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A dynamical analysis of the proposed HU Aquarii planetary system

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ABSTRACT

It has recently been suggested that the eclipsing polar HU Aquarii is a host to at least two giant planets. We have performed highly detailed dynamical analyses of the orbits of those planets and showed that the proposed system is highly unstable on time-scales of $<5 \times 10^3$ yr. For the coplanar orbits suggested in the discovery Letter, we find stable orbital solutions for the planetary system only if the outer body moves on an orbit that brings it no closer to the host star than ∼6 au. The required periastron distance for the outer planet lies approximately 5 Hill radii beyond the orbit of the inner planet, and well beyond the 1σ error bars placed on the orbit of the outer planet in the discovery Letter. If the orbits of the proposed planets are significantly inclined with respect to one another, the median stability increases slightly, but such systems still become destabilized on astronomically minute time-scales (typically within a few 10^4 yr). Only in the highly improbable scenario where the outer planet follows a retrograde but coplanar orbit (i.e. inclined by 180° to the orbit of the inner planet) is there any significant region of stability within the original 1σ orbital uncertainties. Our results suggest that, if there is a second (and potentially, a third planet) in the HU Aquarii system, its orbit is dramatically different from that suggested in the discovery Letter, and that more observations are critically required in order to constrain the nature of the suggested orbital bodies.

Key words: planets and satellites: dynamical evolution and stability – binaries: close – binaries: eclipsing – stars: individual: HU Aquarii – planetary systems – white dwarfs.

1 INTRODUCTION

Precise measurements of timing variations of strictly periodic events have been successfully used to infer the existence bodies orbiting around distant stars. Perhaps the best-known examples are the pulsar planets of Wolszczan & Frail (1992), a system of three extremely-low-mass planets orbiting the millisecond pulsar PSR 1257+12. Pulsating subdwarf B stars have also been found to host planets, for example, V391 Peg (Silvotti et al. 2007) and HW Vir (Lee et al. 2009). The same technique has recently been applied to eclipsing polars, using the egress of the small, bright accretion spot as a precise ‘clock’. Unseen orbiting bodies can cause small shifts in the timing of eclipses (ranging from ∼10 s to a few minutes) due to the light-traveltime differences imposed by the gravitational influence of the orbiting bodies on the system barycentre. Qian et al. (2010) discovered the first such planet orbiting the eclipsing polar DP Leo by combining their observed eclipse timings with a long-term data set from Schwope et al. (2002); the combined data set revealed a sinusoidal variation indicative of a 6.3$M_{\rm Jup}$ planet with a period of 23.8 yr. The timing method has shown that planets can orbit stars which are wildly different from the main-sequence solar-type stars most commonly targeted by Doppler and transit planet-search programmes.

In a recent Letter, Qian et al. (2011) announced the discovery of two (and potentially more) giant planets orbiting the eclipsing polar HU Aquarii (hereinafter HU Aqr). The authors provided fits to the orbits of those planets, placing them at the orbital radii of 3.6 and 5.4 au, from the system barycentre, and ascribed the minimum masses of 5.9 and 4.5$M_{\rm Jup}$ (respectively) to the two bodies. In this Letter, we perform a detailed dynamical analysis of the HU Aqr planetary system in order to assess the stability of the proposed planet candidates.

2 THE HU AQR PLANETARY SYSTEM

In their study of the eclipsing polar system HU Aqr, Qian et al. (2011) consider the temporal variation in the observed eclipse timings (‘O’) as compared to predicted timings (‘C’) that would be expected from a linear ephemeris. By plotting a simple O−C diagram, they show that the O−C residuals for HU Aqr contain two cyclical signals superposed on a longer period curvature. Each signal can be modelled as a Keplerian orbit to determine the planetary parameters. We reproduce the parameter estimates of Qian et al. (2011) in Table 1. As for planets detected with the radial velocity

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Table 1. The orbits of the HU Aqr exoplanets (Qian et al. 2011).

Parameter	HU Aqr (AB)b	HU Aqr (AB)c
Eccentricity	0.0	0.51 ± 0.15
Orbital period (yr)	6.54 ± 0.01	11.96 ± 1.41
Orbital radius (au)	3.6 ± 0.8	5.4 ± 0.9
Minimum mass (M_{Jup})	5.9 ± 0.6	4.5 ± 0.5

method, only the radial component of the planet's influence on the host star is detectable. Here, only the line-of-sight light-traveltime differences are observed, so the mass estimates for the HU Aqr planets are given as minimum values. Qian et al. (2011) note that the HU Aqr system inclination is 85°, so, if the planets orbit in the same plane as the stars, then their true masses would only be 0.4 per cent larger than the minimum values given in Table 1.

3 A DYNAMICAL SEARCH FOR STABLE ORBITS

In order to examine the potential dynamical stability of the two planets suggested for the HU Aqr system, we performed a large number of detailed dynamical simulations using the Hybrid integrator within the N-body dynamical package MERCURY (Chambers 1999). Following the strategy employed to analyse the stability of the HR 8799 system (Marshall, Horner & Carter 2010), we held the orbit of the inner planet constant (with $a = 3.6$ au and $e = 0.0$) and varied the orbital elements of the outer planet across a range corresponding to ± 3 times the discovery Letter's quoted uncertainties in the semi-major axis, a, and eccentricity, e. We initially considered the scenario described in Qian et al. (2011), where the planets were considered to be coplanar. In other words, we set the orbital inclinations, i, of the two planets to be 0° at the start of our integrations. We treat the central stars, a 0.88-$M_\odot$ white dwarf and a 0.2-$M_\odot$ secondary, as a single point mass. Since the stars orbit each other with a period of only 2.08 h (i.e. with a separation of 0.004 au), and the bodies of interest are believed to orbit at the distances of 3.6 and 5.4 au, this treatment is dynamically justified. We give each planet the minimum mass estimated in Qian et al. (2011). We note in passing that, if the masses of the proposed planets are significantly greater than those detailed in Qian et al. (2011), then this could only have a deleterious effect on the stability of their proposed orbits.

Fixing the orbit of the inner planet, we simulated a total of 9261 planetary systems. In each simulated system, the inner planet began on the same orbit, but the orbital elements of the outer planet were chosen such that each simulation sampled a unique set of possible parameters. We distributed the orbital elements of the outer planet such that we tested 21 values of the semi-major axis, spread evenly across $\pm 3\sigma$ from the value ($a = 5.4$ au) given in Qian et al. (2011). For each value of the semi-major axis, we tested 21 values of orbital eccentricity, again spread evenly across $\pm 3\sigma$ from the value ($e = 0.51$) given in that work. Finally, at each of these 441 (a, e) locations, we carried out 21 tests, with the initial location of the planet distributed across a range of $\pm 3\sigma$ from the nominal mean anomaly of the outer planet (calculated from fig. 2 of Qian et al. 2011). These 9261 unique models, based on the HU Aqr system parameters, were integrated using the Hybrid integrator within MERCURY for a period of 100 Myr, following the evolution and final fates of the two postulated planets in the system. A planet was deemed ejected from the system upon reaching a distance of 1000 au from the barycentre, and all mutual collisions were recorded. This yielded a lifetime from each individual integration in the range 0–100 Myr, defined as the time until one or of the planets was removed from the system through either collision or ejection.

To explore the 1σ parameter range in greater detail, we launched a second suite of integrations, again using 9261 test systems. The setup was performed exactly as described above, except that the orbital parameters of the outer planet were varied within a 1σ range, rather than the 3σ distribution carried out previously.

These two suites of integrations yielded 17 493 distinct tests of the stability of the HU Aqr system, with 9261 of these performed in the central $\pm 1\sigma$ of the element space described in Qian et al. (2011) and the other 8232 distributed in the range 1–3σ. The results of these integrations are shown in Fig. 1.

Once these simulations had been completed, we carried out equivalent suites for scenarios where the outer planet was moving on an orbit inclined to that of the inner planet. Following the procedure detailed above, we tested systems in which the outer planet's orbit was inclined by 5°, 15° and 45° with respect to the inner's orbit, in order to examine the influence of mutual orbital inclinations on the stability of the system. We then considered further scenarios in which the outer planet was moving in a retrograde sense, with respect to the inner body, with the inclinations of 135° and 180°. For simplicity, we kept the masses of the two planets constant through these runs

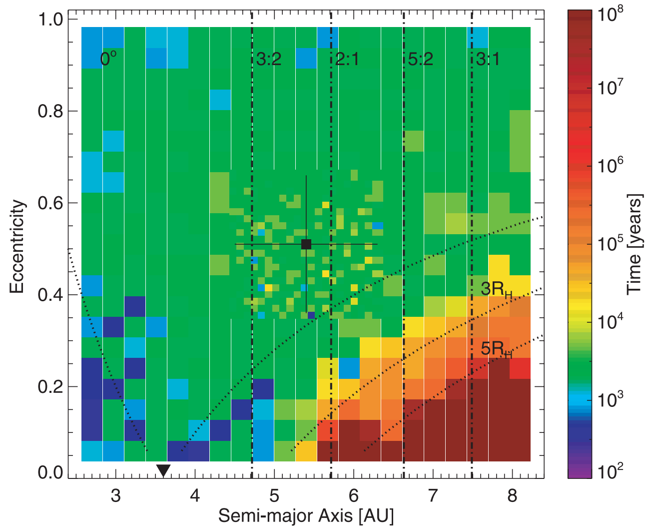

Figure 1. Lifetime stability plot of the HU Aqr system using the parameters of Qian et al. (2011) with the planets on coplanar orbits. The simulations detailed cover the 3σ parameter space in a grid of 21 × 21 points. The central 1σ parameter space is covered by an additional and denser set of 21 × 21 points. Each grid point is the median lifetime of 21 simulated HU Aqr systems with the outer planet's initial a and e. The location of the inner planet is denoted by the point of the solid triangle on the x-axis, while the nominal orbit of the outer planet is marked by the filled square. The solid lines show the extent of the 1σ errors in a and e suggested by Qian et al. (2011). The vertical dot–dashed lines show the location of the strongest MMRs in relation to the orbit of the inner planet. The two dotted lines radiating from the location of the inner planet connect all orbits that have either their periastron (outward curving line) or apastron (inward curving line) at the location of the planet. As such, all orbits in the region bounded by these lines cross the orbit of the inner planet. The dotted lines labelled 3 and $5R_{\mathrm{H}}$ connect all orbits that pass periastron at a distance of 3 or $5R_{\mathrm{H}}$ beyond the orbit of the inner planet. It can readily be seen that the planetary system shows extreme instability, aside from in a small region of the a–e space to the lower right-hand side of the figure, corresponding to orbits of the outer planet that remain at a barycentric distance beyond ∼6 au.

at the lowest values suggested in Qian et al. (2011). Although it is true that significantly inclined orbits for the planets would result in larger real masses for them, we note that a significant mutual inclination between the planets does not necessarily mean that it is the outermost planet that is inclined to the line of sight, while the innermost planet is in that plane. Rather than attempting to shift the planetary mass by the free parameter of potential inclination, we instead took the lowest masses possible. We remind the reader that this essentially means that we have allowed the planetary system the best possible chance of being stable – as the mass of the planets increases, so does their gravitational reach, increasing the strength of any mutual interactions.

4 RESULTS

The results of our $i = 0°$ (i.e. coplanar) dynamical integrations are shown in Fig. 1. Each colour box in that figure shows the median lifetime obtained from 21 independent integrations performed with the outer planet placed on an orbit with that particular combination of a and e. The location of the inner planet is marked with a hollow circle, whilst the location of the nominal orbit for the outer planet given in Qian et al. (2011) is shown by a hollow square, with the 1σ error bars given in that work denoted by the solid lines stretching from that square. The vertical dot–dashed lines show the locations of the strongest mean-motion resonances (hereinafter MMRs) with the orbit of the innermost planet. The curved dotted lines that meet at the location of the inner planet join all orbits whose periastron (outward curving line) or apastron (inward curving line) lies at a distance equal to the orbital radius of the inner planet. Each point in the region above these lines denotes orbits for the outer planet that cross that of the inner planet. The two dotted lines labelled 3 and $5R_{\rm H}$ connect orbits which pass periastron at a distance equal to the orbital radius of the inner planet plus three and five times that of the planet's Hill radius ($R_{\rm H}$), respectively, where $R_{\rm H}$ is defined as

$$R_{\rm H} = a_{\rm p} \left(\frac{M_{\rm p}}{3M_{\rm s}} \right)^{1/3}. \tag{1}$$

The Hill radius, $R_{\rm H}$, is commonly used in studies of orbital dynamics as a proxy for the dynamical 'reach' of a given body (Horner et al. 2003; Horner, Evans & Bailey 2004a,b). Close encounters between two massive bodies are typically defined as those that occur at a distance closer than $3R_{\rm H}$, although some particularly conservative studies consider $5R_{\rm H}$ a sufficiently close approach to be labelled as such. These lines therefore show the limits at which the outer planet approaches the inner one within the prescribed number of $R_{\rm H}$ – those orbits outside the lines are too widely separated at the start of the integrations to undergo close encounters, while all those within the region bounded by the periastron and apastron lines for the orbital radius of the inner planet are orbits which cross that of the planet.

It is immediately apparent in Fig. 1 that the great majority of possible orbits suggested in Qian et al. (2011) are extremely unstable, with just a small region at low eccentricities and high semi-major axes (i.e. below the $3R_{\rm H}$ line in the bottom right-hand corner) showing any significant long-term stability. This is not a surprising result – the two planets in question have particularly large masses and therefore have very large dynamical reaches, and so must be widely separated in order that they do not strongly perturb one another. Indeed, it is clear from that figure that the regions of stability and instability for the HU Aqr system are a strong function of the periastron and apastron distances of the initial orbits of the outer planet. The only orbits that display relatively strong stability all have periastron at distances greater than $5R_{\rm H}$ beyond the orbit of the inner planet. Any closer, and the encounters between the planets are strongly disruptive. Orbits of the outer planet which approach the inner planet to a distance between 3 and $5R_{\rm H}$ are clearly more stable than those which come closer to that planet, but still display significant instability on astronomically short time-scales.

However, what of mutual inclinations between the two planets? Could the system be stabilized by the planets moving on orbits that are significantly inclined with respect to one another? This question can be answered by examination of Fig. 2, which presents the results for all six scenarios considered in this work.

As the orbital inclination of the outer planet is increased to 5° and then to 15°, the overall stability of the system appears to increase somewhat, with the yellow and orange colours that denote moderate stability spreading across the entire plot. However, the effect of increased inclination on the region of greatest stability, those orbits with median lifetimes greater than one million years, is negligible. Indeed, by the time the inclination is increased to 15°, that region appears to shrink somewhat, with particular destabilization occurring just inwards of the location of the 5:2 MMR with the orbit of the inner planet. Once the orbital inclination of the outer planet is increased to 45° (top right-hand panel), this effect becomes far more pronounced, with only small regions of stability remaining in low-eccentricity orbits around the 2:1 MMR, around the 5:2 MMR and a larger region beyond the location of the 3:1 MMR. Indeed, far from enhancing the stability of the planetary system, as might be expected, the 3:1 MMR acts to destabilize the orbits of the planets in this more highly inclined scenario.

The middle right-hand panel of Fig. 2 shows the scenario for which the orbit of the outer planet is inclined by 135° to that of the inner planet. That scenario displays a remarkable lack of stability across the entire 3σ region of plausible orbits for the outer planet.

The only scenario in which orbits within the 1σ uncertainty range for the outer planet display strong stability is shown in the lower right-hand hand panel of Fig. 2 (i =180°), when the orbit is both retrograde and coplanar. In that scenario, almost all orbits for the outer planet not crossing that of the inner are dynamically stable on long time-scales. In that extreme scenario, even some configurations in which the orbits of the two planets cross show significant stability. Those scenarios are the only ones of our entire suite of almost 105 000 test integrations in which orbits of the HU Aqr planets situated within the 1σ error bounds stipulated in Qian et al. (2011) can display any long-term stability.

5 CONCLUSIONS AND DISCUSSION

We have investigated the stability of the recently discovered planetary system around HU Aqr across the $\pm 3\sigma$ uncertainty ranges for the orbit of the outer planet. In the simplest, coplanar, case, we find that the system is unstable on time-scales $<5 \times 10^3$ yr, except for scenarios in which the mutual separation between the two planets is greater than $5R_{\rm H}$ at the outer planet's periastron. When the orbit of the outer planet is set such that it reaches periastron between 3 and $5R_{\rm H}$ beyond the orbit of the inner planet, the orbital stability is somewhat greater than scenarios where the minimum separation between the two planets is smaller than $3R_{\rm H}$, but the planetary system still falls apart on time-scales too short to inspire any confidence that the tested orbits represent the true state of the HU Aqr system. We see a steady increase in the median lifetime of the system across much of the tested phase space as the mutual inclination of the orbits climbs from the coplanar case, through 5° to 15° to 45°, as expected from

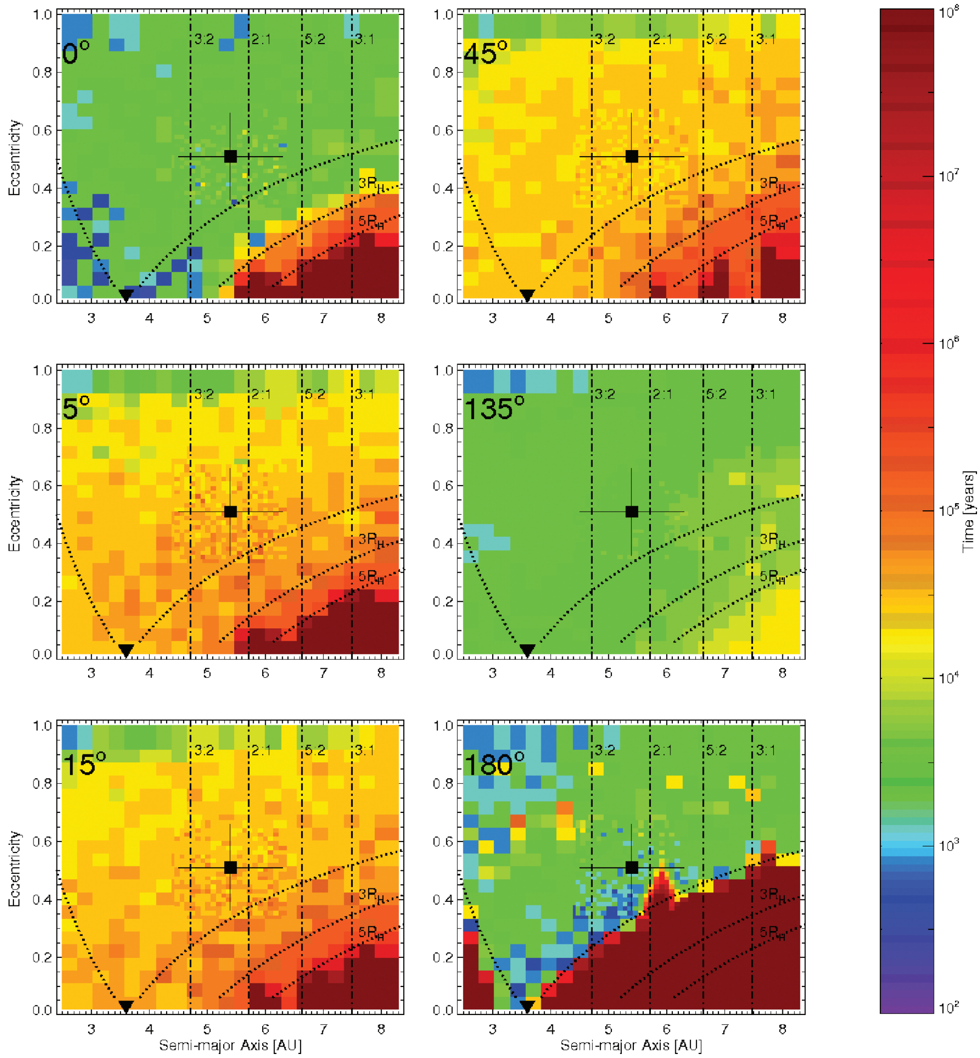

Figure 2. Lifetime *a–e* stability plots of the HU Aqr system using the parameters of Qian et al. (2011). The top left-hand panel reproduces the results shown in Fig. 1, detailing the scenario in which both planets are considered to be moving on coplanar, prograde orbits. The middle left-hand panel shows the results for the scenario where the outer planet has an orbit inclined by 5° to that of the inner planet, with the lower left-hand, top right-hand, middle right-hand and lower right-hand panels showing the results for the initial orbital inclinations of 15°, 45°, 135° and 180°, respectively. The various features plotted in each panel are the same as those shown in Fig. 1.

the resulting reduction in the amount of time the two planets spend in close proximity. However, even in the most extreme prograde case investigated ($i = 45°$), the lifetime of systems within the 1σ parameter space did not exceed 10^5 yr. This is much shorter than the expected system age, suggesting that we have either caught the system during a period of significant dynamical instability in which the planets are undergoing a rearrangement/ejection or that the orbital solution described in Qian et al. (2011) does not represent the true state of the system. Interestingly, although an increase in the mutual inclination of the planetary orbits causes those orbits in the unstable region to become slightly more stable, it also results in a reduction in the stable region at distances greater than $5R_{\rm H}$ from the inner planet.

If the orbit of the outer planet is retrograde compared to that of the inner planet, but the orbits remain coplanar (i.e. the mutual inclination of the two orbits is 180°), then we find that the system could be stable across a wide range of parameter space, including some scenarios within the 1σ errors quoted for the orbit of the outer planet. However, it seems difficult to comprehend how the proposed planets could have evolved into such orbits. By contrast, if the orbits of the two planets have mutual inclinations of 135°, then we find that no region of the tested *a–e* phase space is stable on time-scales greater than $\sim 10^4$ yr.

The results of our dynamical simulations raise a number of interesting possibilities. Assuming that the detection of the planets by Qian et al. (2011) is robust, and that the orbits of the planets at the current epoch are exactly as described in that work, the planetary system must be going through a dramatic period of dynamical instability, which will in short order result in the loss of one (or both) of the planets therein. However, the incredibly short lifetimes we find for the proposed planets suggest that, statistically, this is unlikely (i.e. the odds of observing a planetary system during the last few thousand years of a multibillion-year lifetime seem remarkably small). On the other hand, if we assume that the detection of the planets is robust, but the orbital parameters given are not a good measure of the true state of the system, we suggest that it is most likely that the outer planet is moving on a low-eccentricity orbit far from the central bodies. Such an orbit would allow that planet to remain sufficiently far from the inner planet to be dynamically stable on multimillion-year time-scales and therefore seems a much more reasonable solution for the dynamics of the system. If the two planets detected by Qian et al. (2011) have significant mutual orbital inclinations, then we find that a slightly wider range of orbits are possible for the outer planet to display moderate stability, but that at the same time the region of greatest stability for that planet's orbit actually reduces in size. In any case, in such scenarios, the stable regions remain relatively restrictive in terms of both orbital eccentricity and semi-major axis – keeping the planet well beyond the inner's sphere of influence.

The only way in which the orbits of the two planets can be induced to show significant stability is to consider a scenario in which they are coplanar, but with the orbit of the outer planet being retrograde with respect to that of the inner. In such a scenario, a wide area of the studied a–e phase space becomes dynamically stable, including some solutions within the 1σ uncertainties detailed by Qian et al. (2011). Whilst this might appear promising, we note that it is hard to envision a way in which the planets could evolve into such an unusual configuration without significantly destabilizing one another's orbits.

When one examines the residuals from the O$-$C diagram in the Qian et al. (2011) work, a first glance suggests that a low-eccentricity orbit for the outer planet is incompatible with the observed data. However, it is possible that the removal of the long-term quadratic trend by Qian et al. (2011) could have resulted in an artificially enhanced eccentricity for the outer planet, reducing its stability. On the other hand, detailed simulations of Doppler velocity data by Anglada-Escudé, López-Morales & Chambers (2010) demonstrated that two planets in circular orbits can mimic the signal of a single planet on an eccentric orbit, so long as those planets move on mutually resonant orbits. This possibility was recently explored by Tinney et al. (2011) for the case of HD 38283b, an eccentric planet in a 1-year orbit. If, rather than a highly eccentric second planet, we have a scenario where the system contains at least three massive planets, each moving on a low-eccentricity orbit, it might be possible to explain the suggested shape of the O$-$C diagram in Qian et al. (2011) whilst placing the planets on orbits that are dynamically stable. Indeed, Qian et al. (2011) suggest that there might be a third massive body at a large barycentric distance, based on the archaic Titius–Bode law. While the use of that law is not generally encouraged as a predictive tool in exoplanetary science, the presence of a more distant third body would allow orbital fits with the second planet moving on a low-eccentricity orbit. Although we have not dynamically simulated such a speculative scenario, it seems reasonable to assume that if a third planet were located at an orbital radius at least $5R_{\mathrm{H}}$ beyond that of the second planet, the system could display long-term dynamical stability without violating the observed variations in the egress of the accretion hotspot from the eclipse by the stellar secondary body.

A more detailed statistical analysis of this highly fascinating exoplanetary system is clearly necessary in order to disentangle the true nature of the proposed planetary system. Such work will undoubtedly throw fresh light on one of the most peculiar planetary systems detected to date.

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