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Research Article

Synthesis of Relativistic Wave Equations: The Noninteracting Case

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We study internal structure of the Duffin-Kemmer-Petiau equations for spin 0 and spin 1 mesons. We show that in the noninteracting case full covariant solutions of the $s = 0$ and $s = 1$ DKP equations are generalized solutions of the Dirac equation.

1. Introduction

The Duffin-Kemmer-Petiau (DKP) equations [1–3], describing spin 0 and spin 1 mesons, are becoming increasingly useful due to their applications to problems in particle and nuclear physics [4–13]. It is well known that the DKP equations contain redundant components since only $2(2s+1)$ components are needed to describe free spin s particles with nonzero rest masses [14, 15] while $s = 0$ and $s = 1$ DKP equations contain 5 and 10 components, respectively. The presence of redundant components in DKP equations leads for some interactions to such nonphysical effects as superluminal velocities [16, 17] (see also [18–20] for $s = 3/2, 2$ cases). On the other hand, physically acceptable equations for arbitrary spin can be obtained by removing redundant components with use of additional covariant condition [14, 15]. It seems that presence of redundant components means that the DKP equations have internal structure.

The motivation of this work is our recent discovery that solutions of subequations of the $s = 0$ and $s = 1$ DKP equations fulfill the Dirac equation [21, 22]. On the other hand, these solutions do not contain all spinor components and are thus noncovariant solutions of covariant equations. We studied this problem in [23, 24]. In the present work, we show that, in the free case, full covariant solutions of the $s = 0$ and $s = 1$ DKP equations are generalized solutions of the Dirac equation. This finding may provide a basis for a synthesis of covariant particle equations, alternative to the classical Foldy programme [25].

The paper is organized as follows. In Section 2 the Dirac equation and the Duffin-Kemmer-Petiau equations for $s = 0$ and $s = 1$ are described. It is shown in Section 3 that in the noninteracting case solutions of these DKP equations are generalized matrix solutions of the Dirac equation. We discuss our findings in the last section.

2. Relativistic Wave Equations

In what follows we use conventions and definitions introduced in [22]. The Dirac equation, describing spin 1/2 elementary particles, is

$$\gamma_\mu p^\mu \Psi = m\Psi, \tag{1}$$

where γ^μ are 4×4 matrices fulfilling [26, 27]

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} I_{4\times 4}, \tag{2}$$

where $g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ and $I_{4\times 4}$ is 4×4 unit matrix. In the spinor representation of the Dirac matrices we have $\Psi = (\xi^A, \eta_{\dot{B}})^T$ [28], where T denotes transposition of a matrix.

The DKP equations for spin 0 and spin 1 mesons are written as

$$\beta_\mu p^\mu \Psi = m\Psi, \tag{3}$$

where β^μ are 5×5 and 10×10 matrices, respectively, obeying the following commutation relations [1–3]:

$$\beta^\lambda \beta^\mu \beta^\nu + \beta^\nu \beta^\mu \beta^\lambda = g^{\lambda\mu} \beta^\nu + g^{\nu\mu} \beta^\lambda. \tag{4}$$

In the case of $s = 0$ representation equation (3) can be written as

$$\begin{aligned} p^{\mu}\psi &= m\psi^{\mu}, \\ p_{\nu}\psi^{\nu} &= m\psi, \end{aligned} \tag{5}$$

with Ψ in (3) defined as

$$\Psi = \left(\psi^{\mu}, \psi\right)^{T} = \left(\psi^{0}, \psi^{1}, \psi^{2}, \psi^{3}, \psi\right)^{T}. \tag{6}$$

In the case of $s = 1$ (3) reduces to

$$\begin{aligned} p^{\mu}\psi^{\nu} - p^{\nu}\psi^{\mu} &= m\psi^{\mu\nu}, \\ p_{\mu}\psi^{\mu\nu} &= m\psi^{\nu}, \end{aligned} \tag{7}$$

with Ψ in (3) equalling

$$\begin{aligned} \Psi &= \left(\psi^{\mu\nu}, \psi^{\lambda}\right)^{T} \\ &= \left(\psi^{01}, \psi^{02}, \psi^{03}, \psi^{23}, \psi^{31}, \psi^{12}, \psi^{0}, \psi^{1}, \psi^{2}, \psi^{3}\right)^{T}, \end{aligned} \tag{8}$$

where ψ^{λ} are real and $\psi^{\mu\nu}$ are purely imaginary (alternatively, $-\partial^{\mu}\psi^{\nu} + \partial^{\nu}\psi^{\mu} = m\psi^{\mu\nu}$ and $\partial_{\mu}\psi^{\mu\nu} = m\psi^{\nu}$, where ψ^{λ} and $\psi^{\mu\nu}$ are real). The $s = 1$ condition, $p_{\nu}\psi^{\nu} = 0$, follows from the second equation of (7) due to antisymmetry of tensor $\psi^{\mu\nu}$. Equations for spin 1 bosons (7) were first written by Proca [29].

3. From the DKP Equations to Generalized Solutions of the Dirac Equation

3.1. Spin 0. Equations (5) can be written within spinor formalism as

$$\begin{aligned} p_{A\dot{B}}\psi &= m\psi_{A\dot{B}}, \\ p^{A\dot{B}}\psi_{A\dot{B}} &= 2m\psi. \end{aligned} \tag{9}$$

Splitting the last equation of (9), $p^{A\dot{B}}\psi_{A\dot{B}} = p^{1\dot{1}}\psi_{1\dot{1}} + p^{1\dot{2}}\psi_{1\dot{2}} + p^{2\dot{1}}\psi_{2\dot{1}} + p^{2\dot{2}}\psi_{2\dot{2}} = 2m\psi$; we obtain two sets of equations involving components $\psi_{1\dot{1}}, \psi_{1\dot{2}}, \psi$ and $\psi_{2\dot{1}}, \psi_{2\dot{2}}, \psi$, respectively:

$$\begin{aligned} p_{1\dot{1}}\psi &= m\psi_{1\dot{1}}, \\ p_{1\dot{2}}\psi &= m\psi_{1\dot{2}}, \\ p^{1\dot{1}}\psi_{1\dot{1}} + p^{1\dot{2}}\psi_{1\dot{2}} &= m\psi, \end{aligned} \tag{10}$$

$$\begin{aligned} p_{2\dot{1}}\psi &= m\psi_{2\dot{1}}, \\ p_{2\dot{2}}\psi &= m\psi_{2\dot{2}}, \\ p^{2\dot{1}}\psi_{2\dot{1}} + p^{2\dot{2}}\psi_{2\dot{2}} &= m\psi, \end{aligned} \tag{11}$$

each of which describes particle with mass m (we check this substituting, e.g., $\psi_{1\dot{1}}$ and $\psi_{1\dot{2}}$ or $\psi_{2\dot{1}}$ and $\psi_{2\dot{2}}$, into the third equations). The splitting preserving $p_{\mu}p^{\mu}\psi = m^2\psi$ is possible due to spinor identities, $p_{1\dot{1}}p^{1\dot{1}} + p_{2\dot{1}}p^{2\dot{1}} = p_{\mu}p^{\mu}$ and $p_{1\dot{2}}p^{1\dot{2}} + p_{2\dot{2}}p^{2\dot{2}} = p_{\mu}p^{\mu}$ (cf. [22]). Thus (10) and (11) are equivalent to DKP equations (9). We described similar equations in [21]. From each of (10) and (11) an identity follows:

$$p_{1\dot{2}}\psi_{1\dot{1}} = p_{1\dot{1}}\psi_{1\dot{2}}, \tag{12a}$$

$$p_{2\dot{2}}\psi_{2\dot{1}} = p_{2\dot{1}}\psi_{2\dot{2}}. \tag{12b}$$

Equation (10) and the identity (12a), as well as (11) and the identity (12b), can be written in form of the Dirac equations:

$$\begin{pmatrix} 0 & 0 & p_{1\dot{1}} & p_{2\dot{1}} \\ 0 & 0 & p_{1\dot{2}} & p_{2\dot{2}} \\ p^{1\dot{1}} & p^{1\dot{2}} & 0 & 0 \\ p^{2\dot{1}} & p^{2\dot{2}} & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_{1\dot{1}} \\ \psi_{1\dot{2}} \\ \psi \\ 0 \end{pmatrix} = m \begin{pmatrix} \psi_{1\dot{1}} \\ \psi_{1\dot{2}} \\ \psi \\ 0 \end{pmatrix}, \tag{13}$$

$$\begin{pmatrix} 0 & 0 & p_{1\dot{1}} & p_{2\dot{1}} \\ 0 & 0 & p_{1\dot{2}} & p_{2\dot{2}} \\ p^{1\dot{1}} & p^{1\dot{2}} & 0 & 0 \\ p^{2\dot{1}} & p^{2\dot{2}} & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_{2\dot{1}} \\ \psi_{2\dot{2}} \\ 0 \\ \psi \end{pmatrix} = m \begin{pmatrix} \psi_{2\dot{1}} \\ \psi_{2\dot{2}} \\ 0 \\ \psi \end{pmatrix}, \tag{14}$$

respectively, with one component equalling zero. Since in (13) and (14) there is the same differential operator we can write these equations as a single Dirac equation. Substituting explicit formulae for the spinors $p^{A\dot{B}}$ and $p_{A\dot{B}}$ (see [22]), we have

$$\left(p^{0}\gamma^{0} - p^{1}\gamma^{1} - p^{2}\gamma^{2} - p^{3}\gamma^{3}\right)\mathbb{A} = m\mathbb{A}, \tag{15}$$

where $\mathbb{A} = \begin{pmatrix} \psi_{1\dot{1}} & \psi_{2\dot{1}} \\ \psi_{1\dot{2}} & \psi_{2\dot{2}} \\ \psi & 0 \\ 0 & \psi \end{pmatrix}$ is a generalized (matrix) wavefunction and γ^{μ} matrices read

$$\begin{aligned} \gamma^{0} &= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \\ \gamma^{1} &= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \\ \gamma^{2} &= \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \\ \gamma^{3} &= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{16}$$

Let us note that this is the modified spinor representation with $\gamma^i \rightarrow -\gamma^i$ ($i = 1, 2, 3$) and $\Psi = (\xi^A, \eta_{\dot{B}})^T \rightarrow \Psi = (\eta_{\dot{B}}, \xi^A)^T$ with respect to [28]. In what follows we will use a shorthand $\mathbb{A} = (\psi_{A\dot{B}}, \psi I_{2\times 2})^T$, where $I_{2\times 2}$ is the 2×2 unit matrix.

3.2. Spin 1. We will now describe in two steps splitting of the DKP equations. To achieve first level of splitting we write DKP equations (7) in spinor notation as [30, 31]

$$\begin{aligned} p_A^{\dot{B}} \zeta_{C\dot{B}} + p_C^{\dot{B}} \zeta_{A\dot{B}} &= 2m\eta_{AC}, \\ p_{\dot{B}}^{A} \zeta_{A\dot{D}} + p_{\dot{D}}^{A} \zeta_{A\dot{B}} &= 2m\chi_{\dot{B}\dot{D}}, \\ p_A^{\dot{C}} \chi_{\dot{B}\dot{C}} + p_{\dot{B}}^{C} \eta_{AC} &= -2m\zeta_{A\dot{B}}. \end{aligned} \tag{17}$$

Equations (17) are now splitted to yield two separate equations for spinors $\chi_{\dot{B}\dot{D}}$ and $\zeta_{A\dot{B}}$ and η_{AC} and $\zeta_{A\dot{B}}$:

$$\begin{aligned} p_A^{\dot{B}} \zeta_{C\dot{B}} &= m\eta_{AC}, \quad \eta_{AC} = \eta_{CA}, \\ p_{\dot{B}}^{C} \eta_{AC} &= -m\zeta_{A\dot{B}}, \end{aligned} \tag{18}$$

$$\begin{aligned} p_{\dot{B}}^{A} \zeta_{A\dot{D}} &= m\chi_{\dot{B}\dot{D}}, \quad \chi_{\dot{B}\dot{D}} = \chi_{\dot{D}\dot{B}}, \\ p_A^{\dot{D}} \chi_{\dot{B}\dot{D}} &= -m\zeta_{A\dot{B}}, \end{aligned} \tag{19}$$

respectively. The splitting was achieved due to appropriate spinor identities; see equation (11) in [22]. Indeed, solutions of (18) and (19) obey the DKP equations (17). This derivation was described in [22] and is included here for the sake of completeness.

Equations (18) and (19), first written by Dirac [32, 33], are known to describe spin 1 bosons where spinors η_{CA} and $\chi_{\dot{D}\dot{B}}$ correspond to self-dual or anti-self-dual antisymmetric tensors $\psi^{\mu\nu}$, respectively [34]. Each of the above equations is covariant except from space reflection but (18) and (19) considered together are fully covariant. These equations written in tensor form, $\beta^\mu p_\mu \Psi = m\Psi$, $\Psi = [\psi_{01}, \psi_{02}, \psi_{03}, \psi_0, \psi_1, \psi_2, \psi_3]^T$, where $\psi_{\mu\nu}$ are self-dual or anti-self-dual antisymmetric tensors, with 7×7 matrices β^μ fulfilling the DKP algebra (4), are the Hagen-Hurley equations [35–38]. Explicit formulae for these β^μ matrices are given in [38].

We will now split the $s = 1$ Dirac equations (18). Substituting expressions for $p_A^{\dot{B}}$ and $p_{\dot{B}}^{C}$ (cf. [22]), we obtain a system of eight equations:

$$\begin{aligned} -\left(p^1 + ip^2\right)\zeta_{1\dot{1}} - \left(p^0 - p^3\right)\zeta_{1\dot{2}} &= m\eta_{11}, \\ \left(p^0 + p^3\right)\zeta_{1\dot{1}} + \left(p^1 - ip^2\right)\zeta_{1\dot{2}} &= m\eta_{21}, \\ -\left(p^1 - ip^2\right)\eta_{11} - \left(p^0 - p^3\right)\eta_{12} &= -m\zeta_{1\dot{1}}, \\ \left(p^0 + p^3\right)\eta_{11} + \left(p^1 + ip^2\right)\eta_{12} &= -m\zeta_{1\dot{2}}, \end{aligned} \tag{20a}$$

$$\begin{aligned} -\left(p^1 + ip^2\right)\zeta_{2\dot{1}} - \left(p^0 - p^3\right)\zeta_{2\dot{2}} &= m\eta_{12}, \\ \left(p^0 + p^3\right)\zeta_{2\dot{1}} + \left(p^1 - ip^2\right)\zeta_{2\dot{2}} &= m\eta_{22}, \\ -\left(p^1 - ip^2\right)\eta_{21} - \left(p^0 - p^3\right)\eta_{22} &= -m\zeta_{2\dot{1}}, \\ \left(p^0 + p^3\right)\eta_{21} + \left(p^1 + ip^2\right)\eta_{22} &= -m\zeta_{2\dot{2}}, \end{aligned} \tag{20b}$$

where all equations are arranged into two subsets (20a) and (20b) and we have not assumed yet that $\eta_{12} = \eta_{21}$.

Demanding now $\eta_{12} = \eta_{21} \equiv \eta$ we achieve splitting of (20a) and (20b) obtaining two Dirac-like equations:

$$\left(p^0\tilde{\gamma}^0 - p^1\tilde{\gamma}^1 - p^2\tilde{\gamma}^2 - p^3\tilde{\gamma}^3\right)\Psi = m\Psi, \tag{21a}$$

with $\Psi = (\zeta_{1\dot{1}}, \zeta_{1\dot{2}}, \eta_{11}, \eta)^T$ and

$$\left(p^0\tilde{\gamma}^0 - p^1\tilde{\gamma}^1 - p^2\tilde{\gamma}^2 - p^3\tilde{\gamma}^3\right)\widetilde{\Psi} = m\widetilde{\Psi}, \tag{21b}$$

with $\widetilde{\Psi} = (\zeta_{2\dot{1}}, \zeta_{2\dot{2}}, \eta, \eta_{22})^T$, where $\tilde{\gamma}^\mu$ matrices are expressed by γ^μ matrices (16):

$$\begin{aligned} \tilde{\gamma}^0 &= i\gamma^2, \\ \tilde{\gamma}^1 &= -\gamma^3, \\ \tilde{\gamma}^2 &= i\gamma^0, \\ \tilde{\gamma}^3 &= \gamma^1. \end{aligned} \tag{22}$$

The Dirac-like equations (21a) and (21b) are highly nonstandard because of several reasons: they contain higher-order spinors η_{AB} and $\zeta_{A\dot{B}}$ and one common component ($\eta_{12} = \eta_{21} \equiv \eta$) and their solutions, Ψ and $\widetilde{\Psi}$, are not fully covariant since, considered separately, they do not involve all components of the spinors η_{AB} and $\zeta_{A\dot{B}}$. On the other hand, these equations are fully covariant when considered as a whole.

Since in (21a) and (21b) there is the same differential operator we can write these equations as a single Dirac equation. We note, however, that the Dirac matrices in (16) and (22) are different. We thus first transform $\tilde{\gamma}^\mu$ matrices unitarily to get γ^μ defined in (16):

$$\gamma^\mu = V\tilde{\gamma}^\mu V^\dagger, \quad V = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \tag{23}$$

Now, we can write (21a) and (21b) as a single Dirac equation and with the same representation of γ^μ matrices as in (15):

$$\left(p^0\gamma^0 - p^1\gamma^1 - p^2\gamma^2 - p^3\gamma^3\right)\mathbb{B} = m\mathbb{B}, \tag{24}$$

where $\mathbb{B} = V \begin{pmatrix} \zeta_{1\dot{1}} & \zeta_{2\dot{1}} \\ \zeta_{1\dot{2}} & \zeta_{2\dot{2}} \\ \eta_{11} & \eta \\ \eta & \eta_{22} \end{pmatrix} = \begin{pmatrix} \zeta_{1\dot{1}} & \zeta_{2\dot{1}} \\ \zeta_{1\dot{2}} & \zeta_{2\dot{2}} \\ -\eta_1^1 & -\eta_2^1 \\ -\eta_1^2 & -\eta_2^2 \end{pmatrix}$ is a generalized matrix wavefunction. In what follows we will write $\mathbb{B} = (\zeta_{A\dot{B}}, -\eta_C^D)^T$.

4. Discussion

We have demonstrated that, in the noninteracting case, full covariant solutions of the $s = 0$ and $s = 1$ DKP equations are generalized solutions of the same Dirac equation; see (1), (15), and (24). More exactly, if we choose the modified spinor representation of the Dirac matrices defined in (16) then the following functions $\Psi = (\eta_{\dot{B}}, \xi^A)^T$, $\mathbb{A} = (\psi_{A\dot{B}}, \psi I_{2\times 2})^T$, and $\mathbb{B} = (\zeta_{A\dot{B}}, -\eta_C^D)^T$ with $\eta_{CD} = \eta_{DC}$ are solutions of the same Dirac equation and correspond to $s = 1/2$, $s = 0$, and $s = 1$ cases, respectively. We note that $\mathbb{A}$ and $\mathbb{B}$ are generalized (matrix) solutions. It follows that these solutions of the Dirac equation provide synthesis of relativistic equations for $s = 1/2$ and $s = 0, 1$.

Similar generalized solutions exist also in the interacting case. Indeed, although, in the $s = 0$ case, (3) and (15) are not equivalent in general fields, they are equivalent in crossed fields [39]. Similarly, the Dirac equations (18) or (19) are not equivalent to the $s = 1$ DKP equations (17) in general fields. It remains to determine if the $s = 1$ Dirac equations are equivalent to the $s = 1$ DKP equations in some special fields.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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