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Research Article

Distributed H_∞ Sampled-Data Filtering over Sensor Networks with Markovian Switching Topologies

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This paper considers a distributed H_∞ sampled-data filtering problem in sensor networks with stochastically switching topologies. It is assumed that the topology switching is triggered by a Markov chain. The output measurement at each sensor is first sampled and then transmitted to the corresponding filters via a communication network. Considering the effect of a transmission delay, a distributed filter structure for each sensor is given based on the sampled data from itself and its neighbor sensor nodes. As a consequence, the distributed H_∞ sampled-data filtering in sensor networks under Markovian switching topologies is transformed into H_∞ mean-square stability problem of a Markovian jump error system with an interval time-varying delay. By using Lyapunov Krasovskii functional and reciprocally convex approach, a new bounded real lemma (BRL) is derived, which guarantees the mean-square stability of the error system with a desired H_∞ performance. Based on this BRL, the topology-dependent H_∞ sampled-data filters are obtained. An illustrative example is given to demonstrate the effectiveness of the proposed method.

1. Introduction

A sensor network is composed of a large number of sensor nodes which are usually distributed in a spatial region. These sensor nodes are capable of cooperatively achieving some special tasks by communicating with neighbour nodes. In recent years, lots of attentions have been paid to sensor networks due to their wide applications such as environment monitoring and forecasting, object tracking, infrastructure safety, intelligent traffic system, and space exploration. An important problem in sensor networks is to observe the states of a system via the information exchange among sensor nodes. Since sensor nodes are spatially assigned in a large scale domain, it is practical to design a distributed algorithm for state estimation or filtering in sensor networks. Recently, the distributed filtering or state estimation over sensor networks has drawn considerable interests of many researchers. For example, a distributed filtering for sensor network was addressed in [1], where a consensus algorithm was introduced to make the estimate of each sensor asymptotically converge to the average estimation of these sensors. The distributed Kalman filtering algorithm in [1] was further improved in [2]. In [3], pining observers were designed under the condition that sensor can only observe partial states of the target. In [4], a novel distributed estimation scheme was proposed based on local Luenberger-like observers with a consensus strategy, where network-induced delays and package dropouts were considered.

In addition, H_∞ filtering has been widely investigated in the past two decades due to its capability of minimizing the highest energy gain of the estimation error for all initial conditions and noise; see [5–7]. The aim of H_∞ filtering is to design a stable filter by using the measurements outputs to estimate the system states or a combination of them. Such an approach has been recently applied to the distributed filtering for sensor networks. To mention some, in [8], the distributed filtering problem for sensor networks with multiple missing measurements was investigated, where the concept of H_∞ consensus performance was defined to quantify the consensus degree over a finite horizon. By using the vector dissipativity, a novel approach to the design of distributed robust H_∞ consensus filters was given in [9]. The H_∞ filtering problem was investigated in [10] for class of nonlinear systems with randomly occurring incomplete information including both sensor saturations and missing measurements. The distributed H_∞ filtering problem in sensor networks for discrete-time systems with missing measurements and communication link failure was studied in [11].

With the rapid development of digital technologies, the H_∞ filtering based on sampled data has been exploited in recent years; see, for example, [12–16]. It is well known that sampled-data control can bring some advantages for multiagent systems such as flexibility, robustness, low cost, and energy saving. Due to the limited energy power of sensor networks, it is therefore more practical and significant to design distributed filters by using the sampled data of each sensor. In [17], a stochastic sampled-data approach to the analysis and design of distributed H_∞ filters for sensor networks was proposed. It is worth pointing out that the H_∞ sampled-data filtering in sensor networks has not yet been fully investigated. In addition, the communication topologies of sensor network may randomly change duo to the effect of link failures, packet dropouts, external disturbances, channel fading, task execution alteration, and so forth. In [18], the consensus-based distributed filtering problem for a discrete-time linear system was solved where the communication topology was time-varying with a stochastic process. Considering lossy sensor networks, the distributed finite-horizon filtering problem for a class of time-varying systems was investigated in [19]. In [20], a distributed robust estimation over sensor network with Markovian randomly varying topology was addressed, where the sufficient conditions were given to guarantee a suboptimal H_∞ level of disagreement of estimates. Notice that these results with stochastically switching topologies are mainly concerned with the distributed filtering in pure continuous-time or discrete-time setting; however, there is little related work done for the distributed H_∞ filtering in a sampled-data setting over sensor network with Markovian switching topologies, which motivates the current study.

In this paper, we aim to investigate the distributed H_∞ sampled-data filtering in sensor networks, where the communication topologies are switched by a Markov chain. A new filter structure for each sensor node is given with the sampled data of the output measurements from itself and its neighbour sensor nodes, where the communication delay is taken into consideration. Based on this filter structure, the H_∞ sampled-data filtering problem is transformed into the H_∞ control problem of Markovian jump systems with a time-varying delay. Then, a new BRL for the system is obtained by employing Lyapunov-Krasovskii functional and reciprocally convex approach. Correspondingly, based on this the appropriate topology-dependent H_∞ sampled-data filters are derived by solving a set of linear matrix inequalities, whose effectiveness is illustrated by a numerical example.

Notation. $\mathbb{N}$ represents the set of natural numbers. $I_n \in \mathbb{N}^{n\times n}$ is the identity matrix. $\mathbf{1}_n \in \mathbb{R}^n$ and $\mathbf{0}_n \in \mathbb{R}^n$ represent vectors whose entries are ones and zeros, respectively. $\|\cdot\|$ denotes the Euclidean norm. $\text{diag}\{a_i\}$ is a diagonal matrix with diagonal entries a_i. $P > 0$ means that matrix P is symmetric positive definite. The symbol $*$ denotes the symmetric terms in a symmetric matrix.

2. Problem Statement

In this paper, we consider that the sensor network with N sensor nodes is spatially distributed, whose topology is represented by a directed weighted graph $\mathscr{G} = \{\Delta, \mathscr{E}, \mathscr{W}\}$ of order N, where $\Delta = \{v_1, v_2, \dots, v_N\}$ and $\mathscr{E} \subseteq \Delta\times\Delta$ are the set of nodes and edges, $\mathscr{W} = [w_{ij}] \in \mathbb{R}^{N\times N}$ represents a weighted adjacency matrix with nonnegative adjacency elements w_{ij}. An edge defined as $\varepsilon_{ij} = (v_i, v_j)$ implies that node v_i can receive information from node v_j. Node v_j is considered as a neighbor of node v_i if $\varepsilon_{ij} \in \mathscr{E}$. For all $i \in \Delta$, $w_{ii} > 0$, and $w_{ij} > 0$ if $\varepsilon_{ij} \in \mathscr{E}$; otherwise, $w_{ij} = 0$.

Consider continuous-time systems as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bv_1(t), \\ z(t) &= Ex(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $z(t) \in \mathbb{R}^p$ are the state vector and the output signal to be estimated, respectively; $v_1(t) \in \mathbb{R}^m$ is exogenous disturbance belonging to $\mathscr{L}_2[0,\infty)$; A, B, and E are known constant matrices of appropriate dimensions. The state x of the system (1) is observed by a network of N sensors, where each sensor is given as

$$y_i(t) = C_i x(t) + D_i v_2(t), \quad i = 1, 2, \dots, N, \tag{2}$$

where $y_i(t) \in \mathbb{R}^l$ is the measured output received by the sensor i from the plant, $v_2(t) \in \mathbb{R}^m$ is output measurement noise belonging to $\mathscr{L}_2[0,\infty)$, and C_i and D_i are known constant matrices of appropriate dimensions.

Due to the possible occurrence of random events in sensor networks, we consider a group of directed graph $\mathscr{G}(\gamma(t)) \in \{\mathscr{G}_1, \mathscr{G}_2, \dots, \mathscr{G}_q\}$, where $\gamma(t)$ is a continuous-time Markov process with values in a finite set $\mathscr{S} = \{1, 2, \dots, q\}$. The transition probabilities are defined as

$$\text{prob}\{\gamma(t+\Delta t) = s \mid \gamma(t) = r\} = \begin{cases} \pi_{rs}\Delta t + o(\Delta t), & r \neq s \\ 1 + \pi_{rr} + o(\Delta t), & r = s, \end{cases} \tag{3}$$

where $\Delta t > 0$, $o(\Delta t) \to 0$ as $\Delta t \to 0$, and π_{rs} is the transition rate from mode r to mode s, which satisfies $\pi_{rs} \geq 0$ for $r \neq s$ and $\pi_{rr} = -\sum_{s=1, s\neq r}^{m} \pi_{rs}$ for $r \in \mathscr{S}$. Then, it is easily known that $L(\gamma(t)) \in \{L_1, L_2, \dots, L_q\}$.

Remark 1. In many practical applications, for example, the marine oil pervasion monitoring systems deployed in an oceanic area, see in [21], the topologies of the mobile sensor network may vary (switch) with the coverage area and the spatial distribution of the pervading oil, which is affected by some random factors such as wind, sea wave, and temperature. In this case, it is suitable to model the randomly switching network topologies as a Markov process [21]. Another example for Markovian switching topology can be seen in [22]. Hence, in this paper we focus on the filtering problem in the sensor network with Markovian switching topologies.

In this paper, we investigate the filter design issue for sensor networks in a sampled-data setting. Denote $\hat{x}_i \in \mathbb{R}^n$ as the estimate of the plant's state $x(t)$ at sensor node i and define the output estimation error for sensor node i as

$$\tilde{y}_i(t) = y_i(t) - C_i\hat{x}_i(t). \tag{4}$$

It is assumed that the sampler is time-driven and the output estimation error $\tilde{y}_i(t)$ of sensor i is first sampled at each sampling instant s_k, and then the sampled output estimation error $\tilde{y}_i(s_k)$ will be sent to the ZOHs (zero order hold) of itself and its neighbors for the filter design through a communication network. Then, the ZOH of sensor i is used to collect the sampled data from itself and its neighbor sensor j and keep them constant until a new sampled data arrives. All the collected sampled data of the ZOH is then sent to filter i for the estimate of the state $x(t)$. Considering the negative effects of network uncertainty, all the sampled-data transmitted via communication network is assumed to suffer an expected communication delay τ, where τ is constant and larger than zero. Also, we assume that there exists a constant $h > 0$ such that $0 < s_{k+1} - s_k = h_{k+1} \le h$. Based on the above analysis, the filter to be designed for sensor i is given as follows:

$$\begin{aligned}\dot{\hat{x}}_i(t) &= A\hat{x}_i(t) - F_i(\gamma(s_k))\sum_{j=1}^{N} w_{ij}(\gamma(s_k))\,\tilde{y}_j(s_k),\\ \hat{z}_i(t) &= E\hat{x}_i(t), \quad t \in [s_k+\tau, s_{k+1}+\tau),\end{aligned} \tag{5}$$

where $\hat{x}_i(0) = 0$, $\hat{z}_i \in \mathbb{R}^p$ is the estimate of $z(t)$ at sensor node i, and matrix $F_i(\gamma(s_k))$ are parameters of filter i to be designed later.

Remark 2. In fact, due to the introduction of a communication network, sampled-data information during transmission may suffer the network uncertainty such as communication delay, data packet dropouts, and disorders. In order to make the analysis easier, we only take the effect of communication delay into account. Considering the effect of communication delays, data packet dropouts and disorders simultaneously will be investigated in the future work.

Let $e_i(t) = x - \hat{x}_i$ and $\tilde{z}_i(t) = z - \hat{z}_i$ be the local estimate error and the local filtering error at sensor i. From (1), (2), and (5), we have the following filtering error system for sensor node i:

$$\begin{aligned}\dot{e}_i(t) &= Ae_i(t) + F_i(\gamma(s_k))\sum_{j=1}^{N} w_{ij}(\gamma(s_k))\,C_je_j(s_k)\\ &\quad + Bv_1(t) + F_i(\gamma(s_k))\sum_{j=1}^{N} w_{ij}(\gamma(s_k))\,D_jv_2(s_k),\\ \tilde{z}_i(t) &= Ee_i(t), \quad t \in [s_k+\tau, s_{k+1}+\tau).\end{aligned} \tag{6}$$

Defining $e(t) = [e_1^T(t), e_2^T(t), \dots, e_N^T(t)]^T$ and $\tilde{z}(t) = [\tilde{z}_1^T(t), \tilde{z}_2^T(t), \dots, \tilde{z}_N^T(t)]^T$, we have

$$\begin{aligned}\dot{e}(t) &= \overline{A}e(t) + F_rL_rCe(s_k) + \overline{B}v_1(t) + F_rL_r\overline{D}v_2(s_k),\\ \tilde{z}(t) &= \overline{E}e(t), \quad t \in [s_k+\tau, s_{k+1}+\tau),\end{aligned} \tag{7}$$

where $\overline{A} = I_N \otimes A$, $C = \operatorname{diag}\{C_i\}$, $\overline{B} = \mathbf{1}_N \otimes B$, $\overline{D} = [D_1^T, D_2^T, \dots, D_N^T]^T$, $L_r = \mathscr{W} \otimes I_l$, $\overline{E} = I_N \otimes E$, and $F_r = \operatorname{diag}\{F_{ir}\}$ for each fixed $\gamma(t_k) = r$.

Define an "artificial delay" as $d_k(t) = t - s_k$, $t \in [s_k + \tau, s_{k+1} + \tau)$. Apparently, $d_k(t)$ is piecewise-linear with the derivative $\dot{d}_k(t) = 1$ at $t \ne s_k + \tau$ and is discontinuous at $t = s_k + \tau$. Then, it is clear that $\tau \le d_k(t) < h_{k+1} + \tau < h + \tau$, $t \in [s_k + \tau, s_{k+1} + \tau)$. Thus, the system (7) can be written as

$$\begin{aligned}\dot{e}(t) &= \overline{A}e(t) + F_rL_rCe(t - d_k(k)) + \overline{B}v_1(t)\\ &\quad + F_rL_r\overline{D}v_2(t - d_k(k)),\\ \tilde{z}(t) &= \overline{E}e(t), \quad t \in [s_k+\tau, s_{k+1}+\tau).\end{aligned} \tag{8}$$

The initial condition of state $e(t)$ is supplemented as $e(\theta) = \phi(\theta)$, $\theta \in [-\tau - h, 0]$, with $\phi(0) = e(0) = [e_1^T(0), e_2^T(0), \dots, e_N^T(0)]^T$ and $\phi \in W$, where W denotes the Banach space of absolutely continuous functions $[-\tau - h, 0] \to \mathbb{R}^N$ with square-integrable derivatives and the norm $\|\phi\|_W = \max_{\theta\in[-h-\tau,0]}\|\phi(\theta)\| + [\int_{-\tau-h}^{0}\|\dot{\phi}(s)\|^2 ds]^{1/2}$.

Next, we need to introduce the following definition.

Definition 3. The system (8) with $v_1(t) = v_2(t) = 0$ is said to be exponentially mean-square stable if there exist constants $\lambda > 0$ and $\beta > 0$ such that

$$\mathbb{E}\left\{\|e(t)\|^2\right\} \le \lambda e^{-\beta t}\sup_{-\tau-h\le\theta\le 0}\mathbb{E}\left\{\|\phi(\theta)\|^2\right\}. \tag{9}$$

The distributed H_∞ sampled-data filtering problem under consideration in the paper is to determine the parameters F_{ir} of the filter (5) such that the following requirements are simultaneously satisfied:

(i) the filtering error system (8) with $v_1(t) = v_2(t) = 0$ is exponentially mean-square stable;

(ii) under the zero-initial condition, for a prescribed H_∞ performance level γ, the filtering error $\tilde{z}(t)$ satisfies

$$\begin{aligned}&\mathbb{E}\left\{\int_0^\infty \|\tilde{z}(t)\|^2 dt\right\}\\ &\quad\le \gamma^2\int_0^\infty \|v_1(t)\|^2 + \|v_2(t - d_k(t))\|^2 dt\end{aligned} \tag{10}$$

for any nonzero $v_1(t), v_2(t) \in \mathscr{L}_2[0,\infty)$.

Before ending this section, the following lemmas are very useful for the proofs of the main results.

Lemma 4 (Schur complement [23]). *Let S be a symmetric real matrix represented by $\begin{bmatrix} s_{11} & s_{12}\\ s_{12}^T & s_{22}\end{bmatrix}$, where s_{22} is square and nonsingular. Then $S > 0$ if and only if $s_{22} > 0$ and $s_{11} - s_{12}s_{22}^{-1}s_{12} > 0$.*

Lemma 5 (see [24]). *For any constant matrix $R > 0 \in \mathbb{R}^{n\times n}$, scalar $\tau > 0$, and vector function $\dot{z} : [-\tau, 0] \rightarrow \mathbb{R}^n$ such that the following integration is well defined, then*

$$\begin{aligned} -\tau \int_{t-\tau}^{t} \dot{e}^T(s) R\dot{e}(s)\, ds \\ \leq -[e(t) - e(t-\tau)]^T R [e(t) - e(t-\tau)]. \end{aligned} \tag{11}$$

Lemma 6 (see [25]). *For given positive integers n, m, a scalar η in the interval $(0,1)$, and a given matrix $R > 0$, two matrices W_1 and W_2, define the function $f(\eta, R)$ for all vector $\varphi(t)$ in $\mathbb{R}^m$ as*

$$\begin{aligned} f(\eta, R) = \frac{1}{\eta}\varphi^T(t) W_1^T R W_1 \varphi(t) \\ + \frac{1}{1-\eta}\varphi^T(t) W_2^T R W_2 \varphi(t). \end{aligned} \tag{12}$$

If there exists a matrix $Y \in \mathbb{R}^{n\times n}$ such that $\left[\begin{smallmatrix} R & Y \\ * & R \end{smallmatrix}\right] \geq 0$, then the following inequality holds:

$$\min_{\eta\in(0,1)} f(\eta, R) \geq \varphi^T(t) \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}^T \begin{bmatrix} R & Y \\ * & R \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \varphi(t). \tag{13}$$

3. Main Results

3.1. H_∞ Filtering Analysis. In this subsection, we will derive a BRL for the filtering error system (8). For this purpose, we first choose a Lyapunov-Krasovskii functional as

$$V(t, e_t, \dot{e}_t) = \sum_{i=1}^{3} V_i(t, e_t, \dot{e}_t), \tag{14}$$

where

$$\begin{aligned} V_1(t, e_t, \dot{e}_t) &= e^T(t) P_r e(t), \\ V_2(t, e_t, \dot{e}_t) &= \int_{t-\tau}^{t} e^T(s) Q_1 e(s)\, ds \\ &\quad + \int_{t-h-\tau}^{t-\tau} e^T(s) Q_2 e(s)\, ds, \\ V_3(t, e_t, \dot{e}_t) &= \tau \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}^T(s) R_1 \dot{e}(s)\, ds\, d\theta \\ &\quad + \int_{-h-\tau}^{-\tau}\int_{t+\theta}^{t} \dot{e}^T(s) R_2 \dot{e}(s)\, ds\, d\theta, \end{aligned} \tag{15}$$

with $e_t(\theta) = e(t+\theta)$, $\theta \in [-h-\tau, 0]$, $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$. For the sake of simplicity, let $\psi(t) = [e^T(t), e^T(t-d_k(t)), e^T(t-\tau), e^T(t-h-\tau)]^T$, $\breve{\psi}(t) = [\psi^T(t), v_1^T(t), v_2^T(t-d_k(t))]^T$ and $\mathscr{E}_i$ be a block entry matrix with $\mathscr{E}_{ij} = \mathscr{E}_i - \mathscr{E}_j$. For example, $\mathscr{E}_1 = [I, 0, 0, 0, 0, 0]$ and $\mathscr{E}_{35} = [0, 0, I, 0, -I, 0]$.

Now, we state and establish the following result.

Theorem 7. *For given scalars $h > 0$, τ, and filter parameters F_{ir}, the filtering error system (8) with $v_1(t) = v_2(t) = 0$ is exponentially mean-square stable if there exist real matrices $P_r > 0$ $(r = 1, 2, \ldots, q)$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$ and some matrices Y of appropriate dimensions such that*

$$\begin{bmatrix} R_2 & Y \\ * & R_2 \end{bmatrix} \geq 0, \tag{16}$$

$$\Psi_r = \begin{bmatrix} \Gamma_r & \Upsilon_r^T \left(\tau^2 R_1 + h^2 R_2\right) \\ * & -\left(\tau^2 R_1 + h^2 R_2\right) \end{bmatrix} < 0 \tag{17}$$

for $r = 1, 2, \ldots, q$, where

$$\begin{aligned} \Gamma_r &= \mathscr{E}_1^T \left[\sum_{s=1}^{q} \pi_{rs} P_s + Q_1\right] \mathscr{E}_1 + \mathscr{E}_1^T P_r \Upsilon_r \\ &\quad + \Upsilon_r P_r \mathscr{E}_1 + \mathscr{E}_3^T (Q_2 - Q_1) \mathscr{E}_3 - \mathscr{E}_4^T Q_2 \mathscr{E}_4 \\ &\quad - \mathscr{E}_{13}^T R_1 \mathscr{E}_{13} - \begin{bmatrix} \mathscr{E}_{24} \\ \mathscr{E}_{32} \end{bmatrix}^T \begin{bmatrix} R_2 & Y \\ Y & R_2 \end{bmatrix} \begin{bmatrix} \mathscr{E}_{24} \\ \mathscr{E}_{32} \end{bmatrix} \\ \Upsilon_r &= \overline{A}\mathscr{E}_1 + F_r L_r C \mathscr{E}_2. \end{aligned} \tag{18}$$

Proof. Define the weak infinitesimal operator $\mathfrak{L}$ of $V(t, z_t, \dot{z}_t)$ along the trajectory (8) with respect to $t \in [s_k + \tau, s_{k+1} + \tau)$ as

$$\begin{aligned} \mathfrak{L}V(t, z_t, \dot{z}_t) = \lim_{\Delta\to 0^+} \frac{1}{\Delta} \{\mathbb{E}\{V(t+\Delta, z_{t+\Delta}, \dot{z}_{t+\Delta}) \mid z_t\} \\ - V(t, z_t, \dot{z}_t)\}. \end{aligned} \tag{19}$$

Then, along with (14), we have

$$\mathfrak{L}V(t, z_t, \dot{z}_t) = \sum_{i=1}^{3} \mathfrak{L}V_i(t, z_t, \dot{z}_t), \tag{20}$$

where

$$\begin{aligned} \mathfrak{L}V_1(t, z_t, \dot{z}_t) &= 2\psi^T(t) \mathscr{E}_1^T \left[P_r \Upsilon_r + \frac{1}{2}\sum_{s=1}^{q} \pi_{rs} P_s \mathscr{E}_1\right] \psi(t), \\ \mathfrak{L}V_2(t, z_t, \dot{z}_t) &= \psi^T(t) \left\{\mathscr{E}_1^T Q_1 \mathscr{E}_1 + \mathscr{E}_3^T (Q_2 - Q_1) \mathscr{E}_3 \right. \\ &\quad \left. - \mathscr{E}_4^T Q_2 \mathscr{E}_4\right\} \psi(t), \\ \mathfrak{L}V_3(t, z_t, \dot{z}_t) &= \psi^T(t) \left\{\Upsilon_r^T \left[\tau^2 R_1 + h^2 R_2\right] \Upsilon_r\right\} \psi(t) + \eta_1 + \eta_2, \end{aligned} \tag{21}$$

with $\eta_1 = -\tau \int_{t-\tau}^{t} \dot{e}^T(s) R_1 \dot{e}(s) ds$ and $\eta_2 = -h \int_{t-h-\tau}^{t-\tau} \dot{e}^T(s) R_2 \dot{e}(s) ds$. Applying Lemma 5, we have

$$\eta_1(t) \leq -\psi^T(t) \mathscr{E}_{13}^T R_1 \mathscr{E}_{13} \psi(t). \tag{22}$$

Notice that

$$\begin{aligned}\eta_2 &= -h\int_{t-\tau-h}^{t-d_k(t)} \dot{e}^T(s) R_2 \dot{e}(s)\,ds \\ &\quad - h\int_{t-d_k(t)}^{t-\tau} \dot{e}^T(s) R_2 \dot{e}(s)\,ds \\ &\le -\frac{h}{\tau+h-d_k(t)}\psi^T(t)\mathscr{E}_{24}^T R_2 \mathscr{E}_{24}\psi(t) \\ &\quad - \frac{h}{d_k(t)-\tau}\psi^T(t)\mathscr{E}_{32}^T R_2 \mathscr{E}_{32}\psi(t).\end{aligned} \tag{23}$$

Using Lemma 6, we obtain

$$\eta_2 \le -\psi^T(t)\begin{bmatrix}\mathscr{E}_{24}\\ \mathscr{E}_{32}\end{bmatrix}^T \begin{bmatrix}R_2 & Y\\ Y & R_2\end{bmatrix}\begin{bmatrix}\mathscr{E}_{24}\\ \mathscr{E}_{32}\end{bmatrix}\psi(t). \tag{24}$$

Taking the mathematical expectation on both sides of (20) and from (20)–(24), we have for $t \in [s_k+\tau, s_{k+1}+\tau)$

$$\mathbb{E}\{\mathfrak{L}V(t,z_t,\dot{z}_t)\} \le \mathbb{E}\{\psi^T(t)\Sigma_r\psi(t)\}, \tag{25}$$

where $\Sigma_r = \Gamma_r + \Upsilon_r^T(\tau^2R_1+h^2R_2)\Upsilon_r$. It is clear that if $\Sigma_r < 0$, then there exists a small enough constant $c_1 > 0$ such that $\mathbb{E}\{\mathscr{L}V(t,e_t,\dot{e}_t)\} \le -c_1\mathbb{E}\{\|e(t)\|^2)\}$, which means that the system (8) is mean-square stable.

Define a new function $\mathscr{V} = e^{-\varepsilon t}V(t,z_t,\dot{z}_t)$, where $\varepsilon > 0$ is a scalar to be determined. Then, we have

$$\mathfrak{L}\mathscr{V} = e^{\varepsilon t}\left[\varepsilon V(t,z_t,\dot{z}_t) + \mathfrak{L}V(t,z_t,\dot{z}_t)\right]. \tag{26}$$

Integrating both sides of (26) from 0 to $T > 0$ and taking the expectation, one can obtain that

$$\begin{aligned}&\mathbb{E}\{e^{\varepsilon T}V(T,e_T,\dot{e}_T)\} - \mathbb{E}\{V(0,e_0,\dot{e}_0)\} \\ &= \int_0^T \varepsilon e^{\varepsilon t}\mathbb{E}\{V(t,e_t,\dot{e}_t)\}\,dt + \int_0^T e^{\varepsilon t}\mathbb{E}\{\mathfrak{L}V(t,e_t,\dot{e}_t)\}\,dt.\end{aligned} \tag{27}$$

Using the similar method in [26], we can know that there exists a scalar $\lambda > 0$ such that for $T \ge 0$

$$\mathbb{E}\{V(T,e_T,\dot{e}_T)\} \le \lambda e^{-\varepsilon T}\sup_{-2(\tau+h)\le\theta\le 0}\mathbb{E}\{\|\phi(\theta)\|^2\}. \tag{28}$$

Due to the fact that $V(T,z_T,\dot{z}_T) \ge \kappa z^T(T)z(T)$, where $\kappa = \min_{r=1,2,\dots,q}\{\lambda_{\min}(P_r)\}$,

$$\mathbb{E}\{\|e(T)\|^2\} \le \frac{\lambda}{\kappa}e^{-\varepsilon T}\sup_{-2(\tau+h)\le\theta\le 0}\mathbb{E}\{\|\phi(\theta)\|^2\}. \tag{29}$$

Therefore, it can be concluded from Definition 3 that the error system (8) is exponentially mean-square stable. Applying Lemma 4 to Γ_r, one can arrive at (17). The proof is completed. □

Next, we are in a position to obtain a sufficient condition that guarantees the H_∞ performance in (11) for the filtering error system (8).

Theorem 8. *For given scalars $h > 0$, τ, and filter parameters F_{ir}, the filtering error system* (8) *is exponentially mean-square stable with a prescribed H_∞ performance level γ if there exist real matrices $P_r > 0$ $(r = 1,2,\dots,q)$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$ and some matrices Y of appropriate dimensions such that*

$$\begin{bmatrix}R_2 & Y\\ * & R_2\end{bmatrix} \ge 0, \tag{30}$$

$$\breve{\Psi}_r = \begin{bmatrix}\breve{\Gamma}_r & \breve{\Upsilon}_r^T(\tau^2R_1+h^2R_2)\\ * & -(\tau^2R_1+h^2R_2)\end{bmatrix} < 0 \tag{31}$$

for $r = 1,2,\dots,q$, where

$$\begin{aligned}\breve{\Gamma}_r &= \mathscr{E}_1^T\left[\sum_{s=1}^q \pi_{rs}P_s + Q_1 + \overline{E}^T\overline{E}\right]\mathscr{E}_1 + \mathscr{E}_1^TP_r\breve{\Upsilon}_r + \breve{\Upsilon}_rP_r\mathscr{E}_1 \\ &\quad + \mathscr{E}_3^T(Q_2-Q_1)\mathscr{E}_3 - \mathscr{E}_4^TQ_2\mathscr{E}_4 - \mathscr{E}_{13}^TR_1\mathscr{E}_{13} \\ &\quad - \begin{bmatrix}\mathscr{E}_{24}\\ \mathscr{E}_{32}\end{bmatrix}^T\begin{bmatrix}R_2 & Y\\ Y & R_2\end{bmatrix}\begin{bmatrix}\mathscr{E}_{24}\\ \mathscr{E}_{32}\end{bmatrix} - \gamma^2\mathscr{E}_5^T\mathscr{E}_5 - \gamma^2\mathscr{E}_6^T\mathscr{E}_6, \\ \breve{\Upsilon}_r &= \overline{A}\mathscr{E}_1 + F_rL_rC\mathscr{E}_2 + \overline{B}\mathscr{E}_5 + F_rL_r\overline{D}\mathscr{E}_6.\end{aligned} \tag{32}$$

Proof. First, it is easily known that if the inequality (31) holds, then the inequality (17) is satisfied, which ensures that the filtering error system (8) with $\nu_1(t) = \nu_2(t) = 0$ is exponentially mean-square stable. The remaining proof is to guarantee that under zero initial conditions, the filtering error system (8) satisfies the H_∞ performance (11). Similar to the proof of Theorem 7, we can calculate

$$\begin{aligned}\dot{V}(t,e_t,\dot{e}_t) &\le \breve{\psi}^T(t)\breve{\Sigma}\breve{\psi}(t) - z(t)^Tz(t) \\ &\quad + \gamma^2\nu_1(t)^T\nu_1(t) \\ &\quad + \gamma^2\nu_2(t-d_k(t))^T\nu_2(t-d_k(t)),\end{aligned} \tag{33}$$

where $\breve{\Sigma} = \breve{\Gamma} + \breve{\Upsilon}_r^T(\tau^2R_1+h^2R_2)\breve{\Upsilon}_r$. It is clear from Lemma 4 that the inequality (31) leads to $\breve{\Sigma} < 0$. Under the zero initial condition, it follows from (33) that the inequality (11) holds. The proof of Theorem 8 is completed. □

3.2. H_∞ Filter Design. Based on Theorem 8, we will derive a sufficient condition on the existence of the topology dependent H_∞ filter parameters F_{ir} as follows.

Theorem 9. *For given scalars $h > 0$, τ, μ, the distributed filtering problem is solvable by the filters* (5) *if there exist real matrices $\widehat{P}_r = \mathrm{diag}\{\widehat{P}_i\} > 0$ $(r = 1,2,\dots,q)$, $\widehat{F} = \mathrm{diag}\{\widehat{F}_i\}$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$ and some matrices Y of appropriate dimensions such that*

$$\begin{bmatrix}R_2 & Y\\ * & R_2\end{bmatrix} \ge 0, \tag{34}$$

$$\widehat{\Psi}_r = \begin{bmatrix}\widehat{\Gamma}_r & \widehat{\Upsilon}_r^T\\ * & \mu^2(\tau^2R_1+h^2R_2) - 2\mu\widehat{P}_r\end{bmatrix} < 0 \tag{35}$$

for $r = 1, 2, \ldots, q$, *where*

$$\begin{aligned}\widehat{\Gamma}_r &= \mathscr{E}_1^T \left[\sum_{s=1}^{q} \pi_{rs} P_s + Q_1 + \overline{E}^T \overline{E}\right] \mathscr{E}_1 + \mathscr{E}_1^T \widehat{\Upsilon}_r + \widehat{\Upsilon}_r \mathscr{E}_1 \\ &\quad + \mathscr{E}_3^T (Q_2 - Q_1) \mathscr{E}_3 - \mathscr{E}_4^T Q_2 \mathscr{E}_4 - \mathscr{E}_{13}^T R_1 \mathscr{E}_{13} \\ &\quad - \begin{bmatrix}\mathscr{E}_{24} \\ \mathscr{E}_{32}\end{bmatrix}^T \begin{bmatrix} R_2 & Y \\ Y & R_2\end{bmatrix} \begin{bmatrix}\mathscr{E}_{24} \\ \mathscr{E}_{32}\end{bmatrix} - \gamma^2 \mathscr{E}_5^T \mathscr{E}_5 - \gamma^2 \mathscr{E}_6^T \mathscr{E}_6, \\ \widehat{\Upsilon}_r &= \widehat{P}_r \overline{A} \mathscr{E}_1 + \widehat{F}_r L_r C \mathscr{E}_2 + \widehat{P}_r \overline{B} \mathscr{E}_5 + \widehat{F}_r L_r \overline{D} \mathscr{E}_6.\end{aligned} \tag{36}$$

Moreover, the filter gain F_{ir} *is given by*

$$F_{ir} = \widehat{P}_{ir}^{-1} \widehat{F}_{ir}. \tag{37}$$

Proof. First, for any real symmetric matrix R and a given scalar $\mu > 0$, we have that $(\mu R - P)^T R^{-1} (\mu R - P) \geq 0$, which implies that

$$-PR^{-1}P \leq \mu^2 R - 2\mu P. \tag{38}$$

Let $P_r = \widehat{P}_r = \mathrm{diag}\{\widehat{P}_{ir}\}$ for the conditions (17) in Theorem 7, and define $\widehat{F}_{ir} = \widehat{P}_{ir} F_{ir}$, $J_r = \mathrm{diag}\{I, I, I, I, I, I, (\tau^2 R_1 + h^2 R_2)^{-1} P_r, I\}$. Pre- and postmultiplying both sides of (31) by J, respectively, and using Lemma 4, one can obtain (35), where the term $P_r (\tau^2 R_1 + h^2 R_2)^{-1} P_r$ is dealt with by the inequality (38). The proof of Theorem 9 is completed. □

Remark 10. It should be pointed out that, to simplify the analysis, we assume that the communication delay is constant and larger than zero. In fact, the design method proposed in this paper can be extended to the more general case that the communication delay $\tau(t)$ is time-varying with an upper bound.

4. A Numerical Example

In this section, a numerical example is given to illustrate the effectiveness of the proposed method.

Example 1. Consider a continuous linear system as

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} 0.1 & -1 \\ 0 & -0.5 \end{bmatrix} x(t) + \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix} v_1(t), \\ z(t) &= \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} x(t),\end{aligned} \tag{39}$$

with the initial state $x(0) = [0, 0]^T$ and the exogenous disturbance $v_1(t) = (\sin(0.5t))/(1+5t)$. The state of plant (39) is estimated by a sensor network with 4 sensor nodes, where the parameters of sensor i are given by

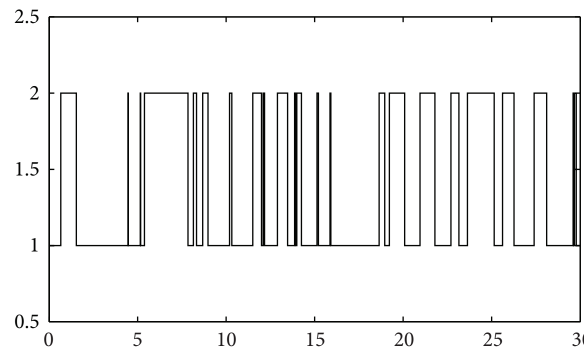

Figure 1: Markov chain.

$$\begin{aligned} C_1 &= [1, 0]; \\ C_2 &= [0.25, -0.75]; \\ C_3 &= [-0.5, -1.5]; \\ C_4 &= [-1, 0.1]; \\ D_1 &= 0.3; \\ D_2 &= 0.1; \\ D_3 &= 0.15; \\ D_4 &= 0.06 \end{aligned} \tag{40}$$

and output measurement noise is given by $v_2(t) = 3e^{-t}\cos(0.2 + 0.3\pi)t$. The network topologies of the sensor network are given by two directed graphs, whose adjacency matrices are given by

$$\mathscr{W}_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}; \qquad \mathscr{W}_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}. \tag{41}$$

The topology switching is trigged by a Markov chain, whose transition rate is given by

$$\pi = \begin{bmatrix} -0.5 & 0.5 \\ 0.8 & -0.8 \end{bmatrix}. \tag{42}$$

Then, the switching rule of Markov chain is shown in Figure 1.

By Theorem 9, we design the filter parameters F_{ir}. Set the sampling period

$$h_k = \begin{cases} 0.1\ \mathrm{s} & k = 1, 3, 5, \ldots \\ 0.05\ \mathrm{s} & k = 2, 4, 6, \ldots. \end{cases} \tag{43}$$

Then, it is easily known that the maximum sampling period $h = 0.1$ s. It is assumed that all the sampled data is subject to a communication delay $\tau = 0.05$ in the process of transmission. By choosing the performance level $\gamma = 1$ and the parameter

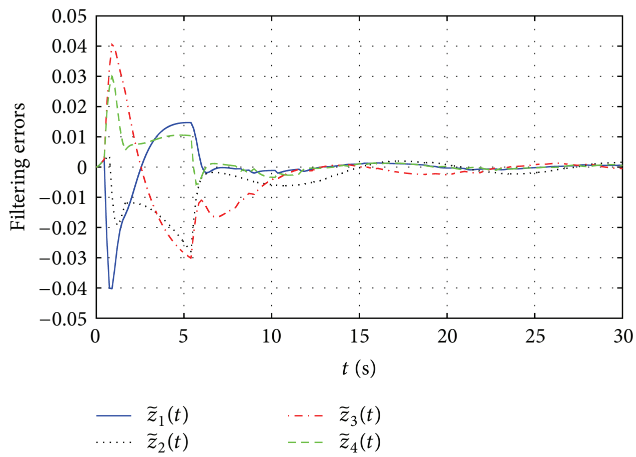

Figure 2: Filtering errors $\tilde{z}_i(t)$.

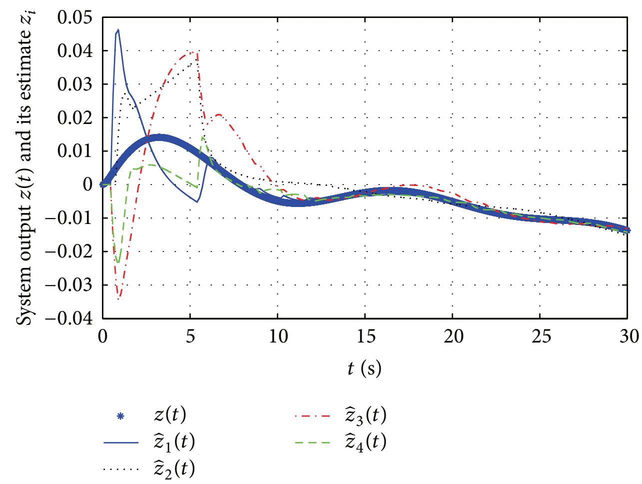

Figure 3: Output of system $z(t)$ and its estimation $\hat{z}_i(t)$.

$\mu = 10$, we employ Theorem 9 to obtain the suitable filer gains F_{ir} as follows

$$
\begin{aligned}
F_{11} &= \begin{bmatrix} -1.8108 \\ -0.4434 \end{bmatrix}, \\
F_{21} &= \begin{bmatrix} -0.0236 \\ -0.0177 \end{bmatrix}, \\
F_{31} &= \begin{bmatrix} 2.4356 \\ 0.2352 \end{bmatrix}, \\
F_{41} &= \begin{bmatrix} 1.0779 \\ 0.2406 \end{bmatrix}, \\
F_{21} &= \begin{bmatrix} -2.1748 \\ -0.5260 \end{bmatrix}, \\
F_{22} &= \begin{bmatrix} -7.6882 \\ 1.1413 \end{bmatrix}, \\
F_{32} &= \begin{bmatrix} 2.7854 \\ 0.1978 \end{bmatrix}, \\
F_{42} &= \begin{bmatrix} 3.1132 \\ 0.8868 \end{bmatrix}.
\end{aligned}
\tag{44}
$$

Based on the derived filter gains, we depict the filtering errors of node i in Figure 2 and the system output and its estimates from filter i in Figure 3. It follows from all the figures that the method proposed in the paper can effectively solve the distributed sampled-data filtering problem in sensor networks with a desired performance level γ.

5. Conclusion

We have proposed a distributed H_∞ sampled-data filtering in sensor networks with Markovian switching topologies. Each sensor node can receive sampled data of output measurement from itself and its neighbouring nodes for filter design. In this scheme, the distributed H_∞ sampled-data filtering problem in sensor networks can be converted into the H_∞ mean-square stable problem of a Markovian jump system with an interval time-varying delay. By using Lyapunov-Krasovskii functional approach, we have given a new BRL to guarantee the mean-square stability of the transformed system with a desired performance index γ. Based on this BRL, we have derived the filter gains corresponding to the network topology switching by solving a set of LMIs. Finally, the effectiveness of the proposed method has been illustrated by a numerical example.

In the future research, we will investigate network-based H_∞ filtering for sensor networks like [27]. Another possible research direction is to extend the proposed method to sensor networks under energy constraints [28].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] R. Olfati-Saber, “Distributed Kalman filter with embedded consensus filters,” in *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference (CDC-ECC ’05)*, pp. 8179–8184, IEEE, Seville, Spain, December 2005.

[2] R. Olfati-Saber, “Distributed Kalman filtering for sensor networks,” in *Proceedings of the 46th IEEE Conference on Decision and Control (CDC ’07)*, pp. 5492–5498, IEEE, New Orleans, La, USA, December 2007.

[3] W. Yu, G. Chen, Z. Wang, and W. Yang, “Distributed consensus filtering in sensor networks,” *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 39, no. 6, pp. 1568–1577, 2009.

[4] P. Millán, L. Orihuela, C. Vivas, and F. R. Rubio, "Distributed consensus-based estimation considering network induced delays and dropouts," *Automatica*, vol. 48, no. 10, pp. 2726–2729, 2012.

[5] Z. Duan, J. Zhang, C. Zhang, and E. Mosca, "Robust H_2 and H_∞ filtering for uncertain linear systems," *Automatica*, vol. 42, no. 11, pp. 1919–1926, 2006.

[6] S. Xu, J. Lam, and X. Mao, "Delay-dependent H_∞ control and filtering for uncertain Markovian jump systems with time-varying delays," *IEEE Transactions on Circuits and Systems I*, vol. 54, no. 9, pp. 2070–2077, 2007.

[7] X.-M. Zhang and Q.-L. Han, "Robust H_∞ filtering for a class of uncertain linear systems with time-varying delay," *Automatica*, vol. 44, no. 1, pp. 157–166, 2008.

[8] B. Shen, Z. Wang, and Y. S. Hung, "Distributed H_∞-consensus filtering in sensor networks with multiple missing measurements: the finite-horizon case," *Automatica*, vol. 46, no. 10, pp. 1682–1688, 2010.

[9] V. Ugrinovskii, "Distributed robust filtering with H_∞ consensus of estimates," *Automatica*, vol. 47, no. 1, pp. 1–13, 2011.

[10] Z. Wang, B. Shen, and X. Liu, "H_∞ filtering with randomly occurring sensor saturations and missing measurements," *Automatica*, vol. 48, no. 3, pp. 556–562, 2012.

[11] H. Yu, Y. Zhuang, and W. Wang, "Distributed H_∞ filtering in sensor networks with randomly occurred missing measurements and communication link failures," *Information Sciences*, vol. 222, pp. 424–438, 2013.

[12] P. Shi, "Filtering on sampled-data systems with parametric uncertainty," *IEEE Transactions on Automatic Control*, vol. 43, no. 7, pp. 1022–1027, 1998.

[13] D. Yue, Q.-L. Han, and J. Lam, "Network-based robust H_∞ control of systems with uncertainty," *Automatica*, vol. 41, no. 6, pp. 999–1007, 2005.

[14] Z. Wang, B. Huang, and P. Huo, "Sampled-data filtering with error covariance assignment," *IEEE Transactions on Signal Processing*, vol. 49, no. 3, pp. 666–670, 2001.

[15] M. Liu, J. You, and X. Ma, "H_∞ filtering for sampled-data stochastic systems with limited capacity channel," *Signal Processing*, vol. 91, no. 8, pp. 1826–1837, 2011.

[16] X.-M. Zhang and Q.-L. Han, "Network-based H_∞ filtering using a logic jumping-like trigger," *Automatica*, vol. 49, no. 5, pp. 1428–1435, 2013.

[17] B. Shen, Z. Wang, and X. Liu, "A stochastic sampled-data approach to distributed H_∞ filtering in sensor networks," *IEEE Transactions on Circuits and Systems I*, vol. 58, no. 9, pp. 2237–2246, 2011.

[18] I. Matei and J. S. Baras, "Consensus-based linear distributed filtering," *Automatica*, vol. 48, no. 8, pp. 1776–1782, 2012.

[19] H. Dong, Z. Wang, and H. Gao, "Distributed filtering for a class of time-varying systems over sensor networks with quantization errors and successive packet dropouts," *IEEE Transactions on Signal Processing*, vol. 60, no. 6, pp. 3164–3173, 2012.

[20] V. Ugrinovskii, "Distributed robust estimation over randomly switching networks using H_∞ consensus," *Automatica*, vol. 49, no. 1, pp. 160–168, 2013.

[21] G. Guo, "Linear systems with medium-access constraint and Markov actuator assignment," *IEEE Transactions on Circuits and Systems I*, vol. 57, no. 11, pp. 2999–3010, 2010.

[22] G. Guo, Z. Lu, and Q.-L. Han, "Control with Markov sensors/actuators assignment," *IEEE Transactions on Automatic Control*, vol. 57, no. 7, pp. 1799–1804, 2012.

[23] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, vol. 15 of *SIAM Studies in Applied Mathematics*, SIAM, Philadelphia, Pa, USA, 1987.

[24] Q.-L. Han, "Absolute stability of time-delay systems with sector-bounded nonlinearity," *Automatica*, vol. 41, no. 12, pp. 2171–2176, 2005.

[25] P. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.

[26] X. Mao, "Robustness of exponential stability of stochastic differential delay equations," *IEEE Transactions on Automatic Control*, vol. 41, no. 3, pp. 442–447, 1996.

[27] L. Ding, Q.-L. Han, and G. Guo, "Network-based leader-following consensus for distributed multi-agent systems," *Automatica*, vol. 49, no. 7, pp. 2281–2286, 2013.

[28] G. Guo, Y. Zhao, and G. Yang, "Cooperation of multiple mobile sensors with minimum energy cost for mobility and communication," *Information Sciences*, vol. 254, pp. 69–82, 2014.