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A novel design of physical layer network coding in strong asymmetric two-way relay channels

Hao Wei^{1*}, Baoyu Zheng¹ and Xiaodong Ji^{1,2}**Abstract**

The quality of two-way links is always asymmetric in practical two-way relay channels (TWRC) and therefore, the capacity of TWRC will be limited by the weaker link. An asymmetric modulation scheme, with physical layer network coding, was presented in order to improve the transmission reliability in TWRC. This makes full use of the stronger link to improve the overall transmission rate and also ensures the reliability of the weaker link. The simulation results show that the proposed asymmetric modulation scheme in the case of strongly asymmetric channels, compared to the symmetric transmission, enhances the system capacity significantly and also guarantees the system reliability.

Keywords: Network coding; Asymmetric modulation; Two-way relay channel; Subset constellation

1. Introduction

Network coding (NC) was originally proposed to improve the performance of multicast throughput in wired networks by Ahlswede et al. in 2000 [1]. Recently, the broadcast nature of wireless channels has attracted a lot of research activities on the application of NC in wireless networks.

The two-way relay channel (TWRC) is a typical scenario in wireless communications. Physical layer network coding (PLNC) was proposed to improve the throughput of TWRC [2], which maps the superimposition of the signals received simultaneously to a digital bit stream. PLNC improves the system performance by making use of the interference, instead of avoiding it. PLNC can further be classed into two categories - PNCF (PNC over finite field) and PNCFI (PNC over infinite field) - according to whether the network coding field adopted is finite or infinite [3]. The capacity of TWRC with PLNC is higher than the traditional communication strategies [4,5]. The design of modulation suited for TWRC with PLNC can be BPSK or QPSK [6], and an unconventional 5-ary modulation which is optimized according to the channel condition [7]. In addition, PLNC can be combined with channel code

to improve bit error rate (BER) performance of the system [8].

The research works reported above are all based on the same assumption that the transmission rate of two end nodes is symmetric. In practical TWRC, however, the quality of two-way links is always asymmetric. Therefore, the capacity of TWRC will be limited by the weaker link. Thus, in the symmetric rate transmission of TWRC, in order to ensure the reliability of the weaker link, the stronger link has to transmit and receive with low-order modulation as same as the weaker link. For this reason, the stronger link does not take advantage of its good channel conditions to improve the overall transmission rate, which lowers the validity.

Asymmetric modulation is a method to realize the asymmetric rate transmission. The power matching ratio of the two end nodes in TWRC is corresponding to the performance of symbol error rate in the multiple access phase [9,10]. In the broadcast phase of TWRC, under the same BER constraint, the weaker link can decode at lower signal noise ratio (SNR) compared to the stronger link by exploiting *a priori* bit information in each transmit symbol [11].

This paper investigates the asymmetric rate transmission both in the multiple access phase and broadcast phase of the two-phase TWRC by designing an asymmetric modulation scheme with PLNC. The simulation results show that the proposed scheme not only

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improves the transmission validity by increasing the system capacity but also guarantees the transmission reliability.

The rest of this paper is organized as follows. Section 2 introduces the system model. Section 3 presents the new scheme of asymmetric modulation with PLNC. Section 4 analyzes the performance of the asymmetric modulation scheme. Simulation experiments and performance comparisons between the symmetric transmission mode and the proposed scheme will be discussed in Section 5. Finally, Section 6 concludes this paper.

2. System model

We consider the two-phase TWRC system as shown in Figure 1, in which two independent end nodes n_1 and n_2 exchange information with each other via the relay node n_R . As illustrated in Figure 1, $B_i, i \in \{1, 2, R\}$ denotes the bit information of node n_i which is modulated to symbol X_i . The coefficient $h_{ij}, i, j \in \{1, 2, R\}$ is the complex path-loss coefficient for the channel from n_i to n_j . W_i is the noise at n_i to be zero-mean complex Gaussian variable with variance σ_i^2 . P_i is the transmitting power of n_i . In this paper, the following assumptions are made: (1) all nodes operate in a half-duplex manner, i.e., a node cannot transmit and receive simultaneously; (2) symbol-level time synchronization is assumed; (3) there is no direct link between n_1 and n_2 ; (4) channel coefficient is quasi-static and possess reciprocity, i.e., $h_{1R} = h_{R1} = h_1, h_{2R} = h_{R2} = h_2$; (5) nodes can perfectly estimate the channel state information (CSI) to realize the phase synchronization and amplitude pre-equalization; (6) the energy of symbol X_i is normalized to unit 1; (7) power constrains $P_1 + P_2 = 1, P_R = 1$.

The process of information exchange comprises of two phases:

1. Multiple access phase (MAC). First, n_1 and n_2 modulate B_1 and B_2 respectively to X_1 and X_2 . Second, n_1 and n_2 transmit X_1 and X_2 to the relay node n_R simultaneously. Then n_R demodulates the superimposition of the signal Y_R to $B_R = B_1 \oplus B_2$.
2. Broadcast phase (BRC). The relay node n_R modulates B_R to X_R and broadcasts it to n_1 and n_2 , simultaneously. Then n_1 demodulates Y_1 to B_1 and get the bit information of n_2 by using exclusive or (XOR) operation $B_2 = B_1 \oplus B_R$. Similarly,

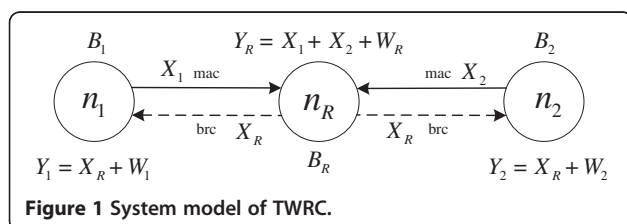


Figure 1 System model of TWRC.

n_2 demodulates Y_2 to B_R and get the bit information of n_1 by using XOR operation $B_1 = B_2 \oplus B_R$.

So, n_1 and n_2 can finish the information exchange only by two phases. The importance of the information exchange is the design of modulation and demodulation at the relay node n_R . In multiple access phase, n_R is supposed to realize the demodulation of $Y_R \rightarrow B_R = B_1 \oplus B_2$. In broadcast phase, n_R has to realize the modulation of $B_R \rightarrow X_R$. The performance of the system depends on the design at n_R , which is a key point in this paper.

3. Asymmetric rate transmission

The design of the modulation and demodulation for n_1, n_2 , and n_R will be presented in this section to realize the asymmetric rate transmission both in the multiple access phase and broadcast phase. For simplicity and without loss of generality, we assume $\sigma_{h_1}^2 > \sigma_{h_2}^2$ (respectively denote the variances of h_1 and h_2) which means that the stronger link C_1 (the channel between n_1 and n_R) has better quality than the weaker link C_2 (the channel between n_2 and n_R).

3.1 Design of asymmetric modulation

The asymmetric modulation is proposed by utilizing the asymmetric channel quality to make n_1 and n_2 transmit and receive at different rates. For simplicity and clarity, QPSK and 16QAM will be as examples to be depicted for the design of the asymmetric modulation, which are respectively 2 bit/symbol and 4 bit/symbol.

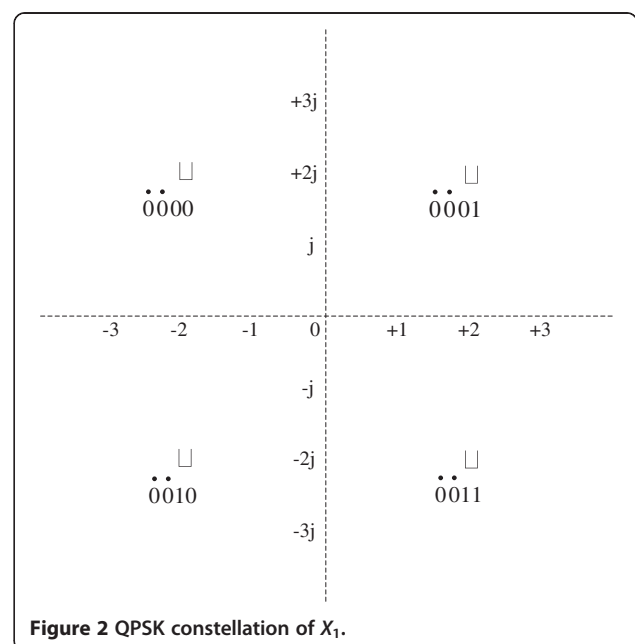


Figure 2 QPSK constellation of X_1 .

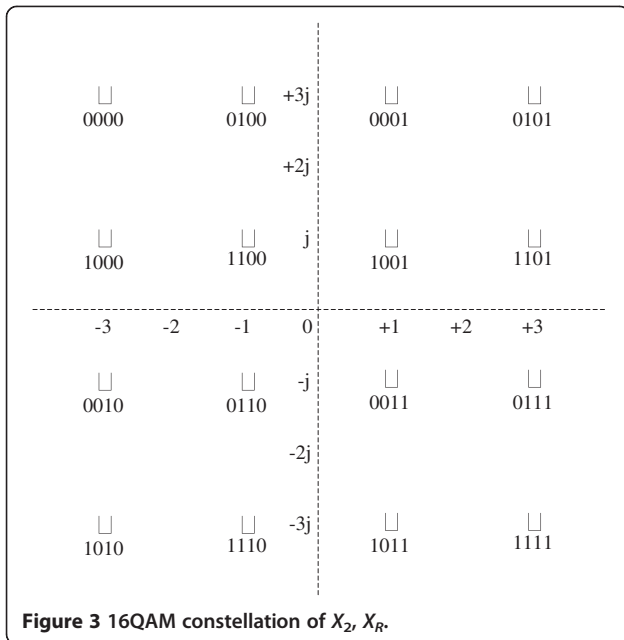


Figure 3 16QAM constellation of X_2, X_R .

and known to both n_1 and n_2); Then n_1 modulates B_1 to X_1 by QPSK, and n_2 modulates B_2 to X_2 by 16QAM (the constellation of X_1 and X_2 are shown in Figures 2 and 3. Finally, n_1 and n_2 transmit X_1 and X_2 to n_R , respectively. So in the same symbol period, n_1 transmits two-bit information while n_2 transmits four bit information.

As illustrated in Figures 2 and 3, the stronger link C_1 transmits signals by low-order modulation (QPSK), while the weaker link C_2 transmits signals by high-order modulation (16QAM). This kind of design might be opposite to our straight thinking, but it lets the system make use of the stronger link C_1 to improve the overall transmission rate and also guarantee the transmission reliability of the weaker link C_2 in the BRC phase (the specific statement is next). When the transmitting power ratio is $P_{\text{QPSK}}:P_{\text{16QAM}} = 4:5$, the BER performance of the QPSK-16QAM superimposition signals is optimal in the MAC phase, which is demonstrated in [9]. Note that there are 36 points in the QPSK-16QAM constellation as shown in Figure 4.

3.1.1 MAC phase

Before transmitting, n_1 first inserts two dummy zeros '00' for each two bits in B_1 to obtain B_1 making '00;01;10; and '11' become '0000;0001;0010; and '0011' (these dummy zeros contain no information and their positions are fixed

3.1.2 BRC phase

In the BRC phase, n_R first modulates B_R to X_R . Then n_R broadcasts X_R to n_1 and n_2 . The constellation of X_R is shown in Figure 3, which is same as the constellation of X_2 in the MAC phase.

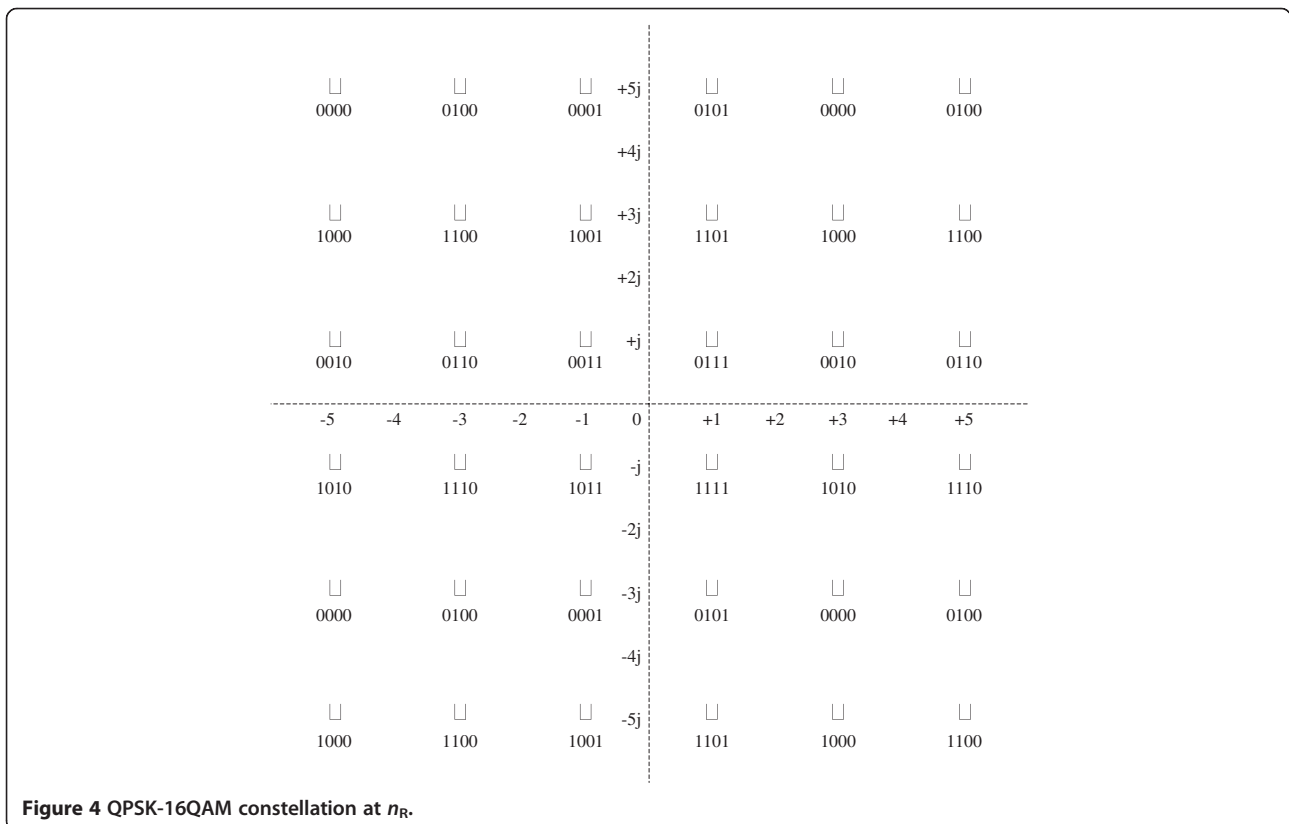


Figure 4 QPSK-16QAM constellation at n_R .

Due to the fore 2 bit '00' in each 4 bit of B_1 which is inserted by n_1 as the dummy zeros, the fore 2 bits of B_2 are kept unchanged in B_R after the XOR operation at n_R . So n_2 can exploit these as a priori information and does not need to decode every bit in B_R . Node n_2 can discard the known bits and decode only the latter 2 bits of B_R according to the subset of 16QAM constellation (QPSK), which is illustrated in Figure 5. Then n_2 can obtain the bit information of n_1 by executing the XOR operation $B_1 = B_2 \oplus B_R$.

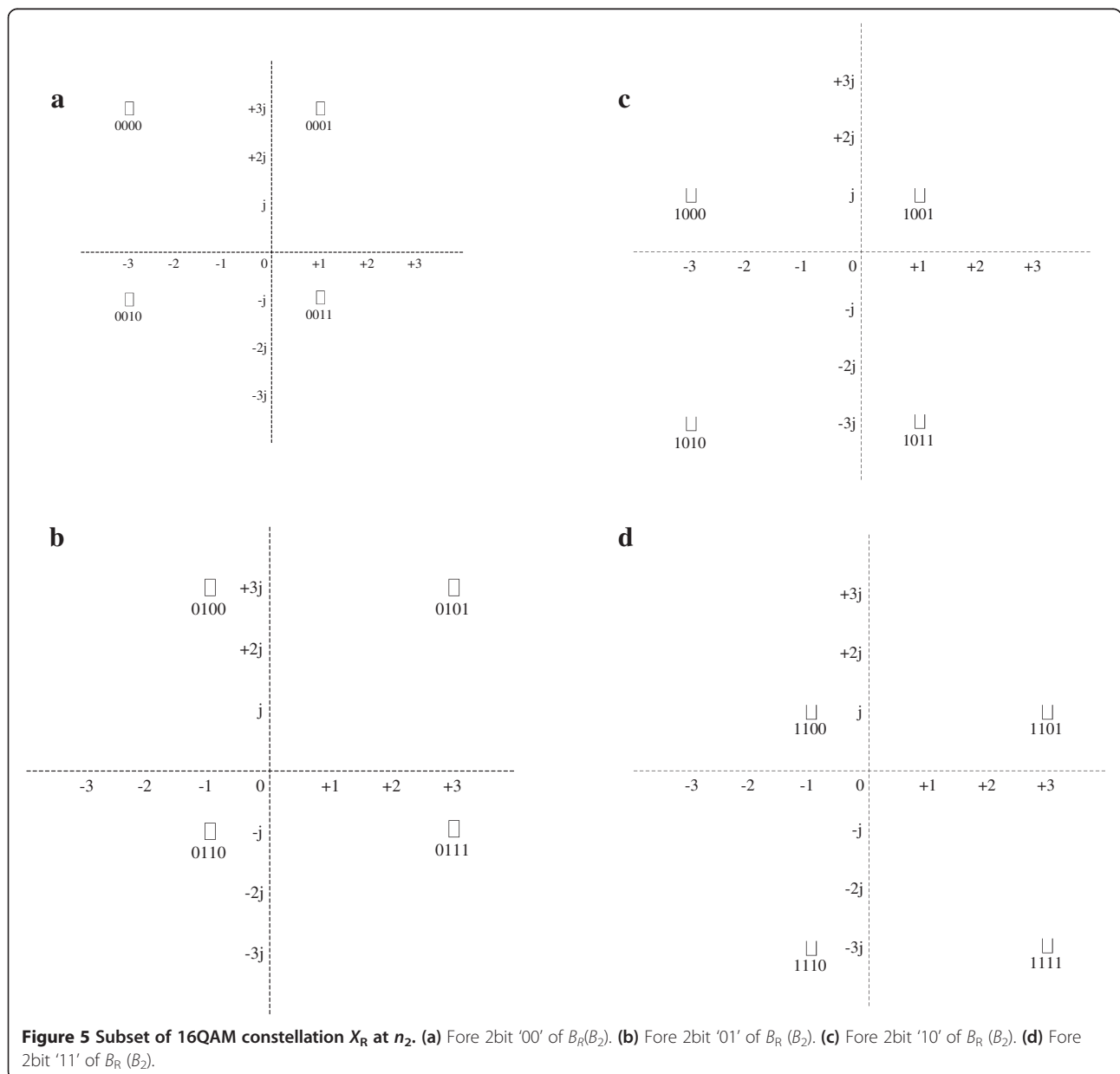
As illustrated in Figure 5, the constellation used to demodulate signals by n_2 is QPSK, which is a subset of 16QAM constellation. In this way, the distance of the

adjacent points is increased, so the BER performance is improved and the transmission reliability is guaranteed.

By using this scheme, in BRC phase, the signal modulated by high-order modulation (16QAM) is transmitted through the stronger link C_1 to improve the transmission rate, while the signal modulated by low-order modulation (QPSK, which is the subset of 16QAM constellation) is transmitted through the weaker link C_2 to ensure the transmission reliability.

3.2 Symmetric modulation for comparison

In order to show up the advantages of the proposed asymmetric modulation scheme, we use the symmetric



modulation QPSK-QPSK as examples for a simple comparison.

By using symmetric modulation, the stronger link C_1 has to adopt the same low-order modulation as the weaker link C_2 to transmit and receive signals. Both n_1 and n_2 adopt QPSK modulation to transmit and receive signals, and n_R adopt QPSK modulation to transmit the XOR signals. The constellations are shown in Figures 6 and 7.

4. Performance analysis

In this section, we will analyze the performance of the proposed asymmetric modulation and compare it to that of the symmetric modulation. As we know, C_1 is better than C_2 . Without loss of generality, we assume $\mu > 1$. For simplicity, we assume that the noises at the three nodes have the same variance, $\sigma_1^2 = \sigma_2^2 = \sigma_R^2 = \sigma^2$.

4.1 BER analysis

4.1.1 MAC phase

Here, we use d_{\min} to denote the minimum distance between the adjacent points in the constellation of the superimposition of the signals received at n_R (as shown in Figures 4 and 7). Thus, we have

$$\begin{aligned} d_{\text{QPSK-16QAM min}} &= 2/3 \approx 0.667 \\ d_{\text{QPSK-QPSK min}} &= \sqrt{2} \approx 1.414 \end{aligned} \quad (1)$$

Then the approximation of the BER can be obtained according to [12,13] as

$$\begin{aligned} \text{BER}_{\text{QPSK-16QAM}} &\approx \frac{5}{\log_2 16} Q\left(\sqrt{\frac{2}{5}}\gamma_R\right) \\ \text{BER}_{\text{QPSK-QPSK}} &\approx \frac{3}{\log_2 4} Q\left(\sqrt{\gamma_R}\right) \end{aligned} \quad (2)$$

Where γ_R is SNR of the signals received at n_R .

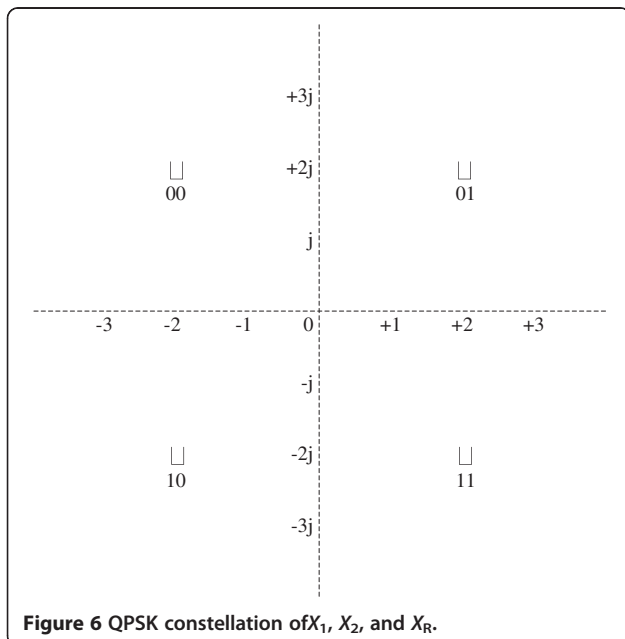


Figure 6 QPSK constellation of X_1 , X_2 , and X_R .

4.1.2 BRC phase

Also, we use d_{\min} to denote the minimum distance between the adjacent points in the constellation of the signals received at n_1 and n_2 from n_R (as shown in Figures 3, 5, and 6). Then we have

$$\begin{aligned} d_{n_1-16\text{QAM}_{\text{ASY min}}} &= 2\sqrt{5}/5 \approx 0.894 & d_{n_2-\text{QPSK}_{\text{ASY min}}} &= 4\sqrt{5}/5 \approx 1.789 \\ d_{n_1-\text{QPSK}_{\text{SYM min}}} &= 2 & d_{n_2-\text{QPSK}_{\text{SYM min}}} &= 2 \end{aligned} \quad (3)$$

Further, the approximation of the BER can be obtained according to [12,13] as

$$\begin{aligned} \text{BER}_{n_1-16\text{QAM}_{\text{ASY}}} &\approx \frac{4}{\log_2 16} Q\left(\sqrt{\frac{2}{5}}\gamma_1\right) & \text{BER}_{n_2-\text{QPSK}_{\text{ASY}}} &\approx \frac{2}{\log_2 16} Q\left(\sqrt{2 \times \frac{4}{5}}\gamma_2\right) \\ \text{BER}_{n_1-\text{QPSK}_{\text{SYM}}} &\approx \frac{2}{\log_2 4} Q\left(\sqrt{2}\gamma_1\right) & \text{BER}_{n_2-\text{QPSK}_{\text{SYM}}} &\approx \frac{2}{\log_2 4} Q\left(\sqrt{2}\gamma_2\right) \end{aligned} \quad (4)$$

where γ_1 and γ_2 denote SNR of the signals received at n_1 and n_2 , respectively.

4.3 Capacity analysis

According to [14,15], the AWGN channel in MAC phase and BRC phase can be regarded as an equivalent virtual channel C_V shown in Figure 8.

The input of C_V is X and the output is $Y = X + W$, where W is a zero-mean complex Gaussian random variable $W \sim CN(0, \sigma^2)$. C_V is a discrete-input and continuous-output channel according to [15]. The input X comprises symbols selected from a finite and discrete input alphabet $X = x_k$ ($k = 0, 1, \dots, q - 1$), and the output is continuous $Y = \{-\infty, +\infty\}$. For a given X , it follows that Y is a complex Gaussian random variable with mean x_k and variance σ^2 , that is,

$$p(y|X = x_k) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(y-x_k)^2/2\sigma^2} \quad (5)$$

For any given input sequence X_i ($i = 1, 2, \dots, n$), there is a corresponding output sequence of

$$Y_i = X_i + W_i \quad (i = 1, 2, \dots, n). \quad (6)$$

The condition that the channel is memoryless can be expressed as

$$\begin{aligned} p(y_1, y_2, \dots, y_n | X_1 = u_1, X_2 = u_2, \dots, X_n = u_n) \\ = \prod_{i=1}^n p(y_i | X_i = u_i). \end{aligned} \quad (7)$$

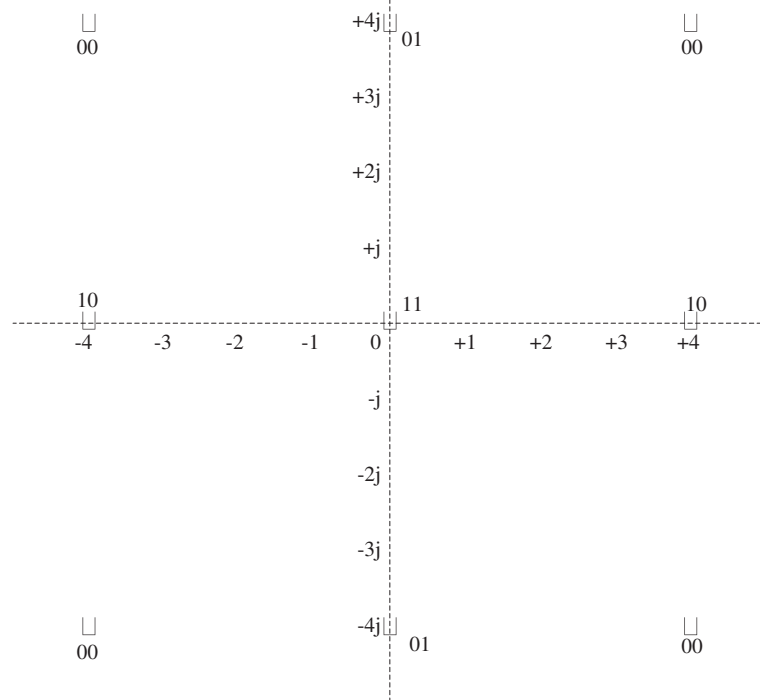


Figure 7 QPSK-QPSK constellation at n_R .

So, the capacity of C_V is defined as

$$\begin{aligned}
 C_V &= \max_{p(x_i)} I(X; Y) \\
 &= \max_{p(x_i)} \sum_{i=0}^{q-1} \int_{-\infty}^{+\infty} p(y|x_i) p(x_i) \log \frac{p(y|x_i)}{p(y)} dy \\
 &= \max_{p(x_i)} \sum_{i=0}^{q-1} \int_{-\infty}^{+\infty} p(y|x_i) p(x_i) \log \frac{p(y|x_i)}{\sum_{k=0}^{q-1} p(y|x_k) p(x_k)} dy.
 \end{aligned}
 \tag{8}$$

The input X are equally probable symbols, and the probability $p(x_i)$ can be obtained according to the corresponding modulation constellation. Substituting $p(x_i)$ and Equation 7 into Equation 8, we can obtain the capacity of the equivalent virtual channel C_V .

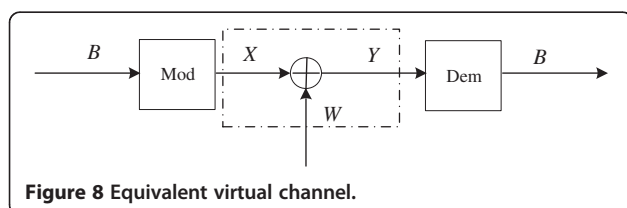


Figure 8 Equivalent virtual channel.

5. Simulation results

In this section, simulation results are presented to demonstrate the performance of our proposed scheme and to verify the accuracy of our analytical analysis in Section 4. In the simulation, all the numerical results are calculated with averaging over 10,000,000 packets, and the number of bits contained by each packet is equal to the bits contained by each symbol in the corresponding constellation. For simplicity and without loss of generality, we consider two scenarios of $\mu = 4$ and $\mu = 8$. The SNR of the stronger link C_1 is 6 and 9 dB higher than that of the weaker link C_2 .

5.1 BER performance

Here, the simulation results are presented to demonstrate the BER performance of the asymmetric modulation at n_1 and n_2 .

Figures 9 and 10 respectively show the BER performance of n_1 and n_2 when the asymmetric levels of the two-way links are 6 and 9 dB. As can be seen in Figures 9 and 10, when the asymmetric modulation QPSK-16QAM is adopted, although the modulation is different (one is QPSK, another is 16QAM), the BER performance of n_1 is close to n_2 . Moreover, the closeness of the BER performance for the symmetric modulation QPSK-QPSK is similar with the asymmetric modulation QPSK-16QAM. When the asymmetric level of the two-way

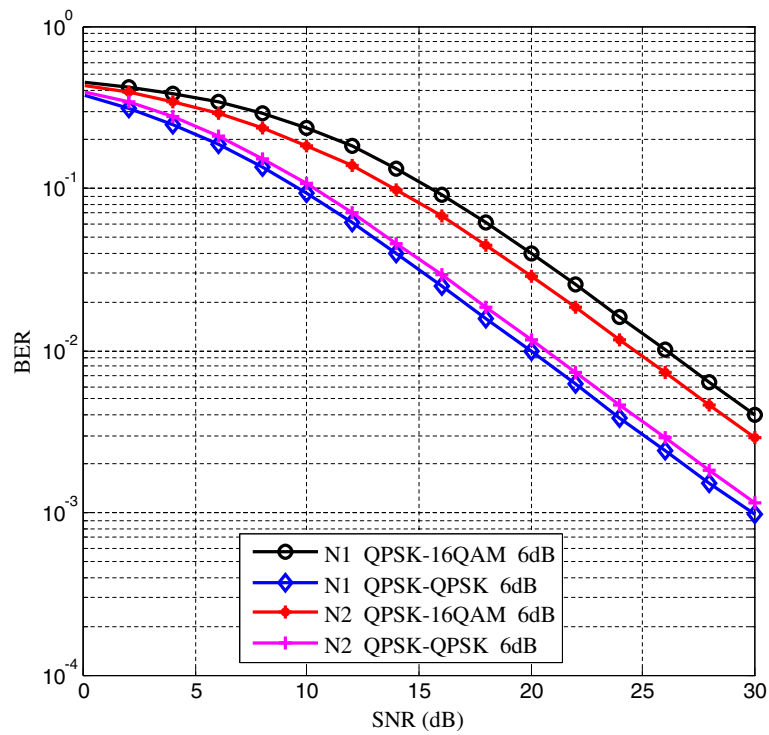


Figure 9 BER performance (asymmetric level is 6 dB).

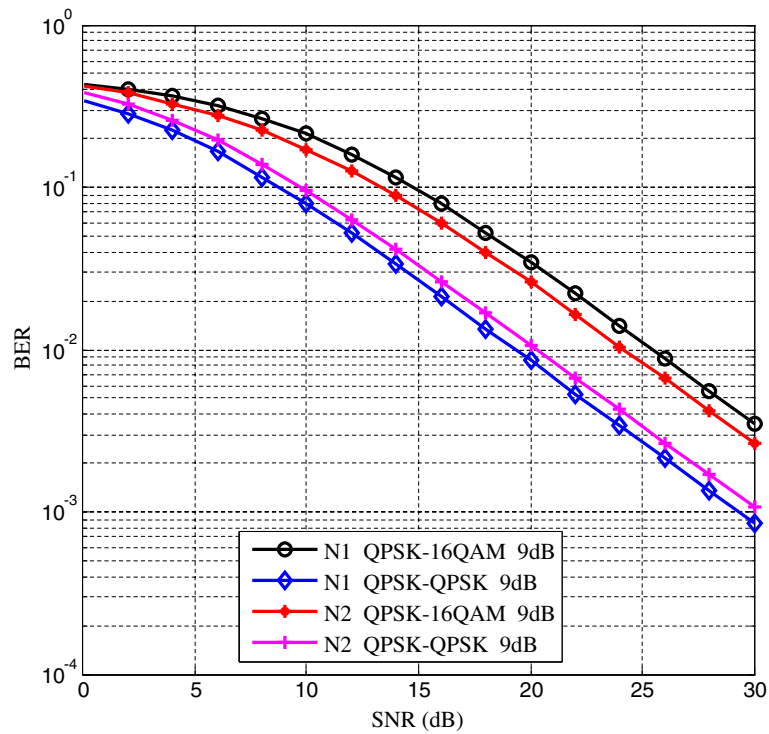


Figure 10 BER performance (asymmetric level is 9 dB).

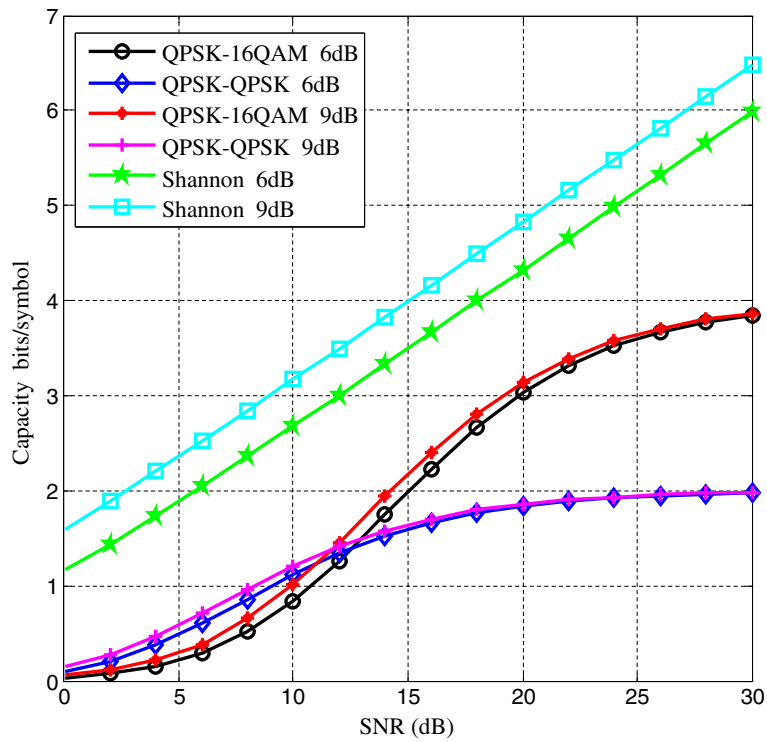


Figure 11 Capacity at n_1 .

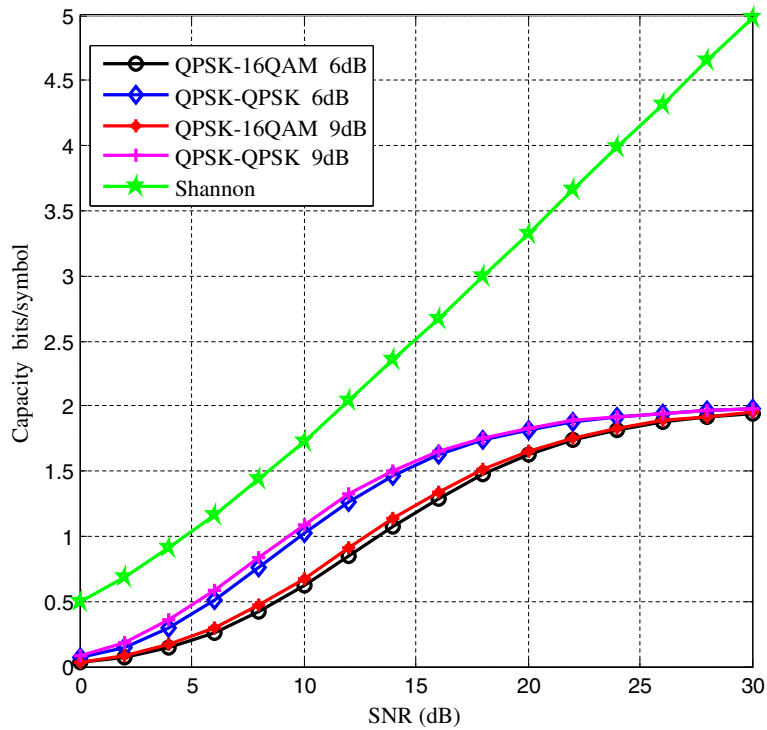


Figure 12 Capacity at n_2 .

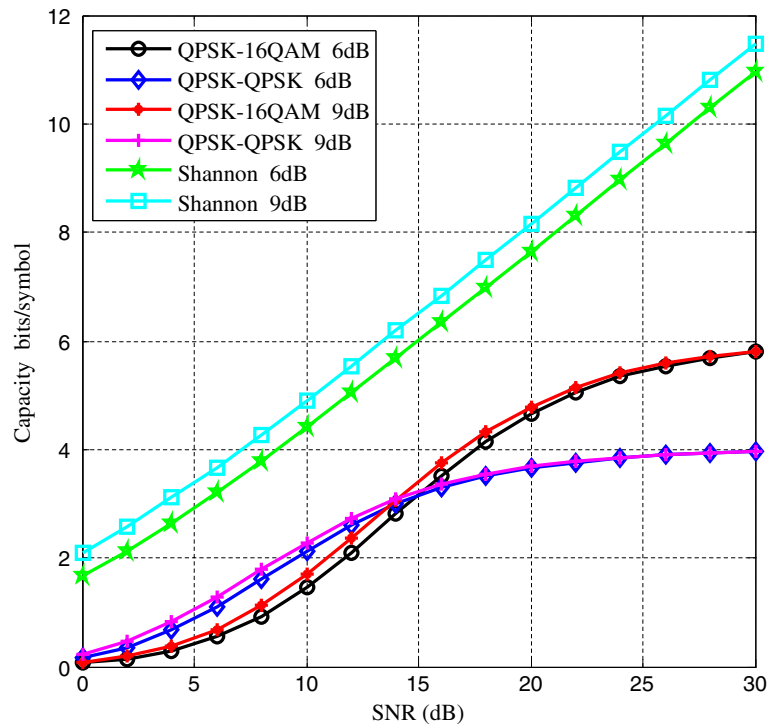


Figure 13 Sum of capacity.

links increases (from 6 to 9 dB), the trend remains unchanged.

5.2 Capacity

Here, the simulation results are presented to demonstrate the capacity of the asymmetric modulation at the end nodes n_1 and n_2 . The sum of capacity is also illustrated here.

Figures 11 and 12 respectively show the capacity of n_1 and n_2 . The x -axis is the SNR of C_2 in the two figures above, which needs to be paid attention to. As shown in Figure 11, when the SNR of the stronger link C_1 is 6 dB higher than that of the weaker link C_2 and the SNR is higher than 13 dB, the capacity of n_1 by using the asymmetric modulation QPSK-16QAM is higher than by using the symmetric modulation QPSK-QPSK. When the SNR of the stronger link C_1 is 9 dB higher than that of the weaker link C_2 , the situation is similar when SNR is higher than 11 dB. As shown in Figure 12, when the channel quality is asymmetric, the capacity of n_2 by using the asymmetric modulation QPSK-16QAM is close to the capacity of n_2 by using the symmetric modulation QPSK-QPSK.

Figure 13 shows the total capacity of n_1 and n_2 when the asymmetric level of the two-way links is 6 and 9 dB. As shown in Figure 13, when the SNR of the stronger link C_1 is 6 dB higher than that of the weaker link C_2 and the SNR is higher than 15 dB, the capacity of n_1 by

using the asymmetric modulation QPSK-16QAM is higher than by using the symmetric modulation QPSK-QPSK. When the SNR of the stronger link C_1 is 9 dB higher than that of the weaker link C_2 , the situation is similar when SNR is higher than 13 dB. Overall, under higher SNR, compared with the symmetric modulation, the total capacity has been greatly improved by using the asymmetric modulation.

6. Conclusions

In this paper, an asymmetric modulation scheme with PLNC in TWRC is proposed, which aims to improve both the validity and reliability in two-way relay transmissions. The proposed asymmetric modulation scheme realized the asymmetric rate transmission both in MAC phase and BRC phase of TWRC. In MAC phase, the BER performance at the relay is improved. In BRC phase, the capacity is boosted by making full use of the stronger link, and the BER performance is guaranteed by exploiting *a priori* bit information to demodulate for the weaker link. We derived the approximated BER expressions for the scheme proposed, which were also demonstrated by simulation experiments. Through the comparisons of the symmetric modulation scheme, it is found that by using the proposed asymmetric modulation scheme, the total capacity is improved significantly under the asymmetric level of the two-way links.

In addition, as well-known channel coding possesses the correcting ability and can improve the BER performance further, combining channel coding, network coding, and modulation for asymmetric transmissions in TWRC will be our future researches.

Competing interests

The authors declare that they have no competing interests.

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