$, approximation by straight line segments and mirror imaging to form a complete period.

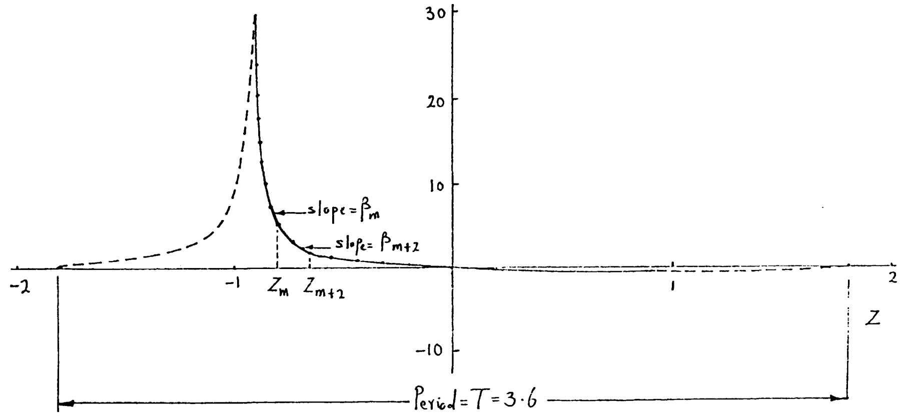

FIGURE 2 (b) The function $\Psi(z)$ of Fig. 1(b) after removing the offset at $Z = 0$, approximating by straight line segments and mirror imaging to form a complete period.

and

$$\delta_o = 1 + \frac{1}{T}\left(\frac{1}{2} z_2 \Psi_2 + \frac{1}{2}(z_M - z_{M-1})\,\Psi_{M-1} + \sum_{m=2}^{M-2}\left((z_{m+1} - z_m)\Psi_{m+1} - \frac{1}{2}(z_{m+1} - z_m)\,(\Psi_{m+1} - \Psi_m)\right)\right) \tag{15}$$

$$\zeta_k = \frac{-T}{2(k\pi)^2}\left(\sum_{m=1}^{M-2}(\beta_{m+1} - \beta_m)\sin\left(\frac{2k\pi}{T} z_{m+1}\right)\right) \tag{16}$$

$$\delta_k = \frac{-T}{2(k\pi)^2}\left(\beta_1 - \beta_{M-1} + \sum_{m=1}^{M-2}(\beta_{m+1} - \beta_m)\cos\left(\frac{2k\pi}{T} z_{m+1}\right)\right) \tag{17}$$

where T is the period of the periodic functions of Fig. 2, and Θ_m and Ψ_m, $m = 2, 3, \ldots, M$ are the values of the functions $\Theta(z)$ and Ψ_m at z_m, $m = 2, 3, \ldots, M$.

From (13), (14), (16), and (17), one can see that calculation of the parameters γ_k, η_k, δ_k, and ζ_k requires only simple mathematical operations. Also, inspection of (13), (14), (16), and (17) suggests that as k becomes infinite, the parameters η_k, γ_k, ζ_k, and δ_k always approach zero. For numerical computation using mainframe or personal computers, there is no reason to avoid increasing the number of terms in (10) and (11) until the inclusion of the next term is seen to make a negligible contribution towards a best fit criterion; for example the minimum relative-root-mean square (RRMS) error. Tables I and II show the first 72 terms for approximating the nonlinear terms of (8) and (9).

Using the parameters of Tables I and II and equations (10) and (11), calculations were made and are shown in Fig. 1 from which it is obvious that the proposed Fourier-series approximations accurately represents the nonlinear terms of (8) and (9).

CURRENT COMPONENTS

One of the potential applications of the approximations of (10) and (11) is in the prediction of the amplitudes of the current components generated in a parametric amplifier/mixer built around a reverse-biased p-n junction. Combining (7), (10), and (11), normalized capacitor current can be expressed as

TABLE I

First 36 terms of, γ_k and η_k, of equation (10) for fitting (8). γ_o = 1.18709, T = 3.6 and RRMS error = 0.00096.

k	1	2	3	4	5	6	7	8
γ_k	0.0	−0.26986	0.0	0.132702	0.0	−0.08156	0.0	0.055283
k	9	10	11	12	13	14	15	16
γ_k	0.0	−0.04022	0.0	0.030071	0.0	−0.02337	0.0	0.018590
k	17	18	19	20	21	22	23	24
γ_k	0.0	−0.01467	0.0	0.011898	0.0	−0.00947	0.0	0.007517
k	25	26	27	298	29	30	31	32
γ_k	0.0	−0.00595	0.0	0.004513	0.0	−0.00326	0.0	0.002073
k	33	34	35	36				
γ_k	0.0	−0.00093	0.0	0.0				

k	1	2	3	4	5	6	7	8
η_k	−0.59367	0.0	0.196152	0.0	−0.10787	0.0	0.069291	0.01
k	9	10	11	12	13	14	15	16
η_k	−0.04857	0.0	0.035856	0.0	−0.02715	0.0	0.021501	0.0
k	17	18	19	20	21	22	23	24
η_k	−0.01696	0.0	0.13576	0.0	−0.01097	0.0	0.008674	0.0
k	25	26	27	28	29	30	31	32
η_k	−0.00694	0.0	0.005397	0.0	−0.00404	0.0	0.002806	0.0
k	33	34	35	36				
η_k	−0.00162	0.0	0.000484	0.0				

TABLE II
First 36 terms of, δ_k and ζ_k, of equation (11) for fitting (9). δ_o = 2.9046, T = 3.6 and RRMS error = 0.002616.

k	1	2	3	4	5	6	7	8
δ_k	0.0	−3.13189	0.0	2.110289	0.0	−1.54190	0.0	1.175436
k	9	10	11	12	13	14	15	16
δ_k	0.0	−0.91910	0.0	0.729496	0.0	−0.58434	0.0	0.470100
k	17	18	19	20	21	22	23	24
δ_k	0.0	−0.37747	0.0	0.300864	0.0	−0.23664	0.0	0.182373
k	25	26	27	28	29	30	31	32
δ_k	0.0	−0.13596	0.0	0.095954	0.0	−0.06168	0.0	0.032974
k	33	34	35	36				
δ_k	0.0	−0.01084	0.0	0.0				
k	1	2	3	4	5	6	7	8
ζ_k	−4.21618	0.0	2.556632	0.0	−1.80006	0.0	1.345299	0.0
k	9	10	11	12	13	14	15	16
δ_k	−1.03958	0.0	0.819487	0.0	−0.65361	0.0	0.525017	0.0
k	17	18	19	20	21	22	23	24
ζ_k	−0.42223	0.0	0.338034	0.0	−0.26786	0.0	0.208808	0.0
k	25	26	27	28	29	30	31	32
ζ_k	−0.15865	0.0	0.115509	0.0	−0.07838	0.0	0.046829	0.0
k	33	34	35	36				
ζ_k	−0.02113	0.0	0.003442	0.0				

$$y = \cos\Omega_s\left(\gamma_o + \sum_{k=1}^{K}\left(\gamma_k\cos\left(\frac{2k\pi}{T}x\sin\Omega_p\right) + \eta_k\sin\left(\frac{2k\pi}{T}x\sin\Omega_p\right)\right)\right)$$
$$-\frac{1}{2}\frac{\omega_p}{\omega_s}x\sin\Omega_s\cos\Omega_p\left(\delta_o + \sum_{k=1}^{K}\left(\delta_k\cos\left(\frac{2k\pi}{T}x\sin\Omega_p\right)\right.\right.$$
$$\left.\left.+\ \zeta_k\sin\left(\frac{2k\pi}{T}x\sin\Omega_p\right)\right)\right) \qquad (18)$$

Now using the trigonometric identities

$$\sin(z\sin\omega t) = 2\sum_{l=0}^{\infty} J_{2l+1}(z)\sin(2l+1)\omega t$$

$$\cos(z\sin\omega t) = J_o(z) + 2\sum_{l=1}^{\infty} J_{2l}(z)\cos 2l\omega t$$

where $J_l(z)$ is the Bessel function of order l, and after simple mathematical manipulations, (18) reduces to

$$y = \left(\gamma_o + \sum_{k=1}^{K} \gamma_k J_o \left(\frac{2k\pi}{T} x \right) \right) \cos \Omega_s + \sum_{k=1}^{K} \gamma_k \sum_{l=1}^{\infty} J_{2l} \left(\frac{2k\pi}{T} x \right) (\cos(2l\Omega_p + \Omega_s)$$
$$+ \cos(2l\Omega_p - \Omega_s))$$
$$+ \sum_{k=1}^{K} \eta_k \sum_{l=0}^{\infty} J_{2l+1} \left(\frac{2k\pi}{T} x \right) (\sin((2l + 1)\Omega_p + \Omega_s) + \sin((2l + 1)\Omega_p - \Omega_s)) - \frac{1}{4} \frac{\omega_p}{\omega_s} x$$
$$\left(\delta_o + \sum_{k=1}^{K} \delta_k J_o \left(\frac{2k\pi}{T} x \right) \right) (\sin(\Omega_p + \Omega_s) - \sin(\Omega_p - \Omega_s))$$
$$- \frac{1}{4} \frac{\omega_p}{\omega_s} x \sum_{k=1}^{K} \delta_k \sum_{l=1}^{\infty} J_{2l} \left(\frac{2k\pi}{T} x \right) (\sin((2l + 1)\Omega_p + \Omega_s) - \sin((2l - 1)\omega_p - \Omega_s)$$
$$+ \sin((2l - 1)\Omega_p + \Omega_s) - \sin((2l + 1)\Omega_p - \Omega_s))$$
$$- \frac{1}{4} \frac{\omega_p}{\omega_s} x \sum_{k=1}^{K} \zeta_k \sum_{l=0}^{\infty} J_{2l+1} \left(\frac{2k\pi}{T} x \right) (\cos(2l\Omega_p - \Omega_s) - \cos((2l + 2)\Omega_p + \Omega_s)$$
$$+ \cos((2l + 2)\Omega_p - \Omega_s) - \cos(2l\Omega_p + \Omega_s)) \tag{19}$$

Using (19), it is easy to show that the normalized-current component of frequency Ω_s will be given by

$$y_1(t) = \left(\gamma_o + \sum_{k=1}^{K} \gamma_k J_o \left(\frac{2k\pi}{T} x \right) \right) \cos\Omega_s \tag{20}$$

the normalized current component of frequency $\Omega_p - \Omega_s$, the idler frequency, will be given by

$$y_{1,-1}(t) = \left(\sum_{k=1}^{K} \eta_k J_1 \left(\frac{2k\pi}{T} x \right) + \frac{1}{4} \frac{\omega_p}{\omega_s} x \left(\delta_o + \sum_{k=1}^{K} \delta_k \left(J_o \left(\frac{2k\pi}{T} x \right) \right.\right.\right.$$
$$\left.\left.\left. + J_2 \left(\frac{2k\pi}{T} x \right) \right) \right) \right) \sin(\Omega_p - \Omega_s) \tag{21}$$

and the normalized current component of frequency $\Omega_p + \Omega_s$ will be given by

$$y_{1,+1}(t) = \left(\sum_{k=1}^{K} \eta_k J_1 \left(\frac{2k\pi}{T} x \right) - \frac{1}{4} \frac{\omega_p}{\omega_s} x \left(\delta_o + \sum_{k=1}^{K} \delta_k \left(J_o \left(\frac{2k\pi}{T} x \right) \right.\right.\right.$$
$$\left.\left.\left. + J_2 \left(\frac{2k\pi}{T} x \right) \right) \right) \right) \cos(\Omega_p + \Omega_s) \tag{22}$$

Using (20)–(22), the amplitudes of the normalized current components of frequencies ω_s, $\omega_p - \omega_s$ and $\omega_p + \omega_s$ can be calculated in terms of the ordinary Bessel functions available in most mainframe computers. However, for users of programmable pocket calculators, the approximations of [4]–[6] may be useful. Moreover, for sufficiently small values of x, the Bessel functions can be approximated by

$$J_l(z) \cong (z/2)^l/l!$$

and (20)–(22) reduce to

$$y_1(t) = \left(\gamma_o + \sum_{k=1}^{K} \gamma_k \right) cos\Omega_s \tag{23}$$

$$y_{1,-1}(t) = x\left(\frac{\pi}{T}\sum_{k=1}^{K} k\eta_k + \frac{1}{4}\frac{\omega_p}{\omega_s}\left(\delta_o + \sum_{k=1}^{K} \delta_k \left(1 + \frac{k^2}{2!}\left(\frac{\pi}{T}x\right)^2 \right)\right)\right)$$
$$\sin(\Omega_p - \Omega_s) \tag{24}$$

and

$$y_{1,+1}(t) = x\left(\frac{\pi}{T}\sum_{k=1}^{K} k\eta_k - \frac{1}{4}\frac{\omega_p}{\omega_s}\left(\delta_o + \sum_{k=1}^{K} \delta_k \left(1 + \frac{k^2}{2!}\left(\frac{\pi}{T}x\right)^2 \right)\right)\right)$$
$$\sin(\Omega_p + \Omega_s) \tag{25}$$

Using (23)–(25), and ignoring x^3 terms, the current components of frequency ω_s, $\omega_p - \omega_s$ and $\omega_p + \omega_s$ can be expressed as

$$i_1(t) \cong C_o\omega_s V_s \left(\gamma_o + \sum_{k=1}^{K} \gamma_k \right)\cos(\omega_s t + \phi_s) \tag{26}$$

$$i_{1,-1}(t) = C_o\omega_s V_s \frac{V_p}{V_o + V_{Ro}}\left(\frac{\pi}{T}\sum_{k=1}^{K} k\eta_k + \frac{1}{4}\frac{\omega_p}{\omega_s}(\delta_o + \sum_{k=1}^{K}\delta_k) \right)\sin((\omega_p - \omega_s)$$
$$t + \phi_p - \phi_s) \tag{27}$$

and

$$i_{1,+1}(t) = C_o\omega_s V_s \frac{V_p}{V_o + V_{Ro}}\left(\frac{\pi}{T}\sum_{k=1}^{K} k\eta_k - \frac{1}{4}\frac{\omega_p}{\omega_s}\left(\delta_o + \sum_{k=1}^{K}\delta_k\right) \right)\sin((\omega_p + \omega_s)$$
$$t + \phi_p + \phi_s) \tag{28}$$

In order to compare the results obtained here with previously published results, here we recall eqn. (9.2.9) of Reference [1], from which the current components of frequency ω_s, $\omega_p - \omega_s$ and $\omega_p + \omega_s$ can be expressed as

$$i_1(t) = C_o V_s \omega_s \cos(\omega_s t + \phi_s) \tag{29}$$

$$i_{1,-1}(t) = \frac{1}{4} C_o V_s \frac{V_p}{V_o + V_{Ro}} (\omega_p - \omega_s) \sin((\omega_p - \omega_s)t + \phi_p - \phi_s) \tag{30}$$

and

$$i_{1,+1}(t) = \frac{1}{4} C_o V_s \frac{V_p}{V_o + V_{Ro}} (\omega_p + \omega_s) \sin((\omega_p + \omega_s)t + \phi_p + \phi_s) \tag{31}$$

Using values of T, γ_o, δ_o and γ_k, η_k, δ_k, $k = 1, 2, 3, \ldots, 36$, (26)–(28) can be rewritten as

$$i_1(t) \cong 1.0004 C_o \omega_s V_s \cos(\omega_s t + \phi_s) \tag{32}$$

$$i_{1,-1}(t) \cong \frac{1.003}{4} C_o V_s \frac{V_p}{V_o + V_{Ro}} (\omega_p - \omega_s) \sin((\omega_p - \omega_s)t + \phi_p - \phi_s) \tag{33}$$

and

$$i_{1,+1}(t) \cong \frac{1.003}{4} C_o V_s \frac{V_p}{V_o + V_{Ro}} (\omega_p + \omega_s) \cos((\omega_p + \omega_s)t + \phi_p + \phi_s) \tag{34}$$

Inspection of (29)–(34) shows that the small signal results obtained by Ishii [1] can be obtained, with excellent accuracy, as a special case from the general large-signal analysis presented here.

CONCLUSION

In this paper, approximations using the Fourier-series have been presented for the nonlinear terms of the current-voltage relationship of the parametric mixing/amplification using the varactor diode. The Fourier-series coefficients can be evaluated using simple calculations without recourse to numerical integration. The analytical expressions obtained for the amplitudes of the current components are in terms of the ordinary Bessel functions, with arguments proportional to the amplitude of the input voltages, and can be easily evaluated using programmable hand calculators. The special case of relatively small-amplitude input voltages was considered in detail and the results obtained in this paper are in excellent agreement with previously published results.

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