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Relational Structure of Measurement with Application on Specification of Free-form Surface

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A contradiction is shown in this paper that, for contact surface measurement, if a measured surface profile is exactly coincident with the USL (upper specification limit), the measured result may still be out of specification. To understand and avoid this contradiction, a relational construction of measurement is proposed bases on the representational measurement theory. By observing the connection between measurement and inverse problem, measurement is modeled as a mapping from the preordered set of measurands (objects to be measured) to the partially ordered set of measured values. Thereby, a desired property of the specifications limits is derived, and a correction of the USL of surface profile is proposed.

NOMENCLATURE

M = preordered set of measurands (objects to be measured)
X = specified numerical relational system (NRS)
D = partially ordered set of observed data
ψ = homomorphism, a structure preserving mapping
φ = deterministic measurement process
h = forward mapping of the measurement, $h\psi = \varphi$
g = the inverse or pseudo-inverse of h
D_S = (function processing) dilation filter
E_S = (function processing) erosion filter
C_S = (function processing) closing filter

1. Introduction

In surface metrology, the upper and lower specification limits (USL and LSL) of a free-form surface profile are defined in ISO 1101 (2005 [1]) as two curves enveloping circles of certain diameter t, the centers of which are situated on the nominal surface profile. Let l_o, l_T, l_B be the functions representing the nominal profile, the USL and the LSL respectively in a specified interval I, $l_B(x) \le l_o(x) \le l_T(x)$ for all $x \in I$. Then it can be observed that, by taking the circle of diameter t as a structuring element of morphological operation [2], l_T and l_B are respectively the dilation and erosion of l_o. A partial order $\preceq$ between the functions can be defined as $l_1 \preceq l_2$, if and only if (iff) $l_1(x) \le l_2(x)$ for all $x \in I$. Let l be a function representing the real surface profile of a work piece fabricated according to the nominal profile, then it is within specification if $l_B \preceq l \preceq l_T$.

The canonical method of measuring surface profile is contact measurement by moving a tactile stylus along the surface to be measured to obtain the locus of the centre point of the stylus tip, called the traced profile (see figure 1). Let c be a function representing the traced profile and assume the stylus tip is an ideal circular disk S in the plane of the surface profile. Then c is the dilation of l, and the real surface profile l can be estimated by the erosion of c with S as the structuring element, which is called as real mechanical profile in [3], denoted as l'. Denote the dilation filter as $D_S : l \mapsto c$ and the erosion filter as $E_S : c \mapsto l'$, the combination of D_S followed by E_S is a closing filter [2], denoted as $C_S : l \mapsto l'$. Since C_S is not an identical operation, the estimated profile is not always the same as the real surface profile.

Assume there is a real surface profile l which is exactly the same as the USL l_T, i.e. l is marginally within specification. By the extensive property of a closing filter [4], we have $l \preceq C_S(l)$, thus the estimated profile is always above the real surface profile. Hence we have $l = l_T \preceq l'$, which shows a contradiction that the measurement result of l can be out of specification.

To be able to understand this contradiction, a framework of measurement is proposed base on representational measurement theory in section 2. For solving this contradiction, a desired property and a correction of the specification of free-form surface profile is proposed in section 3.

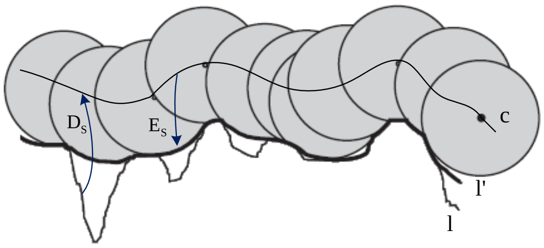

Fig. 1 Measurement of surface profile with tactile stylus

2. Relational Structure of Measurement

2.1 Relational construction

In representational measurement theory a measurement is possible only if there is a structure-preserving mapping ψ (called homomorphism) from the empirical relational system (ERS) to a specified numerical relational system (NRS) [5]. An ERS consists of a set of measurands with certain relational structures (e.g. concatenation structure, conjoint structure) called empirical structure. There are always certain ordered relations between the measurands which are determined by the 'value' of the common attribute(s) to be measured. For different empirical structures, there are many types of ordered relations, like simple order, weak order and partial order, but normally they all belong to a type of very general relation called preorder, which is transitive and reflexive. Both the ERS and the specified NRS are preordered, denoted as M and X respectively.

A practical measurement process can be modeled as a mapping between the ERS to a NRS of observed data, denoted as $\varphi : \mathrm{M} \to \mathrm{D}$, but due to the limitation of measurement resolution and measurement error, normally φ is not a homomorphism. Moreover the output of a measurement process is not necessarily the measured values (of the measurands), since in many cases the measured values cannot be directly observed. For simplicity, we assume no random error is involved in the observed data, which means the measurement process φ is deterministic. To be precise, φ is deterministic iff for any measurands $\mathrm{a}, \mathrm{b} \in \mathrm{M}$, $\mathrm{a} \sim \mathrm{b} \Rightarrow \varphi(\mathrm{a}) = \varphi(\mathrm{b})$, where $\mathrm{a} \sim \mathrm{b}$ means a is equivalent to b.

Take the measurands true values and observed data as preordered sets M, X and D respectively, the following connection between X and D can be derived. For a deterministic measurement process $\varphi : \mathrm{M} \to \mathrm{D}$, let $\psi : \mathrm{M} \to \mathrm{X}$ be a homomorphism between the ERS M and a specified NRS X, there exists a unique mapping $\mathrm{h} : \mathrm{X} \to \mathrm{D}$, such that $\mathrm{h}\psi = \varphi$ (see figure 2, proof omitted). The derived diagram is called the relational construction of measurement.

$$\begin{tikzcd} & & \mathrm{D} \\ \mathrm{M} \arrow[urr, "\varphi"] \arrow[rr, "\psi"'] & & \mathrm{X} \arrow[u, dashed, "\mathrm{h}"'] \end{tikzcd}$$

Fig. 2 Relational construction of measurement

2.2 Inverse problems of measurement

In many cases the measurands are measured indirectly by another related quantity, e.g. electrical resistance of a resistor can be measured by the observed data of electric current under a certain voltage. To estimate the true values of the measurands from the observed data is an inverse problem [6], and its forward mapping is the mapping h in the relational construction.

The inverse or pseudo-inverse of h, denoted as $\mathrm{g} : \mathrm{D} \to \mathrm{X}$ can be used to find the inverse solution. The output of g is thus the measured value. Hence a deterministic measurement can be taken as a mapping $\theta : \mathrm{M} \to \mathrm{X}$, and $\theta = \mathrm{g}\varphi$.

For the measurement of a surface profile mentioned in section 1, the forward mapping is $\mathrm{D_S}$, which is not invertible, and its pseudo-inverse is $\mathrm{E_S}$, in the sense that $\mathrm{D_S E_S D_S} = \mathrm{D_S}$ [4]. The essential reason of the contradiction is that $\mathrm{D_S}$ is not a one-to-one mapping, and thus the inverse solution is not unique.

3. A Correction in Specification of free-form surface

We expect that if the true value of a measurand is in the specification, its measured value is also within specification. Hence the following desired property of specification (P1) should be satisfied. Let a be a specification limit, $a \in X$, then $gh(a) = a$. Moreover, when P1 is satisfied, the measurement resolution can be reflected by the specification limits. Just like from 3.00+/-0.5mm, we can see the measurement resolution is expected to be 0.01mm.

The problem is how to make sure P1 is satisfied. For the measurement of surface profile, since the closing filter is idempotent [4], i.e. $\mathrm{C_S C_S} = \mathrm{C_S}$, and $\mathrm{C_S} = \mathrm{E_S D_S}$, if the USL and LSL are in the range of $\mathrm{C_S}$, we have $\mathrm{E_S D_S(a)} = \mathrm{a}$, $a = l_B$ or l_T, thus P1 is satisfied. Therefore, the closing filter can be used as a correction for the specification limits of surface profiles defined in ISO 1101 (2005). The contradiction can be solved by correcting the USL from l_T to $C_S(l_T)$. For the LSL, it is the erosion of the nominal profile by a disk of diameter t, denoted as $\mathrm{D_T(l_o)}$. By the basic properties of erosion and dilation [4], we have $\mathrm{E_S D_S E_S} = \mathrm{E_S}$, and t is normally bigger than the diameter of the stylus S, so $\mathrm{E_T(l_o)} = \mathrm{E_S D_S E_T(l_o)} = \mathrm{C_S E_T(l_o)}$. That means, $l_B = C_S(l_B)$, the LSL does not need to be corrected.

4. Conclusions

The relational structure derived in this paper is useful for understanding measurement in the perspective of inverse problems. The correction of a contradiction in the specification of free-form surface is demonstrated as an application of the theory. The next stage of research is to model measurement with uncertainty involved as a system of mappings base on the deterministic framework.

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