Multiproduct Firms, Income Distribution, and Trade

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Abstract

We develop a general equilibrium model of multiproduct firms with quality differentiated goods. Households are characterized by an heterogeneous taste for the differentiated good and their income level. The use of non-homothetic preferences and vertical product differentiation (product quality) enables us to analyze how distributional changes in income affect the number of vertically differentiated firms, their product range and prices in the presence of strategic interaction across firms. The implications of lowering the barriers to trade within this setting are considered as well.

Keywords: Multiproduct Firms; Endogenous Product Scope; Product Quality; Income Distribution; Discrete Choice ; Trade Liberalization; Oligopoly;

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1 Introduction

In this paper we complement the growing literature of multi-product firms in international trade by introducing non-homothetic preferences and vertical product differentiation (product quality) in a general equilibrium framework. This allows us to analyze how income distribution affects the number of vertically differentiated firms, their product range and prices. The study of such income distributional effects has been neglected in the literature of multi-product firms due to the reliance on the assumption that preferences are (quasi)-homothetic.[1] Our paper therefore provides an important contribution that will help understand changes in the industrial structure within an international environment.

There is ample empirical evidence that shows that the income elasticity of demand varies across vertically differentiated products – a feature inherently ignored under homothetic preferences. Broda and Romalis (2009), for example, show that poorer households consume disproportionately more goods of low quality. Moreover, there is a growing body of empirical research in international trade that documents systematic patterns of vertical specialization.[2] Not only do richer countries export disproportionately more goods of high quality, they also import disproportionately more high quality goods (Hallak, 2006 and Khandelwal, 2010). Moreover, price variations within the same product categories between rich and poor countries reveal that richer countries command higher unit values for their exports in comparison to exports of poor countries (Schott, 2004 and Hallak and Schott, 2011), suggesting that the quality of exports is correlated with the per capita income of the home country.

In the demand framework considered in this paper, households consume two goods: a homogeneous and a vertically differentiated good (see also Fajgelbaum et al., 2011). The decision process of a household to buy one unit of the differentiated good among a set of mutually exclusive alternatives is modeled using the theory of discrete choice based on a random utility formulation (McFadden, 1978 and Ben-Akiva et al., 1993). Specifically, the process of buying a specific brand is modeled as a sequence of nested-logit models with the quality choice at the first stage and the choice of firm and particular brand from that firm at subsequent stages;[3] multi-product firms are assumed to offer more than one product of

[1] Early work on multi-product firms can be mostly found in the industrial organization literature, for example, Brander and Eaton (1984), Anderson et al. (1992), Johnson and Myatt (2003), and Allanson and Montagna (2005).

[2] Earlier studies include Hunter and Markusen, (1987), Hunter (1991) and more recently Fieler (2011).

[3] Similar ideas were used by Berry, Levinsohn and Pakes (1995), Goldberg (1995) and Verboven (1996) to estimate the demand in various car markets.

the same quality level.[4] The multinomial logit structure allows for differences in the degree of substitutability across goods with consumers considering varieties from a particular firm as closer substitutes than varieties with the same quality from other firms.[5]

We assume an oligopolistic market structure (see also Eckel and Neary, 2010 and Anderson and De Palma, 1992) where firms have to choose both the price and range of brands. We allow for free entry and exit into the differentiated goods market. The implied strategic interaction across firms within the same quality sector ensures that the equilibrium number of firms, the number of products per firm, and the price of a variety is responsive to changes in the income distribution. In the absence of strategic interaction (monopolistic competition), any changes in the environment are exclusively linked to changes in the number of firms (Schafgans and Stibora, 2012). We complement the strategic inter-firm competition with strategic intra-firm competition – also known as the cannibalization effect – one of the most defining feature of multi-product firms, where firms coordinate their pricing decision across its product range.[6]

The coexistence of multi-product firms that internalize demand linkages in the presence of non-homothetic preferences permit a fruitful discussion of changes in industrial structure to income changes and size of the economy. Specifically, we show that in autarky an increase in income (first order stochastic dominance) and a mean preserving spread lead to a larger number of high-quality firms, each producing a larger number of products at a smaller scale with lower prices in the long-run equilibrium. The opposite occurs in the low-quality sector and the differentiated goods composition therefore clearly shifts towards high-quality brands. When the number of low-quality firms exceeds that of high-quality firms, an increase in the size of the economy is more likely to be accompanied by a shift in its composition to the high-quality firms the more dissimilar consumers perceive the brands and firms of high and low-quality goods to be. In the open economy setting, under complete specialization in quality, lower trade costs for a particular differentiated good unambiguously increases the number of firms and brands of that differentiated product with the firm expansion effect dominating the product line expansion when the initial number of firms is sufficiently large. In the case where both countries produce the same

[4] In Schafgans and Stibora (2013) we develop a model that considers the alternative strategy in which firms produce multiple products of different quality.

[5] Dhingra (2013) also allows for differences in the degree of substitutability using a variation of the linear quadratic utility function. In a multi-product model, Bernard et al. (2011) use CES preferences for this purpose.

[6] Cannibalization is also present in Eckel and Neary (2010) and Dhingra (2013), arising respectively from strategic inter-firm interaction in a oligopolistic market and strategic intra firm competition in a monopolistic competitive market.

differentiated product, our analysis is much more complex (and consequently yields more ambiguous findings) in part due to the so called relative price effect. It measures the impact arising from a change in relative prices of domestic and foreign competitors as a result of the relative change of the number of brands in both countries. Despite its analytical intricacies, these results are of empirical relevance.

The remainder of the paper is structured as follows. In Section 2 we develop the framework in autarky. In Section 3 we discuss the properties of the short and long-run autarky equilibrium and provide conditions for a unique and stable long run equilibrium in which firms produce multiple varieties in low and high quality goods markets. We also discuss the welfare implications of a change in the population and the income distribution. In Section 4 we extend the model by considering an open economy with two countries that, in the presence of trading cost, may engage in trade in the differentiated products. Section 5 concludes. Technical details are provided in Appendix A (autarky) and B (open economy).

2 Model

Let us analyze a general equilibrium model of multi-product firms with vertically differentiated goods and households that differ in income and taste in a closed economy setting.

Individuals consume two goods: a homogeneous good (z) and an optimally selected good from a finite set of mutually exclusive differentiated goods. We let the differentiated goods sector consist of two different qualities: a high-quality, q_H, and a low-quality, q_L where $q_H > q_L$. In each quality there are $n_i > 1$ firms each producing $m_i \geq 1$ varieties, for $i = H, L$.

Labor is the only factor of production and is inelastically supplied in a competitive market. The homogeneous good is produced with one unit of effective labor per unit of output which is also supplied in a perfectly competitive market. The unit price of the homogeneous good therefore equals the wage rate and we use the wage rate as the numeraire.

On the demand side, we assume there is a continuum of households each endowed with a different skill level generating a non-degenerate income distribution. Each household is assumed to have sufficient income to purchase the homogeneous good and at least the lowest quality of the differentiated good. We denote the income distribution by $\mathcal{F}_y(y)$, so that $\mathcal{F}_y(y)$ is the fraction of the N households with income less than or equal to y and $N \int_{y_{\min}}^{\infty} y dF_y(y)$ the total supply of labor. We next consider how a consumer selects one unit of a variety of quality q_i, for $i = H, L$, from the total set of varieties in order to maximize utility, given prices and characteristics of all available commodities.

2.1 Demand

Consider household h that consumes z units of a homogeneous good and one unit of a differentiated good k of quality q_i, with $i = H, L$. Following Fajgelbaum et al. (2011) the utility attained from consuming a combination of the homogeneous and differentiated good is assumed to have the following form[7]

$$u_{ik}^h = zq_i + \nu_{ik}^h. \tag{1}$$

The term ν_{ik}^h is a residual that describes among others the idiosyncratic valuation of household h of product k with quality i. It is specified as

$$\nu_{ik}^h = \mu_i \varepsilon_i^h + \mu_{if} \varepsilon_{if}^h + \mu_{ik} \varepsilon_{ifk}^h,$$

where the subscript f refers to firms and k to a brand provided by firm f. The ε^h terms are household-specific shocks of which ε_i is a quality shock; ε_{if} is a firm-specific shock and ε_{ifk} denotes a taste-for-brand shock. The μ terms measure the degree of heterogeneity: μ_i measures the degree of heterogeneity between the two quality groups; μ_{if} measures the degree of heterogeneity among firm in the same quality group; μ_{ik} measures the heterogeneity among products from the same firm. The larger μ_i, the greater is the degree of heterogeneity among different quality goods and when μ_i approaches zero, consumers consider products of different quality as perfect substitutes.

Given household h's income, y^h, and the price firm f charges for good k of quality q_i, p_{ifk}, a household chooses the differentiated variety of quality q_i that yields the highest utility. The remaining income $(y^h - p_{ifk})$ is spend on the homogeneous good z. The deterministic part of the utility function exhibits a complementarity between the homogeneous and differentiated good that leads to the non-homotheticity in the aggregate demand: the marginal utility of quality increases with higher consumption of the homogeneous good z. A richer household that spends a larger fraction of its income on the homogeneous good z experiences a larger marginal utility of quality.

We assume that the distribution of $\varepsilon_i^h, \varepsilon_{if}^h$ and ε_{ifk}^h is such that the disturbances $\mu_i \varepsilon_{ifk}^h$, $\mu_{if} \varepsilon_{if}^h + \mu_{ik} \varepsilon_{ifk}^h$ and $\mu_i \varepsilon_i^h + \mu_{if} \varepsilon_{if}^h + \mu_{ik} \varepsilon_{ifk}^h$ are distributed independently and identically across the population according to a generalized extreme value distribution. As shown by

[7] Di Comte et al. (2014) consider also idiosyncratic consumer taste using a quasi-linear model where the differentiated good enters with a quadratic sub-utility allowing for horizontal and vertical product differentiation. However, their focus is very different from ours.

McFadden (1978) and Ben Akiva et al. (1993), the distributional assumptions about the ε's allow us to model consumer's choice as a sequential process in which first the quality level q_i (or also known as 'nest') is chosen with (marginal) probability ρ_i, then, conditional on the choice of quality, a particular firm is chosen with probability $\rho_{f|i}$, and, finally, conditional on the choice of firm, a particular brand is selected with probability $\rho_{k|i,f}$. Hence, the joint probability that a consumer endowed with income y chooses brand k of quality q_i sold by firm f can be expressed as the product of two conditional probabilities and the marginal probability, namely,

$$\rho_{ifk}(y) = \rho_{k|i,f} \cdot \rho_{f|i} \cdot \rho_i(y). \tag{2}$$

The three choice levels are described by logit models and are given by

$$\rho_{k|i,f} = \frac{e^{-p_{ifk}\cdot q_i/\mu_{ik}}}{\sum_{h=1}^{m_{if}} e^{-p_{ifh}\cdot q_i/\mu_{ik}}} = \frac{e^{-p_{ifk}\cdot q_i/\mu_{ik}}}{e^{I_{if}/\mu_{ik}}} \tag{3}$$

$$\rho_{f|i} = \frac{e^{I_{if}/\mu_{if}}}{\sum_{j=1}^{n_i} e^{I_{ij}/\mu_{if}}} = \frac{e^{I_{if}/\mu_{if}}}{e^{I_i/\mu_{if}}} \tag{4}$$

$$\rho_i(y) = \frac{e^{(y\cdot q_i + I_i)/\mu_i}}{\sum_{l=1}^{\omega} e^{(y\cdot q_l + I_l)/\mu_l}}, \tag{5}$$

with m_{if} representing the number of products of firm f with quality i, n_i denoting the number of firms in quality class i and ω the number of quality classes (as $i = H, L$ we have $\omega = 2$). The sequential decision process is illustrated in Figure 1.[8] I_{if} and I_i are so-called inclusive values that are given by

$$I_{if} \equiv \mu_{ik} \ln \left[\sum_{h=1}^{m_{if}} e^{-p_{ifh}\cdot q_i/\mu_{ik}} \right], \quad I_i \equiv \mu_{if} \ln \left[\sum_{j=1}^{n_i} e^{I_{ij}/\mu_{if}} \right] \quad i = H, L. \tag{6}$$

The inclusive values convey information from the lower level (nest), for example, the choice of a particular brand from firm f, to the higher level, the choice of firm of quality q_i. For example, I_{if} measures the expected utility that household h receives from the choice among alternative brands offered by firm f. Likewise, I_i is the expected benefit household h attains from the choice among the alternative brands offered by firms offering goods of quality q_i.[9]

The various parameters denoted by μ measure the degree of correlation among alternatives within a subgroup, with a larger μ indicating less correlation between the error term ε. The nested logit model is consistent with random-utility maximization for

[8] Even though households may not actually make decisions sequentially, the structure generates reasonable correlation patterns among unobserved (by the econometrician) factors across the alternatives needed to study household decision processes, see Goldberger (1995).

[9] See Train (2003), chapter 4 for a detailed discussion.

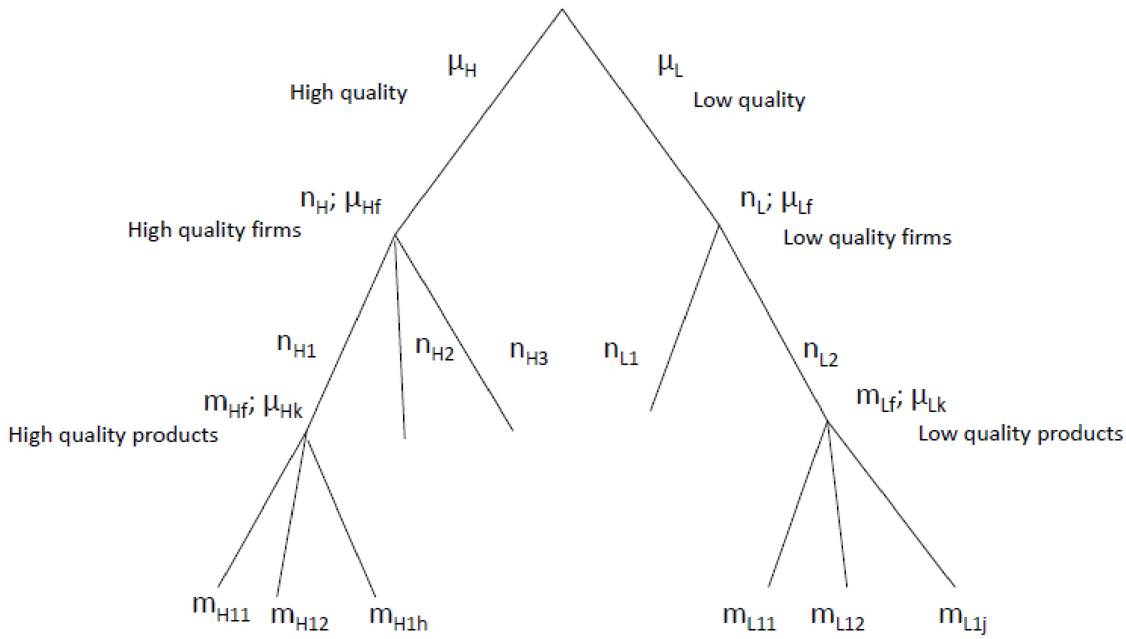

Figure 1: Demand side

$0 \leq \mu_{ik} \leq \mu_{if} \leq \mu_i \leq 1$. Without loss of generality we set $\mu_i = 1$, $i = H, L$. The parameters μ_{ik} and μ_{if} can be interpreted as measures of *intra*- and *inter*-firm heterogeneity respectively and can be inferred from the calculations of two cross price elasticities. Assuming there are n_i firms in quality class i, each selling m_i varieties at price p_i, the cross price elasticity of demand for brands from different firms in quality class i is $p_i q_i / n_i m_i \mu_{if}$, while the cross price elasticity of demand for brands from the same firm in quality class i equals $(p_i q_i / n_i m_i)[n_i/\mu_{ik} - (n_i - 1)/\mu_{if}]$. The two cross price elasticities are equal if $\mu_{if} = \mu_{ik}$. In this case, brands and firms cannot be distinguished $-$ equal *intra*- and *inter*-firm heterogeneity $-$ and the nested logit model reduces in essence to the model of Fajgelbaum et al. (2011). When $\mu_{if} > \mu_{ik}$, the latter elasticity is larger than the former implying that consumers consider brands from the same firm as closer substitutes to each other than brands of the same quality offered by a competitor.

The error term ν_{ik}^h in (1) implies that not every rich household chooses to buy a higher quality good nor does every poor household fancy a low-quality one. However, one should expect that richer households on average consume more the higher quality good (i.e., the probability of consuming the higher quality good rises with income). Looking at equation

(2) this key property of non-homothetic preferences is satisfied if

$$\begin{aligned}\frac{1}{\rho_{Hfk}(y)}\frac{\partial\rho_{Hfk}(y)}{\partial y} &= \frac{1}{\rho_H(y)}\frac{\partial\rho_H(y)}{\partial y} > 0 \\ &\Leftrightarrow\ q_H - q_a(y) > 0,\end{aligned} \tag{7}$$

where $q_a(y) = \rho_H(y)q_H + \rho_L(y)q_L$ is the average quality consumed by households with income y. The share of households who purchase the higher quality good, therefore, increases with increasing income at all income levels if and only if q_H is larger than the average quality consumed by households in this income group. Since we consider two quality classes ($q_H > q_L$), we require $q_H > q_a(y) > q_L$ for all y.

With N the total number of households in the economy, the aggregate demand for variety k of quality q_i offered by firm f is

$$\begin{aligned}d_{ifk} &= N\int_{y_{\min}}^{\infty}\rho_{ifk}(y)d\mathcal{F}_y(y) \\ &= N\rho_{k|i,f}\cdot\rho_{f|i}\cdot\int_{y_{\min}}^{\infty}\rho_i(y)\cdot d\mathcal{F}_y(y) \\ &= N\rho_{k|i,f}\cdot\rho_{f|i}\cdot E[\rho_i(y)]\end{aligned} \tag{8}$$

where $y_{\min}$ is the minimum income of the poorest household in the country. By definition $E[\rho_i(y)] \equiv \int_{y_{\min}}^{\infty}\rho_i(y)\cdot d\mathcal{F}_y(y)$ denotes the expected value of $\rho_i(y)$ with respect to the income distribution $\mathcal{F}_y(y)$.

2.2 Costs and profits of multi-product firms

The nested logit model is used to characterize the demand perceived by a firm which sells multiple products of the same quality. We assume that varieties offered by the same firm are closer substitutes than varieties of the same quality offered by different firms. To this end we require the *intra*-firm heterogeneity, μ_{ik}, to be smaller than the *inter*-firm heterogeneity, μ_{if}, for all i.[10]

There are n_i firms in each of the differentiated goods industries, for $i = H, L$. A firm in industry i can produce any number $m_{if} \geq 1$ of varieties subject to three types of cost, all measured in terms of labor. First, there is a fixed overhead cost of K_i the firm must bear irrespective of the number of products offered. Second, a firm has to pay a brand-variety fixed cost of F_i. This cost might represent the price of acquiring a patent or the marketing

[10] See Dhingra (2013) for references citing empirical evidence: e.g., using supermarket data Broda and Weinstein (2010) find that various brands of the same company are closer substitutes than brands across different companies.

of the product. Finally, a firm incurs a constant marginal cost c_i. The total profit function of a firm producing m_{if} variants of quality q_i at price p_{ifk} per variety therefore is

$$\Pi_{if} = \sum_{k=1}^{m_{if}} (p_{ifk} - c_i)\, d_{ifk} - F_i m_{if} - K_i \quad \text{for } i = H, L \tag{9}$$

where d_{ifk} is defined by (8). Each firm has to make a decision about the price structure and the number of products maximizing profits.

Next, we derive the multi-product firm equilibrium given the nested demand structure. We make use of results obtained in Anderson and de Palma (1992) for multi-product firms.

2.3 Product range and pricing of multi-product firms

We assume that firms play a sequential game: in the first stage a firm decides to enter one of the two quality markets; in the second stage, active firms decide how many variants to produce before the price structure is determined.[11] We solve the game recursively starting with the short-run equilibrium analysis were the number of firms in each market is taken as given. In the long run, firms are allowed to enter or exit depending on their profitability. The analysis is simplified by considering a symmetric equilibrium in which firms of quality q_i supply the same number of varieties.

Starting with the short-run equilibrium analysis we make two assumptions with regard to the price setting behavior of firms. First, we assume that a firm coordinates its pricing decision across its product range, which is also known as cannibalization. Second, we assume that the market structure within each industry is oligopolistic but that there is no strategic interaction between markets of different quality. This implies that firms take $\rho_i(y)$ as given.[12]

Removing the strategic interaction among firms producing goods of the same quality, that is assuming monopolistic competition instead, generates an equilibrium of limited interest because neither a change in the income distribution nor a change in the country size, N, would impact the price or the range of varieties produced per firm. While we assume strategic interaction within markets, across markets of different quality there is no strategic interaction which allows us to apply the results of Anderson and de Palma (1992) for $i = H, L$ separately. We provide the necessary details here to fit our general equilibrium framework.

[11] Eckel and Neary (2010), on the other hand, assume a single-stage Cournot game.

[12] Bernard et al. (2011) and Mayer et al. (2014) assume there is no strategic interaction within and between firms. Eckel and Neary (2010), in a one sector economy, allow for strategic interaction across firms in a single market.

To solve for the price subgame, we make use of the partial derivatives: $\partial\rho_{k|i,f}/\partial p_{ifk} = -q_i/\mu_{ik}[\rho_{k|i,f}(1-\rho_{k|i,f})]$, $\partial\rho_{f|i}/\partial p_{ifk} = (q_i\rho_{f|i}(\rho_{f|i}-1)\rho_{k|i,f})/\mu_{if}$, and $\partial\rho_{h|i,f}/\partial p_{ifk} = q_i/\mu_{ik}[\rho_{k|i,f}\rho_{h|i,f}]$. The first order condition of (9) with respect to p_{ifk} yields

$$(p_{ifk} - c_i) = \frac{\mu_{ik}}{q_i} + \left[1 - \frac{\mu_{ik}}{\mu_{if}}\left(1-\rho_{f|i}\right)\right]\sum_{h=1}^{m_{if}}\left(p_{ifh}-c_i\right)\rho_{h|i,f} \tag{10}$$

for $k \in m_{if}$, $f \in n_i$, and $i = H, L$. Since $\mu_{if} > \mu_{ik}$, the mark-up of the price over marginal cost is positive. Clearly, the mark-up is the same for all brand varieties, so that $p_{ifk} = p_{if}$. The presence of the probability weighted summation of all brand mark-ups in (10) reflects the cannibalization effect: a reduction in the price of one brand leads a reduction in the prices for all other brands supplied by the same firm.

Since we are interested in a symmetric equilibrium in the second stage, we consider the case where all firms except firm f have $m_i \geq 1$ brands while firm f has $m_{if} > 0$ brands so that the mark-up over marginal cost simplifies to

$$(p_{if} - c_i) = \frac{\mu_{if}}{q_i}\frac{1}{(1-\rho_{f|i})} \tag{11}$$

$$(p_{ij} - c_i) = \frac{\mu_{if}}{q_i}\frac{1}{(1-\rho_{j|i})}, \quad j = 1,..,n_i \text{ with } j \neq f \tag{12}$$

for $i = H, L$. In contrast to Fajgelbaum et al. (2011), the absolute mark-up depends not only on μ_{if} and the quality class q_i, but also on $\rho_{f|i}$, the conditional probability that firm f will be chosen given that a consumer decided to purchase quality i. The presence of $\rho_{f|i}$ in the pricing strategy implies that the mark-up depends on the number of firms producing quality i. The number of active firms in the differentiated sector i, in turn, depends on the income distribution, as we will demonstrate further below.

As is apparent from equations (11) and (12), the parameters μ_{if} and q_i affect the mark-up in opposite direction. On the one hand, a smaller value of μ_{if} leads to a reduction in the mark-up over marginal cost as variants from firms in the same quality group are considered to become closer substitutes in the eyes of consumers, rendering varieties more price sensitive, all else being the same. On the other hand, a lower q_i, ceteris paribus, reduces the marginal utility from consuming the homogeneous goods (as implied by equation (1)), rendering differentiated varieties less price elastic.

Following Anderson and de Palma (1992), we recognize $p_{ij} = p_i$ for all $j \neq f$ and express the conditional probabilities $\rho_{f|i}$ and $\rho_{j|i}$ used in (11) and (12) as

$$\rho_{f|i} = \frac{(m_{if})^{\mu_{ik}/\mu_{if}} \exp[-p_{if}q_i/\mu_{if}]}{(n_i-1)(m_i)^{\mu_{ik}/\mu_{if}} \exp[-p_i q_i/\mu_{if}] + (m_{if})^{\mu_{ik}/\mu_{if}} \exp[-p_{if}q_i/\mu_{if}]} \tag{13}$$

$$\rho_{j|i} = \frac{(m_i)^{\mu_{ik}/\mu_{if}} \exp[-p_i q_i/\mu_{if}]}{(n_i-1)(m_i)^{\mu_{ik}/\mu_{if}} \exp[-p_i q_i/\mu_{if}] + (m_{if})^{\mu_{ik}/\mu_{if}} \exp[-p_{if}q_i/\mu_{if}]}, \tag{14}$$

$j \in n_i$ with $j \neq f$, for $i = H, L$. Equation (11) then provides a unique p_{if} given $p_{ij} = p_i$, while equation (12) provides a unique p_i for given p_{if}. The proof of existence of a unique price equilibrium (p_{if}, p_i) of the subgame at which firm f has m_{if} variants and all other firms have m_i variants then directly follows from Anderson and de Palma (1992). Their proof is based on the fact that the difference between (12) and (11) can be expressed as

$$\varkappa_i = \left\{ \frac{1}{(n_i-2) + M_i \exp \varkappa_i} - \frac{M_i \exp \varkappa_i}{(n_i-1)} \right\} \tag{15}$$

with $\varkappa_i \equiv (p_i - p_{if})q_i/\mu_{if}$ and $M_i \equiv (m_{if}/m_i)^{(\mu_{ik}/\mu_{if})}$, which has a unique solution in $\varkappa_i$ as the left hand side of (15) is increasing in $\varkappa_i$ while the right hand side is decreasing in $\varkappa_i$.

We now turn to a firm's optimal choice of number of varieties. To this end, consider firm f that sells m_{if} variants at price p_{if}, while its closest $(n_i - 1)$ competitors offer m_i brands at a price p_i, for $i = H, L$. As before, we abstain from strategic interaction between sectors of different quality and can therefore treat each quality $i = H, L$, symmetrically. At the second stage, the profit function of firm f is

$$\widetilde{\pi}_{if} = (p_{if} - c_i)\, N \rho_{f|i} E[\rho_i(y)] - m_{if}F_i - K_i, \tag{16}$$

where $\rho_i(y)$, using the above notation, simplifies to

$$\rho_{f|i} = \frac{M_i \exp \varkappa_i}{(n_i-1) + M_i \exp \varkappa_i}. \tag{17}$$

Given the pricing strategy (11), the profit function can be expressed as

$$\widetilde{\pi}_{if} = \frac{N}{(n_i-1)} \frac{\mu_{if}}{q_i} E[\rho_i(y)] \left(\frac{m_{if}}{m_i}\right)^{(\mu_{ik}/\mu_{if})} \exp \varkappa_i - m_{if}F_i - K_i. \tag{18}$$

The first order condition with respect to the scope of production, m_{if}, is:

$$\frac{N}{(n_i-1)} \frac{\mu_{if}}{q_i} E[\rho_i(y)] \left(\frac{m_{if}}{m_i}\right)^{(\mu_{ik}/\mu_{if})} \exp \varkappa_i \left[\frac{\mu_{ik}}{\mu_{if}} \frac{1}{m_{if}} + \frac{\partial \varkappa_i}{\partial m_{if}}\right] - F_i = 0,$$

where $\partial \varkappa_i / \partial m_{if}$ captures the effect a change in m_{if} has on competitors' prices. Evaluating the first order condition at a symmetric equilibrium, where $m_{if} = m_i$, yields

$$N \frac{\mu_{ik}}{q_i} E[\rho_i(y)] \left[\frac{n_i - 1}{n_i^2 - n_i + 1}\right] = m_i F_i. \tag{19}$$

Since $E[\rho_i(y)]$ is a function of m_L and m_H, equation (19) for $i = H, L$ provides a system of equations that implicitly defines the equilibrium number of brands per firm given the number of firms.

3 Autarky Equilibrium

3.1 Short-run autarky equilibrium

Analogous to Anderson and de Palma (see proof in Appendix 7.10.4 of Anderson et al. 1992), it can be shown that $m_{Lf} = m_L$ is the number of brands that maximizes firm f's profits for quality q_L, given all other low quality firms choose m_L and all high quality firms choose m_H. At a symmetric equilibrium, where firms offer the same range of brands and charge the price for each brand, aggregate demand for a typical brand of quality q_i offered by firm f can be expressed as

$$d_{if} = \frac{N}{m_i n_i} E[\rho_i(y)] \tag{20}$$

with

$$E[\rho_i(y)] = E\left[\frac{n_i^{\mu_{if}} m_i^{\mu_{ik}} \phi_i(y, n_i)}{n_H^{\mu_{Hf}} m_H^{\mu_{Hk}} \phi_H(y, n_H) + n_L^{\mu_{Lf}} m_L^{\mu_{Lk}} \phi_L(y, n_L)}\right], \tag{21}$$

where

$$\phi_i(y, n_i) \equiv \exp\left[(y - c_i)q_i - \mu_{if} \tfrac{n_i}{(n_i - 1)}\right], \tag{22}$$

and corresponding equilibrium prices (here $\rho_{f|i}$ simplifies to $1/n_i$)

$$p_i = c_i + \frac{\mu_{if}}{q_i} \frac{n_i}{n_i - 1}. \tag{23}$$

The term $\phi_i(y, n_i)$ captures the effect of income and price on the probability of choosing a good of quality q_i, for $i = H, L$. The price effect works through a change in the number of firms. An increase in the number of firms producing quality, say q_L, lowers the price for these varieties relative to the price of high-quality brands thereby making it more likely that the fraction of households purchasing low-quality goods increases.[13] This contrasts with Fajgelbaum et al. (2011), where this additional price effect is absent due to their assumption of monopolistic competition.

The short-run autarky equilibrium of the two stage product range and the subsequent price game, with n_i firms each choosing m_i varieties given by (19) and then charging a price given by (23), is illustrated in Figure 2. The short-run equilibrium is unique with

[13] For strictly positive μ_{if}/q_i, as $n_i \to \infty$, we are back in the monopolistic competition setting, where the mark-up of the price over marginal cost is constant (positive).

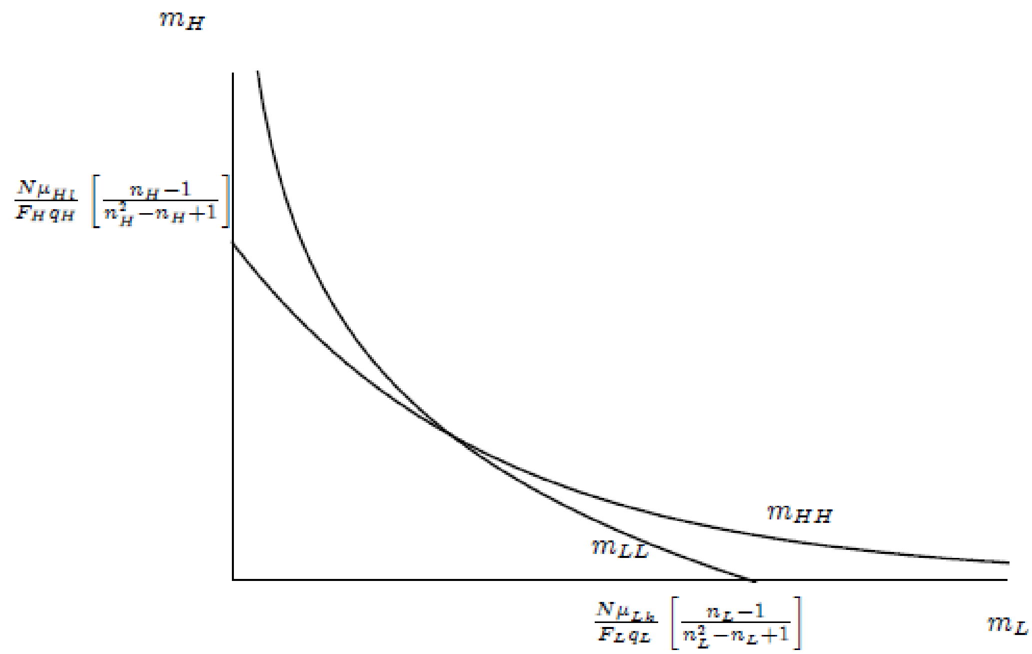

Figure 2: Short-run autarky equilibrium

$m_H > 0$ and $m_L > 0$.[14] The m_{LL} and m_{HH} curves show combinations of m_L and m_H for which the first order conditions (19) are satisfied. The two curves are negatively sloped as an expansion in the product range of a firm in one quality sector requires the reduction in the product range of a firm in the other product class to preserve profitability.

Before turning to the long-run equilibrium, where the number of firms are endogenous, we first discuss how the short-run equilibrium is related to the population size, the income distribution and the number of firms.

An increase in the population size, N, will lead to an increase in the optimal scope m_i. In terms of Figure 2, both m_{LL} and m_{HH} curves shift to the right. When $\mu_{Hk} = \mu_{Lk}$, the number of brands, m_i, increases equiproportionally since the perceived differences among the various brands across quality classes are the same, while the number of brands offered by firms with low-quality increases relatively more, $\widehat{m}_H < \widehat{m}_L$, when $\mu_{Hk} < \mu_{Lk}$.[15] The quantitative effects are given in equations (A.7) and (A.8) in Appendix A.

To evaluate the effect of a change in the income distribution, we consider two popular scenarios: first, we shift the cumulative income distribution, $\mathcal{F}_y(y)$, downwards (so that the new distribution has first-order stochastic dominance over the former) and second we shift

[14] The short-run equilibrium is unique and stable as the determinant of the coefficient matrix given in equation (A.3) in Appendix A is always positive.

[15] A circumflex above a variable denotes a proportional change, i.e. $\widehat{m} = dm/m$.

the cumulative income distribution in such a way that the weight is shifted from the center towards the tails while holding the mean constant (also called mean preserving spread).[16]

When we shift $\mathcal{F}_y(y)$ to the right, the fraction of the population with an income less than or equal to y decreases for all income levels; the economy becomes richer. For a given population size and number of firms in each quality class, demand shifts away from low-quality goods to high-quality goods. The change in the relative profitability in favour of high-quality goods ensures that firms within that quality class increase their product range while the opposite takes place in low-quality sector, i.e. $\widehat{m}_H > 0$ and $\widehat{m}_L < 0$. The relative increase in the number of brands with quality q_H is smaller than the relative fall in the number of brands with quality q_L, i.e. $|\widehat{m}_H| < |\widehat{m}_L|$. In terms of Figure 2, the $m_{LL}-$curve shifts to the left while the $m_{HH}-$curve shifts to the right, with the latter shift relatively larger than the former. See equations (A.9) and (A.10) in Appendix A.

With a mean preserving spread, the variance of income increases without changing the expectation implying an increase in income inequality (as reflected by the Lorenz curve). Starting from an economy in which the number of households in all income classes demand more low-quality goods relative to high-quality goods, i.e. $\rho_L(y) > \rho_H(y)$ for all y, this increase in income inequality will influence the product range of both sectors in the same way as the previous scenario. See equations (A.13) and (A.14) in Appendix A.

Quantitatively the differential impact associated with these changes in the income distribution is determined by the magnitude they have on the fraction of households purchasing the high-quality good $(E(\rho_H(y))$, in the case of first stochastic dominance given by $E(\rho_L(y)\rho_H(y))(q_L - q_H)$, versus $dE(\rho_H(y))/d\beta$ in the case of a mean preserving spread, with β parameterizing the mean preserving spread.

The effect of an increase in the number of firms producing low-quality goods on the range of brands per firm turns out to be ambiguous:

$$\frac{\widehat{m}_L}{\widehat{n}_L} = \frac{1}{D_1}\mu_{Lf}\frac{\rho_{HL}}{\rho_L}\left(1+\tfrac{n_L}{(n_L-1)^2}\right) - \frac{1}{D_1}\left(1-\mu_{Hk}\frac{\rho_{HL}}{\rho_H}\right)\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)} \tag{24}$$

$$\frac{\widehat{m}_H}{\widehat{n}_L} = -\frac{1}{D_1}\frac{\rho_{HL}}{\rho_H}\mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right) + \frac{1}{D_1}\frac{\rho_{HL}}{\rho_H}\mu_{Lk}\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}, \tag{25}$$

where $D_1 > 0$, $\rho_i \equiv E(\rho_i(y))$, $i = L, H$, and $\rho_{HL} \equiv E(\rho_L(y)\rho_H(y))$; Jensen's inequality ensures $\rho_{HL} \leq \rho_H\rho_L$. We can decompose these effects in a selection (first term) and competition (second term) effect.

The selection effect reflects the impact a higher n_L exerts on the probability of choosing respectively a low-quality good and a high-quality good. It consists of two components. On

[16] We also say, the new distribution is second-order stochastically dominated, or Lorenz dominated by the former. Stochastic dominance and its relation to inequality is discussed, e.g., in Davidson (2008).

the one hand, having more firms in the low-quality industry q_L, makes it more likely that a firm and one of its products will be selected. This provides an incentive for these firms to increase the range of varieties in this quality class while reducing the probability that a good from the high-quality industry is chosen. On the other hand, a higher n_L reduces the price of low-quality goods relative to high quality goods, providing consumers at all income levels with an incentive to substitute away from the high-quality goods towards the low-quality goods. Overall, the selection effect ensures that an increase in n_L raises m_L and lowers m_H.[17] The competition effect reflects the impact a higher n_L exerts on the profitability among low (high) quality firms. Having more firms in the market for goods of quality q_L, reduces (increases) the profitability of low (high) quality firms which leads to a decrease (increase) in the range of low (high) quality brands, i.e. $dm_L < 0$ and $dm_H > 0$.

Which of the two effects dominate depends on the interaction of the number of firms and the value of the two parameters measuring the degree of heterogeneity, μ_{Lf} and μ_{Lk}: the smaller the difference between $(\mu_{Lf} - \mu_{Lk}) > 0$, the larger n_L has to be for the selection effect to dominate the competition effect and the overall effect to be negative on the number of high-quality varieties ($dm_H < 0$). Intuitively the larger the number of n_L firms active in the market for given $\mu_{Lf} > \mu_{Lk}$, the smaller the competition effect and the price effect when an additional firms enters in the low-quality range; this increases the net selection effect thereby reducing the high-quality firm's range of products $dm_H < 0$. While the net effect on m_L is ambiguous, a sum of probability weighted relative changes is unambiguously negative:

$$\rho_L \frac{\widehat{m}_L}{\widehat{n}_L} + \rho_H \frac{\widehat{m}_H}{\widehat{n}_L} = -\rho_L \tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)} < 0.$$

Since the relative change in m_H is negative for n_L sufficiently high and $\mu_{Lf} > \mu_{Lk}$, the range of low-quality brands m_L can increase with an increasing number of firms in q_L.[18] In Appendix A we show that for the long run equilibrium to be stable, the selection effect cannot dominate the competition effect.

3.2 Long-run autarky equilibrium

So far we have treated the number of firms as being given. In the long-run, however, n_i is determined by the free entry zero profit condition for firm q_i. Substituting (19) into (18)

[17] Note, the selection effect is decreasing in n_i and converges towards one as n_i approaches infinity.

[18] In case the market structure is monopolistic competitive, both m_L and m_H unambiguously fall with a higher number of firms in quality class i, for $i = H, L$, see Schafgans and Stibora (2012).

yields the following profit functions:

$$\Pi_i(n_H, n_L) = \frac{N}{q_i} E[\rho_i(y)] \left[\frac{(\mu_{if} - \mu_{ik})(n_i - 1)^2 + \mu_{if} n_i}{(n_i - 1)(n_i^2 - n_i + 1)} \right] - K_i, \tag{26}$$

for $i = H, L$, where $\mu_{if} > \mu_{ik}$ by assumption. We will use these profit functions to determine the effective number of firms producing in equilibrium. As long as profits are positive firms will enter the market for quality good q_i; they exit otherwise. The flow of firms in and out of each industry can be described by two differential equations of the form

$$\dot{n}_i = n_i \Pi_i(n_H, n_L),$$

where a dot above a variable indicates differentiation with respect to time, i.e., $\dot{n}_i \equiv dn_i/dt$. In the long run, or in steady state, $\dot{n}_i = 0$ and $n_H > 1$ and $n_L > 1$ as $\Pi_H(n_H, n_L) = \Pi_L(n_H, n_L) = 0$.[19] Once the number of firms in each quality class is known, the number of brands per firm of quality q_i is determined from (19) and subsequently the price and sales of each brand is established. This allows us to determine the total sales per quality group, equalling $n_i m_i d_{if}$. Since each household purchases only one unit of the differentiated good, the aggregate sales of all differentiated products satisfies the condition $n_H m_H d_{Hf} + n_L m_L d_{Lf} = N$.

Labor market clearing, furthermore, requires that the demand for effective labor in differentiated and homogeneous good production equals the economy's aggregate supply:

$$\sum_{i=H,L} n_i \left[m_i \left(N c_i d_{if} + F_i \right) + K_i \right] + L_z = N \int_{y_{\min}}^{\infty} y d\mathcal{F}_y(y),$$

where L_z denotes the effective labor used in the homogeneous good industry. Given n_i and m_i this gives us the labor demand for the differentiated goods sector. Taking the difference between the total labor supply and the demand of labor for the differentiated goods yields the labor demand for the homogeneous sector. Applying Walras' Law allows us to concentrate on the long-run market equilibrium for differentiated products, to which we turn next.

For a firm producing m_i brands of quality q_i to be active, total output has to cover the fixed headquarters cost, K_i; in the long-run a firm has to break even. Let $m_i x_i$ define the total quantity of output of quality q_i a firm has to produce in order to break-even when its products are priced according to (23) and the optimal scope is given by (19). Total quantity of output then has to satisfy:

$$m_i x_i = \frac{q_i K_i}{n_i} \left[\frac{(n_i - 1)(n_i^2 - n_i + 1)}{(\mu_{if} - \mu_{ik})(n_i - 1)^2 + \mu_{if} n_i} \right], \tag{27}$$

[19] As is standard practice in the trade literature we ignore the integer constraint on n_i and m_i.

for $i = H, L$. The break-even output depends on the fixed headquarters cost K_i and crucially on the number of firms n_i: a higher number of firms reduces the price of varieties thereby requiring a larger total output for a firm to break even. Firms that produce goods of quality q_i base their entry-exit decision on the comparison of the break-even volume with the expected demand for their products. Firms will not produce if demand falls short of supply, $m_i d_{if} < m_i x_i$. In equilibrium total output of all differentiated products produced has to be equal the population size N, or

$$\sum_{i=L,H} n_i m_i x_i = N, \tag{28}$$

indicating that at least n_H or n_L has to be positive. In light of (20), if $m_i n_i \to 0$ while $m_{i'} n_{i'} > 0$, $i \neq i'$, the demand d_{if} approaches infinity. This means that a firm producing m_i brands of quality q_i will certainly be able to produce the break-even output when the number of its competitors offering a similar quality is sufficiently small as the break even output per brand falls while at the same time the price of the brand increases, see (23). As a result, in the autarky equilibrium both quality types exists with $n_i m_i > 0$, for $i = H, L$. Market equilibrium in the differentiated good sector then requires that $m_i x_i = m_i d_{if}$, for $i = H, L$, which combined with (20) can equivalently be expressed as

$$m_i x_i = \frac{N}{n_i} E\left[\frac{n_i^{\mu_{if}} m_i^{\mu_{ik}} \phi_i(y, n_i)}{n_H^{\mu_{Hf}} m_H^{\mu_{Hk}} \phi_H(y, n_H) + n_L^{\mu_{Lf}} m_L^{\mu_{Lk}} \phi_L(y, n_L)}\right], \tag{29}$$

for $i = H, L$. The autarky equilibrium is described by the seven equations (19), (27), (28), (29) six of of which are independent. These six equilibrium conditions determine the six endogenous variables: n_H, n_L, m_H, m_L, x_H, and x_L, which subsequently determine all remaining variables.

In Appendix A we provide the technical details of the following propositions:

Proposition 1 *There exits a stable and unique autarky equilibrium with $n_H > 2$ and $n_L > 2$ and $m_H \geq 1$ and $m_L \geq 1$ as long as (i) the competition effect dominates the quality selection effect (see, A.19) and (ii) the effect of a change in n_i on the operating profits of a firm from the same sector i dominates the effect on the operating profits from a change in the number of firms in the other sector, n_j, for $i, j = H, L$, and $i \neq j$ (see, A.20).*

Part (i) of the claim can best be illustrated by considering Figure 3. Figure 3 depicts the profit function (26) for a firm producing q_i. The operating profits are driven by a selection effect (the probability of selecting quality q_i) and a competition effect (the term in large square brackets in equation (26)). Suppose current profits $\Pi_i(n_H, n_L) > 0$. This will cause

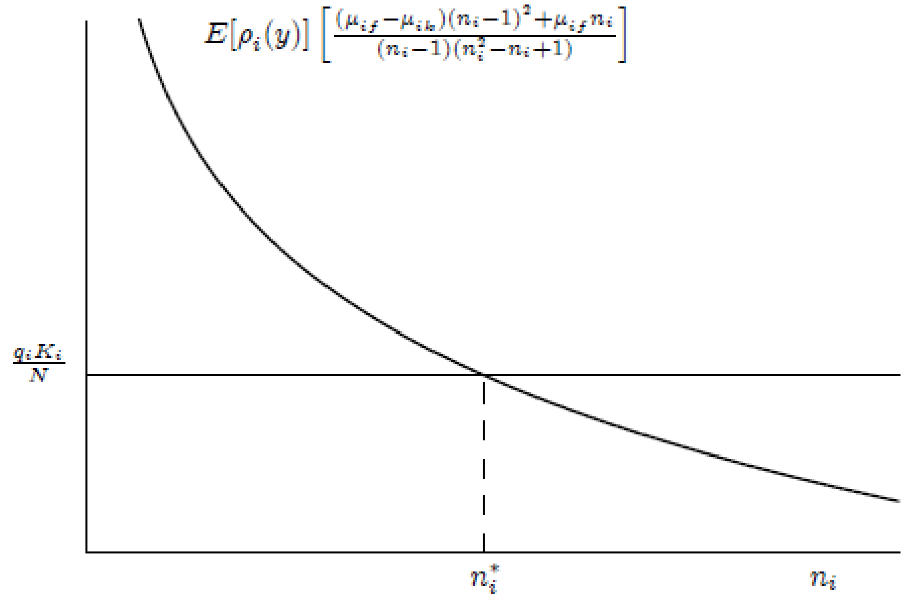

Figure 3: Long-run autarky equilibrium

new firms to enter that want to exploit these profit opportunities. With a higher number of firms in sector i, competition increases and operating profits decrease, ceteris paribus. As the number of firms increases it becomes more likely that a consumer selects a brand of quality q_i, which increases operating profits, given n_i. As long as the competition effect dominates the selection effect will there be a unique equilibrium as shown in Figure 3.[20]

We are now in the position to analyze how a change in the population size, N, and the income distribution, $\mathcal{F}_y(y)$, will affect the long-run equilibrium, that is how it will affect the number of firms, each firm's scope and scale of products with its corresponding price effects.

When considering an increase in the population size we have to discern two effects. First, a higher N provides an incentive for incumbent firms in both sectors to increase their range of products, $dm_i > 0$. Second, it provides an incentive for new firms to enter, $dn_i > 0$. Both, a larger range of products per firm (given the number of firms) and the entry of new firms (given the range of products per firm) reduce profitability in each sector, thereby diminishing the incentive for new firms to enter or for established firms to increase their product line, respectively. The number of new firms could increase to the point where

[20]The stable and unique long run equilibrium can be represented in a graph like Figure 2 by replacing m_i for n_i on the axes, relabelling the curves m_{ii} to n_{ii}. The stability analysis, provided in the Appendix, demonstrates the negative slope of the curves.

established firms have no incentive to offer a wider product line.[21] In Appendix A we show that in a two-stage product line price game with nested logit demand and free entry and exit both the equilibrium number of brands per firm and the equilibrium number of firms increase:

$$\frac{\widehat{n}_i}{\widehat{N}} = \frac{D_1}{D_2}\left\{\varphi_i - \mu_{ik}\frac{\rho_{HL}}{\rho_H\rho_L}\left(\varphi_i - \frac{n_i^2(n_i-2)}{(n_i^2-n_i+1)(n_i-1)}\right) - \mu_{if}\frac{\rho_{HL}}{\rho_H\rho_H}\left(1+\frac{n_i}{(n_i-1)^2}\right)\right\} > 0,$$

with $D_1 > 0$ and $D_2 > 0$; and

$$\frac{\widehat{m}_i}{\widehat{N}} = \left(\varphi_i - \frac{n_i^2(n_i-2)}{(n_i^2-n_i+1)(n_i-1)}\right)\frac{\widehat{n}_i}{\widehat{N}} > 0,$$

for $n_i > 2$, $i = H, L$. The relative change in the range of products and number of firms is proportional, with the factor of proportionality given by $\varphi_i - \frac{n_i^2(n_i-2)}{(n_i^2-n_i+1)(n_i-1)}$. The term $\varphi_i \geq 1$ captures the effect of firm entry on net total profits, see (A.18); the term $\frac{n_i^2(n_i-2)}{(n_i^2-n_i+1)(n_i-1)}$ measures the effect of firm entry on product level profits, see (19). Which of the two effects dominates depends on the number of firms active in industry i. When $n_i > 3$, this difference (positive) is smaller than 1, ensuring $\hat{n}_i > \hat{m}_i > 0$; in contrast, when $n_i = 2$ the effect on the range of products always dominates, i.e., $\hat{m}_i > \hat{n}_i > 0$. For $n_i = 3$, the effect on firms dominates as long as $\frac{\mu_{if}-\mu_{ik}}{\mu_{if}}$ is sufficiently large.[22] With a larger range of brands per firm and a larger number of firms, the scale of production decreases for firms to break even (A.22). At the same time the larger number of firms increases competition forcing prices per brand to fall (A.23). The results are summarized in the following proposition.

Proposition 2 *An increase in the size of the economy in the long run leads to more firms in both industries, each producing larger product lines with a smaller scale and lower prices. That is $\widehat{n}_i, \widehat{m}_i > 0$, $\widehat{x}_i < 0$, and $dp_i < 0$ for $i = H, L$. The firms expansion effect dominates the product line expansion when there are already multiple (>3) firms in the industries; when only two firms exist at the outset, the product line expansion dominates.*

The increase in the size of the economy furthermore generates a shift in the composition of the differentiated products. There is a shift in composition of differentiated goods toward

[21] If the assumed market structure would be monopolistic competitive, an increase in the population size would increase the number of firms producing in each sector to such an extend that the range of products per brand remains constant. That is, the number of firms would change but not the number of brands, see Schafgans and Stibora (2012).

[22] With $n_i = 3$, $\hat{n}_i > \hat{m}_i > 0$ as long as $\frac{\mu_{if}-\mu_{ik}}{\mu_{if}} > 3.45$.

the high-quality brands and firms, if the sign of

$$\begin{aligned}
&\varphi_H \frac{\widehat{n}_H}{\widehat{N}} - \varphi_L \frac{\widehat{n}_L}{\widehat{N}} \qquad (30)\\
= \ &\frac{D_1}{D_2} \frac{\rho_{HL}}{\rho_H \rho_L} \varphi_H \varphi_L \left\{ \left[\frac{\mu_{Hf}}{\varphi_H} \left(1 + \frac{n_H}{(n_H-1)^2} \right) - \frac{\mu_{Lf}}{\varphi_L} \left(1 + \frac{n_L}{(n_L-1)^2} \right) \right] + \right. \\
&\left. \left[\frac{\mu_{Hk}}{\varphi_H} \left(\varphi_H - \frac{n_H^2 (n_H - 2)}{(n_H^2 - n_H + 1)(n_H - 1)} \right) - \frac{\mu_{Lk}}{\varphi_L} \left(\varphi_L - \frac{n_L^2 (n_L - 2)}{(n_L^2 - n_L + 1)(n_L - 1)} \right) \right] \right\}.
\end{aligned}$$

is positive. The relative changes in n_H and n_L are multiplied by respectively φ_H and φ_L to take into account the fact that the marginal impact is not constant but depends on the initial number of firms active in each industry. If the number of active firms is the same in both industries (i.e., $n_H = n_L$) and all φ terms are equal an increase in the size of the economy will shift the composition of differentiated goods towards the high-quality brands and firms given $\mu_{Hk} > \mu_{Lk}$ and $\mu_{Hf} > \mu_{Lf}$ (that is assuming that households consider brands and firms of low-quality as closer substitutes than brands and firms of high-quality). A more reasonable set of assumptions is that there is a larger number of firms with low-quality brands ($n_L > n_H$) and a relatively smaller marginal impact of an additional low-quality firm ($\varphi_L < \varphi_H$). As a result of these different marginal impacts, larger differences $\mu_{Hk} - \mu_{Lk}$ and $\mu_{Hf} - \mu_{Lf}$ are needed to ensure that the composition effect continuous to favour the high quality sector. The difference in marginal impacts offset the selection and competition effects which both favour the high quality brand. We summarize this result in the following proposition.

Proposition 3 *When $n_L > n_H$, an increase in the size of the economy is more likely to be accompanied by a shift in its composition to the high-quality firms the more dissimilar consumers perceive the brands and firms of high and low-quality goods. When $n_L = n_H$, an increase in the size of the economy is always accompanied by a shift towards high-quality firms and products.*

Next we consider the consequences of changing the income distribution. First, we shift the income distribution $\mathcal{F}_y(y)$ downwards, in the sense of first order stochastic dominance. For given population size N, with a higher income in each income class consumers are more likely to choose a high-quality brand instead of a low-quality one. The shift in demand away from low-quality brands towards high-quality ones provides an incentive for new firms to enter the high-quality sector and for firms producing low-quality brands to exit

$$\begin{aligned}
\frac{\widehat{n}_H}{dy} &= -\frac{\varphi_L D_1}{D_2} \frac{\rho_{HL}}{\rho_H} \left[q_L - q_H \right] > 0 \\
\frac{\widehat{n}_L}{dy} &= \frac{\varphi_H D_1}{D_2} \frac{\rho_{HL}}{\rho_L} \left[q_L - q_H \right] < 0,
\end{aligned}$$

while at the same time for firms producing high (low) quality goods to offer a wider (smaller) range of brands

$$\frac{\widehat{m}_i}{dy} = \left(\varphi_i - \tfrac{n_i^2(n_i-2)}{(n_i^2-n_i+1)(n_i-1)}\right)\frac{\widehat{n}_i}{dy},$$

for $i = H, L$, with the relative change in the range of products and number of firms proportional as before. The additional competition brought about by the entrance of new firms pushes down the prices of high-quality goods relative to the prices in the low-quality sector:

$$\frac{dp_i}{dy} = -\frac{\mu_{if}}{qi}\frac{n_i}{(n_i-1)^2}\frac{\widehat{n}_i}{dy}.$$

Associated with the increased number of high-quality firms each offering a wider range of brands, the scale of production will be reduced so as to break even (reverse argument holding for low-quality firms):

$$\frac{\widehat{x}_i}{dy} = \tfrac{(2n_i-1)}{(n_i^2-n_i+1)(n_i-1)}\frac{\widehat{n}_i}{dy}.$$

We summarize these results in the following proposition.

Proposition 4 *An increase in income (first order stochastic dominance) leads to more high-quality firms, each producing a larger number of variants at a smaller scale with lower prices in the long-run equilibrium. The opposite occurs in the low-quality sector and the compositional of differentiated goods therefore clearly shifts towards high-quality brands. That is* $\widehat{n}_L < 0 < \widehat{n}_H$, $\widehat{m}_L < 0 < \widehat{m}_H$, $\widehat{x}_H < 0 < \widehat{x}_L$, $dp_L > 0$, *and* $dp_H < 0$.

The effect of an increase in income inequality has similar consequences under the assumption that $\rho_H(y)$ is a convex increasing function of y. The qualitative difference emanates as before from the effect these income distributional changes have on the fraction of households purchasing the high-quality good.

Proposition 5 *If* $\rho_L(y) > \rho_H(y)$ *for all* y, *an increase in income inequality (second order stochastic dominance) leads to more high-quality firms, each producing a larger number of variants at a smaller scale with lower prices in the long-run equilibrium. The opposite occurs in the low-quality sector and the compositional of differentiated goods therefore clearly shifts towards high-quality brands. That is* $\widehat{n}_L < 0 < \widehat{n}_H$, $\widehat{m}_L < 0 < \widehat{m}_H$, $\widehat{x}_H < 0 < \widehat{x}_L$, $dp_L > 0$, *and* $dp_H < 0$.

3.3 Welfare

We next turn to the welfare implications associated with the previous analysis. In our model, the expected maximum utility of a household with income y, $E[\max U^*(y)]$, is given by[23]

$$\ln\left(n_L^{\mu_{Lf}} m_L^{\mu_{Lk}} \exp\left((y-p_L)\, q_L\right) + n_H^{\mu_{Hf}} m_H^{\mu_{Hk}} \exp\left((y-p_H)\, q_H\right)\right).$$

The expected maximum utility is increasing with income and welfare therefore is enhanced as a result of a first order stochastic dominating change in the income distribution. Since the expected marginal utility of income is increasing, $\partial^2 E(\max U^*(y))/\partial^2 y > 0$, the mean preserving spread improves welfare as well.[24]

These welfare implications ignore the fact that $\{n_H, n_L, m_H, m_L\}$ will change as well, and the associated compositional changes in the relative number of brands and firms due to a distributional change in income and the population size (discussed in detail in the previous section) may affect income groups differently. To analyze this, we turn now to the expected welfare for a household with income y. With

$$v(y) \equiv \left[n_H^{\mu_{Hf}} m_H^{\mu_{Hk}} \phi_H(y, n_H)\right] + \left[n_L^{\mu_{Lf}} m_L^{\mu_{Lk}} \phi_L(y, n_L)\right],$$

these effects are provided by $\hat{v}(y)$, which we derive in Appendix A. Acknowledging that the relative changes in m_i are proportional to the relative changes in n_i, we decompose the change in expected welfare for a household with income y into a pure *scale* and pure *composition effect* as Fajgelbaum et al. (2011):

$$\begin{aligned}
\hat{v}(y) \;=\; & \left\{ \begin{array}{l} \frac{\rho_H(y)}{\rho_H \varphi_H}\left[\mu_{Hf}\left(1+\frac{n_H}{(n_H-1)^2}\right)+\mu_{Hk}\left(\varphi_H-\frac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right)\right] \\ +\frac{\rho_L(y)}{\rho_L \varphi_L}\left[\mu_{Lf}\left(1+\frac{n_L}{(n_L-1)^2}\right)+\mu_{Lk}\left(\varphi_L-\frac{n_L^2(n_L-1)}{(n_L^2-n_L+1)(n_L-1)}\right)\right] \end{array} \right\} \widehat{N} \qquad (31) \\
& +\rho_H \rho_L \left\{ \begin{array}{l} \left[\frac{\rho_H(y)}{\rho_H}\frac{\mu_{Hf}}{\varphi_H}\left(1+\frac{n_H}{(n_H-1)^2}\right)-\frac{\rho_L(y)}{\rho_L}\frac{\mu_{Lf}}{\varphi_L}\left(1+\frac{n_L}{(n_L-1)^2}\right)\right] \\ +\left[\frac{\rho_H(y)}{\rho_H}\frac{\mu_{Hk}}{\varphi_H}\left(\varphi_H-\frac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right)-\frac{\rho_L(y)}{\rho_L}\frac{\mu_{Lk}}{\varphi_L}\left(\varphi_L-\frac{n_L^2(n_L-1)}{(n_L^2-n_L+1)(n_L-1)}\right)\right] \end{array} \right\} \\
& \times\left(\varphi_H \widehat{n}_H-\varphi_L \widehat{n}_L\right),
\end{aligned}$$

where $\rho_i(y)$ denotes the fraction of consumers with income y who purchase differentiated brand q_i and ρ_i is the fraction of all households purchasing differentiated brand with quality q_i, respectively.

The pure *scale effect* is denoted by the first term on the right hand side of (31): holding the relative number of brands and firms constant ($\hat{n}_L = \hat{n}_H = 0$), a larger market results in

[23] Uses results from McFadden (1978).

[24] Assuming social welfare to be the sum of individual utilities, a decrease in inequality is only welfare improving if there is diminishing marginal (social) utility of income, see e.g., Davidson (2008).

a larger number of firms each offering more brands thereby increasing the likelihood that a household finds his or her preferred brand. The pure scale effect is non-negative and, consequently, no income group is worse off following an increase in the size of the economy. There is no pure scale effect when considering the welfare implications associated with changes in the income distribution.

The pure *composition effect* is denoted by the second term on the right hand side of (31). It captures the welfare impact arising from changes in the relative number of firms and brands of different quality, for given population size ($\hat{N} = 0$). The sign of this term is ambiguous and depends on the interaction of the parameters measuring degree of *intra*- and *inter*-firm heterogeneity, the number of firms active in each differentiated goods' sector, the purchasing behavior of households with income y relative to all households, the selection effect, and the strength of the competition effects.

If the initial number of firms in both industries is large then the welfare analysis (31) reduces to that of Fajgelbaum et al. (2011). In this limiting case of monopolistic competition both the change in the number of brands and the change in the price of brands are absent in the change of the expected welfare as m_i and p_i are not affected by changes in the size of the population or the income distribution (which is the interesting contribution in our paper). As a consequence, their composition effect is purely driven by the parameter measuring inter-firm heterogeneity, μ_{if}, and the purchasing behavior of particular income groups relative to that of the total population, as measured by $\rho_i(y)/\rho_i$, for $i = H, L$. In our case, the intra-firm heterogeneity parameters, μ_{ik}, the competition effect (represented by φ_i and $n_i^2(n_i - 2)/[(n_i^2 - n_i + 1)(n_i - 1)]$) and the selection effect (represented by $1 + n_i/(n_i - 1)^2$) enter as well.

Our welfare analysis simplifies if we assume $n_L = n_H$ (all φ terms are the same). In that case, it is clear that the term in brackets associated with the composition effect is more likely to be positive for the richest income groups in the economy given $\mu_{Hk} > \mu_{Lk}$ and $\mu_{Hf} > \mu_{Lf}$. The richest income groups in the economy with income y_{max} buy a larger fraction of high-quality brands and a smaller fraction of the low-quality brands than the average household, i.e. $\rho_H(y_{\max})/\rho_H > 1 > \rho_L(y_{\max})/\rho_L$. Consequently any change which induces the composition of firms to favour the high quality firms, i.e., for which $\varphi_H \widehat{n}_H - \varphi_L \widehat{n}_L > 0$, will benefit the rich more than the poor. The poor, who are more likely than average to purchase the low-quality good and less likely than average to consume the high-quality good, are more likely to benefit if consumers perceive the brands and firms of high and low-quality goods as more dissimilar. When $n_L > n_H$, a larger brand and firm dissimilarity is needed to ensure even the rich benefit.

The condition that ensures that both the pure scale and pure composition welfare effect benefit the rich more than the poor is automatically satisfied by the income distributional changes considered (first order stochastic dominance and mean preserving spread) as both favour the high-quality sector more than the low-quality sector. In fact, while the poor will see their welfare increase by these income changes, their pure composition effect may be negative (in which case the positive pure scale effect dominates the negative pure composition effect); a negative composition effect for the poor is less likely when brands and firms of high and low-quality are perceived to be very dissimilar. A similar discussion follows regarding the welfare implications – favouring the rich more than the poor – of an increase in the size of the economy, when it also favours the high-quality sector more than the low-quality sector, see Proposition (3).

4 Open economy

In this section we evaluate how, in the presence of non-homothetic preferences and vertical product differentiation, trade liberalization affects the number of firms and brands per firm. To this end, we extend the previous framework to allow for two countries that can trade with one another. We abstract from supply side explanations, thereby allowing us to concentrate on the trade liberalization effects in the presence of non-homothetic preferences and oligopolistic market structure.

We consider the countries a and b. In each country, there are N^a (N^b) households with identical preferences given by (1). As before, we assume that the homogeneous good's industry is perfectly competitive, producing under constant returns to scale with one unit of effective labor per unit of output in both countries. Their output is traded freely. With regard to the differentiated goods' sectors, firms can sell their brands in both countries but produce only in their home country, thereby abstracting from multinationals. Assuming that the supply of labor in each country is sufficient to produce both the homogeneous and the differentiated goods, the wage rate is the same in both countries and is independent of the equilibrium in the differentiated goods' sector.

In line with a large body of empirical literature, firms have to incur cost when trading in differentiated goods. For simplicity, we assume that each brand of quality q_i faces the same transportation cost, but that transportation costs differ across quality classes. More specifically, it takes τ_i units of effective labor to trade one unit of a brand with quality q_i from one country to the other, for $i = H, L$. The presence of transportation cost drives a wedge between the consumer and producer price of a brand. We assume that each firm considers markets to be segmented so that a firm chooses a different price of the same brand in each market. Hence, the profit function of firm f from country a and b are given by

$$\Pi^a_{if} = \sum_{k=1}^{m^a_{if}} \left[\left(p^a_{ifk} - c_i\right) d^a_{ifk} + \left(p^{ab}_{ifk} - c_i - \tau_L\right) d^{ab}_{ifk} \right] - F_i m^a_{if} - K_i \tag{32}$$

$$\Pi^b_{if} = \sum_{k=1}^{m^b_{if}} \left[\left(p^b_{ifk} - c_i\right) d^b_{ifk} + \left(p^{ba}_{ifk} - c_i - \tau_L\right) d^{ba}_{ifk} \right] - F_i m^b_{if} - K_i, \tag{33}$$

respectively, where d^a_{ifk} (d^b_{ifk}) is the aggregate demand from local households for brand k produced by firm f and d^{ab}_{ifk} (d^{ba}_{ifk}) is the aggregate demand for the same brand from foreign households. The aggregate demand functions are specified as

$$d^a_{ifk} = N^a \rho^a_{k|i,f} \cdot \rho^a_{f|i} \cdot E^a[\rho^a_i(y)] \tag{34}$$

$$d^{ab}_{ifk} = N^b \rho^a_{k|i,f} \cdot \rho^{ab}_{f|i} \cdot E^b[\rho^b_i(y)], \tag{35}$$

where E^a (E^b) denotes the expectation with respect to the income distribution of country a (b). The conditional probability $\rho^a_{k|i,f}$ is defined as in (3). The conditional probability that a domestic brand from firm f is purchased by consumers from country a (b) is $\rho^a_{f|i}$ ($\rho^b_{f|i}$); similarly the conditional probability that consumers from country b (a) select firm f originating from country a (b) is $\rho^{ab}_{f|i}$ ($\rho^{ba}_{f|i}$), see also (4) and (5).

Since the analytical complexity increases considerably, we consider two particular cases. In the first case, we assume that the rich country produces both high-quality and low-quality goods, while the poor country is specialized in the production of low-quality goods only. We characterize its associated short-run equilibrium. In the second case, we assume that both countries are completely specialized – with the rich country producing the high-quality goods and the poor country the low quality goods. Here we characterize both the short- and long-run equilibrium. Both production patterns can occur in our framework if the income distribution of the rich country (country a) first (second)-order stochastically dominates the income distribution of the poor country (country b) given market sizes, that is $\mathcal{F}^a_y(y) \leq \mathcal{F}^b_y(y)$, for all y.[25]

4.1 Trade with incomplete specialization

Let us assume that the rich country produces both types of differentiated goods while the poor country only produces the low-quality good. When a firm exports a unit of output it has to pay $\tau_i > 0$ units of transportation, $i = H, L$. As before, we adopt the symmetric equilibrium of the sequential game where firms first decide on the range of products before determining the price structure.

[25] Hallak (2006) and Hallak and Schott (2011) show that the quality level is correlated with development.

First, we evaluate the price setting for products sold in country a assuming that all firms in a except firm f, have $m_i^a \geq 1$ brands while firm f has m_{if}^a brands, and all firms from country b have $m_L^b \geq 1$ brands ($m_H^b = 0$ since only firms in country a produce the H good). The mark-up over marginal cost of firms selling in a can be expressed as

$$\left(p_{if}^a - c_i\right) = \frac{\mu_{if}}{q_i} \frac{1}{(1-\rho_{f|i}^a)}; \tag{36}$$

$$\left(p_{ij}^a - c_i\right) = \frac{\mu_{if}}{q_i} \frac{1}{(1-\rho_{j|i}^a)} \qquad j = 1, .., n_i^a \quad j \neq f; \tag{37}$$

$$\left(p_{Lj}^{ba} - c_L - \tau_L\right) = \frac{\mu_{Lf}}{q_L} \frac{1}{(1-\rho_{j|L}^{ba})} \qquad j = 1, .., n_L^b. \tag{38}$$

With the differences of prices denoted by

$$\begin{aligned} \chi_i^a &= \left(p_i^a - p_{if}^a\right) q_i/\mu_{if}, \qquad i = H, L \\ \chi_L^{ba} &= \left(p_L^a - p_L^{ba}\right) q_L/\mu_{Lf}, \end{aligned}$$

(recognizing that $p_{ij} = p_i$, $j \neq f$ and $p_{Lj}^{ba} = p_L^{ba}$), the conditional probabilities can be expressed as

$$\begin{aligned} \rho_{f|i}^a &= \frac{M_i^a \exp\left(\chi_i^a\right)}{(n_i^a - 1) + M_i^a \exp\left(\chi_i^a\right) + n_i^b M_i^{ba} \exp\left(\chi_i^{ba}\right)} \\ \rho_{j|i}^a &= \frac{1}{(n_i^a - 1) + M_i^a \exp\left(\chi_i^a\right) + n_i^b M_i^{ba} \exp\left(\chi_i^{ba}\right)} \\ \rho_{j|L}^{ba} &= \frac{M_L^{ba} \exp\left(\chi_L^{ba}\right)}{(n_L^a - 1) + M_L^a \exp\left(\chi_L^a\right) + n_L^b M_L^{ba} \exp\left(\chi_L^{ba}\right)}, \end{aligned}$$

where $M_i^a \equiv (m_{if}^a/m_i^a)^{\frac{\mu_{ik}}{\mu_{if}}}$ and $M_L^{ba} \equiv (m_L^b/m_L^a)^{\frac{\mu_{Lk}}{\mu_{Lf}}}$ ($M_H^{ba} = 0$ because only firms in a produce H). As in the closed economy setting, there is a unique solution for χ_H^a yielding a unique price equilibrium ($p_H^a = p_{Hf}^a$). In Appendix B, it is shown that the resulting system of equations in χ_L^a and χ_L^{ba} (see (B.5) and (B.6)) also has a unique solution yielding a unique price equilibrium ($p_L^a = p_{Lf}^a, p_L^{ba}$). Equally, a unique price equilibrium for products sold in b is found: for the H good ($p_H^{ab} = p_{Hf}^{ab}$) and for the L good $\left(p_L^{ab} = p_{Lf}^{ab}, p_L^b\right)$.

Next, we consider the optimal choice of number of varieties for firm f in country a. Given the above pricing strategy, firm f maximizes his profits, i.e.,

$$\max_{m_{if}^a} \pi_{if}^a = \frac{\mu_{if}}{q_i} \left[\frac{\rho_{f|i}^a}{(1-\rho_{f|i}^a)} N^a E^a[\rho_i^a(y)] + \frac{\rho_{f|i}^{ab}}{(1-\rho_{f|i}^{ab})} N^b E^b[\rho_i^b(y)]\right] - F_i m_{if}^a - K_i, \; i = H, L.$$

For high quality firms in country a, the profit function simplifies to

$$\pi_{Hf}^a = \frac{\mu_{Hf}}{q_H\left(n_H^a - 1\right)} \left[N^a E^a[\rho_H^a(y)] + N^b E^b[\rho_H^b(y)]\right] \left(\frac{m_{Hf}^a}{m_H^a}\right)^{\mu_{Hk}/\mu_{Hf}} \exp\left(\chi_H^a\right) - F_H m_{Hf}^a - K_H,$$

as $\rho^a_{f|H} = \rho^{ab}_{f|H}$ (χ^a_H defined as in (15)). Evaluating its first order condition at a symmetric equilibrium with $m^a_{Hf} = m^a_H$, yields a condition similar to that of the closed economy, specifically

$$\frac{q_H}{\mu_{Hk}} m^a_H F_H = \left(N^a E^a \left[\rho^a_H(y)\right] + N^b E^b \left[\rho^b_H(y)\right]\right) \left[\frac{n^a_H - 1}{\left(n^a_H\right)^2 - n^a_H + 1}\right]. \tag{39}$$

For low quality firms in country a, the choice of the optimal range of brands that maximizes profits becomes more complex. The first order condition with respect to m^a_{Lf} is:

$$\begin{aligned} 0 \quad = \quad & \frac{N^a \rho^a_{f|L}}{(1-\rho^a_{f|L})} E^a[\rho^a_L(y)] \frac{\mu_{Lf}}{q_L} \left\{ \frac{\mu_{Lk}}{\mu_{Lf}} \frac{1}{m^a_{Lf}} + \frac{d\chi^a_L}{dm^a_{Lf}} - \frac{\rho^{ba}_{j|L}}{(1-\rho^a_{f|L})} n^b_L \frac{d\chi^{ba}_L}{dm^a_{Lf}} \right\} \\ & + \frac{N^b \rho^{ab}_{f|L}}{(1-\rho^{ab}_{f|L})} E^b[\rho^b_L(y)] \frac{\mu_{Lf}}{q_L} \left\{ \frac{\mu_{Lk}}{\mu_{Lf}} \frac{1}{m^a_{Lf}} + \frac{d\chi^{ab}_L}{dm^a_{Lf}} - \frac{\rho^b_{j|L}}{(1-\rho^{ab}_{f|L})} n^b_L \frac{d\chi^b_L}{dm^a_{Lf}} \right\} - F_L, \end{aligned}$$

where the terms $d\chi^a_L/dm^a_{Lf}$ and $d\chi^{ba}_L/dm^a_{Lf}$ capture the effect a change in m^a_{Lf} has on the price differential between local and foreign competitors in the home market. Similarly, $d\chi^{ab}_L/dm^a_{Lf}$ and $d\chi^b_L/dm^a_{Lf}$ capture the effect a change in m^a_{Lf} has on the price differential between local and foreign competitors in the foreign market. As shown in Appendix B, evaluating the first order condition at a symmetric equilibrium where $m^a_{Lf} = m^a_L$ yields

$$\begin{aligned} \frac{q_L}{\mu_{Lk}} m^a_L F_L \quad = \quad & N^a E^a[\rho^a_L(y)] \left[\frac{\tilde{n}^a_L - 1}{\left(\tilde{n}^a_L\right)^2 - \tilde{n}^a_L + 1}\right] \left[1 + s^a(m^a_L, m^b_L)\right] \\ & + N^b E^b[\rho^b_L(y)] \left[\frac{\tilde{n}^{ab}_L - 1}{\left(\tilde{n}^{ab}_L\right)^2 - \tilde{n}^{ab}_L + 1}\right] \left[1 + s^{ab}(m^a_L, m^b_L)\right] \end{aligned} \tag{40}$$

where $\tilde{n}^a_L = n^a_L + \theta^a n^b_L$ and $\tilde{n}^{ab}_L = n^a_L + \theta^{ab} n^b_L$, and

$$\theta^a = M^{ba}_L \exp\left(\chi^{ba}_L\right) \text{ and } \theta^{ab} = M^{ba}_L \exp\left(\chi^b_L\right).$$

The $(\tilde{n}^a_L, \tilde{n}^{ab}_L)$ terms can be interpreted as the effective competitors in the domestic and foreign market; they reflect trade costs and differences in number of brands among domestic and foreign firms. The $s^a(m^a_L, m^b_L)$ and $s^{ab}(m^a_L, m^b_L)$ terms are defined in (B.16) and (B.17), they reflect direct relative price effects arising from foreign competition (see more below).

In the extreme setting of high trade costs: $\tilde{n}^a_L = n^a_L$ ($\theta^a \to 0$ and the probability of selling domestically, ρ^a_f, equals $1/n^a_L$) and $\tilde{n}^{ab}_L \to \infty$ ($\theta^{ab} \to \infty$ and $\rho^{ab}_f \to 0$, as if the effective competitors from the foreign market are too large). In the other extreme setting of zero trade costs, when $m^a_L = m^b_L$: $\tilde{n}^a_L = \tilde{n}^{ab}_L = n^a_L + n^b_L$ ($\theta^a \to 1, \theta^{ab} \to 1$ and the probability of selling domestically or abroad is equal $1/(n^a_L + n^b_L)$). In this case countries a and b can be seen to be one big country. In both cases the terms (s^a, s^{ab}) in (40) vanish; in

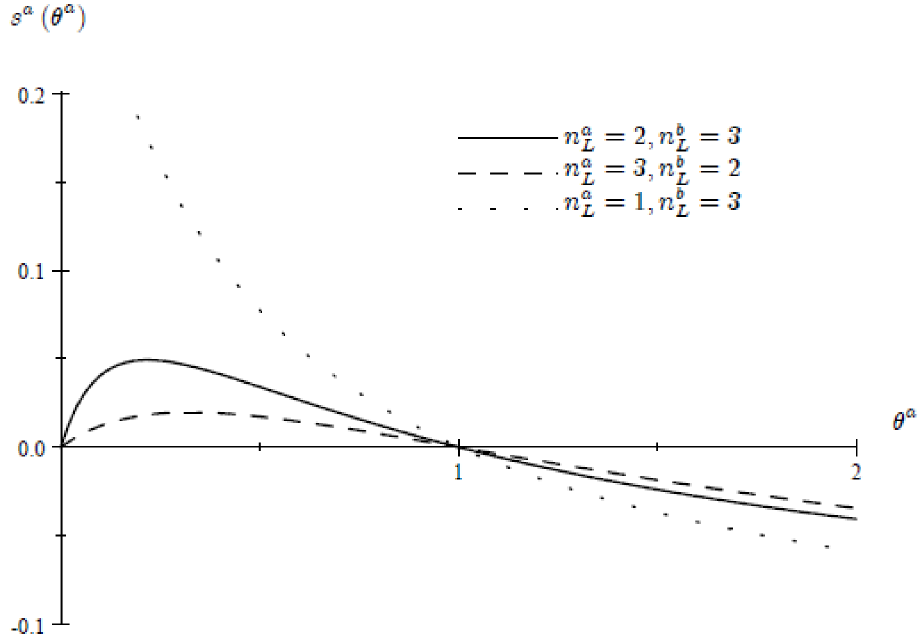

Figure 4: $s^a(m_L^a, m_L^b)$ as a function of $\theta^a \equiv (\frac{m_L^b}{m_L^a})^{\frac{\mu_{Lk}}{\mu_{Lf}}} \exp\left(\chi_L^{*ba}\right)$

the high trade cost setting we are back in the autarky setting while in the zero trade cost setting with equality of number of brands we get a first order condition similar to equation (39). In general, though, we need to consider the impact of the terms (s^a, s^{ab}).

The term $s^a(m_L^a, m_L^b)$ measures a direct relative price effect of the additional competition faced by firm f in the home market brought about by foreign competitors via changes in the product range; likewise $s^{ab}(m_L^a, m_L^b)$ measures a direct relative price effect foreign firms face from country a firms brought about by changes in the range of products. Depending on their signs the impact on the revenue is strengthened (positive) or weakened (negative) in the presence of foreign competition; it is influenced by the share of foreign effective competitors (in addition to trade costs).

Figure 4 depicts the relationship between $s^a(m_L^a, m_L^b)$ and θ^a for various values of $\tilde{n}_L^a$ and $\tilde{n}_L^{ab}$. In most cases, the relationship is negative (unless $m_L^a >> m_L^b$). For sufficiently large n_L^a and n_L^b, the condition determining the sign of the terms (s^a, s^{ab}) can be reduced to whether $\left(\theta^a, \theta^{ab}\right)$ is larger (smaller) than one, rendering θ negative (positive).

For positive but not prohibitively large transportation cost, $0 < s^a(m_L^a, m_L^b) < 1$ and $s^{ab}(m_L^a, m_L^b) < 0$ so that marginal profits from an additional brand are larger in the home market than the marginal profits attained in the foreign market, ceteris paribus.

The subgame of a foreign firm (assuming that all firms in b except firm f, have $m_L^b \geq 1$ brands while firm f has m_{Lf}^b brands, and all firms from country a have $m_L^a \geq 1$ brands) is derived analogous to (40). Evaluating its first order condition at a symmetric equilibrium where $m_{Lf}^b = m_L^b$ yields

$$\begin{aligned} \frac{q_L}{\mu_{Lk}} m_L^b F_L &= N^b E^b[\rho_L^b(y)] \left[\frac{\tilde{n}_L^b - 1}{\left(\tilde{n}_L^b\right)^2 - \tilde{n}_L^b + 1} \right] \left[1 + s^b(m_L^a, m_L^b)\right] \\ &\quad + N^a E^a[\rho_L^a(y)] \left[\frac{\tilde{n}_L^{ba} - 1}{\left(\tilde{n}_L^{ba}\right)^2 - \tilde{n}_L^{ba} + 1} \right] \left[1 + s^{ba}(m_L^a, m_L^b)\right], \end{aligned} \tag{41}$$

where $\tilde{n}_L^b = n_L^b + \theta^b n_L^a$ and $\tilde{n}_L^{ba} = n_L^b + \theta^{ba} n_L^a$.[26]

At equilibrium prices,

$$\begin{aligned} & E^a[\rho_L^a(y)] \\ = \; & E^a \left[\frac{\left[(n_L^a)^{\mu_{Lf}} (m_L^a)^{\mu_{Lk}} + \left(\lambda_L v_L^a n_L^b\right)^{\mu_{Lf}} \left(m_L^b\right)^{\mu_{Lk}}\right] \phi_L(y, \tilde{n}_L^a)}{\left[(n_L^a)^{\mu_{Lf}} (m_L^a)^{\mu_{Lk}} + \left(\lambda_L v_L^a n_L^b\right)^{\mu_{Lf}} \left(m_L^b\right)^{\mu_{Lk}}\right] \phi_L(y, \tilde{n}_L^a) + (n_H^a)^{\mu_{Hf}} (m_H^a)^{\mu_{Hk}} \phi_H(y, n_H^a)} \right] \\ & \text{with } \lambda_L = \exp\left(-\tau_L q_L / \mu_{Lf}\right) \text{ and } v_L^a = \exp\left(\tfrac{\tilde{n}_L^a}{(\tilde{n}_L^a - 1)} - \tfrac{\tilde{n}_L^a}{(\tilde{n}_L^a - \theta^a)} \right) \end{aligned} \tag{42}$$

and

$$\begin{aligned} & E^b[\rho_L^b(y)] \\ = \; & E^b \left[\frac{\left[\left(\lambda_L v_L^b n_L^a\right)^{\mu_{Lf}} (m_L^a)^{\mu_{Lk}} + \left(n_L^b\right)^{\mu_{Lf}} \left(m_L^b\right)^{\mu_{Lk}}\right] \phi_L(y, \tilde{n}_L^a)}{\left[\left(\lambda_L v_L^b n_L^a\right)^{\mu_{Lf}} (m_L^a)^{\mu_{Lk}} + \left(n_L^b\right)^{\mu_{Lf}} \left(m_L^b\right)^{\mu_{Lk}}\right] \phi_L(y, \tilde{n}_L^a) + (\lambda_H n_H^a)^{\mu_{Hf}} (m_H^a)^{\mu_{Hk}} \phi_H(y, n_H^a)} \right] \\ & \text{with } \lambda_H = \exp\left(-\tau_H q_H / \mu_{Hf}\right) \text{ and } v_L^b = \exp\left(\tfrac{\tilde{n}_L^b}{(\tilde{n}_L^b - 1)} - \tfrac{\tilde{n}_L^b}{(\tilde{n}_L^b - \theta^b)} \right). \end{aligned} \tag{43}$$

These expectations resemble equation (21) obtained in autarky with $\phi_i(y, n_i)$ defined as before (see (22)). The term λ_i, $i = H, L$, accounts for trade costs differentials between the two countries. When λ_i is close to one (zero), trade costs associated with quality q_i are small (high) which will increase (decrease) the aggregate demand for its export. In the extreme setting of prohibitive trade costs: $\lambda_i = 0$ for $i = L, H$ and $E^a\left[\rho_L^a(y)\right]$ is equal to the autarky solution and $E^b[\rho_L^b(y)] = 1$ as b only produces L and H is not traded.

While consumers in country a will face the transport cost for the imported low-quality commodity only, consumers from country b face transport cost associated with both the high-quality and the low-quality commodity. The λ's therefore discount the demands for different quality goods for the associated differences in trade costs.

The equations (39), (40) and (41) provide a system of equations that determine (m_L^a, m_L^b, m_H) in the symmetric equilibrium given the number of firms.

[26]In Appendix B, θ^b, θ^{ba}, $s^b(m_L^a, m_L^b)$ and $s^{ba}(m_L^a, m_L^b)$ are defined explicitly.

4.1.1 Short-run open economy equilibrium

We now analyze how changes in trade cost affect the range of brands per firm, holding the number of firms constant. To do this we recognize that using the total differential of equations (39), (40) and (41), we can characterize this by three individual effects: (i) the selection effect associated with the direct impact on $E^a[\rho_i^a(y)]$ and $E^b[\rho_i^b(y)]$ $i = H, L$, (ii) the competition effect associated with the impact on the effective number of competitors $[\tilde{n}_L^a, \tilde{n}_L^{ba}, \tilde{n}_L^b, \tilde{n}_L^{ba}]$ and (iii) the direct relative price effect associated with the impact on the $\left(s^a, s^{ab}, s^b, s^{ba}\right)$ terms. Since these three effects are not unidirectional, the net effect on the range of products of either quality turns out to be ambiguous and depend on their relative strengths.[27]

The analysis, details of which can be found in Appendix B, is notationally quite demanding. We have introduced the following notation: related to the selection effect, e.g., $\delta_{m_L^a \rho_L^a}$ and $\delta_{m_L^b \rho_L^a}$ measure the effect of a change in m_L^a and m_L^b, respectively, on ρ_L^a;[28] related to the competition effect we have

$$h_L^a \equiv \left[\omega_L^{aa} \frac{w_L^a}{(\tilde{n}_L^a - 1)} \frac{(\tilde{n}_L^a)^2 (\tilde{n}_L^a - 2)}{\left[(\tilde{n}_L^a)^2 - \tilde{n}_L^a + 1\right]} + \omega_L^{ab} \frac{w_L^{ab}}{\left(\tilde{n}_L^{ab} - 1\right)} \frac{\left(\tilde{n}_L^{ab}\right)^2 (\tilde{n}_L^{ab} - 2)}{\left[\left(\tilde{n}_L^{ab}\right)^2 - \tilde{n}_L^{ab} + 1\right]}\right] > 0$$

$$h_L^b \equiv \left[\omega_L^{bb} \frac{w_L^b}{\left(\tilde{n}_L^b - 1\right)} \frac{\left(\tilde{n}_L^b\right)^2 (\tilde{n}_L^b - 2)}{\left[\left(\tilde{n}_L^b\right)^2 - \tilde{n}_L^b + 1\right]} + \omega_L^{ba} \frac{w_L^{ba}}{\left(\tilde{n}_L^{ba} - 1\right)} \frac{\left(\tilde{n}_L^{ba}\right)^2 (\tilde{n}_L^{ba} - 2)}{\left[\left(\tilde{n}_L^{ba}\right)^2 - \tilde{n}_L^{ba} + 1\right]}\right] > 0;$$

and related to the direct price effect we have

$$e_L^a \equiv \left[\omega_L^{aa} \frac{s^a}{(1 + s^a)} \varepsilon_{s^a} + \omega_L^{ab} \frac{s^{ab}}{(1 + s^{ab})} \varepsilon_{s^{ab}}\right]$$

$$e_L^b \equiv \left[\omega_L^{bb} \frac{s^b}{(1 + s^b)} \varepsilon_{s^b} + \omega_L^{ba} \frac{s^{ba}}{(1 + s^{ba})} \varepsilon_{s^{ba}}\right].$$

The h_L^a (h_L^b) terms measure the impact changes in the relative number of brands (m_L^a, m_L^b) have through changes in the appropriately weighted effective competitiveness in the domestic- and foreign market, $\tilde{n}_L^a$ and $\tilde{n}_L^{ab}$ ($\tilde{n}_L^b$ and $\tilde{n}_L^{ba}$), respectively .[29] The e_L^a (e_L^b) terms similarly

[27] Note, we start from the premises that a symmetric equilibrium for the product range game exists without providing a proof. In order to keep the comparative-statics analysis tractable we take into account the direct effect of changes in τ_i for $i = H, L$ but ignore second round effects on $\tilde{n}_L^a$ and $\tilde{n}_L^b$ implying that χ_L^b (χ_L^{*a}) and χ_L^{ba} (χ_L^{*ab}) are constant.

[28] The δ parameters are defined in Appendix B, see equations (B.19) and (B.20).

[29] The weights $\omega_L^{aa} = 1 - \omega_L^{ab}$ and $\omega_L^{ba} = 1 - \omega_L^{bb}$ are defined in (B.21) and (B.22) and denote the share of domestic (foreign) marginal profits in hands of firms from country a; and w_L^a and w_L^b are defined in (B.15) and (B.18) denoting the share of effective foreign (home) brands in the home (foreign) market.

measure the impact changes in the relative number of brands (m_L^a, m_L^b) have through a change in relative prices of domestic and foreign competitors s^a and s^{ab} (s^b and s^{ba}), respectively; ε_{s^a} denotes the θ^a elasticity of $s_L^a(m_L^a, m_L^b)$ and $\varepsilon_{s^{ab}}$ the θ^{ab} elasticity of $s_L^{ab}(m_L^a, m_L^b)$.

The signs of h_L^a and h_L^b are unambiguously positive. The signs of e_L^a and e_L^b are ambiguous, though, as the elasticities are negative for a large range of (m_L^a, m_L^b) combinations as reflected by the θ's (as suggested by Figure 4) and $0 < s^a < 1$, $0 < s^b < 1$, $s^{ab} < 0$, and $s^{ba} < 0$.

In the discussion of the short term impacts, it will be important to decide whether we think that the price effect (if positive) can outweigh the competition effect, as these terms appear as $h_L^a - e_L^a$ and $h_L^b - e_L^b$, and how their joint impact compares to the selection effect.

We concentrate here on a discussion of the effect of a reduction in transportation cost of high-quality goods, τ_H, or equivalently an increase in λ_H. In Appendix B, we derive the following short-run impacts: for the high quality brand we have

$$\frac{\hat{m}_H^a}{\hat{\lambda}_H} = \frac{\delta_{\lambda_H \rho_L^b}}{D_{IS}} \left\{ \begin{array}{l} \omega_H^b \frac{\rho_L^b}{\rho_H^b} \left[1 - \omega_L^{aa} \delta_{m_L^a \rho_L^a} - \omega_L^{ba} \delta_{m_L^b \rho_L^a}\right] + \omega_H^a \frac{\rho_L^a}{\rho_H^a} \left[\omega_L^{ab} \delta_{m_L^a \rho_L^a} + \omega_L^{bb} \delta_{m_L^b \rho_L^a}\right] \\ -\left(h_L^b - e_L^b\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_H^b \frac{\rho_L^b}{\rho_H^b} \left[1 - \omega_L^{aa} \left(\delta_{m_L^a \rho_L^a} + \delta_{m_L^b \rho_L^a}\right)\right] + \omega_H^a \frac{\rho_L^a}{\rho_H^a} \omega_L^{ab} \left(\delta_{m_L^a \rho_L^a} + \delta_{m_L^b \rho_L^a}\right)\right] \\ -\left(h_L^a - e_L^a\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_H^b \frac{\rho_L^b}{\rho_H^b} \left[1 - \omega_L^{ba} \left(\delta_{m_L^a \rho_L^a} + \delta_{m_L^b \rho_L^a}\right)\right] + \omega_H^a \frac{\rho_L^a}{\rho_H^a} \omega_L^{bb} \left(\delta_{m_L^a \rho_L^a} + \delta_{m_L^b \rho_L^a}\right)\right] \end{array} \right\}, \tag{44}$$

where $D_{IS} > 0$, and ω_H^a $(= 1 - \omega_H^b)$ denotes the fraction of consumers from country a who buy high-quality goods; for the low quality brand in country a and b, respectively, we calculate

$$\frac{\hat{m}_L^a}{\hat{\lambda}_H} = -\frac{\delta_{\lambda_H \rho_L^b}}{D_{IS}} \left\{ \begin{array}{l} \omega_H^b \frac{\rho_L^b}{\rho_H^b} \omega_{LL}^a + (1 - \omega_{HH}^a) \omega_L^{ab} + \delta_{m_L^b \rho_L^a} \left(\omega_L^{aa} - \omega_L^{ba}\right) \left[\omega_H^b \frac{\rho_{HL}^b}{\rho_H^b} + \omega_H^a \frac{\rho_{HL}^a}{\rho_H^a} + (1 - \omega_{HH}^a)\right] \\ -\left(h_L^b - e_L^b\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_{LL}^a \omega_H^b \frac{\rho_L^b}{\rho_H^b} + (1 - \omega_{HH}^a) \omega_L^{ab}\right] \\ -\left(h_L^a - e_L^a\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_{LL}^b \omega_H^b \frac{\rho_L^b}{\rho_H^b} + (1 - \omega_{HH}^a) \omega_L^{bb}\right] \end{array} \right\} \tag{45}$$

$$\frac{\hat{m}_L^b}{\hat{\lambda}_H} = -\frac{\delta_{\lambda_H \rho_L^b}}{D_{IS}} \left\{ \begin{array}{l} \omega_H^b \frac{\rho_L^b}{\rho_H^b} \omega_{LL}^b + (1 - \omega_{HH}^a) \omega_L^{bb} + \delta_{m_L^a \rho_L^a} \left(\omega_L^{ba} - \omega_L^{aa}\right) \left[\omega_H^b \frac{\rho_{HL}^b}{\rho_H^b} + \omega_H^a \frac{\rho_{HL}^a}{\rho_H^a} + (1 - \omega_{HH}^a)\right] \\ -\left(h_L^b - e_L^b\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_{LL}^a \omega_H^b \frac{\rho_L^b}{\rho_H^b} + (1 - \omega_{HH}^a) \omega_L^{ab}\right] \\ -\left(h_L^a - e_L^a\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_{LL}^b \omega_H^b \frac{\rho_L^b}{\rho_H^b} + (1 - \omega_{HH}^a) \omega_L^{bb}\right] \end{array} \right\}, \tag{46}$$

where $\omega_{HH}^a, \omega_{LL}^a$, and ω_{LL}^b are suitable weights (see Appendix). The first line of each impact reflects the selection effect, while the second and third line relate to the competition and relative price effects.

Starting from an initial short-run equilibrium, lower trade costs of high-quality goods reduce the price of those goods relative to low-quality goods for consumers in country b. As a consequence, the likelihood that a high quality good is chosen in country b is increasing ($\delta_{\lambda_H \rho_L^b} > 0$), thereby raising the profitability of high-quality products, ceteris paribus. This higher profitability of high-quality goods in the export market is an incentive for producers of high quality goods to increase their range of high-quality brands on offer (the selection effect is positive on $\hat{m}_H^a$). In addition, low-quality goods' producing firms in both countries are forced to prune their product lines, i.e., $\widehat{m}_L^a < 0$ and $\widehat{m}_L^b < 0$ (the selection effect is negative on $\hat{m}_L^a$ and $\hat{m}_L^b$). The last term of the selection effect on $\hat{m}_L^a$ and $\hat{m}_L^b$, related to the market share difference (ω_L^{aa} versus ω_L^{ba}), assigns a stronger selection effect on the product line in the market that has the larger share - overall we expect these components of the selection effect to be of secondary importance.

Associated with the pruning of product lines of low-quality goods and expanding product lines of high-quality goods (due to the selection effect) comes a change in competitiveness and relative prices. Holding the number of firms constant, the smaller range of low-quality products available reduces the degree of competition among low-quality producers in country a and b thereby increasing the profitability which has the effect of augmenting the range of low-quality goods (the competition effect is positive on $\hat{m}_L^a$ and $\hat{m}_L^b$). The competition effect has the opposite impact on the range of high-quality goods, $\hat{m}_H$ (the competition effect is negative on $\hat{m}_H^a$). The competition effects are the suitably weighted h_L^a and h_L^b terms in the second and third lines of (44)–(46) (the terms in squared brackets are positive). Finally, to the extend that the number of low-quality brands in country a and b are impacted differently, there is a direct relative price effect associated with the reduced trade cost arising from this additional competition. This effect enters in the same way as the competition effect (the suitably weighted e_L^a and e_L^b terms). While the direct relative price effect is ambiguous, we expect it to be dominated by the first order competition effect.[30]

The impact of a reduction in transportation cost of low-quality goods, τ_L, or equivalently an increase in λ_L is even more complex. In Appendix B, we show that it can be written as a suitably weighted sum of the effect of changes in income of country a and b plus an additional competition and direct price effect brought about by τ_L. In view of the added complexity, together with the intractability of the long-run equilibrium, we now consider the complete specialization in quality setting.

[30] In Appendix B, it is shown that the impact of an increase of income in country b can be easily obtained as by proportionality $\delta_{y^b \rho_L^b} dy^b = -\delta_{\lambda_H \rho_L^b} \hat{\lambda}_H$. Similar relations are obtained for an increase of income in country a.

4.2 Trade with complete specialization

Here we assume that the rich country, country a, produces the high quality goods while the poor country, country b, only produces the low-quality good. This simplification allows us to analyze the short-run and the long-run behavior and to look at the distributional aspects of lower trade cost, which was not possible in the previous section. When a firm exports a unit of output it again has to pay $\tau_i > 0$ units of transportation, $i = H, L$.

We consider only the decision problem of a firm from a, where $i = H$. The mark-ups of firm f and the remaining $(n_i - 1)$ competitors in the domestic market are

$$\left(p_{if}^{a} - c_i\right) = \frac{\mu_{if}}{q_i}\frac{1}{(1-\rho_{f|i}^{a})} \tag{47}$$

$$\left(p_{ij}^{a} - c_i\right) = \frac{\mu_{if}}{q_i}\frac{1}{(1-\rho_{j|i}^{a})} \quad j = 1, .., n_i \quad j \neq f \tag{48}$$

while in the foreign market their mark-ups are

$$\left(p_{if}^{ab} - c_i - \tau_i\right) = \frac{\mu_{if}}{q_i}\frac{1}{(1-\rho_{f|i}^{a})} \tag{49}$$

$$\left(p_{ij}^{ab} - c_i - \tau_i\right) = \frac{\mu_{if}}{q_i}\frac{1}{(1-\rho_{j|i}^{a})} \quad j = 1, .., n_i \quad j \neq f. \tag{50}$$

In comparison to (38), note that $\rho_{j|i}^{a} = \rho_{j|i}^{ab}$ as there are no foreign competitors.

4.2.1 Short-run open economy equilibrium

The first order condition evaluated at a symmetric equilibrium with $m_{if} = m_i$ yields

$$\frac{\mu_{ik}}{q_i}\left(N^a E^a[\rho_i^a(y)] + N^b E^b[\rho_i^b(y)]\right)\left[\frac{n_i - 1}{n_i^2 - n_i + 1}\right] = m_i F_i. \tag{51}$$

This equation implicitly determines the optimal number of brands per firm with $i = H$ for country a. A similar analysis for firms from country b provides, implicitly, the optimal number of brands per firm with $i = L$.

At a symmetric equilibrium, where firms produce the same range of brands and charge the same price for each brand, aggregate demand by domestic and foreign households for a typical brand of quality q_H produced in a is, respectively,

$$d_{Hf}^{a} = \frac{N^a}{m_H n_H} E^a[\rho_H^a(y)] \quad \text{and} \quad d_{Hf}^{ab} = \frac{N^b}{m_H n_H} E^b[\rho_H^b(y)].$$

By symmetry, aggregate demand from domestic and foreign households for a typical brand of quality q_L produced in b is, respectively,

$$d_{Lf}^{b} = \frac{N^b}{m_L n_L} E^b[\rho_L^b(y)] \quad \text{and} \quad d_{Lf}^{ba} = \frac{N^a}{m_L n_L} E^a[\rho_L^a(y)].$$

The conditional probabilities (42) and (43) simplify to

$$\begin{aligned}
E^a[\rho_H^a(y)] &= E^a\left(\frac{n_H^{\mu_{Hf}}(m_H)^{\mu_{Hk}}\phi_H(y,n_H)}{[(n_H)^{\mu_{Hf}}(m_H)^{\mu_{Hk}}\phi_H(y,n_H)]+[(\lambda_L n_L)^{\mu_{Lf}}(m_L)^{\mu_{Lk}}\phi_L(y,n_L)]}\right)\\
E^b[\rho_H^b(y)] &= E^b\left(\frac{(n_H\lambda_H)^{\mu_{Hf}}(m_H)^{\mu_{Hk}}\phi_H(y,n_H)}{[(\lambda_H n_H)^{\mu_{Hf}}(m_H)^{\mu_{Hk}}\phi_H(y,n_H)]+[(n_L)^{\mu_{Lf}}(m_L)^{\mu_{Lk}}\phi_L(y,n_L)]}\right),
\end{aligned}$$

with λ_i and ϕ_i for $i = H, L$, defined as above.

Evaluating (51) at the short-run equilibrium, shows that a fall in trade cost (both for high and low quality goods) increases the profitability of firms whose products belong to that quality class and consequently provides an incentive to increase unambiguously their range of brands, while firms producing the other product quality prune their range of products:

$$\frac{\widehat{m}_L}{\widehat{\lambda}_H} = -\frac{1}{D_S}\mu_{Hf}\omega_L^b\frac{\rho_{HL}^b}{\rho_L^b} < 0 \tag{52}$$

$$\frac{\widehat{m}_H}{\widehat{\lambda}_H} = \frac{1}{D_S}\mu_{Hf}\omega_H^b\frac{\rho_{HL}^b}{\rho_H^b} > 0, \tag{53}$$

with $D_S > 0$ and ω_i^b denotes the fraction of consumers from country b who buy goods of quality q_i, $i = H, L$, and $\rho_{HL}^b \equiv E^b(\rho_H^b(y)\rho_L^b(y))$ and $\rho_H^b \equiv E^b(\rho_H^b(y))$.

4.2.2 Long-run open economy equilibrium

In the long-run the number of firms producing in each industry is determined by the free entry zero-profit condition. In this case the profit functions are given by

$$\Pi_i(n_H, n_L) = \frac{1}{q_i}\left(N^a E[\rho_i^a(y)] + N^b E[\rho_i^b(y)]\right)\left[\frac{(\mu_{if}-\mu_{ik})(n_i-1)^2+\mu_{if}n_i}{(n_i-1)(n_i^2-n_i+1)}\right] - K_i,$$

for $i = H, L$ and as long as profits are positive (negative) firms will enter (exit) the market. Market clearing for each brand requires

$$\begin{aligned}
n_H m_H x_H &= N^a E^a[\rho_H^a(y)] + N^b E^b[\rho_H^b(y)]\\
n_L m_L x_L &= N^a E^a[\rho_L^a(y)] + N^b E^b[\rho_L^b(y)],
\end{aligned}$$

where $m_i x_i$ is the break-even output of a firm producing quality q_i as given by (29) and n_i is the equilibrium number of firms active in differentiated industry q_i. The long-run open economy equilibrium is stable and unique under conditions analogous to those derived for the closed economy, see Appendix B for details. We can now readily analyze the effects of lower transportation cost on the number of firms, its product range and prices.

A reduction in the transportation cost of low-quality products (associated with an increase in λ_L) leads to a larger number of firms producing brands of quality q_L and a smaller number of firms producing brands of quality q_H:

$$\begin{aligned} \frac{\widehat{n}_L}{\widehat{\lambda}_L} &= \frac{1}{\widetilde{D}_S}\varphi_H D_S \mu_{Lf}\omega_L^a \frac{\rho_{HL}^a}{\rho_L^a} > 0 \\ \frac{\widehat{n}_H}{\widehat{\lambda}_L} &= -\frac{1}{\widetilde{D}_S}\varphi_L D_S \mu_{Hf}\omega_H^a \frac{\rho_{HL}^b}{\rho_L^b} < 0, \end{aligned}$$

with $\widetilde{D}_S > 0$ and $\varphi_i \geq 1$, capturing the effect of firm entry on net total profits, defined as in (A.18). The number of brands of quality q_L (q_H) changes proportional to the number of firms n_L (n_H) according to

$$\begin{aligned} \frac{\widehat{m}_L}{\widehat{\lambda}_L} &= \left(\varphi_L - \tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right)\frac{\widehat{n}_L}{\widehat{\lambda}_L} > 0 \\ \frac{\widehat{m}_H}{\widehat{\lambda}_L} &= -\left(\varphi_H - \tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right)\frac{\widehat{n}_H}{\widehat{\lambda}_L} < 0, \end{aligned}$$

for $n_i > 2$, $i = H, L$. Analogous to the autarky case, the relative change in m_i in comparison to the relative change in n_i depends on initial number of firms active in the market (see Proposition 2).

The price of low quality goods falls in both countries but more so in the importing country a than country b due to the direct effect of the transport cost. The price of high-quality goods increases in both countries. A reduction in the trade cost of high quality goods leads to analogous results: the number of firms and the number of products per firm decreases for low quality and expands in the high quality sector.

4.2.3 Welfare

We next turn to the welfare implications associated with the long run impact of lower trade costs. To this end, consider respectively the expected welfare of a household with income y from country a and country b:

$$\begin{aligned} v^a(y) &\equiv \left[n_H^{\mu_{Hf}} m_H^{\mu_{Hk}}\phi_H(y, n_H)\right] + \left[(\lambda_L n_L)^{\mu_{Lf}} m_L^{\mu_{Lk}}\phi_L(y, n_L)\right] \\ v^b(y) &\equiv \left[(\lambda_H n_H)^{\mu_{Hf}} m_H^{\mu_{Hk}}\phi_H(y, n_H)\right] + \left[n_L^{\mu_{Lf}} m_L^{\mu_{Lk}}\phi_L(y, n_L)\right]. \end{aligned}$$

Differentiating $v^a(y)$ and $v^b(y)$ yields expressions that decompose the change in expected welfare of a household with income y into a cost-savings effect and a composition effect (similar to Fajgelbaum et al., 2011). For a household from country a we obtain

$$
\begin{aligned}
\hat{v}^{a}(y) &= \mu_{Lf}\rho_{L}^{a}(y)\widehat{\lambda}_{L} \\
&+\overline{\rho}_{L}\overline{\rho}_{H}\left\{\begin{array}{c}\left[\frac{\rho_{H}^{a}(y)}{\overline{\rho}_{H}}\frac{\mu_{Hf}}{\varphi_{H}}\left(1+\frac{n_{H}}{(n_{H}-1)^{2}}\right)-\frac{\rho_{L}^{a}(y)}{\overline{\rho}_{L}}\frac{\mu_{Lf}}{\varphi_{L}}\left(1+\frac{n_{L}}{(n_{L}-1)^{2}}\right)\right] \\ +\left[\frac{\rho_{H}^{a}(y)}{\overline{\rho}_{H}}\frac{\mu_{Hk}}{\varphi_{H}}\left(\varphi_{H}-\zeta_{H}\right)-\frac{\rho_{L}^{a}(y)}{\overline{\rho}_{L}}\frac{\mu_{Lk}}{\varphi_{L}}\left(\varphi_{L}-\zeta_{L}\right)\right]\end{array}\right\}\left(\varphi_{H}\widehat{n}_{H}-\varphi_{L}\widehat{n}_{L}\right),
\end{aligned}
\tag{54}
$$

where $\overline{\rho}_{i}=\frac{N^{a}}{N^{a}+N^{b}}\rho_{i}^{a}+\frac{N^{b}}{N^{a}+N^{b}}\rho_{i}^{b}$, $i=H,L$, is a population weighted fraction of consumers from both countries that consume a brand of quality q_i; $\rho_{i}^{a}(y)$ denotes the fraction of consumers from country a that purchase a brand of quality q_i. Similarly, for a household from country b we get

$$
\begin{aligned}
\hat{v}^{b}(y) &= \mu_{Hf}\rho_{H}^{b}(y)\widehat{\lambda}_{H} \\
&+\overline{\rho}_{L}\overline{\rho}_{H}\left\{\begin{array}{c}\left[\frac{\rho_{H}^{b}(y)}{\overline{\rho}_{H}}\frac{\mu_{Hf}}{\varphi_{H}}\left(1+\frac{n_{H}}{(n_{H}-1)^{2}}\right)-\frac{\rho_{L}^{b}(y)}{\overline{\rho}_{L}}\frac{\mu_{Lf}}{\varphi_{L}}\left(1+\frac{n_{L}}{(n_{L}-1)^{2}}\right)\right] \\ +\left[\frac{\rho_{H}^{b}(y)}{\overline{\rho}_{H}}\frac{\mu_{Hk}}{\varphi_{H}}\left(\varphi_{H}-\zeta_{H}\right)-\frac{\rho_{L}^{b}(y)}{\overline{\rho}_{L}}\frac{\mu_{Lk}}{\varphi_{L}}\left(\varphi_{L}-\zeta_{L}\right)\right]\end{array}\right\}\left(\varphi_{H}\widehat{n}_{H}-\varphi_{L}\widehat{n}_{L}\right),
\end{aligned}
\tag{55}
$$

with $\rho_{i}^{b}(y)$ denoting the fraction of consumers from country b that purchase a brand of quality q_i. The first line in each expression represents the cost-savings effect while the second line represents the composition effect. The latter is similar to the one discussed in relation to the welfare implications under autarky, see (31).

A decrease in the trade cost of low-quality goods reduces the cost of imports and hence is beneficial to all consumers in country a, irrespective of their income level, while leaving the consumers in country b unaffected. Holding constant the relative number of brands and firms, lower cost of trading low-quality goods raises the profitability of low-quality products leading to an increase in the number of firms and brands in country b. The composition effect captures the associated welfare impact emanating from changes in the relative number of firms and brands of different qualities. Analogous to the autarky setting, the composition effect impacts households differently depending on their income and will favour the poor households in both countries more than the rich households given $(\varphi_{H}\widehat{n}_{H}-\varphi_{L}\widehat{n}_{L})<0$. The discussion of a reduction in the trade cost of high-quality goods is analogous: it is beneficial to all consumers in country b because of the reduced cost of imports, while the associated composition effect will favour the rich households in both countries more than the poor ones given $(\varphi_{H}\widehat{n}_{H}-\varphi_{L}\widehat{n}_{L})>0$.

5 Concluding Remarks

In this paper we have developed a general equilibrium model in the presence of vertical product differentiation (product quality) and non-homothetic preferences. We assume that the market structure in the differentiated goods' sector is oligopolistic. The associated strategic interaction between firms producing brands of the same quality render both the number of firms and the number of brands per firm in equilibrium responsive to changes in the income distribution in the presence of non-homothetic preferences. These income distributional effects – with empirical evidence to support them – have been neglected in the literature of multiproduct firms due to the reliance on the assumption that preferences are (quasi)-homothetic. We derive conditions under which a general equilibrium exists and is stable with both types of firms producing more than one brand and extend the framework to two countries where trade in differentiated goods is subject to trading costs.

Our results are revealing compared to those in the literature (either in the absence of multi-products or with multi-product firms producing under monopolistic competition). In particular, in autarky we show how income changes (and changes in the size of the economy) impact the number of firms and number of brands per firm for each quality differently. Two opposing effects are operative: a quality selection and a competition effect with their relative magnitude depending on the initial number of firms active in the market. Our model predicts that (i) a larger country sustains a larger number of firms that produce a larger product range at a smaller scale and lower prices for each quality; and (ii) a richer country (first order stochastic dominance) supports a larger number of higher-quality goods, each producing a larger number of variants at a smaller scale with lower prices. Also, in the open economy setting under incomplete specialization an additional effect is operative: the relative price effect. It accounts for the fact that the number of brands in both countries are affected differently which impacts the competitiveness. Under complete specialization this relative price effect is absent and lower trade costs for a particular differentiated good unambiguously increases the number of firms and brands of that differentiated product. Depending on the initial number of firms either the firm expansion- or the product line expansion effect dominates.

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A Closed Economy

A.1 Comparative Statics: Short-run equilibrium

We start by total differentiation of the short-run equilibrium conditions (19) for $i = H, L$. Differentiation of the expectation term on the lhs, $E(\rho_i(y))$, yields for $i = L$:

$$\begin{aligned} & d(E\rho_L(y)) \\ = \; & \mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)\rho_{HL}\widehat{n}_L + \mu_{Lk}\rho_{HL}\widehat{m}_L - \mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)\rho_{HL}\widehat{n}_H \\ & -\mu_{Hk}\rho_{HL}\widehat{m}_H + \rho_{HL}\left[q_L - q_H\right]dy, \end{aligned}$$

where we define $\rho_{HL}(=\rho_{LH}) \equiv E(\rho_L(y)\rho_H(y))$.[31] From Jensen's inequality it follows that $E(\rho_H(y)\rho_L(y)) \leq E(\rho_H(y))E(\rho_L(y))$ or $\rho_{HL} \leq \rho_H\rho_L$; furthermore $\rho_{HL} \leq \rho_H$ and $\rho_{HL} \leq \rho_L$ (because $0 \leq \rho_i(y) \leq 1$). Expressed in relative changes, $E(d\rho_L(y))/\rho_L$, we have

$$\begin{aligned} \widehat{\rho}_L = \; & \frac{\rho_{HL}}{\rho_L}\{\mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)\widehat{n}_L + \mu_{Lk}\widehat{m}_L - \mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)\widehat{n}_H \\ & -\mu_{Hk}\widehat{m}_H + \left[q_L - q_H\right]dy\}. \end{aligned} \tag{A.1}$$

Similarly

$$\begin{aligned} \widehat{\rho}_H = \; & \frac{\rho_{HL}}{\rho_H}\{\mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)\widehat{n}_H + \mu_{Hk}\widehat{m}_H - \mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)\widehat{n}_L \\ & -\mu_{Lk}\widehat{m}_L - \left[q_L - q_H\right]\}dy. \end{aligned} \tag{A.2}$$

After substituting the two previous equations into the total differential of (19) for $i = H, L$, we obtain, in matrix notation

$$\begin{bmatrix} a_{L1} & a_{L2} \\ a_{H1} & a_{H2} \end{bmatrix}\begin{bmatrix} \widehat{m}_L \\ \widehat{m}_H \end{bmatrix} = \begin{bmatrix} a_{L3} & a_{L4} & -a_{L5} & L_y \\ a_{H3} & -a_{H4} & a_{H5} & H_y \end{bmatrix}\begin{bmatrix} \widehat{N} \\ \widehat{n}_L \\ \widehat{n}_H \\ dy \end{bmatrix} \tag{A.3}$$

with

$a_{L1} = \left[1 - \mu_{Lk}\frac{\rho_{HL}}{\rho_L}\right]$

$a_{L2} = \mu_{Hk}\frac{\rho_{HL}}{\rho_L}$

$a_{L3} = 1$

$a_{L4} = \mu_{Lf}\frac{\rho_{HL}}{\rho_L}\left[1+\frac{n_L}{(n_L-1)^2}\right] - \frac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}$

$a_{L5} = \mu_{Hf}\frac{\rho_{HL}}{\rho_L}\left[1+\frac{n_H}{(n_H-1)^2}\right]$

$L_y = \frac{\rho_{HL}}{\rho_L}\left[q_L - q_H\right]$

$a_{H1} = \mu_{Lk}\frac{\rho_{HL}}{\rho_H}$

$a_{H2} = \left[1 - \mu_{Hk}\frac{\rho_{HL}}{\rho_H}\right]$

$a_{H3} = 1$

$a_{H4} = \mu_{Lf}\frac{\rho_{HL}}{\rho_H}\left[1+\frac{n_L}{(n_L-1)^2}\right]$

$a_{H5} = \mu_{Hf}\frac{\rho_{HL}}{\rho_H}\left[1+\frac{n_H}{(n_H-1)^2}\right] - \frac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}$

$H_y = -\frac{\rho_{HL}}{\rho_H}\left[q_L - q_H\right].$

[31] The order of integration and differentation can be exchanged.

Solving for the relative change in m_L and m_H gives

$$\begin{bmatrix} \widehat{m}_L \\ \widehat{m}_H \end{bmatrix} = \frac{1}{D_1} \begin{bmatrix} a_{H2} & -a_{L2} \\ -a_{H1} & a_{L1} \end{bmatrix} \begin{bmatrix} a_{L3} & a_{L4} & -a_{L5} & L_y \\ a_{H3} & -a_{H4} & a_{H5} & H_y \end{bmatrix} \begin{bmatrix} \widehat{N} \\ \widehat{n}_L \\ \widehat{n}_H \\ dy \end{bmatrix}. \tag{A.4}$$

The sign of the determinant D_1 is unambiguously positive:

$$D_1 = 1 - \mu_{Lk}\frac{\rho_{HL}}{\rho_L} - \mu_{Hk}\frac{\rho_{HL}}{\rho_H} > 0.$$

This follows as we can decompose D_1 as

$$\begin{aligned} &(1-\mu_{Hk})(1-\mu_{Lk}) + (1-\mu_{Hk})\,\mu_{Lk}\left(1-\frac{\rho_{HL}}{\rho_H}\right) + (1-\mu_{Lk})\,\mu_{Hk}\left(1-\frac{\rho_{HL}}{\rho_L}\right) \\ &+\mu_{Hk}\mu_{Lk}\left[\left(1-\frac{\rho_{HL}}{\rho_H}\right)\left(1-\frac{\rho_{HL}}{\rho_L}\right) - \frac{\rho_{HL}\rho_{HL}}{\rho_L\rho_H}\right], \end{aligned}$$

where

$$\left[\left(1-\frac{\rho_{HL}}{\rho_H}\right)\left(1-\frac{\rho_{HL}}{\rho_L}\right) - \frac{\rho_{HL}\rho_{HL}}{\rho_L\rho_H}\right] = \left(1-\frac{\rho_{HL}}{\rho_L\rho_H}\right) > 0.$$

A.1.1 Figure 2

The two short-run equilibrium conditions can be written as

$$\frac{E[\rho_L(y)]}{m_i}\frac{(n_L-1)}{(n_L^2-n_L+1)}\frac{\mu_{Lk}}{q_i} = \frac{F_L}{N} \tag{A.5}$$

$$\frac{E[\rho_H(y)]}{m_H}\frac{(n_H-1)}{(n_H^2-n_H+1)}\frac{\mu_{Hk}}{q_H} = \frac{F_H}{N}. \tag{A.6}$$

Analog to Fajgelbaum et al. (2011), they are depicted in Figure 2 as m_{LL} and m_{HH}, respectively. They show combinations of number of brands offered by a high-quality producing firm and a low-quality producing firm satisfying the first order conditions for profit maximization, given the number of firms. We can determine uniqueness of m_L and m_H by considering various limits. Consider (A.5): if $m_H \to 0$, $m_L = \frac{\mu_{Lk}}{q_L}\frac{N}{F_L}\frac{(n_L-1)}{(n_L^2-n_L+1)}$; as $m_H \to \infty$, $m_L \to 0$. Similarly for (A.6): as $m_L \to 0$, $m_H = \frac{\mu_{Hk}}{q_H}\frac{N}{F_H}\frac{(n_H-1)}{(n_H^2-n_H+1)}$; as $m_L \to \infty$, $m_H \to 0$.

A.1.2 Change in population size N

From (A.4) we obtain

$$\frac{\widehat{m}_H}{\widehat{N}} = \frac{1}{D_1}\left[1-\mu_{Lk}\frac{\rho_{HL}}{\rho_H\rho_L}\right] > 0 \tag{A.7}$$

$$\frac{\widehat{m}_L}{\widehat{N}} = \frac{1}{D_1}\left[1-\mu_{Hk}\frac{\rho_{HL}}{\rho_H\rho_L}\right] > 0 \tag{A.8}$$

with $D_1 = 1 - \mu_{Lk}\frac{\rho_{HL}}{\rho_L} - \mu_{Hk}\frac{\rho_{HL}}{\rho_H}$.

A.1.3 Change in the income distribution

First order stochastic dominance To evaluate the effect of an increase in income on the short-run equilibrium, we consider the effect of changing the cumulative distribution of income, $\mathcal{F}_Y(y)$, to one that stochastically dominates the initial distribution of income. From (A.4), the increase in income, will cause $\widehat{m}_H$ to increase and $\widehat{m}_L$ to decrease as

$$\frac{\widehat{m}_H}{dy} = -\frac{1}{D_1}\frac{\rho_{HL}}{\rho_H}\left[q_L - q_H\right] > 0 \tag{A.9}$$

$$\frac{\widehat{m}_L}{dy} = \frac{1}{D_1}\frac{\rho_{HL}}{\rho_L}\left[q_L - q_H\right] < 0. \tag{A.10}$$

Mean-preserving spread Similar to Fajgelbaum et al. (2011), a mean-preserving spread in $\mathcal{F}_y(y)$, ceteris paribus, leads to an increase in m_H and a fall in m_L, when for all y, $\rho_L(y) > \rho_H(y)$. This statement relies on the fact that this condition ensures that $\rho_H(y)$ is a convex increasing function of income; that is $\partial\rho_H(y; m_H, m_L)/\partial y \equiv \rho'_H(y) = \rho_H(y)\left(q_H - q_a(y)\right)$ and $\partial^2\rho_H(y; m_H, m_L)/\partial^2 y = (1 - 2\rho_H(y))\rho'_H(y)\left(q_H - q_L\right) > 0$.[32] The mean preserving spread will then cause $E[\rho_H(y; m_H, m_L)]$ to increase holding m_H and m_L constant (Gravelle and Rees, 2004). For the two short-order equilibrium conditions to remain satisfied, m_L and m_H need to respond. Intuitively, the increase in $E[\rho_H(y; m_H, m_L)]$ will need to be compensated to ensure $E[\rho_H(y)]/m_H$ remains constant. An obvious solution for given m_L is to increase m_H; whereas this would further increase $E[\rho_H(y)]$ (since the partial derivative of ρ_H with respect to m_H is positive) it cannot dominate the direct effect increasing m_H has on the ratio $E[\rho_H(y)]/m_H$); equally given m_H an increase in m_L would establish this compensation. The mean preserving spread therefore moves the m_{HH} curve towards the right. The reverse holds true for the m_{LL} curve, which shifts towards the left, yielding the overall effect of an increase in m_H and fall in m_L (see Figure 2).

In order to quantify the optimal responses to the mean preserving spread let $f(y;\beta)$ denote the probability density function of income, i.e., $d\mathcal{F}_y(y;\beta)/dy$, with β parameterizing this mean preserving spread (see also Gravelle and Rees, 2004). Furthermore, let ρ'_{H,m_H} and $\rho'_{H_{m_L}}$ denote the partial derivatives of $\rho_H = E[\rho_H(y)]$ with respect to m_H and m_L

$$\begin{aligned}\rho'_{H,m_H} &= \frac{\mu_{Hk}\rho_{HL}}{m_H} > 0\\ \rho'_{H,m_L} &= -\frac{\mu_{Lk}\rho_{HL}}{m_H} < 0\end{aligned}$$

and

$$\psi_i = \frac{q_i}{\mu_{ik}}\frac{n_i^2 - n_i + 1}{n_i - 1}\frac{F_i}{N},\ i = H, L. \tag{A.11}$$

[32] By symmetry $\rho_L(y)$ is a concave decreasing function of income.

As argued before, the mean preserving spread is associated with an increase in $E[\rho_H(y; m_H, m_L)]$, or

$$\int_{y_{\min}}^{\infty} \rho_H(y; m_H, m_L) f_\beta(\rho y; \beta) dy > 0, \tag{A.12}$$

where $f_\beta(y;\beta)$ is the shift in the probability density function as β varies. We denote the optimal responses $m_H^*(\beta)$ and $m_L^*(\beta)$ which for notional convenience we call m^*. The optimal responses to the mean preserving spread are described by the total differentiation of (A.6), simplified here as $\rho_H = \psi_H m_H$,

$$\int_{y_{\min}}^{\infty} \rho_H(y; m^*) f_\beta(y;\beta) dy + \left(\rho'_{H,m_H}(m^*) - \psi_H\right) \frac{dm_H}{d\beta} + \rho'_{H,m_L}(m^*) \frac{dm_L}{d\beta} = 0$$

and the requirement that both short-run equilibrium conditions are satisfied. This ensures that $\frac{dm_L}{d\beta} / \frac{dm_H}{d\beta} = -\psi_H/\psi_L$.[33] Then

$$\frac{dm_H}{d\beta}\left(-\rho'_{H,m_H}(m^*) + \psi_H + \frac{\psi_H}{\psi_L}\rho'_{H,m_L}(m^*)\right) = \frac{d\rho_H}{d\beta}.$$

We therefore have

$$\begin{aligned}
\frac{dm_H}{d\beta} &= \left[\psi_H - \frac{\mu_{Hk}\rho_{HL}(m^*)}{m_H^*} - \frac{\psi_H}{\psi_L}\frac{\mu_{Lk}\rho_{HL}(m^*)}{m_H^*}\right]^{-1} \frac{d\rho_H}{d\beta} \\
&= \frac{m_H^*}{\rho_H(m^*)}\left[1 - \mu_{Hk}\frac{\rho_{HL}(m^*)}{\rho_H(m^*)} - \mu_{Lk}\frac{\rho_{HL}(m^*)}{\rho_L(m^*)})\right]^{-1} \frac{d\rho_H}{d\beta} > 0 \qquad \text{(A.13)} \\
\frac{dm_L}{d\beta} &= -\frac{m_L^*}{\rho_L(m^*)}\left[1 - \mu_{Hk}\frac{\rho_{HL}(m^*)}{\rho_H(m^*)} - \mu_{Lk}\frac{\rho_{HL}(m^*)}{\rho_L(m^*)})\right]^{-1} \frac{d\rho_H}{d\beta} < 0, \qquad \text{(A.14)}
\end{aligned}$$

where the inequalities follow since the term in square brackets equals D_1 which is positive.

A.1.4 Change in number of firms

For the cross effect we obtain from (A.4) an unambiguous negative effect for $n_L > 1$. Specifically

$$\frac{\widehat{m}_H}{\widehat{n}_L} = -\frac{1}{D_1}\frac{\rho_{HL}}{\rho_H}\left\{\mu_{Lf}\left(1 + \tfrac{n_L}{(n_L-1)^2}\right) - \mu_{Lk}\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right\} = -\frac{1}{D_1}\frac{\rho_{HL}}{\rho_H} b_{2nL} < 0.$$

The terms in parenthesis can be written as

$$\left\{\mu_{Lf}\left(1 + \tfrac{n_L}{(n_L-1)^2} - \tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right) + \left(\mu_{Lf} - \mu_{Lk}\right)\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right\},$$

and both components are positive with $\mu_{Lf} - \mu_{Lk} > 0$, rendering the total effect negative.

[33] If we sum the two short-run equilibrium conditions, $E(\rho_H(y)) = \psi_H m_H$ and $E(\rho_L(y)) = \psi_L m_L$, we have $1 = \psi_H m_H + \psi_L m_L$ from which this statement directly follows.

The own effect is related as

$$\begin{aligned}\frac{\widehat{m}_L}{\widehat{n}_L} &= \frac{1}{D_1}\left\{\mu_{Lf}\frac{\rho_{HL}}{\rho_L}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)-\left[1-\mu_{Hk}\frac{\rho_{HL}}{\rho_H}\right]\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right\}\\ &= -\frac{\rho_H}{\rho_L}\frac{\widehat{m}_H}{\widehat{n}_L}-\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}.\end{aligned}$$

Clearly

$$\rho_L\frac{\widehat{m}_L}{\widehat{n}_L}+\rho_H\frac{\widehat{m}_H}{\widehat{n}_L}=-\rho_L\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}<0.$$

The own effect can be positive, but not too strong (the sum of the probability weighted own and cross effect remains negative).

The results are symmetric for $\frac{\widehat{m}_L}{\widehat{n}_H}$ and $\frac{\widehat{m}_H}{\widehat{n}_H}$:

$$\begin{aligned}\frac{\widehat{m}_L}{\widehat{n}_H} &= -\frac{1}{D_1}\frac{\rho_{HL}}{\rho_L}\left\{\mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)-\mu_{Hk}\tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right\}<0,\\ \frac{\widehat{m}_H}{\widehat{n}_H} &= -\frac{\rho_L}{\rho_H}\frac{\widehat{m}_H}{\widehat{n}_H}-\tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}.\end{aligned}$$

A.2 Comparative Statics: Long-run autarky equilibrium

We solve the long-run equilibrium by substituting (A.4) into (A.1) and (A.2), which allows us to express the relative change $\widehat{\rho}_i$ in terms of the relative change $\hat{N}$, $\hat{n}_H$, $\hat{n}_L$, and in terms of dy:

$$\begin{aligned}\widehat{\rho}_H = & -\frac{\rho_{HL}}{\rho_H}\frac{1}{D_1}\{\,[\mu_{Lk}-\mu_{Hk}]\,\widehat{N}+(q_L-q_H)\,dy\\ &+\left[\mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)-\mu_{Lk}\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right]\widehat{n}_L\\ &-\left[\mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)-\mu_{Hk}\tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right]\widehat{n}_H\}\end{aligned}\tag{A.15}$$

$$\begin{aligned}\widehat{\rho}_L = & \frac{\rho_{HL}}{\rho_L}\frac{1}{D_1}\{\,[\mu_{Lk}-\mu_{Hk}]\,\widehat{N}+[q_L-q_H]\,dy\\ &+\left[\mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)-\mu_{Lk}\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right]\widehat{n}_L\\ &-\left[\mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)-\mu_{Hk}\tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right]\widehat{n}_H\}.\end{aligned}\tag{A.16}$$

A.2.1 Stability and uniqueness of the long-run autarky equilibrium

Assume that firms are short-sighted: they enter when profits are positive and exit otherwise, with profits given by

$$\Pi_i(n_H,n_L)=\frac{N}{q_i}E[\rho_i(y)]\left\{\frac{(\mu_{if}-\mu_{ik})\,(n_i-1)^2+\mu_{if}n_i}{(n_i-1)\,(n_i^2-n_i+1)}\right\}-K_i \qquad \text{for } i=H,L. \tag{A.17}$$

The flow of firms into each industry can be described by two differential equations of the following form

$$\dot{n}_i = n_i \Pi_i(n_H, n_L).$$

In the steady state, $\dot{n}_i = 0$ and $n_H > 0$ and $n_L > 0$ as $\Pi_H(n_H, n_L) = \Pi_L(n_H, n_L) = 0$. To find conditions for a locally stable equilibrium consider the Jacobian matrix evaluated around the steady state:

$$J = \begin{bmatrix} n_L \Pi_{Ln_L} & n_L \Pi_{Ln_H} \\ n_H \Pi_{Hn_L} & n_H \Pi_{Hn_H} \end{bmatrix}$$

(subscripts refers to the derivative with respect to n_H and n_L).

Consider, e.g.,

$$\begin{aligned} \Pi_{Ln_L} &= \frac{\partial \left[\frac{N}{q_L} \rho_L \left\{ \frac{(\mu_{Lf} - \mu_{Lk})(n_L-1)^2 + \mu_{Lf} n_L}{(n_L-1)(n_L^2 - n_L + 1)} \right\} - K_L \right]}{\partial n_L} \\ &= \frac{N}{q_L} \left\{ \frac{(\mu_{Lf} - \mu_{Lk})(n_L-1)^2 + \mu_{Lf} n_L}{(n_L-1)(n_L^2 - n_L + 1)} \right\} \frac{\partial \rho_L}{\partial n_L} + \frac{N}{q_L} \rho_L \frac{\partial \left\{ \frac{(\mu_{Lf} - \mu_{Lk})(n_L-1)^2 + \mu_{Lf} n_L}{(n_L-1)(n_L^2 - n_L + 1)} \right\}}{\partial n_L}. \end{aligned}$$

The first term represents the selection effect on profits and the second term the competition effect. We denote $\frac{\partial \left\{ \frac{(\mu_{Lf} - \mu_{Lk})(n_L-1)^2 + \mu_{Lf} n_L}{(n_L-1)(n_L^2 - n_L + 1)} \right\}}{\partial n_L} = -\frac{(\mu_{Lf} - \mu_{Lk})(n_L-1)^2 + \mu_{Lf} n_L}{(n_L-1)(n_L^2 - n_L + 1)} \varphi_L / n_L$, with

$$\begin{aligned} \varphi_i &= \frac{n_i^2 (n_i - 1)}{(n_i - 1)(n_i^2 - n_i + 1)} + \frac{\mu_{if} n_i (n_i^3 + 1)}{\left[\left(\mu_{if} - \mu_{ik} \right)(n_i-1)^2 + \mu_{if} n_i \right](n_i - 1)(n_i^2 - n_i + 1)} \\ &\geq 1, \end{aligned} \tag{A.18}$$

$i = H, L$.[34] Using (A.16), we obtain

$$\Pi_{Ln_L} = -\frac{K_L}{n_L D_1} \left\{ \varphi_L D_1 - \frac{\rho_{HL}}{\rho_L} \left[\mu_{Lf} \left(1 + \frac{n_L}{(n_{HL}-1)^2} \right) - \mu_{Lk} \frac{n_L^2 (n_L - 2)}{(n_L^2 - n_L + 1)(n_L - 1)} \right] \right\}.$$

Similarly,

$$\begin{aligned} \Pi_{Ln_H} &= \frac{\partial \left[\frac{N}{q_L} \rho_L \left\{ \frac{(\mu_{Lf} - \mu_{Lk})(n_L-1)^2 + \mu_{Lf} n_L}{(n_L-1)(n_L^2 - n_L + 1)} \right\} - K_L \right]}{\partial n_H} \\ &= -\frac{K_L}{n_H D_1} \left\{ \frac{\rho_{HL}}{\rho_L} \left[\mu_{Lf} \left(1 + \frac{n_L}{(n_{HL}-1)^2} \right) - \mu_{Lk} \frac{n_L^2 (n_L - 2)}{(n_L^2 - n_L + 1)(n_L - 1)} \right] \right\}. \end{aligned}$$

There is no competition effect across quality groups. Using (A.15) for the remaining terms, we obtain

$$J = -\frac{1}{D_1} \begin{bmatrix} K_L b_{1nL} & K_L \frac{n_L}{n_H} b_{1nH} \\ K_H \frac{n_H}{n_L} b_{2nL} & K_H b_{2nH} \end{bmatrix}$$

[34] Note, φ_i approaches asymptotically one as $n_i \to \infty$.

with

$$\begin{aligned} b_{1nL} &\equiv \varphi_L D_1 - \frac{\rho_H}{\rho_L} b_{2nL} \\ b_{1nH} &\equiv \frac{\rho_{HL}}{\rho_L}\left[\mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right) - \mu_{Hk}\tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right] > 0 \\ b_{2nL} &\equiv \frac{\rho_{HL}}{\rho_H}\left[\mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right) - \mu_{Lk}\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right] > 0 \\ b_{2nH} &\equiv \varphi_H D_1 - \frac{\rho_L}{\rho_H} b_{1nH}, \end{aligned}$$

for $i = H, L$. Here $D_1 = \left[1 - \mu_{Lk}\frac{\rho_{HL}}{\rho_L} - \mu_{Hk}\frac{\rho_{HL}}{\rho_H}\right]$ as before.

The long-run autarky equilibrium is locally stable if the trace of the Jacobian matrix is negative and its determinant is positive. For the trace we obtain

$$\begin{aligned} tr\,(J) = &\ -\tfrac{1}{D_1}\left[K_L b_{1nL} + K_H b_{2nH}\right] \\ = &\ -\tfrac{K_H}{D_1}\left\{\varphi_H D_1 - \tfrac{\rho_{HL}}{\rho_H}\left[\mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right) - \mu_{Hk}\tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right]\right\} \\ &\ -\tfrac{K_L}{D_1}\left\{\varphi_L D_1 - \tfrac{\rho_{HL}}{\rho_L}\left[\mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right) - \mu_{Lk}\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right]\right\}, \end{aligned}$$

which is negative if

$$\varphi_i D_1 > \frac{\rho_{HL}}{\rho_i}\left[\mu_{if}\left(1+\tfrac{n_i}{(n_i-1)^2}\right) - \mu_{ik}\tfrac{n_i^2(n_i-2)}{(n_i^2-n_i+1)(n_i-1)}\right], \tag{A.19}$$

$i = H, L$, so that $b_{1nL} > 0$ and $b_{2nH} > 0$, ensuring that the competition effect dominates the selection effect. The determinant of the Jacobian matrix is positive if

$$\begin{aligned} \det(J) &= n_H n_L \left(\Pi_{Hn_H}\Pi_{Ln_L} - \Pi_{Hn_L}\Pi_{Ln_H}\right) \\ &= \frac{K_H K_L}{D_1}\left(\varphi_L\varphi_H D_1 - \varphi_L \frac{\rho_L}{\rho_H} b_{1nH} - \varphi_H \frac{\rho_H}{\rho_L} b_{2nL}\right) > 0. \end{aligned} \tag{A.20}$$

This is true when the effect of a change in n_i on the operating profits of a firm from the same sector i dominates the effect on the operating profits from a change in the number of firms from the other sector, n_j, for $i = j = H, L$, and $i \neq j$.

Given the stability and the uniqueness of the long-run autarky equilibrium, we are now in the position to conduct a comparative static analysis. Using (A.15) and (A.16), total differentiation of (A.17) evaluated in the steady state yields

$$\begin{bmatrix} b_{1nL} & b_{1nH} \\ b_{2nL} & b_{2nH} \end{bmatrix}\begin{bmatrix} \widehat{n}_L \\ \widehat{n}_H \end{bmatrix} = \begin{bmatrix} 1 - \mu_{Hk}\frac{\rho_{HL}}{\rho_L\rho_H} & \frac{\rho_{HL}}{\rho_L} \\ 1 - \mu_{Lk}\frac{\rho_{HL}}{\rho_L\rho_H} & -\frac{\rho_{HL}}{\rho_H} \end{bmatrix}\begin{bmatrix} \widehat{N} \\ (q_L - q_H)\,dy \end{bmatrix}.$$

By inversion, we obtain

$$\begin{bmatrix} \widehat{n}_L \\ \widehat{n}_H \end{bmatrix} = \frac{1}{D_2}\begin{bmatrix} b_{2nH} & -b_{1nH} \\ -b_{2nL} & b_{1nL} \end{bmatrix}\begin{bmatrix} 1 - \mu_{Hk}\frac{\rho_{HL}}{\rho_L\rho_H} & \frac{\rho_{HL}}{\rho_L} \\ 1 - \mu_{Lk}\frac{\rho_{HL}}{\rho_L\rho_H} & -\frac{\rho_{HL}}{\rho_H} \end{bmatrix}\begin{bmatrix} \widehat{N} \\ (q_L - q_H)\,dy \end{bmatrix}. \tag{A.21}$$

From our discussion of the stability analysis, $D_2 > 0$ or

$$\frac{D_2}{D_1} = \varphi_L \varphi_H D_1 - \varphi_L \frac{\rho_L}{\rho_H} b_{1nH} - \varphi_H \frac{\rho_H}{\rho_L} b_{2nL} > 0.$$

A.2.2 Change in population size N

From (A.21) we obtain,

$$\begin{aligned}
\frac{\widehat{n}_L}{\widehat{N}} &= \frac{D_1}{D_2} \left\{ \varphi_H \left[1 - \mu_{Hk} \frac{\rho_{HL}}{\rho_H \rho_L} \right] - \frac{b_{1nH}}{\rho_H} \right\} > 0 \\
\frac{\widehat{n}_H}{\widehat{N}} &= \frac{D_1}{D_2} \left\{ \varphi_L \left[1 - \mu_{Lk} \frac{\rho_{HL}}{\rho_H \rho_L} \right] - \frac{b_{2nL}}{\rho_L} \right\} > 0.
\end{aligned}$$

Considering $\widehat{n}_L/\widehat{N}$, we note that the term in the curly brackets can be written as

$$\varphi_H \left[1 - \mu_{Lk} \frac{\rho_{HL}}{\rho_H \rho_L} - \mu_{Hk} \frac{\rho_{HL}}{\rho_H \rho_L} \right] - \frac{b_{1nH}}{\rho_H} + \varphi_H \mu_{Lk} \frac{\rho_{HL}}{\rho_H \rho_L}.$$

Since

$$\left[1 - \mu_{Lk} \frac{\rho_{HL}}{\rho_H \rho_L} - \mu_{Hk} \frac{\rho_{HL}}{\rho_H \rho_L} \right] > D_1,$$

in combination with $\varphi_L D_1 - \frac{\rho_H}{\rho_L} b_{2nL} > 0$, it follows that $\widehat{n}_L/\widehat{N} > 0$. An analogous result follows for $\widehat{n}_H/\widehat{N}$.

The compositional effect associated with an increase in the size of the economy is given by

$$\begin{aligned}
&\varphi_H \frac{\widehat{n}_H}{\widehat{N}} - \varphi_L \frac{\widehat{n}_L}{\widehat{N}} \\
&= \frac{D_1}{D_2} \frac{\rho_{HL}}{\rho_H \rho_L} \varphi_H \varphi_L \left\{ \frac{\mu_{Hk}}{\varphi_H} \left(\varphi_H - \frac{n_H^2 (n_H - 2)}{(n_H^2 - n_H + 1)(n_H - 1)} \right) - \frac{\mu_{Lk}}{\varphi_L} \left(\varphi_L - \frac{n_L^2 (n_L - 2)}{(n_L^2 - n_L + 1)(n_L - 1)} \right) \right. \\
&\quad \left. + \left[\frac{\mu_{Hf}}{\varphi_H} \left(1 + \frac{n_H}{(n_H - 1)^2} \right) - \frac{\mu_{Lf}}{\varphi_L} \left(1 + \frac{n_L}{(n_L - 1)^2} \right) \right] \right\}.
\end{aligned}$$

Using (A.18), the effect on the scope is

$$\frac{\widehat{m}_i}{\widehat{N}} = \left(\varphi_i - \frac{n_i^2 (n_i - 2)}{(n_i^2 - n_i + 1)(n_i - 1)} \right) \frac{\widehat{n}_i}{\widehat{N}} > 0$$

The effect on the scale is :

$$\frac{\widehat{x}_i}{\widehat{N}} = - \left\{ \frac{(2n_i - 1)}{(n_i^2 - n_i + 1)(n_i - 1)} \right\} \frac{\widehat{n}_i}{\widehat{N}} \leq 0 \tag{A.22}$$

since $n_i > 1$, and the effect on prices is:

$$\frac{dp_i}{\widehat{N}} = - \frac{\mu_{if}}{q_i} \frac{n_i}{(n_i - 1)^2} \frac{\widehat{n}_i}{\widehat{N}} < 0. \tag{A.23}$$

A.2.3 Change in the income distribution

First order stochastic dominance From (A.21) the increase in income on the number of firms, is

$$\begin{aligned}\frac{\widehat{n}_L}{dy} &= \frac{\varphi_H D_1}{D_2}\frac{\rho_{HL}}{\rho_L}\left[q_L - q_H\right] < 0 \\ \frac{\widehat{n}_H}{dy} &= -\frac{\varphi_L D_1}{D_2}\frac{\rho_{HL}}{\rho_H}\left[q_L - q_H\right] > 0.\end{aligned}$$

The compositional effect associated with this increase, $\varphi_H \frac{\widehat{n}_H}{dy} - \varphi_L \frac{\widehat{n}_L}{dy}$, is unambiguously positive. The increase will generate a shift in the composition of differentiated goods towards the high-quality goods.

The increase in income has similar effects on the number of brands per firm

$$\frac{\widehat{m}_i}{dy} = \left(\varphi_i - \tfrac{n_i^2(n_i-2)}{(n_i^2-n_i+1)(n_i-1)}\right)\frac{\widehat{n}_i}{dy};$$

the scale

$$\frac{\widehat{x}_i}{dy} = -\left\{\tfrac{(2n_i-1)}{(n_i^2-n_i+1)(n_i-1)}\right\}\frac{\widehat{n}_i}{dy}$$

and the prices

$$\frac{dp_i}{dy} = -\frac{\mu_{if}}{q_i}\frac{n_i}{(n_i-1)^2}\frac{\widehat{n}_i}{dy}.$$

Mean-preserving spread We again assume that $\rho_L(y) > \rho_H(y)$ for all y, to ensure that $\rho_H(y)$ is a convex increasing function of income. The mean preserving spread will then cause $E[\rho_H(y; m_H, m_L, n_L, n_L)]$ to increase holding m_H, m_L, n_H and n_L constant. For the long-run equilibrium to be re-established, m_H, m_L, n_H and n_L need to respond. Let ρ'_{H,m_H}, $\rho'_{H_{m_L}}, \rho'_{H,n_H}$, $\rho'_{H_{n_L}}$ denote the partial derivatives of $\rho_H = E[\rho_H(y)]$ with respect to these variables given by

$$\begin{aligned}\rho'_{H,m_H} &= \frac{\mu_{Hk}\rho_{HL}}{m_H} > 0 \\ \rho'_{H,m_L} &= -\frac{\mu_{Lk}\rho_{HL}}{m_L} < 0 \\ \rho'_{H,n_H} &= \frac{\mu_{Hf}\left[1 + \frac{n_H}{(n_H-1)^2}\right]\rho_{HL}}{n_H} > 0 \\ \rho'_{H,n_L} &= -\frac{\mu_{Lf}\left[1 + \frac{n_L}{(n_L-1)^2}\right]}{n_L}\rho_{HL} < 0.\end{aligned}$$

Recall from Section (A.1.3) that $f(y;\beta)$ denotes the probability density function of income with β parameterizing the mean preserving spread. We use the simplifying notation:

$$\begin{aligned}\psi_i &= \frac{q_i}{\mu_{ik}}\frac{F_i}{N}\frac{n_i^2-n_i+1}{n_i-1} \\ \xi_i &= q_i\frac{K_i}{N}\frac{(n_i-1)(n_i^2-n_i+1)}{\left[(\mu_{if}-\mu_{ik})(n_i-1)^2+\mu_{if}n_i\right]}\end{aligned} \tag{A.24}$$

for $i=H,L$, with $d\psi_i/dn_i=\psi_i\frac{n_i(n_i-2)}{(n_i-1)(n_i^2-n_i+1)}$ and $d\xi_i/dn_i=\xi_i\varphi_i/n_i$. where φ_i is as before.

The optimal responses are denoted $nm^*=\{m_H^*(\beta),m_L^*(\beta),n_H^*(\beta),n_L^*(\beta)\}$. They are described by the total differentiation of the first order conditions associated with optimal scope, simplified here as $E[\rho_H(y)]=m_H\psi_H$, and the zero profit conditions, or $E[\rho_H(y)]=\xi_H$:[35]

$$\begin{aligned}&\textstyle\int_{y_{\min}}^{\infty}\rho_H(y;nm^*)f_\beta(y;\beta)dy+\frac{dm_H}{d\beta}\left[\rho'_{H,m_H}(nm^*)-\psi_H^*\right]+\frac{dm_L}{d\beta}\rho'_{H,m_L}(nm^*) \\ &\quad+\frac{dn_L}{d\beta}\rho'_{H,n_L}(nm^*)+\frac{dn_H}{d\beta}\left[\rho'_{H,n_H}(nm^*)-m_H^*\psi_H^*\frac{n_H^*(n_H^*-2)}{(n_H^*-1)(n_H^{*2}-n_H^*+1)}\right]=0\end{aligned} \tag{A.25}$$

$$\begin{aligned}&\textstyle\int_{y_{\min}}^{\infty}\rho_H(y;nm^*)f_\beta(y;\beta)dy+\frac{dm_H}{d\beta}\rho'_{H,m_H}(nm^*)+\frac{dm_L}{d\beta}\rho'_{H,m_L}(nm^*) \\ &\quad+\frac{dn_L}{d\beta}\rho'_{H,n_L}(nm^*)+\frac{dn_H}{d\beta}\left[\rho'_{H,n_H}(nm^*)-\frac{\xi_H^*\varphi_H^*}{n_H^*}\right]=0\end{aligned} \tag{A.26}$$

together with the requirement that the associated conditions with the decrease in $E[\rho_L(y;m_H,m_L)]$ need to be satisfied as well. The latter ensures that[36]

$$\begin{aligned}&\psi_L^*\frac{dm_L}{d\beta}+\psi_H^*\frac{dm_H}{d\beta}+m_L^*\psi_L^*\frac{n_L^*(n_L^*-2)}{(n_L^*-1)(n_L^{*2}-n_L^*+1)}\frac{dn_L}{d\beta}+ \\ &\qquad m_H^*\psi_H^*\frac{n_H^*(n_H^*-2)}{(n_H^*-1)(n_H^{*2}-n_H^*+1)}\frac{dn_H}{d\beta}=0\end{aligned} \tag{A.27}$$

$$\frac{dn_L}{d\beta}/\frac{dn_H}{d\beta}=-\frac{\xi_H^*\varphi_H^*/n_H^*}{\xi_L^*\varphi_L^*/n_L^*}=-\frac{\rho_H^*\varphi_H^*/n_H^*}{\rho_L^*\varphi_L^*/n_L^*}<0. \tag{A.28}$$

Substracting (A.26) from (A.25), gives

$$\begin{aligned}\frac{dm_H}{d\beta}/\frac{dn_H}{d\beta} &= -\left(m_H^*\psi_H^*\frac{n_H^*(n_H^*-2)}{(n_H^*-1)(n_H^{*2}-n_H^*+1)}-\xi_H^*\varphi_H^*\frac{1}{n_H^*}\right)/\psi_H^* \\ &= \frac{m_H^*}{n_H^*}\left(\varphi_H^*-\frac{n_H^{*2}(n_H^*-2)}{(n_H^*-1)(n_H^{*2}-n_H^*+1)}\right)>0,\end{aligned} \tag{A.29}$$

where the second equality uses $m_H^*\psi_H^*=\xi_H^*$ and the inequality follows from (A.18). Plugging (A.28) and (A.29) into (A.27), gives

$$\frac{dm_L}{d\beta}/\frac{dn_H}{d\beta}=-\frac{\rho_H^*\varphi_H^*/n_H^*}{\rho_L^*\varphi_L^*/n_L^*}\frac{m_L^*}{n_L^*}\left(\varphi_L^*-\frac{n_L^{*2}(n_L^*-2)}{(n_L^*-1)(n_L^{*2}-n_L^*+1)}\right)<0. \tag{A.30}$$

---

[35] Since ψ_i and ξ_i depend on n_i, we will use * to denote their associate values.

[36] As before, the set of conditions associated with optimal scope also yield $1=m_H\psi_H+m_L\psi_L$; summing the two zero-profit conditions, $E(\rho_H(y))=\xi_H$ and $E(\rho_L(y))=\xi_L$, yields $1=\xi_H+\xi_L$. The statement then follows.

Substituting (A.28)–(A.30) into (A.26), given for the number of firms

$$
\begin{aligned}
\frac{dn_H}{d\beta} &= \left\{-\left[\frac{m_H^*}{n_H^*}\left(\varphi_H^* - \tfrac{n_H^{*2}(n_H^*-2)}{(n_H^*-1)(n_H^{*2}-n_H^*+1)}\right)\rho'_{H,m_H} + \rho'_{H,n_H}\right] + \right.\\
&\left.\frac{\rho_H^*\varphi_H^*/n_H^*}{\rho_L^*\varphi_L^*/n_L^*}\left[\frac{m_L^*}{n_L^*}\left(\varphi_L^* - \tfrac{n_L^{*2}(n_L^*-2)}{(n_L^*-1)(n_L^{*2}-n_L^*+1)}\right)\rho'_{H,m_L} + \rho'_{H,n_L}\right] + \frac{\rho_H^*\varphi_H^*}{n_H^*}\right\}^{-1}\frac{d\rho_H}{d\beta}\\
&= \left\{\frac{\rho_H\varphi_H^*}{n_H^*} - \rho_{HL}\left[\mu_{Hk}\frac{1}{n_H^*}\left(\varphi_H^* - \tfrac{n_H^{*2}(n_H^*-2)}{(n_H^*-1)(n_H^{*2}-n_H^*+1)}\right) + \mu_{Hf}\frac{1}{n_H^*}\left(1 + \tfrac{n_H}{(n_H-1)^2}\right)\right] - \right.\\
&\left.\frac{\rho_H\varphi_H^*/n_H^*}{\rho_L\varphi_L^*/n_L^*}\rho_{HL}\left[\mu_{Lk}\frac{1}{n_L^*}\left(\varphi_L^* - \tfrac{n_L^{*2}(n_L^*-2)}{(n_L^*-1)(n_L^{*2}-n_L^*+1)}\right) + \mu_{Lf}\frac{1}{n_L^*}\left(1 + \tfrac{n_L}{(n_L-1)^2}\right)\right]\right\}^{-1}\frac{d\rho_H}{d\beta}\\
&= \frac{\varphi_L^* n_H^*}{\rho_H}\left\{\varphi_L^*\varphi_H^* - \varphi_L^*\left[\mu_{Hk}\frac{\rho_{HL}}{\rho_H}\varphi_H^* + \frac{\rho_{HL}}{\rho_H}\left\{\mu_{Hf}\left(1 + \tfrac{n_H}{(n_H-1)^2}\right) - \mu_{Hk}\tfrac{n_H^{*2}(n_H^*-2)}{(n_H^*-1)(n_H^{*2}-n_H^*+1)}\right\}\right] - \right.\\
&\left.\varphi_H^*\left[\mu_{Lk}\frac{\rho_{HL}}{\rho_L}\varphi_L^* + \frac{\rho_{HL}}{\rho_L}\left\{\mu_{Lf}\left(1 + \tfrac{n_L}{(n_L-1)^2}\right) - \mu_{Lk}\tfrac{n_L^{*2}(n_L^*-2)}{(n_L^*-1)(n_L^{*2}-n_L^*+1)}\right\}\right]\right\}^{-1}\frac{d\rho_H}{d\beta}\\
&= \frac{\varphi_L^* n_H^*}{\rho_H^*}\frac{D_1}{D_2}\frac{d\rho_H}{d\beta}\\
&> 0
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{dn_L}{d\beta} &= -\frac{\rho_H^*\varphi_H^*/n_H^*}{\rho_L^*\varphi_L^*/n_L^*}\frac{\varphi_L^* n_H^*}{\rho_H^*}\frac{D_1}{D_2}\frac{d\rho_H}{d\beta} = -\frac{\varphi_H^*}{\varphi_L^*}\frac{n_L^*}{\rho_L^*}\frac{D_1}{D_2}\frac{d\rho_H}{d\beta} = \frac{\varphi_H^* n_L^*}{\rho_L^*}\frac{D_1}{D_2}\frac{d\rho_L}{d\beta}\\
&< 0,
\end{aligned}
$$

where the inequalities follow as $D_2 > 0$. For the number of brands, we obtain

$$
\begin{aligned}
\frac{dm_H}{d\beta} &= m_H^*\left(\varphi_H^* - \tfrac{n_H^{*2}(n_H^*-2)}{(n_H^*-1)(n_H^{*2}-n_H^*+1)}\right)\frac{\varphi_L^*}{\rho_H^*}\frac{D_1}{D_2}\frac{d\rho_H}{d\beta} > 0\\
\frac{dm_L}{d\beta} &= -m_L^*\left(\varphi_L^* - \tfrac{n_L^{*2}(n_L^*-2)}{(n_L^*-1)(n_L^{*2}-n_L^*+1)}\right)\frac{\varphi_H^*}{\rho_L^*}\frac{D_1}{D_2}\frac{d\rho_H}{d\beta}\\
&= m_L^*\left(\varphi_L^* - \tfrac{n_L^{*2}(n_L^*-2)}{(n_L^*-1)(n_L^{*2}-n_L^*+1)}\right)\frac{\varphi_H^*}{\rho_L^*}\frac{D_1}{D_2}\frac{d\rho_L}{d\beta} < 0.
\end{aligned}
$$

A.3 Welfare

A.3.1 Welfare and second order stochastic dominance

Using McFadden (1978), we note that in our model the expected maximum utility of a household with income y is

$$
E(\max U^*(y)) = \ln\left(n_L^{\mu_{Lf}} m_L^{\mu_{Lk}} \exp\left((y - p_L)\, q_L\right) + n_H^{\mu_{Hf}} m_H^{\mu_{Hk}} \exp\left((y - p_H)\, q_H\right)\right). \quad \text{(A.31)}
$$

Clearly the expected maximum utility is increasing with income as

$$\frac{\partial E(\max U^*(y))}{\partial y} = \rho_L(y)q_L + \rho_H(y)q_H > 0$$

implying that the welfare of everyone increases with income. The expected marginal utility of income is increasing as

$$\begin{aligned}\frac{\partial^2 E(\max U^*(y))}{\partial^2 y} &= \rho_L'(y)q_L + \rho_H'(y)q_H \\ &= \rho_L(y)(q_L - q_a)q_L + \rho_H(y)(q_H - q_a)q_H \\ &= \rho_L(y)(1-\rho_L(y))(q_H - q_L)^2 \\ &> 0,\end{aligned}$$

where $q_a = \rho_L(y)q_L + \rho_H(y)q_H$.

A.3.2 Welfare expression

Here we derive the welfare expression given by equation (31) in the main text. Define $\ln(v(y)) = E(\max U^*(y))$. Using (A.31), the effect changes in $\{n_H, n_L, m_H, m_L\}$ have on expected welfare is then given by

$$\hat{v}(y) = \rho_H(y)\left[\mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)\widehat{n}_H + \mu_{Hk}\widehat{m}_H\right] + \rho_L(y)\left[\mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)\widehat{n}_L + \mu_{Lk}\widehat{m}_L\right]. \tag{A.32}$$

Since the relative changes in m_i are proportional to the relative changes in n_i, with factor of proportionality $(\varphi_i - \zeta_i)$, with $\zeta_i = \frac{n_i^2(n_i-2)}{(n_i^2-n_i+1)(n_i-1)}$, we can re-write (A.32) as

$$\begin{aligned}\hat{v}(y) &= \rho_H(y)\left[\mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right) + \mu_{Hk}(\varphi_H - \zeta_H)\right]\widehat{n}_H \\ &+\rho_L(y)\left[\mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right) + \mu_{Lk}(\varphi_L - \zeta_L)\right]\widehat{n}_L.\end{aligned}$$

In our attempt to obtain the pure scale and composition effect (similar to Fajgelbaum et al. 2011), we first note using (27) and (28) that

$$\widehat{N} = \rho_H\varphi_H\widehat{n}_H + \rho_L\varphi_L\widehat{n}_L, \tag{A.33}$$

with $\rho_i = x_i m_i n_i / N = n_i m_i d_{if}$. Together they provide the relationship

$$\begin{aligned}\hat{v}(y) &= \left(\begin{array}{c}\frac{\rho_H(y)}{\rho_H\varphi_H}\left\{\mu_{Hf}\left(1+\frac{n_H}{(n_H-1)^2}\right)+\mu_{Hk}(\varphi_H-\zeta_H)\right\} \\ +\frac{\rho_L(y)}{\rho_L\varphi_L}\left\{\mu_{Lf}\left(1+\frac{n_L}{(n_L-1)^2}\right)+\mu_{Lk}(\varphi_L-\zeta_L)\right\}\end{array}\right)\widehat{N} \\ &+\rho_H\rho_L\left(\begin{array}{c}\frac{\rho_H(y)}{\varphi_H\rho_H}\left\{\mu_{Hf}\left(1+\frac{n_H}{(n_H-1)^2}\right)+\mu_{Hk}(\varphi_H-\zeta_H)\right\} \\ -\frac{\rho_L(y)}{\varphi_L\rho_L}\left\{\mu_{Lf}\left(1+\frac{n_L}{(n_L-1)^2}\right)+\mu_{Lk}(\varphi_L-\zeta_L)\right\}\end{array}\right)(\varphi_H\widehat{n}_H - \varphi_L\widehat{n}_L)\end{aligned}$$

which is given in the main text.

B Open Economy

B.1 Incomplete specialization

B.1.1 Decision process of an H firm from country a

The first order conditions for H firms selling respectively in country a and b are

$$\left(p_{Hf}^{a}-c_{H}\right) = \frac{\mu_{Hf}}{q_{H}}\frac{1}{\left(1-\rho_{f|H}^{a}\right)} \tag{B.1}$$

$$\left(p_{Hj}^{a}-c_{H}\right) = \frac{\mu_{Hf}}{q_{H}}\frac{\mu_{Hf}/q_{H}}{\left(1-\rho_{j|H}^{a}\right)} \quad j=1,..,n_{H} \quad j\neq f, \tag{B.2}$$

$$\left(p_{Hf}^{ab}-c_{H}-\tau_{H}\right) = \frac{\mu_{Hf}}{q_{H}}\frac{1}{\left(1-\rho_{f|H}^{a}\right)}, \tag{B.3}$$

$$\left(p_{Hf}^{ab}-c_{H}-\tau_{H}\right) = \frac{\mu_{Hf}}{q_{H}}\frac{1}{\left(1-\rho_{j|H}^{a}\right)} \quad j=1,..,n_{H} \quad j\neq f \tag{B.4}$$

since $\rho_{j|H}^{ab} = \rho_{j|H}^{a}$. The first two describe the pricing decision for the home market, the last two for the foreign market. In comparison with the home market, transportation costs increase the price of high-quality goods in the foreign market. Firms assume that markets are segmented. As a result, we can apply the same arguments we applied in the autarky case to prove that the price equilibrium is unique in both markets. The optimal number of brands per firm is implicitly determined once the pricing strategies (B.1)-(B.4) are substituted into the profit function and is given in the main text.

B.1.2 Decision process of an L firm from country a

Firms assume that markets are segregated. Taking ρ_{L}^{a} and ρ_{L}^{b} as given, the first order conditions for firms selling in a are

$$\left(p_{Lf}^{a}-c_{L}\right) = \frac{\mu_{Lf}}{q_{L}}\frac{1}{\left(1-\rho_{f|L}^{a}\right)}$$

$$\left(p_{Lj}^{a}-c_{L}\right) = \frac{\mu_{Lf}}{q_{L}}\frac{1}{\left(1-\rho_{j|L}^{a}\right)} \quad j=1,..,n_{L}^{a} \quad j\neq f,$$

$$\left(p_{Lj}^{ba}-c_{L}-\tau_{L}\right) = \frac{\mu_{Lf}}{q_{L}}\frac{1}{\left(1-\rho_{f|L}^{ba}\right)} \quad j=1,..,n_{L}^{b}.$$

The conditional probabilities are given by

$$\begin{aligned}
\rho^a_{f|L} &= \frac{M^a_L \exp\left(\chi^a_L\right)}{(n^a_L - 1) + M^a_L \exp\left(\chi^a_L\right) + n^b_L M^{ba}_L \exp\left(\chi^{ba}_L\right)} \\
\rho^a_{j|L} &= \frac{1}{(n^a_L - 1) + M^a_L \exp\left(\chi^a_L\right) + n^b_L M^{ba}_L \exp\left(\chi^{ba}_L\right)} \\
\rho^{ba}_{j|L} &= \frac{M^{ba}_L \exp\left(\chi^{ba}_L\right)}{(n^a_L - 1) + M^a_L \exp\left(\chi^a_L\right) + n^b_L M^{ba}_L \exp\left(\chi^{ba}_L\right)}
\end{aligned}$$

where $M^a_L = (\frac{m^a_{Lf}}{m^a_L})^{\frac{\mu_{Lk}}{\mu_{Lf}}}$, $M^{ba}_L = (\frac{m^b_L}{m^a_L})^{\frac{\mu_{Lk}}{\mu_{Lf}}}$ and

$$\chi^a_L = \left(p^a_L - p^a_{Lf}\right) q_L/\mu_{Lf} \text{ and } \chi^{ba}_L = \left(p^a_L - p^{ba}_L\right) q_L/\mu_{Lf}.$$

Note

$$\rho^a_{f|L} + (n^a_L - 1)\rho^a_{j|L} + n^b_L \rho^{ba}_{j|L} = 1.$$

By symmetry, for firms selling in b:

$$\begin{aligned}
\rho^b_{j|L} &= \frac{M^{ba}_L \exp\left(\chi^b_L\right)}{(n^a_L - 1) + M^a_L \exp\left(\chi^{ab}_L\right) + n^b_L M^{ba}_L \exp\left(\chi^b_L\right)} \\
\rho^{ab}_{f|L} &= \frac{M^a_L \exp\left(\chi^{ab}_L\right)}{(n^a_L - 1) + M^a_L \exp\left(\chi^{ab}_L\right) + n^b_L M^{ba}_L \exp\left(\chi^b_L\right)} \\
\rho^{ab}_{j|L} &= \frac{1}{(n^a_L - 1) + M^a_L \exp\left(\chi^{ab}_L\right) + n^b_L M^{ba}_L \exp\left(\chi^b_L\right)}.
\end{aligned}$$

with

$$\chi^b_L = \left(p^{ab}_L - p^b_L\right) q_L/\mu_{Lf} \text{ and } \chi^{ab}_L = \left(p^{ab}_L - p^{ab}_{Lf}\right) q_L/\mu_{Lf}.$$

We obtain the following system of equations concerning the differences in prices:

$$\chi^a_L = \frac{1}{(n^a_L - 2) + M^a_L \exp\left(\chi^a_L\right) + n^b_L M^{ba}_L \exp\left(\chi^{ba}_L\right)} - \frac{M^a_L \exp\left(\chi^a_L\right)}{(n^a_L - 1) + n^b_L M^{ba}_L \exp\left(\chi^{ba}_L\right)} \tag{B.5}$$

$$\chi^{ba}_L = \frac{1}{(n^a_L - 2) + M^a_L \exp\left(\chi^a_L\right) + n^b_L M^{ba}_L \exp\left(\chi^{ba}_L\right)} - \frac{M^{ba}_L \exp\left(\chi^{ba}_L\right)}{(n^a_L - 1) + M^a_L \exp\left(\chi^a_L\right) + \left(n^b_L - 1\right) M^{ba}_L \exp\left(\chi^{ba}_L\right)} - \frac{\tau_L q_L}{\mu_{Lf}} \tag{B.6}$$

$$\chi^b_L = \frac{1}{(n^a_L - 2) + M^a_L \exp\left(\chi^{ab}_L\right) + n^b_L M^{ba}_L \exp\left(\chi^b_L\right)} - \frac{M^{ba}_L \exp\left(\chi^b_L\right)}{(n^a_L - 1) + M^a_L \exp\left(\chi^{ab}_L\right) + \left(n^b_L - 1\right) M^{ba}_L \exp\left(\chi^b_L\right)} + \frac{\tau_L q_L}{\mu_{Lf}} \tag{B.7}$$

$$\chi^{ab}_L = \frac{1}{(n^a_L - 2) + M^a_L \exp\left(\chi^{ab}_L\right) + n^b_L M^{ba}_L \exp\left(\chi^b_L\right)} - \frac{M^a_L \exp\left(\chi^{ab}_L\right)}{(n^a_L - 1) + n^b_L M^{ba}_L \exp\left(\chi^b_L\right)}. \tag{B.8}$$

We next show that the first (last) two equations have a unique solution in χ^a_L (χ^b_L) and χ^{ba}_L (χ^{ab}_L). Consider the first two equations. Total differentiation with respect to m^a_{Lf} yields in matrix form

$$\begin{bmatrix} 1+\frac{\rho^a_{f|L}}{(1-\rho^a_{f|L})^2}+\rho^a_{f|L}\Delta_1 & n^b_L\rho^{ba}_{j|L}\Delta_1 \\ \rho^a_{f|L}\Delta_2 & 1+\frac{\rho^{ba}_{j|L}}{(1-\rho^{ba}_{j|L})^2}+n^b_L\rho^{ba}_{j|L}\Delta_2 \end{bmatrix}\begin{bmatrix} d\chi^a_L/dm^a_{Lf} \\ d\chi^{ba}_L/dm^a_{Lf} \end{bmatrix}$$

$$= -\frac{\mu_{Lk}}{\mu_{Lf}}\frac{1}{m^a_{Lf}}\rho^a_{f|L}\begin{bmatrix} \frac{1}{(1-\rho^a_{f|L})^2}+\Delta_1 \\ \Delta_2 \end{bmatrix},$$

where

$$\Delta_1 \equiv \frac{\rho^a_{j|L}}{(1-\rho^a_{j|L})^2}-\frac{\rho^a_{f|L}}{(1-\rho^a_{f|L})^2} \text{ and } \Delta_2 \equiv \frac{\rho^a_{j|L}}{(1-\rho^a_{j|L})^2}-\frac{\rho^{ba}_{j|L}}{(1-\rho^{ba}_{j|L})^2}. \tag{B.9}$$

By inversion, we obtain

$$\begin{aligned}\frac{d\chi^a_L}{dm^a_{Lf}} &= -\frac{\mu_{Lk}}{\mu_{Lf}}\frac{1}{m^a_{Lf}}\frac{1}{D^a_{J1}}\rho^a_{f|L}\times \\ &\quad \left\{\left(1+\frac{\rho^{ba}_{j|L}}{(1-\rho^{ba}_{j|L})^2}\right)\left(\frac{1}{(1-\rho^a_{f|L})^2}+\Delta_1\right)+\frac{1}{(1-\rho^a_{f|L})^2}n^b_L\rho^{ba}_{j|L}\Delta_2\right\}<0\end{aligned} \tag{B.10}$$

$$\frac{d\chi^{ba}_L}{dm^a_{Lf}} = -\frac{\mu_{Lk}}{\mu_{Lf}}\frac{1}{m^a_{Lf}}\frac{1}{D^a_{J1}}\rho^a_{f|L}\Delta_2. \tag{B.11}$$

It is straightforward to show that the determinant D_{J1} is unambiguously positive:

$$D_{J1} = \left(1+\frac{\rho^a_{f|L}}{(1-\rho^a_{f|L})^2}\right)\left(1+\frac{\rho^{ba}_{j|L}}{(1-\rho^{ba}_{j|L})^2}\right)+\left(1+\frac{\rho^a_{f|L}}{(1-\rho^a_{f|L})^2}\right)n^b_L\rho^{ba}_{j|L}\Delta_2+\left(1+\frac{\rho^{ba}_{j|L}}{(1-\rho^{ba}_{j|L})^2}\right)\rho^a_{f|L}\Delta_1>0,$$

and, consequently, the solution (χ^a_L,χ^{ba}_L) is unique.

Similarly, total differentiation of χ^b_L and χ^{ab}_L with respect to m^a_{Lf} yields

$$\begin{aligned}\frac{d\chi^{ab}_L}{dm^a_{Lf}} &= -\frac{\mu_{Lk}}{\mu_{Lf}}\frac{1}{m^a_{Lf}}\frac{1}{D_{J2}}\rho^{ab}_{f|L}\times \\ &\quad \left\{\left(1+\frac{\rho^b_{j|L}}{(1-\rho^b_{j|L})^2}\right)\left(\frac{1}{(1-\rho^{ab}_{f|L})^2}+\Delta_3\right)+\frac{1}{(1-\rho^{ab}_{f|L})^2}n^b_L\rho^b_{j|L}\Delta_4\right\}<0\end{aligned} \tag{B.12}$$

$$\frac{d\chi^b_L}{dm^a_{Lf}} = -\frac{\mu_{Lk}}{\mu_{Lf}}\frac{1}{m^a_{Lf}}\frac{1}{D_{J2}}\rho^{ab}_{f|L}\Delta_4 \tag{B.13}$$

with

$$\Delta_3 \equiv \frac{\rho^{ab}_{j|L}}{(1-\rho^{ab}_{j|L})^2}-\frac{\rho^{ab}_{f|L}}{(1-\rho^{ab}_{f|L})^2} \text{ and } \Delta_4 \equiv \frac{\rho^{ab}_{j|L}}{(1-\rho^{ab}_{j|L})^2}-\frac{\rho^b_{j|L}}{(1-\rho^b_{j|L})^2} \tag{B.14}$$

and determinant D_{J2}:

$$\begin{aligned}D_{J2} = &\left(1+\frac{\rho^{ab}_{f|L}}{(1-\rho^{ab}_{f|L})^2}\right)\left(1+\frac{\rho^b_{j|L}}{(1-\rho^b_{j|L})^2}\right) \\ &+\left(1+\frac{\rho^{ab}_{f|L}}{(1-\rho^{ab}_{f|L})^2}\right)n^b_L\rho^b_{j|L}\Delta_4+\left(1+\frac{\rho^b_{j|L}}{(1-\rho^b_{j|L})^2}\right)\rho^{ab}_{f|L}\Delta_3>0\end{aligned}$$

and uniqueness for (χ_L^a, χ_L^{ba}) follows.

The first order condition associated with the first stage maximization problem is:

$$\frac{d\widetilde{\pi}_{Lf}^a}{dm_{Lf}^a} =$$

$$N^a E^a[\rho_L^a(y)] \frac{\rho_{f|L}^a}{(1-\rho_{f|L}^a)} \frac{\mu_{Lf}}{q_L} \left\{ \frac{\mu_{Lk}}{\mu_{Lf}} \frac{1}{m_{Lf}^a} + \frac{d\chi_L^a}{dm_{Lf}^a} - \frac{\rho_{j|L}^{ba}}{(1-\rho_{f|L}^a)} n_L^b \frac{d\chi_L^{ba}}{dm_{Lf}^a} \right\}$$
$$+ N^b E^b[\rho_L^b(y)] \frac{\rho_{f|L}^{ab}}{(1-\rho_{f|L}^{ab})} \frac{\mu_{Lf}}{q_L} \left\{ \frac{\mu_{Lk}}{\mu_{Lf}} \frac{1}{m_{Lf}^a} + \frac{d\chi_L^{ab}}{dm_{Lf}^a} - \frac{\rho_{j|L}^b}{(1-\rho_{f|L}^{ab})} n_L^b \frac{d\chi_L^b}{dm_{Lf}^a} \right\} - F_L = 0$$

Making use of (B.10), (B.11), (B.12) and (B.13), the first order condition with respect to the scope of production, m_{Lf}^a, can be rewritten as

$$\begin{aligned} \frac{q_L}{\mu_{Lk}} m_L^a F_L \quad = \quad & N^a E^a \left[\rho_L^a(y)\right] \frac{\rho_{f|L}^a}{(1-\rho_{f|L}^a)} \frac{1}{D_{J1}} \left\{ \left(1 + \frac{\rho_{j|L}^{ba}}{(1-\rho_{j|L}^{ba})^2}\right) + \frac{\rho_{j|L}^{ba}}{(1-\rho_{f|L}^a)} n_L^b \Delta_2 \right\} \\ & + N^b E^b \left[\rho_L^b(y)\right] \frac{\rho_{f|L}^{ab}}{(1-\rho_{f|L}^{ab})} \frac{1}{D_{J2}} \left\{ \left(1 + \frac{\rho_{j|L}^b}{(1-\rho_{j|L}^b)^2}\right) + \frac{\rho_{j|L}^b}{(1-\rho_{f|L}^{ab})} n_L^b \Delta_4 \right\}. \end{aligned}$$

We now want to evaluate the first order condition at the symmetric equilibrium, where $m_{Lf}^a = m_L^a$, given the equilibrium prices. Using (B.9) and (B.14), we get

$$\begin{aligned} \frac{q_L}{\mu_{Lk}} m_L^a F_L \quad = \quad & N^a E^a[\rho_L^a(y)] \left[\frac{\tilde{n}_L^a - 1}{(\tilde{n}_L^a)^2 - \tilde{n}_L^a + 1} \right] \left[1 + s^a(m_L^a, m_L^b)\right] \\ & + N^b E^b[\rho_L^b(y)] \left[\frac{\tilde{n}_L^{ab} - 1}{\left(\tilde{n}_L^{ab}\right)^2 - \tilde{n}_L^{ab} + 1} \right] \left[1 + s^{ab}(m_L^a, m_L^b)\right] \end{aligned} \tag{B.15}$$

where

$$\begin{aligned} s^a(m_L^a, m_L^b) \quad = \quad & \frac{\rho_f^a}{\left(1-\rho_f^a\right)} \frac{w_L^a \Delta_2}{1 + \frac{\rho_j^{ba}}{\left(1-\rho_f^{ba}\right)^2} + w_L^a \Delta_2} \\ = \quad & \frac{1}{\tilde{n}_L^a - 1} \frac{w_L^a \frac{\tilde{n}_L^a}{\left(\tilde{n}_L^a - 1\right)^2} \left[\left(\tilde{n}_L^a - \theta^a\right)^2 - \theta^a \left(\tilde{n}_L^a - 1\right)^2\right]}{\left(\left(\tilde{n}_L^a - \theta^a\right)^2 + \tilde{n}_L^a \theta^a\right) + w_L^a \frac{\tilde{n}_L^a}{\left(\tilde{n}_L^a - 1\right)^2} \left[\left(\tilde{n}_L^a - \theta^a\right)^2 - \theta^a \left(\tilde{n}_L^a - 1\right)^2\right]} \end{aligned} \tag{B.16}$$

$$\begin{aligned} s^{ab}(m_L^a, m_L^b) \quad = \quad & \frac{\rho_j^{ab}}{\left(1-\rho_f^{ab}\right)} \frac{w_L^{ab} \Delta_4}{1 + \frac{\rho_j^b}{\left(1-\rho_f^b\right)^2} + w_L^{ab} \Delta_4} \\ = \quad & \frac{1}{\tilde{n}_L^{ab} - 1} \frac{w_L^{ab} \frac{\tilde{n}_L^{ab}}{\left(\tilde{n}_L^{ab} - 1\right)^2} \left[\left(\tilde{n}_L^{ab} - \theta^{ab}\right)^2 - \theta^{ab} \left(\tilde{n}_L^{ab} - 1\right)^2\right]}{\left(\left(\tilde{n}_L^{ab} - \theta^{ab}\right)^2 + \tilde{n}_L^{ab} \theta^{ab}\right) + w_L^{ab} \frac{\tilde{n}_L^{ab}}{\left(\tilde{n}_L^{ab} - 1\right)^2} \left[\left(\tilde{n}_L^{ab} - \theta^{ab}\right)^2 - \theta^{ab} \left(\tilde{n}_L^{ab} - 1\right)^2\right]} \end{aligned} \tag{B.17}$$

$$\text{and } w_L^a \quad = \quad \frac{\theta^a n_L^b}{n_L^a + \theta^a n_L^b} \text{ and } w_L^{ab} = \frac{\theta^{ab} n_L^b}{n_L^a + \theta^{ab} n_L^b}$$

B.1.3 Decision process of an L firm from country b

By symmetry, the price subgame for firm f of country b, yields

$$\begin{aligned}
\rho^{b}_{j|L} &= \frac{1}{\tilde{n}^{b}_{L}},\ j=1,..,n^{b} \\
\rho^{ba}_{j|L} &= \frac{1}{\tilde{n}^{b}_{L}},\ j=1,..,n^{b} \\
\rho^{ab}_{j|L} &= \frac{\theta^{b}}{\tilde{n}^{b}_{L}},\ j=1,..,n^{b} \\
\text{with } \tilde{n}^{b}_{L} &= n^{b}_{L}+\theta^{b}n^{a}_{L},\ \theta^{b}=M^{ab}_{L}\exp\left(\chi^{*ab}_{L}\right), \\
\tilde{n}^{ba}_{L} &= n^{b}_{L}+\theta^{ba}n^{a}_{L},\ \theta^{ba}=M^{ab}_{L}\exp\left(\chi^{*a}_{L}\right).
\end{aligned}$$

The definitions for χ^{*ab}_{L} and χ^{*a}_{L} are not specified explicitly here. The associated first order condition at this symmetric equilibrium, gives

$$\begin{aligned}
\frac{q_L}{\mu_{Lk}}m^{b}_{L}F_{L} &= N^{b}E^{b}[\rho^{b}_{L}(y)]\left[\frac{\tilde{n}^{b}_{L}-1}{\left(\tilde{n}^{b}_{L}\right)^{2}-\tilde{n}^{b}_{L}+1}\right]\left[1+s^{b}(m^{a}_{L},m^{b}_{L})\right] \\
&\quad +N^{a}E^{a}[\rho^{a}_{L}(y)]\left[\frac{\tilde{n}^{ba}_{L}-1}{\left(\tilde{n}^{ba}_{L}\right)^{2}-\tilde{n}^{ba}_{L}+1}\right]\left[1+s^{ba}(m^{a}_{L},m^{b}_{L})\right]
\end{aligned} \tag{B.18}$$

where

$$\begin{aligned}
s^{b}(m^{a}_{L},m^{b}_{L}) &= \frac{1}{\tilde{n}^{b}_{L}-1}\frac{w^{b}_{L}\frac{\tilde{n}^{b}_{L}}{(\tilde{n}^{b}_{L}-1)^{2}}\left[\left(\tilde{n}^{b}_{L}-\theta^{b}\right)^{2}-\theta^{b}\left(\tilde{n}^{b}_{L}-1\right)^{2}\right]}{\left(\tilde{n}^{b}_{L}-1\right)^{2}\left(\left(\tilde{n}^{b}_{L}\right)^{2}-\tilde{n}^{a}_{L}\theta^{b}+\left(\theta^{b}\right)^{2}\right)+w^{b}_{L}\frac{\tilde{n}^{b}_{L}}{(\tilde{n}^{b}_{L}-1)^{2}}\left[\left(\tilde{n}^{b}_{L}-\theta^{b}\right)^{2}-\theta^{b}\left(\tilde{n}^{b}_{L}-1\right)^{2}\right]} \\
s^{ba}(m^{a}_{L},m^{b}_{L}) &= \frac{1}{\tilde{n}^{ba}_{L}-1}\frac{w^{ba}_{L}\frac{\tilde{n}^{ba}_{L}}{(\tilde{n}^{ba}_{L}-1)^{2}}\left[\left(\tilde{n}^{ba}_{L}-\theta^{ba}\right)^{2}-\theta^{ba}\left(\tilde{n}^{ba}_{L}-1\right)^{2}\right]}{\left(\tilde{n}^{ba}_{L}-1\right)^{2}\left(\left(\tilde{n}^{ba}_{L}\right)^{2}-\tilde{n}^{ba}_{L}\theta^{ba}+\left(\theta^{ba}\right)^{2}\right)+w^{ba}_{L}\frac{\tilde{n}^{ba}_{L}}{(\tilde{n}^{ba}_{L}-1)^{2}}\left[\left(\tilde{n}^{ba}_{L}-\theta^{ba}\right)^{2}-\theta^{ba}\left(\tilde{n}^{ba}_{L}-1\right)^{2}\right]} \\
\text{with } w^{b}_{L} &= \frac{\theta^{b}n^{a}_{L}}{n^{b}_{L}+\theta^{b}n^{a}_{L}} \text{ and } w^{ba}_{L}=\frac{\theta^{ba}n^{a}_{L}}{n^{b}_{L}+\theta^{ba}n^{a}_{L}}.
\end{aligned}$$

B.1.4 Symmetric equilibrium summary

In symmetric equilibrium, the prices are given by

Market a	Market b
$p^{a}_{Hf}=\frac{\mu_{Hf}}{q_H}\frac{n_H}{n_H-1}+c_H$	$p^{ab}_{Hf}=\frac{\mu_{Hf}}{q_H}\frac{n_H}{n_H-1}+c_H+\tau_H$
$p^{a}_{Lf}=\frac{\mu_{Lf}}{q_L}\frac{\tilde{n}^{a}_{L}}{\tilde{n}^{a}_{L}-1}+c_L$	$p^{b}_{Lf}=\frac{\mu_{Lf}}{q_L}\frac{\tilde{n}^{b}_{L}}{\tilde{n}^{b}_{L}-1}+c_L$
$p^{ba}_{Lj}=\frac{\mu_{Lf}}{q_L}\frac{\tilde{n}^{a}_{L}}{\tilde{n}^{a}_{L}-\theta^{a}}+c_L+\tau_L$	$p^{ab}_{Lj}=\frac{\mu_{Lf}}{q_L}\frac{\tilde{n}^{b}_{L}}{\tilde{n}^{b}_{L}-\theta^{b}}+c_L+\tau_L$

and $\chi_L^{*ab} = -\chi_L^b$ and $\chi_L^{*a} = -\chi_L^{ba}$, so that $\theta^a\theta^{ba} = 1$ and $\theta^b\theta^{ab} = 1$. As $\tilde{n}_L^a = \theta^a\tilde{n}_L^{ba}$ and $\tilde{n}_L^b = \theta^b\tilde{n}_L^{ab}$: $\frac{\tilde{n}_L^a}{\tilde{n}_L^a-\theta^a} = \frac{\tilde{n}_L^{ba}}{\tilde{n}_L^{ba}-1}$, $\frac{\tilde{n}_L^b}{\tilde{n}_L^b-\theta^b} = \frac{\tilde{n}_L^{ab}}{\tilde{n}_L^{ab}-1}$.

We assume $\tilde{n}_L^a < \tilde{n}_L^{ab}$ and $\tilde{n}_L^b < \tilde{n}_L^{ba}$, indicative that the effective number of competitors in the presence of transportation cost is smaller in the home market than in the foreign market and $\theta^a < 1 < \theta^{ab}$, $\theta^b < 1 < \theta^{ba}$.

B.1.5 Comparative Statics

To obtain the total differential of equations (39), (40), and (41), we make use of the following results

$$\begin{aligned} d\chi_L^{ba} &= -d\chi_L^{*a} = -\frac{\tau_L q_L}{\mu_{Lf}}\varepsilon_\tau\widehat{\tau}_L \\ d\chi_L^b &= -d\chi_L^{*ab} = \frac{\tau_L q_L}{\mu_{Lf}}\varepsilon_\tau\widehat{\tau}_L \end{aligned}$$

(uses equations (B.5)–(B.8)), where ε_τ denotes the (positive) elasticity of transport cost on prices (through the χ_L terms). For simplicity we assume them to be identical.

Evaluating $E^a[\rho_L^a(y)]$ at the symmetric equilibrium gives

$$\begin{aligned} & E^a[\rho_L^a(y)] \\ = \; & E^a\left[\frac{(n_L^a)^{\mu_{Lf}}(m_L^a)^{\mu_{Lk}}\phi_L(y,\tilde{n}_L^a) \quad + \quad \left(\lambda_L v_L^a n_L^b\right)^{\mu_{Lf}}\left(m_L^b\right)^{\mu_{Lk}}\phi_L(y,\tilde{n}_L^a)}{\left[(n_L^a)^{\mu_{Lf}}(m_L^a)^{\mu_{Lk}} + \left(\lambda_L v_L^a n_L^b\right)^{\mu_{Lf}}\left(m_L^b\right)^{\mu_{Lk}}\right]\phi_L(y,\tilde{n}_L^a) + (n_H^a)^{\mu_{Hf}}(m_H^a)^{\mu_{Hk}}\phi_H(y,n_H^a)}\right] \\ \equiv \; & E^a[\rho_L^{aa}(y)] + E^a[\rho_L^{ab}(y)], \end{aligned}$$

where

$$\lambda_L = \exp\left(-\tau_L q_L/\mu_{Lf}\right) \text{ and } v_L^a = \exp\left(\tfrac{\tilde{n}_L^a}{(\tilde{n}_L^a-1)} - \tfrac{\tilde{n}_L^a}{(\tilde{n}_L^a-\theta^a)}\right)$$

since

$$\begin{aligned} \exp\left[(y-p_H)q_H\right] &= \exp\left[(y-c_H)q_H - \mu_{Hf}\tfrac{n_H}{(n_H-1)}\right] \equiv \phi_H(y,n_H^a) \\ \exp\left[(y-p_L^a)q_L\right] &= \exp\left[(y-c_L)q_L - \mu_{Lf}\tfrac{\tilde{n}_L^a}{(\tilde{n}_L^a-1)}\right] \equiv \phi_L(y,\tilde{n}_L^a) \\ \exp\left[(y-p_L^{ba})q_L\right] &= \exp\left[(y-c_L-\tau_L)q_L - \mu_{Lf}\frac{\tilde{n}_L^a}{\tilde{n}_L^a-\theta^a}\right] \\ &\equiv \phi_L(y,\tilde{n}_L^a)\exp\left(\mu_{Lf}\left(\tfrac{\tilde{n}_L^a}{(\tilde{n}_L^a-1)} - \tfrac{\tilde{n}_L^a}{(\tilde{n}_L^a-\theta^a)}\right)\right)\exp\left(-\tau_L q_L\right). \end{aligned}$$

Similarly

$$\begin{aligned} & E^b[\rho_L^b(y)] \\ = \; & E^b\left[\frac{\left(\lambda_L v_L^b n_L^a\right)^{\mu_{Lf}}(m_L^a)^{\mu_{Lk}}\phi_L(y,\tilde{n}_L^b)) \quad + \quad \left(n_L^b\right)^{\mu_{Lf}}\left(m_L^b\right)^{\mu_{Lk}}\phi_L(y,\tilde{n}_L^b)}{\left[\left(\lambda_L v_L^b n_L^a\right)^{\mu_{Lf}}(m_L^a)^{\mu_{Lk}} + \left(n_L^b\right)^{\mu_{Lf}}\left(m_L^b\right)^{\mu_{Lk}}\right]\phi_L(y,\tilde{n}_L^b) + (\lambda_H n_H^a)^{\mu_{Hf}}(m_H^a)^{\mu_{Hk}}\phi_H(y,n_H^a)}\right] \\ \equiv \; & E^b[\rho_L^{ba}(y)] + E^b[\rho_L^{bb}(y)], \end{aligned}$$

where

$$\lambda_H = \exp\left(-\tau_H q_H/\mu_{Hf}\right) \text{ and } v_L^b = \exp\left(\tfrac{\tilde{n}_L^b}{(\tilde{n}_L^b-1)} - \tfrac{\tilde{n}_L^b}{(\tilde{n}_L^b-\theta^b)}\right)$$

Let us define $\rho_{HL}^{aa} = E^a\left[\rho_L^{aa}(y)\rho_H^a(y)\right], \rho_{HL}^{ab} = E^a\left[\rho_L^{ab}(y)\rho_H^a(y)\right]$ with $\rho_{HL}^{aa}+\rho_{HL}^{ab} = \rho_{HL}^a \equiv E^a\left[\rho_L^a(y)\rho_H^a(y)\right]$, and by symmetry $\rho_{HL}^{ba} = E^b\left[\rho_L^{ba}(y)\rho_H^b(y)\right], \rho_{HL}^{bb} = E^b\left[\rho_L^{bb}(y)\rho_H^b(y)\right]$ with $\rho_{HL}^{ba} + \rho_{HL}^{bb} = \rho_{HL}^b \equiv E^b\left[\rho_L^b(y)\rho_H^b(y)\right]$.

With $\rho_L(y)$ concave in y (i.e., assuming $\rho_L(y) > \rho_H(y)$ for all y as before), we expect $\rho_{HL}^b/\rho_L^b > \rho_{HL}^a/\rho_L^a$, as a denotes the rich country and b the poor country.

Total differentiation of $\rho_L^a(y)$ and $\rho_L^b(y)$ yields

$$\begin{aligned}
\widehat{\rho}_L^a &= \mu_{Lk}\left[\left(1-\frac{\tilde{n}_L^a}{\left(\tilde{n}_L^a-1\right)^2}w_L^a\right)\frac{\rho_{HL}^{aa}}{\rho_L^a} + \frac{\tilde{n}_L^a\theta^a}{\left(\tilde{n}_L^a-\theta^a\right)^2}\left(1-w_L^a\right)\frac{\rho_{HL}^{ab}}{\rho_L^a}\right]\hat{m}_L^a \\
&\quad +\mu_{Lk}\left[\frac{\tilde{n}_L^a}{\left(\tilde{n}_L^a-1\right)^2}w_L^a\frac{\rho_{HL}^{aa}}{\rho_L^a} + \left(1-\frac{\tilde{n}_L^a\theta^a}{\left(\tilde{n}_L^a-\theta^a\right)^2}\left(1-w_L^a\right)\right)\frac{\rho_{HL}^{ab}}{\rho_L^a}\right]\hat{m}_L^b \\
&\quad -\mu_{Hk}\frac{\rho_{HL}^a}{\rho_L^a}\hat{m}_H^a + \frac{\rho_{HL}^a}{\rho_L^a}\left[q_L-q_H\right]dy^a \\
&\quad +\mu_{Lf}\left[\frac{\tilde{n}_L^a}{\left(\tilde{n}_L^a-1\right)^2}w_L^a\varepsilon_\tau\frac{\rho_{HL}^{aa}}{\rho_L^a} + \left(1-\frac{\tilde{n}_L^a\theta^a}{\left(\tilde{n}_L^a-\theta^a\right)^2}\left(1-w_L^a\right)\varepsilon_\tau\right)\frac{\rho_{HL}^{ab}}{\rho_L^a}\right]\widehat{\lambda}_L \\
&\equiv \delta_{m_L^a\rho_L^a}\hat{m}_L^a + \delta_{m_L^b\rho_L^a}\hat{m}_L^b - \delta_{m_H^a\rho_L^a}\hat{m}_H^a + \delta_{y^a\rho_L^a}dy^a + \delta_{\lambda_L\rho_L^a}\widehat{\lambda}_L
\end{aligned} \tag{B.19}$$

$$\begin{aligned}
\widehat{\rho}_L^b &= \mu_{Lk}\left[\frac{\tilde{n}_L^b}{\left(\tilde{n}_L^b-1\right)^2}w_L^b\frac{\rho_{HL}^{bb}}{\rho_L^b} + \left(1-\frac{\tilde{n}_L^b\theta^b}{\left(\tilde{n}_L^b-\theta^b\right)^2}\left(1-w_L^b\right)\right)\frac{\rho_{HL}^{ba}}{\rho_L^b}\right]\hat{m}_L^a \\
&\quad +\mu_{Lk}\left[\left(1-\frac{\tilde{n}_L^b}{\left(\tilde{n}_L^b-1\right)^2}w_L^b\right)\frac{\rho_{HL}^{bb}}{\rho_L^b} + \frac{\tilde{n}_L^b\theta^b}{\left(\tilde{n}_L^b-\theta^b\right)^2}\left(1-w_L^b\right)\frac{\rho_{HL}^{ba}}{\rho_L^b}\right]\hat{m}_L^b \\
&\quad -\mu_{Hk}\frac{\rho_{HL}^b}{\rho_L^b}\hat{m}_H^a + \frac{\rho_{HL}^b}{\rho_L^b}\left[q_L-q_H\right]dy^b - \mu_{Hf}\frac{\rho_{HL}^b}{\rho_L^b}\hat{\lambda}_H \\
&\quad +\mu_{Lf}\left[\frac{\tilde{n}_L^b}{\left(\tilde{n}_L^b-1\right)^2}w_L^b\varepsilon_\tau\frac{\rho_{HL}^{bb}}{\rho_L^b} + \left(1-\frac{\tilde{n}_L^b\theta^b}{\left(\tilde{n}_L^b-\theta^b\right)^2}\left(1-w_L^b\right)\varepsilon_\tau\right)\frac{\rho_{HL}^{ba}}{\rho_L^b}\right]\widehat{\lambda}_L \\
&\equiv \delta_{m_L^a\rho_L^b}\hat{m}_L^a + \delta_{m_L^b\rho_L^b}\hat{m}_L^b - \delta_{m_H^a\rho_L^b}\hat{m}_H^a + \delta_{y^b\rho_L^b}dy^b - \delta_{\lambda_H\rho_L^b}\hat{\lambda}_H + \delta_{\lambda_L\rho_L^b}\widehat{\lambda}_L.
\end{aligned} \tag{B.20}$$

If $w_L^a = w_L^b = 0$ (that is if there is no foreign competition in the home market), the total differential simplifies to that in autarky.

The analysis below makes extensive use of the following notation: $\omega_H^a \equiv \frac{N^a\rho_H^a}{N^a\rho_H^a+N^b\rho_H^b}$, which defines the share of high-quality goods that are bought by consumers from country

a; and

$$\omega_L^{aa} = \frac{N^a\rho_L^a}{N^a\rho_L^a + N^b\rho_L^b\Lambda^{ab}} \text{ with } \Lambda^{ab} = \frac{\left[\frac{\tilde{n}_L^{ab}-1}{\left(\tilde{n}_L^{ab}\right)^2-\tilde{n}_L^{ab}+1}\right]\left[1+s^{ab}(m_L^a,m_L^b)\right]}{\left[\frac{\tilde{n}_L^{a}-1}{\left(\tilde{n}_L^{a}\right)^2-\tilde{n}_L^{a}+1}\right]\left[1+s^{a}(m_L^a,m_L^b)\right]} \tag{B.21}$$

$$\omega_L^{ba} = \frac{N^a\rho_L^a\Lambda^{ba}}{N^a\rho_L^a + N^b\rho_L^b\Lambda^{ba}} \text{ with } \Lambda^{ab} = \frac{\left[\frac{\tilde{n}_L^{ba}-1}{\left(\tilde{n}_L^{ba}\right)^2-\tilde{n}_L^{ba}+1}\right]\left[1+s^{ba}(m_L^a,m_L^b)\right]}{\left[\frac{\tilde{n}_L^{b}-1}{\left(\tilde{n}_L^{b}\right)^2-\tilde{n}_L^{b}+1}\right]\left[1+s^{b}(m_L^a,m_L^b)\right]} \tag{B.22}$$

which, respectively, define the share of domestic and foreign marginal profits belonging to firms from country a. $\omega_L^{ab} = 1-\omega_L^{aa}$ and $\omega_L^{bb} = 1-\omega_L^{ba}$. It is reasonable to expect $\omega_L^{aa} > \omega_L^{ba}$, in fact if $N^b\rho_L^b$ is sufficiently big relative to $N^a\rho_L^a$, we expect $\omega_L^{aa} + \omega_L^{ba} < 1$.

Total differentiation of the short-run equilibrium conditions (39), (40), and (41), then yields, respectively

$$\hat{m}_H^a = \omega_H^a\widehat{\rho}_H^a + (1-\omega_H^a)\widehat{\rho}_H^b = -\omega_H^a\frac{\rho_L^a}{\rho_H^a}\widehat{\rho}_L^a - (1-\omega_H^a)\frac{\rho_L^b}{\rho_H^b}\widehat{\rho}_L^b, \tag{B.23}$$

$$\hat{m}_L^a = \omega_L^{aa}\widehat{\rho}_L^a + \omega_L^{ab}\widehat{\rho}_L^b + (h_L^a - e_L^a)\frac{\mu_{Lk}}{\mu_{Lf}}(\hat{m}_L^a - \hat{m}_L^b) - (h_L^a - e_L^a)\,\varepsilon_\tau\widehat{\lambda}_L, \tag{B.24}$$

and

$$\hat{m}_L^b = \omega_L^{bb}\widehat{\rho}_L^b + \omega_L^{ba}\widehat{\rho}_L^a - \left(h_L^b - e_L^b\right)\frac{\mu_{Lk}}{\mu_{Lf}}(\hat{m}_L^a - \hat{m}_L^b) - \left(h_L^b - e_L^b\right)\varepsilon_\tau\hat{\lambda}_L, \tag{B.25}$$

where

$$\begin{aligned}
h_L^a &= \left[\omega_L^{aa}\frac{w_L^a}{(\tilde{n}_L^a-1)}\frac{(\tilde{n}_L^a)^2(\tilde{n}_L^a-2)}{\left[(\tilde{n}_L^a)^2-\tilde{n}_L^a+1\right]} + \omega_L^{ab}\frac{w_L^{ab}}{(\tilde{n}_L^{ab}-1)}\frac{\left(\tilde{n}_L^{ab}\right)^2(\tilde{n}_L^{ab}-2)}{\left[\left(\tilde{n}_L^{ab}\right)^2-\tilde{n}_L^{ab}+1\right]}\right] > 0\\
e_L^a &= \left[\omega_L^{aa}\frac{s^a}{(1+s^a)}\varepsilon_{s^a} + \omega_L^{ab}\frac{s^{ab}}{(1+s^{ab})}\varepsilon_{s^{ab}}\right]\\
h_L^b &= \left[\omega_L^{bb}\frac{w_L^b}{\left(\tilde{n}_L^b-1\right)}\frac{\left(\tilde{n}_L^b\right)^2(\tilde{n}_L^b-2)}{\left[\left(\tilde{n}_L^b\right)^2-\tilde{n}_L^b+1\right]} + \omega_L^{ba}\frac{w_L^{ba}}{(\tilde{n}_L^{ba}-1)}\frac{\left(\tilde{n}_L^{ba}\right)^2(\tilde{n}_L^{ba}-2)}{\left[\left(\tilde{n}_L^{ba}\right)^2-\tilde{n}_L^{ba}+1\right]}\right] > 0\\
e_L^b &= \left[\omega_L^{bb}\frac{s^b}{(1+s^b)}\varepsilon_{s^b} + \omega_L^{ba}\frac{s^{ba}}{(1+s^{ba})}\varepsilon_{s^{ba}}\right].
\end{aligned}$$

Here ε_{s^a} denotes the θ^a elasticity of $s_L^a(m_L^a, m_L^b)$ and $\varepsilon_{s^{ab}}$ the θ^{ab} elasticity of $s_L^{ab}(m_L^a, m_L^b)$. The definitions of ε_{s^b} and $\varepsilon_{s^{ba}}$ are analogous.

Using (B.19) and (B.20) and recognizing $\hat{\rho}^a_L = -\frac{\rho^a_L}{\rho^a_H}\hat{\rho}^a_H$, we then obtain in matrix form:

$$\begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} \begin{bmatrix} \hat{m}^a_H \\ \hat{m}^a_L \\ \hat{m}^b_L \end{bmatrix}$$

$$= \begin{bmatrix} -\omega^a_H \frac{\rho^a_L}{\rho^a_H} & -\omega^b_H \frac{\rho^b_L}{\rho^b_H} & \omega^b_H \frac{\rho^b_L}{\rho^b_H} & -\omega^a_H \frac{\rho^a_L}{\rho^a_H}\delta_{\lambda_L \rho^a_L} - \omega^b_H \frac{\rho^b_L}{\rho^b_H}\delta_{\lambda_L \rho^b_L} \\ \omega^{aa}_L & \omega^{ab}_L & -\omega^{ab}_L & \omega^{aa}_L \delta_{\lambda_L \rho^a_L} + \omega^{ab}_L \delta_{\lambda_L \rho^b_L} - \left(h^a_L - e^a_L\right)\varepsilon_\tau \\ \omega^{ba}_L & \omega^{bb}_L & -\omega^{bb}_L & \omega^{ba}_L \delta_{\lambda_L \rho^a_L} + \omega^{bb}_L \delta_{\lambda_L \rho^b_L} - \left(h^b_L - e^b_L\right)\varepsilon_\tau \end{bmatrix} \begin{bmatrix} \delta_{y^a \rho^a_L} dy^a \\ \delta_{y^b \rho^b_L} dy^b \\ \delta_{\lambda_H \rho^b_L} \hat{\lambda}_H \\ \widehat{\lambda}_L \end{bmatrix}.$$

The elements of the Jacobian matrix are

$$\begin{aligned} e_{11} &= \left[1 - \left(\omega^a_H \delta_{m^a_H \rho^a_H} + \omega^b_H \delta_{m^a_H \rho^b_H}\right)\right] \equiv 1 - \omega^a_{HH} \\ e_{12} &= -\left(\omega^a_H \delta_{m^a_L \rho^a_H} + \omega^b_H \delta_{m^a_L \rho^b_H}\right) \equiv \left(\omega^a_H \frac{\rho^a_L}{\rho^a_H}\delta_{m^a_L \rho^a_L} + \omega^b_H \frac{\rho^b_L}{\rho^b_H}\delta_{m^a_L \rho^b_L}\right) \\ e_{13} &= -\left(\omega^a_H \delta_{m^b_L \rho^a_H} + \omega^b_H \delta_{m^b_L \rho^b_H}\right) \equiv \left(\omega^a_H \frac{\rho^a_L}{\rho^a_H}\delta_{m^b_L \rho^a_L} + \omega^b_H \frac{\rho^b_L}{\rho^b_H}\delta_{m^b_L \rho^b_L}\right) \end{aligned}$$

$$\begin{aligned} e_{21} &= \left[\omega^{aa}_L \delta_{m^a_H \rho^a_L} + \omega^{ab}_L \delta_{m^a_H \rho^b_L}\right] \equiv \omega^a_{LL} \\ e_{22} &= \left[1 - \left(\omega^{aa}_L \delta_{m^a_L \rho^a_L} + \omega^{ab}_L \delta_{m^a_L \rho^b_L}\right) - \left(h^a_L - e^a_L\right)\frac{\mu_{Lk}}{\mu_{Lf}}\right] \\ e_{23} &= -\left(\omega^{aa}_L \delta_{m^b_L \rho^a_L} + \omega^{ab}_L \delta_{m^b_L \rho^b_L}\right) + \left(h^a_L - e^a_L\right)\frac{\mu_{Lk}}{\mu_{Lf}} \end{aligned}$$

$$\begin{aligned} e_{31} &= \left(\omega^{bb}_L \delta_{m^a_H \rho^b_L} + \omega^{ba}_L \delta_{m^a_H \rho^a_L}\right) \equiv \omega^b_{LL} \\ e_{32} &= -\left(\omega^{bb}_L \delta_{m^a_L \rho^b_L} + \omega^{ba}_L \delta_{m^a_L \rho^a_L}\right) + \left(h^b_L - e^b_L\right)\frac{\mu_{Lk}}{\mu_{Lf}} \\ e_{33} &= \left[1 - \left(\omega^{bb}_L \delta_{m^b_L \rho^b_L} + \omega^{ba}_L \delta_{m^b_L \rho^a_L}\right) - \left(h^b_L - e^b_L\right)\frac{\mu_{Lk}}{\mu_{Lf}}\right]. \end{aligned}$$

and $\omega^a_{HH}, \omega^a_{LL}, \omega^b_{LL} \in (0,1)$.

By inversion we obtain

$$\begin{bmatrix} \hat{m}^a_H \\ \hat{m}^a_L \\ \hat{m}^b_L \end{bmatrix} = \frac{1}{D_{IS}} \begin{bmatrix} (e_{22}e_{33} - e_{32}e_{23}) & -(e_{12}e_{33} - e_{32}e_{13}) & (e_{12}e_{23} - e_{22}e_{13}) \\ -(e_{21}e_{33} - e_{23}e_{31}) & (e_{11}e_{33} - e_{31}e_{13}) & -(e_{11}e_{23} - e_{21}e_{13}) \\ (e_{21}e_{32} - e_{22}e_{31}) & -(e_{11}e_{32} - e_{31}e_{12}) & (e_{11}e_{22} - e_{21}e_{12}) \end{bmatrix} \times$$

$$\begin{bmatrix} -\omega^a_H \frac{\rho^a_L}{\rho^a_H} & -\omega^b_H \frac{\rho^b_L}{\rho^b_H} & \omega^b_H \frac{\rho^b_L}{\rho^b_H} & -\omega^a_H \frac{\rho^a_L}{\rho^a_H}\delta_{\lambda_L \rho^a_L} - \omega^b_H \frac{\rho^b_L}{\rho^b_H}\delta_{\lambda_L \rho^b_L} \\ \omega^{aa}_L & \omega^{ab}_L & -\omega^{ab}_L & \omega^{aa}_L \delta_{\lambda_L \rho^a_L} + \omega^{ab}_L \delta_{\lambda_L \rho^b_L} - \left(h^a_L - e^a_L\right)\varepsilon_\tau \\ \omega^{ba}_L & \omega^{bb}_L & -\omega^{bb}_L & \omega^{ba}_L \delta_{\lambda_L \rho^a_L} + \omega^{bb}_L \delta_{\lambda_L \rho^b_L} - \left(h^b_L - e^b_L\right)\varepsilon_\tau \end{bmatrix} \begin{bmatrix} \delta_{y^a \rho^a_L} dy^a \\ \delta_{y^b \rho^b_L} dy^b \\ \delta_{\lambda_H \rho^b_L} \hat{\lambda}_H \\ \widehat{\lambda}_L \end{bmatrix}$$

When $\omega_{LL}^a = \omega_{LL}^b$, the determinant is given by

$$\begin{aligned} D_{IS} &= (1 - \omega_{HH}^a - \omega_{LL}^a) \times \\ & \left(1 - \left(\omega_L^{aa} - \omega_L^{ba}\right)\left(\delta_{m_L^b \rho_L^b} - \delta_{m_L^a \rho_L^b}\right) - \left(h_L^a - e_L^a + h_L^b - e_L^b\right)\frac{\mu_{Lk}}{\mu_{Lf}}\right). \end{aligned}$$

The first term in the determinant is unambiguously positive. We assume that the differential competition- and price effect $(h_L^a - e_L^a + h_L^b - e_L^b)$ and the differential impact of $\left(m_L^a, m_L^b\right)$ on ρ_L^b $(\delta_{m_L^b \rho_L^b} - \delta_{m_L^a \rho_L^b})$ are not too large to change the sign of the determinant ensuring the determinant is positive.

In general, with $\omega_{LL}^a \neq \omega_{LL}^b$, the determinant is

$$\begin{aligned} D_{IS} &= (1 - \omega_{HH}^a - \omega_{LL}^a) \left(\begin{array}{c} 1 - \frac{\omega_L^{aa} - \omega_L^{ba}}{\omega_L^{aa}} \left[\frac{\rho_{HL}^a}{\rho_L^a} (\delta_{m_L^b \rho_L^b} - \delta_{m_L^b \rho_L^a}) + \left(\frac{\rho_{HL}^a}{\rho_L^a} - \frac{\rho_{HL}^b}{\rho_L^b} \right) \delta_{m_L^b \rho_L^a} \right] \\ - \left[\frac{\omega_{LL}^b}{\omega_{LL}^a} \left(h_L^a - e_L^a \right) + \left(h_L^b - e_L^b \right) \right] \frac{\mu_{Lk}}{\mu_{Lf}} \end{array} \right) \\ &+ \left(1 - \frac{\omega_{LL}^b}{\omega_{LL}^a} \right) \left(\begin{array}{l} \delta_{m_L^b \rho_L^a} \left(\omega_H^a \frac{\rho_L^a}{\rho_H^a} \omega_{LL}^a + \omega_L^{aa} \left(1 - \omega_{HH}^a \right) \right) \\ + \delta_{m_L^b \rho_L^b} \left(\omega_H^b \frac{\rho_L^b}{\rho_H^b} \omega_{LL}^a + \omega_L^{ab} \left(1 - \omega_{HH}^a \right) \right) \\ - \left(1 - \omega_{HH}^a \right) \left(h_L^a - e_L^a \right) \frac{\mu_{Lk}}{\mu_{Lf}} \end{array} \right). \end{aligned}$$

We assume the determinant remains positive. With $\omega_L^{aa} + \omega_L^{ba} < 1$ and $\rho_{HL}^a / \rho_L^a < \rho_{HL}^b / \rho_L^b$, we are guaranteed that $1 - \frac{\omega_{LL}^b}{\omega_{LL}^a} > 0$; as before we assume that the competition (h_L^a, h_L^b) and price (e_L^a, e_L^b) effects (in differences) are not too strong.

Change in trade cost for high quality goods We make use of the following results below:

$$\begin{aligned} (\omega_L^{ab} \omega_{LL}^b - \omega_L^{bb} \omega_{LL}^a) &= -(\omega_L^{aa} - \omega_L^{ba}) \frac{\rho_{HL}^a}{\rho_L^a} \\ (\omega_L^{aa} \omega_{LL}^b - \omega_L^{ba} \omega_{LL}^a) &= (\omega_L^{aa} - \omega_L^{ba}) \frac{\rho_{HL}^b}{\rho_L^b} \\ \omega_L^{aa} \omega_L^{bb} - \omega_L^{ab} \omega_L^{ba} &= \omega_L^{aa} - \omega_L^{ba} = \omega_L^{bb} - \omega_L^{ab}; \end{aligned}$$

and we recall that an increase in λ, is associated with a decrease in transportation cost (τ).

Associated with an increase in λ_H, we obtain:

$$\frac{\hat{m}_H^a}{\hat{\lambda}_H} = \frac{\delta_{\lambda_H \rho_L^b}}{D_{IS}} \left\{ \begin{array}{l} \omega_H^b \frac{\rho_L^b}{\rho_H^b} \left[1 - \omega_L^{aa} \delta_{m_L^a \rho_L^a} - \omega_L^{ba} \delta_{m_L^b \rho_L^a} \right] + \omega_H^a \frac{\rho_L^a}{\rho_H^a} \left[\omega_L^{ab} \delta_{m_L^a \rho_L^a} + \omega_L^{bb} \delta_{m_L^b \rho_L^a} \right] \\ - \left(h_L^b - e_L^b \right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_H^b \frac{\rho_L^b}{\rho_H^b} \left[1 - \omega_L^{aa} \left(\delta_{m_L^a \rho_L^a} + \delta_{m_L^b \rho_L^a} \right) \right] + \omega_H^a \frac{\rho_L^a}{\rho_H^a} \omega_L^{ab} \left(\delta_{m_L^a \rho_L^a} + \delta_{m_L^b \rho_L^a} \right) \right] \\ - \left(h_L^a - e_L^a \right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_H^b \frac{\rho_L^b}{\rho_H^b} \left[1 - \omega_L^{ba} \left(\delta_{m_L^a \rho_L^a} + \delta_{m_L^b \rho_L^a} \right) \right] + \omega_H^a \frac{\rho_L^a}{\rho_H^a} \omega_L^{bb} \left(\delta_{m_L^a \rho_L^a} + \delta_{m_L^b \rho_L^a} \right) \right] \end{array} \right\}$$

$$\frac{\hat{m}_L^a}{\hat{\lambda}_H} = -\frac{\delta_{\lambda_H \rho_L^b}}{D_{IS}} \left\{ \begin{array}{l} \omega_H^b \frac{\rho_L^b}{\rho_H^b} \omega_{LL}^a + (1-\omega_{HH}^a)\omega_L^{ab} + \delta_{m_L^b \rho_L^a} \left(\omega_L^{aa} - \omega_L^{ba}\right) \left[\omega_H^b \frac{\rho_{HL}^b}{\rho_H^b} + \omega_H^a \frac{\rho_{HL}^a}{\rho_H^a} + (1-\omega_{HH}^a)\right] \\ -\left(h_L^b - e_L^b\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_{LL}^a \omega_H^b \frac{\rho_L^b}{\rho_H^b} + (1-\omega_{HH}^a)\omega_L^{ab}\right] \\ -\left(h_L^a - e_L^a\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_{LL}^b \omega_H^b \frac{\rho_L^b}{\rho_H^b} + (1-\omega_{HH}^a)\omega_L^{bb}\right] \end{array} \right\}$$

and

$$\frac{\hat{m}_L^b}{\hat{\lambda}_H} = -\frac{\delta_{\lambda_H \rho_L^b}}{D_{IS}} \left\{ \begin{array}{l} \omega_H^b \frac{\rho_L^b}{\rho_H^b} \omega_{LL}^b + (1-\omega_{HH}^a)\omega_L^{bb} - \delta_{m_L^a \rho_L^a} \left(\omega_L^{aa} - \omega_L^{ba}\right) \left[\omega_H^b \frac{\rho_{HL}^b}{\rho_H^b} + \omega_H^a \frac{\rho_{HL}^a}{\rho_H^a} + (1-\omega_{HH}^a)\right] \\ -\left(h_L^b - e_L^b\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_{LL}^a \omega_H^b \frac{\rho_L^b}{\rho_H^b} + (1-\omega_{HH}^a)\omega_L^{ab}\right] \\ -\left(h_L^a - e_L^a\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_{LL}^b \omega_H^b \frac{\rho_L^b}{\rho_H^b} + (1-\omega_{HH}^a)\omega_L^{bb}\right] \end{array} \right\}.$$

Change in income An increase in income in country b, is easily obtained as by proportionality:

$$\begin{aligned} \frac{\hat{m}_H^a}{dy^b} &= -\frac{\hat{m}_H^a}{\hat{\lambda}_H} \frac{\delta_{y^b \rho_L^b}}{\delta_{\lambda_H \rho_L^b}} \\ \frac{\hat{m}_L^a}{dy^b} &= -\frac{\hat{m}_L^a}{\hat{\lambda}_H} \frac{\delta_{y^b \rho_L^b}}{\delta_{\lambda_H \rho_L^b}} \\ \frac{\hat{m}_L^b}{dy^b} &= -\frac{\hat{m}_L^b}{\hat{\lambda}_H} \frac{\delta_{y^b \rho_L^b}}{\delta_{\lambda_H \rho_L^b}} \end{aligned}$$

An increase in income in country a, gives rise to comparable relations. Specifically

$$\frac{\hat{m}_H^a}{dy^a} = -\frac{\delta_{y^a \rho_L^a}}{D_{IS}} \left\{ \begin{array}{l} \omega_H^a \frac{\rho_L^a}{\rho_H^a} \left[1 - \omega_L^{ab}\delta_{m_L^a \rho_L^b} - \omega_L^{bb}\delta_{m_L^b \rho_L^b}\right] + \omega_H^b \frac{\rho_L^b}{\rho_H^b} \left[\omega_L^{aa}\delta_{m_L^a \rho_L^b} + \omega_L^{ba}\delta_{m_L^b \rho_L^b}\right] \\ -\left(h_L^b - e_L^b\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_H^a \frac{\rho_L^a}{\rho_H^a} \left(1 - \omega_L^{ab}\left(\delta_{m_L^a \rho_L^a} + \delta_{m_L^b \rho_L^a}\right)\right) + \omega_H^b \frac{\rho_L^b}{\rho_H^b} \omega_L^{aa} \left(\delta_{m_L^a \rho_L^a} + \delta_{m_L^b \rho_L^a}\right)\right] \\ -\left(h_L^a - e_L^a\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_H^a \frac{\rho_L^a}{\rho_H^a} \left(1 - \omega_L^{bb}\left(\delta_{m_L^a \rho_L^a} + \delta_{m_L^b \rho_L^a}\right)\right) + \omega_H^b \frac{\rho_L^b}{\rho_H^b} \omega_L^{ba} \left(\delta_{m_L^a \rho_L^a} + \delta_{m_L^b \rho_L^a}\right)\right] \end{array} \right\}$$

$$\frac{\hat{m}_L^a}{dy^a} = \frac{\delta_{y^a \rho_L^a}}{D_{IS}} \left\{ \begin{array}{l} \omega_H^a \frac{\rho_L^a}{\rho_H^a} \omega_{LL}^a + (1-\omega_{HH}^a)\omega_L^{aa} + \delta_{m_L^b \rho_L^a} \left(\omega_L^{aa} - \omega_L^{ba}\right) \left[\omega_H^b \frac{\rho_{HL}^a}{\rho_H^a} + \omega_H^a \frac{\rho_{HL}^b}{\rho_H^b} + (1-\omega_{HH}^a)\right] \\ -\left(h_L^b - e_L^b\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_{LL}^a \omega_H^a \frac{\rho_{HL}^a}{\rho_H^a} + (1-\omega_{HH}^a)\omega_L^{aa} \frac{\rho_{HL}^a}{\rho_L^a}\right] \\ -\left(h_L^a - e_L^a\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_{LL}^b \omega_H^a \frac{\rho_{HL}^a}{\rho_H^a} + (1-\omega_{HH}^a)\omega_L^{ba} \frac{\rho_{HL}^a}{\rho_L^a}\right] \end{array} \right\}$$

and

$$\frac{\hat{m}_L^b}{dy^a} = \frac{\delta_{y^a \rho_L^a}}{D_{IS}} \left\{ \begin{array}{l} \omega_H^a \frac{\rho_L^a}{\rho_H^a} \omega_{LL}^b + (1-\omega_{HH}^a)\omega_L^{ba} + \delta_{m_L^a \rho_L^b} \left(\omega_L^{aa} - \omega_L^{ba}\right) \left[\omega_H^b \frac{\rho_L^b}{\rho_H^b} \frac{\rho_{HL}^b}{\rho_L^b} + \omega_H^a \frac{\rho_L^a}{\rho_H^a} \frac{\rho_{HL}^a}{\rho_L^a} + (1-\omega_{HH}^a)\right] \\ -\left(h_L^b - e_L^b\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_{LL}^a \omega_H^a \frac{\rho_L^a}{\rho_H^a} + (1-\omega_{HH}^a)\omega_L^{aa}\right] \\ -\left(h_L^a - e_L^a\right) \frac{\mu_{Lk}}{\mu_{Lf}} \left[\omega_{LL}^b \omega_H^a \frac{\rho_L^a}{\rho_H^a} + (1-\omega_{HH}^a)\omega_L^{ba}\right] \end{array} \right\}$$

Change in trade cost for low quality goods An increase in λ_L :

$$\begin{aligned}\frac{\hat{m}_H^a}{\widehat{\lambda}_L} &= \left[\frac{\delta_{\lambda_L\rho_H^a}}{\delta_{y^a\rho_L^a}}\frac{\hat{m}_H^a}{dy^a}+\frac{\delta_{\lambda_L\rho_L^b}}{\delta_{y^b\rho_L^b}}\frac{\hat{m}_H^a}{dy^b}\right]+\frac{1}{D_{IS}}\times\\ &\left\{\begin{array}{l} -\left(h_L^a-e_L^a\right)\varepsilon_\tau\left(\omega_H^a\delta_{m_L^a\rho_H^a}+\omega_H^b\delta_{m_L^a\rho_H^b}\right)\\ -\left(h_L^b-e_L^b\right)\varepsilon_\tau\left(\omega_H^a\delta_{m_L^b\rho_H^a}+\omega_H^b\delta_{m_L^b\rho_H^b}\right)\\ -\left(h_L^a-e_L^a\right)\varepsilon_\tau\left(\left(\frac{\rho_L^b}{\rho_H^b}\omega_H^b+\frac{\rho_L^a}{\rho_H^a}\omega_H^a\right)\omega_L^{bb}-\frac{\rho_L^a}{\rho_H^a}\omega_H^a\right)\left(\delta_{m_L^a\rho_L^a}\delta_{m_L^b\rho_L^b}-\delta_{m_L^a\rho_L^b}\delta_{m_L^b\rho_L^a}\right)\\ -\left(h_L^b-e_L^b\right)\varepsilon_\tau\left(\left(\frac{\rho_L^b}{\rho_H^b}\omega_H^b+\frac{\rho_L^a}{\rho_H^a}\omega_H^a\right)\omega_L^{aa}-\frac{\rho_L^a}{\rho_H^a}\omega_H^a\right)\left(\delta_{m_L^a\rho_L^a}\delta_{m_L^b\rho_L^b}-\delta_{m_L^a\rho_L^b}\delta_{m_L^b\rho_L^a}\right)\\ +\left[\left(h_L^a-e_L^a\right)\left(h_L^b-e_L^b\right)\varepsilon_\tau\frac{\mu_{Lk}}{\mu_{Lf}}\right]\left(\frac{\rho_L^a}{\rho_H^a}\left(\delta_{m_L^b\rho_H^a}+\delta_{m_L^a\rho_H^a}\right)+\frac{\rho_L^b}{\rho_H^b}\left(\delta_{m_L^b\rho_H^b}+\delta_{m_L^a\rho_H^b}\right)\right)\end{array}\right\}\end{aligned}$$

$$\begin{aligned}\frac{\hat{m}_L^a}{\widehat{\lambda}_L} &= \left[\frac{\delta_{\lambda_L\rho_H^a}}{\delta_{y^a\rho_L^a}}\frac{\hat{m}_L^a}{dy^a}+\frac{\delta_{\lambda_L\rho_L^b}}{\delta_{y^b\rho_L^b}}\frac{\hat{m}_L^a}{dy^b}\right]+\frac{1}{D_{IS}}\times\\ &\left\{\begin{array}{l} -\left(h_L^a-e_L^a\right)\varepsilon_\tau\left(\left(1-\omega_{HH}^a\right)\left(1-\omega_L^{ba}\delta_{m_L^b\rho_L^a}-\omega_L^{bb}\delta_{m_L^b\rho_L^b}\right)+\omega_{LL}^b\left[\omega_H^a\delta_{m_L^b\rho_H^a}+\omega_H^b\delta_{m_L^b\rho_H^b}\right]\right)\\ -\left(h_L^b-e_L^b\right)\varepsilon_\tau\left(\left(1-\omega_{HH}^a\right)\left(\omega_L^{aa}\delta_{m_L^b\rho_L^a}+\omega_L^{ab}\delta_{m_L^b\rho_L^b}\right)-\omega_{LL}^a\left[\omega_H^a\delta_{m_L^b\rho_H^a}+\omega_H^b\delta_{m_L^b\rho_H^b}\right]\right)\\ +2\left[\left(h_L^a-e_L^a\right)\left(h_L^b-e_L^b\right)\varepsilon_\tau\right]\frac{\mu_{Lk}}{\mu_{Lf}}\left(1-\omega_{HH}^a\right)\end{array}\right\}\end{aligned}$$

$$\begin{aligned}\frac{\hat{m}_L^b}{\widehat{\lambda}_L} &= \left[\frac{\delta_{\lambda_L\rho_H^a}}{\delta_{y^a\rho_L^a}}\frac{\hat{m}_L^b}{dy^a}+\frac{\delta_{\lambda_L\rho_L^b}}{\delta_{y^b\rho_L^b}}\frac{\hat{m}_L^b}{dy^b}\right]+\frac{1}{D_{IS}}\times\\ &\left\{\begin{array}{l} -\left(h_L^a-e_L^a\right)\varepsilon_\tau\left(\left(1-\omega_{HH}^a\right)\left(\omega_L^{ba}\delta_{m_L^a\rho_L^a}+\omega_L^{bb}\delta_{m_L^a\rho_L^b}\right)-\omega_{LL}^b\left[\omega_H^a\delta_{m_L^a\rho_H^a}+\omega_H^b\delta_{m_L^a\rho_H^b}\right]\right)\\ -\left(h_L^b-e_L^b\right)\varepsilon_\tau\left(\left(1-\omega_{HH}^a\right)\left(1-\omega_L^{aa}\delta_{m_L^a\rho_L^a}-\omega_L^{ab}\delta_{m_L^a\rho_L^b}\right)+\omega_{LL}^a\left[\omega_H^a\delta_{m_L^a\rho_H^a}+\omega_H^b\delta_{m_L^a\rho_H^b}\right]\right)\\ +\left[2\left(h_L^a-e_L^a\right)\left(h_L^b-e_L^b\right)\varepsilon_\tau\frac{\mu_{Lk}}{\mu_{Lf}}\right]\left(1-\omega_{HH}^a\right)\end{array}\right\}\end{aligned}$$

B.2 Complete specialization

B.2.1 Decision process of H firms in a and L firms in b

Given complete specialization we can make use of the analysis of the decision process of H firms in a in the incomplete specialization setting for both H firms in a and L firms in b. The conditional probability for low quality goods in country a and b when countries are completely specialized simplifies to

$$\rho_L^a = E^a\left(\frac{(\lambda_L n_L)^{\mu_{Lf}}\,(m_L)^{\mu_{Lk}}\,\phi_L(y,n_L)}{[(\lambda_H n_H)^{\mu_{Hf}}(m_H)^{\mu_{Hk}}\phi_H(y,n_H)]+[(\lambda_L n_L)^{\mu_{Lf}}(m_L)^{\mu_{Lk}}\phi_L(y,n_L)]}\right) \quad \text{(B.26)}$$

$$\rho_L^b = E^b\left(\frac{[(\lambda_L n_L)^{\mu_{Lf}}(m_L)^{\mu_{Lk}}\phi_L(y,n_L)]}{[(\lambda_H n_H)^{\mu_{Hf}}(m_H)^{\mu_{Hk}}\phi_H(y,n_H)]+[(\lambda_L n_L)^{\mu_{Lf}}(m_L)^{\mu_{Lk}}\phi_L(y,n_L)]}\right), \quad \text{(B.27)}$$

where $\lambda_i = \exp[-\tau_i q_i/\mu_{if}]$ (contrast with (42) and (43)).

B.2.2 Short-run analysis

We determine the optimal range of products per firm by total differentiation of the first order conditions, which are

$$m_L F_L - \frac{\mu_{Lk}}{q_L}\left[\frac{(n_L-1)}{(n_L^2-n_L+1)}\right]\left[\begin{array}{c} N^a E^a \left(\frac{(\lambda_L n_L)^{\mu_{Lf}} m_L^{\mu_{Lk}}\phi_L(y)}{\left[n_H^{\mu_{Hf}} m_H^{\mu_{Hk}}\phi_H(y)\right]+\left[(\lambda_L n_L)^{\mu_{Lf}} m_L^{\mu_{Lk}}\phi_L(y)\right]}\right) \\ +N^b E^b \left(\frac{\left[n_L^{\mu_{Lf}} m_L^{\mu_{Lk}}\phi_L(y)\right]}{\left[(\lambda_H n_H)^{\mu_{Hf}} m_H^{\mu_{Hk}}\phi_H(y)\right]+\left[n_L^{\mu_{Lf}} m_L^{\mu_{Lk}}\phi_L(y)\right]}\right)\end{array}\right] = 0 \tag{B.28}$$

$$m_H F_H - \frac{\mu_{Hk}}{q_H}\left[\frac{(n_H-1)}{(n_H^2-n_H+1)}\right]\left[\begin{array}{c} N^a E^a \left(\frac{\left[n_H^{\mu_{Hf}} m_H^{\mu_{Hk}}\phi_H(y)\right]}{\left[n_H^{\mu_{Hf}} m_H^{\mu_{Hk}}\phi_H(y)\right]+\left[(\lambda_L n_L)^{\mu_{Lf}} m_L^{\mu_{Lk}}\phi_L(y)\right]}\right) \\ +N^b E^b \left(\frac{\left[(\lambda_H n_H)^{\mu_{Hf}} m_H^{\mu_{Hk}}\phi_H(y)\right]}{\left[(\lambda_H n_H)^{\mu_{Hf}} m_H^{\mu_{Hk}}\phi_H(y)\right]+\left[n_L^{\mu_{Lf}} m_L^{\mu_{Lk}}\phi_L(y)\right]}\right)\end{array}\right] = 0. \tag{B.29}$$

We make use of

$$\begin{aligned}
\widehat{\rho}_L^a &= \mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)\frac{\rho_{HL}^a}{\rho_L^a}\widehat{n}_L - \mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)\frac{\rho_{HL}^a}{\rho_L^a}\widehat{n}_H + \mu_{Lk}\frac{\rho_{HL}^a}{\rho_L^a}\widehat{m}_L - \mu_{Hk}\frac{\rho_{HL}^a}{\rho_L^a}\widehat{m}_H \\
&\quad +\mu_{Lf}\frac{\rho_{HL}^a}{\rho_L^a}\widehat{\lambda}_L + \frac{\rho_{HL}^a}{\rho_L^a}\left[q_L-q_H\right]dy^a \\
\widehat{\rho}_L^b &= \mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)\frac{\rho_{HL}^b}{\rho_L^b}\widehat{n}_L - \mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)\frac{\rho_{HL}^b}{\rho_L^b}\widehat{n}_H + \mu_{Lk}\frac{\rho_{HL}^b}{\rho_L^b}\widehat{m}_L - \mu_{Hk}\frac{\rho_{HL}^b}{\rho_L^b}\widehat{m}_H \\
&\quad -\mu_{Hf}\frac{\rho_{HL}^b}{\rho_L^b}\widehat{\lambda}_H + \frac{\rho_{HL}^b}{\rho_L^b}\left[q_L-q_H\right]dy^b,
\end{aligned}$$

where as before $\hat{\lambda}_L = -\frac{\tau_L q_L}{\mu_{Lf}}\hat{\tau}_L$ and $\hat{\lambda}_H = -\frac{\tau_H q_H}{\mu_{Hf}}\hat{\tau}_H$. Recognizing that $\widehat{\rho}_H^a = -\frac{\rho_L^a}{\rho_H^a}\widehat{\rho}_L^a$ and $\widehat{\rho}_H^b = -\frac{\rho_L^b}{\rho_H^b}\widehat{\rho}_L^b$, the total differential of the first order conditions for the range of products per firm for $i = H, L$, can then be written in matrix notation as

$$\begin{bmatrix}\widehat{m}_L \\ \widehat{m}_H\end{bmatrix} = \frac{1}{D_S}\begin{bmatrix} c_{H2} & -c_{L2} \\ -c_{H1} & c_{L1}\end{bmatrix} \times$$

$$\begin{bmatrix} c_{L3} & c_{L4} & c_{L5} & -c_{L6} & c_{L7} & -c_{L8} & c_{L9} & c_{L10} \\ c_{H3} & c_{H4} & -c_{H5} & c_{H6} & -c_{H7} & c_{H8} & -c_{H9} & -c_{H10}\end{bmatrix}\begin{bmatrix}\widehat{N}^a \\ \widehat{N}^b \\ \widehat{n}_L \\ \widehat{n}_H \\ \widehat{\lambda}_L \\ \widehat{\lambda}_H \\ \left[q_L-q_H\right]dy^a \\ \left[q_L-q_H\right]dy^b\end{bmatrix}$$

with

$$
\begin{aligned}
c_{L1} &= (1-\omega^a_{LL}) & c_{H1} &= \tfrac{\mu_{Lk}}{\mu_{Hk}}\omega^a_{HH} \\
c_{L2} &= \tfrac{\mu_{Hk}}{\mu_{Lk}}\omega^a_{LL} & c_{H2} &= (1-\omega^a_{HH}) \\
c_{L3} &= \omega^a_L & c_{H3} &= \omega^a_H \\
c_{L4} &= 1-\omega^a_L & c_{H4} &= 1-\omega^a_H \\
c_{L5} &= \left[\omega^a_{LL}\tfrac{\mu_{Lf}}{\mu_{Lk}}(1+\tfrac{n_L}{(n_L-1)^2}) - \tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right] & c_{H5} &= \omega^a_{HH}\tfrac{\mu_{Lf}}{\mu_{Hk}}(1+\tfrac{n_L}{(n_L-1)^2}) \\
c_{L6} &= \omega^a_{LL}\tfrac{\mu_{Hf}}{\mu_{Lk}}\left(1+\tfrac{n_H}{(n_H-1)^2}\right) & c_{H6} &= \left[\omega^a_{HH}\tfrac{\mu_{Hf}}{\mu_{Hk}}(1+\tfrac{n_H}{(n_H-1)^2}) - \tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right] \\
c_{L7} &= \mu_{Lf}\omega^a_L\tfrac{\rho^a_{HL}}{\rho^a_L} & c_{H7} &= \mu_{Lf}\omega^a_H\tfrac{\rho^a_{HL}}{\rho^a_H} \\
c_{L8} &= \mu_{Hf}(1-\omega^a_L)\tfrac{\rho^b_{HL}}{\rho^b_L} & c_{H8} &= \mu_{Hf}(1-\omega^a_H)\tfrac{\rho^b_{HL}}{\rho^b_H} \\
c_{L9} &= \omega^a_L\tfrac{\rho^a_{HL}}{\rho^a_L} & c_{H9} &= \omega^a_H\tfrac{\rho^a_{HL}}{\rho^a_H} \\
c_{L10} &= (1-\omega^a_L)\tfrac{\rho^b_{HL}}{\rho^b_L} & c_{H10} &= (1-\omega^a_H)\tfrac{\rho^b_{HL}}{\rho^b_H},
\end{aligned}
$$

where, as in the incomplete specialization setting,

$$\omega^a_i \equiv \frac{N^a\rho^a_i}{N^a\rho^a_i + N^b\rho^b_i} \equiv 1-\omega^b_i;$$

and

$$
\begin{aligned}
0 &< \omega^a_{HH} = \omega^a_H\left(\mu_{Hk}\tfrac{\rho^a_{HL}}{\rho^a_H}\right) + (1-\omega^a_H)\left(\mu_{Hk}\tfrac{\rho^b_{HL}}{\rho^b_H}\right) \equiv \left(\omega^a_H\delta_{m^a_H\rho^a_H} + \omega^b_H\delta_{m^a_H\rho^b_H}\right) < 1, \\
0 &< \omega^a_{LL} = \omega^a_L\left(\mu_{Lk}\tfrac{\rho^a_{HL}}{\rho^a_L}\right) + (1-\omega^a_L)\left(\mu_{Lk}\tfrac{\rho^b_{HL}}{\rho^b_L}\right) < 1.
\end{aligned}
$$

The determinant D_S, is positive. Specifically, D_S is given by

$$
\begin{aligned}
D_S &= [1-\omega^a_{LL}]\,[1-\omega^a_{HH}] - \omega^a_{LL}\omega^a_{HH} \\
&= 1-\omega^a_{LL}-\omega^a_{HH}
\end{aligned}
$$

which exceeds $1-\left[\omega^a_L\frac{\rho^a_{HL}}{\rho^a_L} + (1-\omega^a_L)\frac{\rho^b_{HL}}{\rho^b_L}\right] - \left[\omega^a_H\frac{\rho^a_{HL}}{\rho^a_H} + (1-\omega^a_H)\frac{\rho^b_{HL}}{\rho^b_H}\right]$ since μ_{Lf} and μ_{Hf} $\in (0,1)$. The latter can be shown to be positive, as using their definitions, it is equivalent to showing

$$
\begin{aligned}
&\left[N^a\rho^a_L + N^b\rho^b_L - (N^a\rho^a_{HL} + N^b\rho^b_{HL})\right]\left[N^a\rho^a_H + N^b\rho^b_H - (N^a\rho^a_{HL} + N^b\rho^b_{HL})\right] \\
&- \left[N^a\rho^a_{HL} + N^b\rho^b_{HL}\right]\left[N^a\rho^a_{HL} + N^b\rho^b_{HL}\right] > 0.
\end{aligned}
$$

This difference can be decomposed in four parts, where

$$
\begin{aligned}
&N^aN^a\left[\rho^a_H\rho^a_L - \rho^a_H\rho^a_{HL} - \rho^a_L\rho^a_{HL}\right] = N^aN^a\left[\rho^a_H\rho^a_L - \rho^a_{HL}\right] > 0 \\
&N^bN^b\left[\rho^b_L\rho^b_H - \rho^b_H\rho^b_{HL} - \rho^b_L\rho^b_{HL}\right] = N^bN^b\left[\rho^b_L\rho^b_H - \rho^b_{HL}\right] > 0 \\
&N^bN^a\left[\rho^b_H\rho^a_L - \rho^b_H\rho^a_{HL} - \rho^a_L\rho^b_{HL}\right] \\
&N^bN^a\left[\rho^a_H\rho^b_L - \rho^a_H\rho^b_{HL} - \rho^b_L\rho^a_{HL}\right]
\end{aligned}
$$

The sum of the latter two terms is positive as well, as rearranging yields

$$
\begin{aligned}
& \left(\rho_H^b \rho_L^a - \rho_H^b \rho_{HL}^a - \rho_L^a \rho_{HL}^b\right) + \left(\rho_H^a \rho_L^b - \rho_H^a \rho_{HL}^b - \rho_L^b \rho_{HL}^a\right) \\
= & \left(\rho_H^b \rho_L^a - \rho_{HL}^b \rho_L^a - \rho_{HL}^b \rho_H^a\right) + \left(\rho_H^a \rho_L^b - \rho_{HL}^a \rho_L^b - \rho_{HL}^a \rho_H^b\right) \\
= & \left(\rho_H^b \rho_L^a - \rho_{HL}^b\right) + \left(\rho_H^a \rho_L^b - \rho_{HL}^a\right) > \rho_H^b \left(\rho_L^a - \rho_L^b\right) + \rho_H^a \left(\rho_L^b - \rho_L^a\right) \\
= & \left(\rho_H^a - \rho_H^b\right)\left(\rho_L^b - \rho_L^a\right) = \left(\rho_L^b - \rho_L^a\right)^2 > 0.
\end{aligned}
$$

It follows that $D_S > 0$.

Change in number of brands: The effect of number of firms on the product range is given by

$$
\begin{aligned}
\frac{\widehat{m}_L}{\widehat{n}_L} &= \frac{1}{\mu_{Lk} D_S}\left[(1 + \tfrac{n_L}{(n_L-1)^2})\mu_{Lf}\omega_{LL}^a - \tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\mu_{Lk}(1-\omega_{HH}^a)\right] \\
\frac{\widehat{m}_H}{\widehat{n}_L} &= -\frac{\omega_{HH}^a}{\mu_{Hk} D_S}\left[(1 + \tfrac{n_L}{(n_L-1)^2})\mu_{Lf} - \tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\mu_{Lk}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\widehat{m}_L}{\widehat{n}_H} &= -\frac{\omega_{LL}^a}{\mu_{Lk} D_S}\left[(1 + \tfrac{n_H}{(n_H-1)^2})\mu_{Hf} - \tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\mu_{Hk}\right] \\
\frac{\widehat{m}_H}{\widehat{n}_H} &= \frac{1}{\mu_{Hk} D_S}\left[(1 + \tfrac{n_H}{(n_H-1)^2})\mu_{Hf}\omega_{HH}^a - \tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\mu_{Hk}\left(1-\omega_{LL}^a\right)\right].
\end{aligned}
$$

Reduction in transportation costs: The effect of lower trade cost (associated with an increase in λ) on product range is given by:

$$
\begin{aligned}
\frac{\widehat{m}_H}{\widehat{\lambda}_L} &= -\frac{\mu_{Lf}}{D_S}\omega_H^a \frac{\rho_{HL}^a}{\rho_H^a} < 0 \\
\frac{\widehat{m}_H}{\widehat{\lambda}_H} &= \frac{\mu_{Hf}}{D_S}\left(1-\omega_H^a\right)\frac{\rho_{HL}^b}{\rho_H^b} > 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\widehat{m}_L}{\widehat{\lambda}_L} &= \frac{\mu_{Lf}}{D_S}\omega_L^a \frac{\rho_{HL}^a}{\rho_L^a} > 0 \\
\frac{\widehat{m}_L}{\widehat{\lambda}_H} &= -\frac{\mu_{Hf}}{D_S}\left(1-\omega_L^a\right)\frac{\rho_{HL}^b}{\rho_L^b} < 0.
\end{aligned}
$$

B.2.3 Long-run analysis

Analogous to the long-run autarky comparative statics, we express $\widehat{\rho}_L^a$ and $\widehat{\rho}_L^b$ in terms of the relative changes in n_i and transportation costs:

$$\begin{aligned}
\widehat{\rho}_L^a &= \frac{\rho_{HL}^a}{\rho_L^a}\frac{1}{D_S}\left[\mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)-\mu_{Lk}\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right]\widehat{n}_L \\
&\quad -\frac{\rho_{HL}^a}{\rho_L^a}\frac{1}{D_S}\left[\mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)-\mu_{Hk}\tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right]\widehat{n}_H \\
&\quad +\mu_{Lf}\frac{\rho_{HL}^a}{\rho_L^a}\frac{1}{D_S}\left[D_S+\mu_{Lk}\omega_L^a\tfrac{\rho_{HL}^a}{\rho_L^a}+\mu_{Hk}\omega_H^a\tfrac{\rho_{HL}^a}{\rho_H^a}\right]\widehat{\lambda}_L \\
&\quad -\mu_{Hf}\frac{\rho_{HL}^a}{\rho_L^a}\frac{1}{D_S}\left[\mu_{Lk}(1-\omega_L^a)\tfrac{\rho_{HL}^b}{\rho_L^b}+\mu_{Hk}(1-\omega_H^a)\tfrac{\rho_{HL}^b}{\rho_H^b}\right]\widehat{\lambda}_H
\end{aligned}\tag{B.30}$$

$$\begin{aligned}
\widehat{\rho}_L^b &= \frac{\rho_{HL}^b}{\rho_L^b}\frac{1}{D_S}\left[\mu_{Lf}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)-\mu_{Lk}\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right]\widehat{n}_L \\
&\quad -\frac{\rho_{HL}^b}{\rho_L^b}\frac{1}{D_S}\left[\mu_{Hf}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)-\mu_{Hk}\tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right]\widehat{n}_H \\
&\quad +\mu_{Lf}\frac{\rho_{HL}^b}{\rho_L^b}\frac{1}{D_S}\left[\mu_{Lk}\omega_L^a\tfrac{\rho_{HL}^a}{\rho_L^a}+\mu_{Hk}\omega_H^a\tfrac{\rho_{HL}^a}{\rho_H^a}\right]\widehat{\lambda}_L \\
&\quad -\mu_{Hf}\frac{\rho_{HL}^b}{\rho_L^b}\frac{1}{D_S}\left[D_S+\mu_{Lk}\left(1-\omega_L^a\right)\tfrac{\rho_{HL}^b}{\rho_L^b}+\mu_{Hk}\left(1-\omega_H^a\right)\tfrac{\rho_{HL}^b}{\rho_H^b}\right]\widehat{\lambda}_H
\end{aligned}\tag{B.31}$$

Making use of (B.30) and (B.31), we obtain the total differential of

$$q_iK_i\left[\frac{(n_i-1)\left(n_i^2-n_i+1\right)}{\left(\mu_{if}-\mu_{ik}\right)(n_i-1)^2+\mu_{if}n_i}\right]=N^aE^a[\rho_i^a(y,n_i)]+N^bE^b[\rho_i^b(y,n_i)]$$

for $i=H,L$. In matrix form we get

$$\begin{bmatrix}\widehat{n}_L\\ \widehat{n}_H\end{bmatrix}=\frac{1}{\widetilde{D}_S}\begin{bmatrix}d_{2nH} & -d_{1nH}\\ -d_{2nL} & d_{1nL}\end{bmatrix}\begin{bmatrix}d_{1\lambda_L} & -d_{1\lambda_H}\\ -d_{2\lambda_L} & d_{2\lambda_H}\end{bmatrix}\begin{bmatrix}\widehat{\lambda}_L\\ \widehat{\lambda}_H\end{bmatrix}$$

where

$$\begin{aligned}
d_{1nL} &= \varphi_LD_S-\omega_{LL}^a\left[\tfrac{\mu_{Lf}}{\mu_{Lk}}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)-\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right]=\varphi_LD_S-\frac{\omega_{LL}^a}{\omega_{HH}^a}d_{2nL}\\
d_{1nH} &= \omega_{LL}^a\left[\tfrac{\mu_{Hf}}{\mu_{Hk}}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)-\tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right]\\
d_{2nL} &= \omega_{HH}^a\left[\tfrac{\mu_{Lf}}{\mu_{Lk}}\left(1+\tfrac{n_L}{(n_L-1)^2}\right)-\tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right]\\
d_{2nH} &= \varphi_HD_S-\omega_{HH}^a\left[\tfrac{\mu_{Hf}}{\mu_{Hk}}\left(1+\tfrac{n_H}{(n_H-1)^2}\right)-\tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right]=\varphi_HD_S-\frac{\omega_{HH}^a}{\omega_{LL}^a}d_{1nH}\\
\widetilde{D}_S &= \left(\varphi_LD_S-\frac{\omega_{LL}^a}{\omega_{HH}^a}d_{2nL}\right)\left(\varphi_HD_S-\frac{\omega_{HH}^a}{\omega_{LL}^a}d_{1nH}\right)-d_{2nL}d_{1nH}
\end{aligned}$$

and

$$
\begin{aligned}
d_{1\lambda_L} &= \mu_{Lf}\omega_L^a \frac{\rho_{HL}^a}{\rho_L^a} \\
d_{1\lambda_H} &= \mu_{Hf}(1-\omega_L^a)\frac{\rho_{HL}^b}{\rho_L^b} \\
d_{2\lambda_L} &= \mu_{Lf}\omega_H^a \frac{\rho_{HL}^a}{\rho_H^a} \\
d_{2\lambda_H} &= \mu_{Hf}(1-\omega_H^a)\frac{\rho_{HL}^b}{\rho_H^b}.
\end{aligned}
$$

Analogous to the previous stability discussion,

$$
\widetilde{D}_S = D_S(\varphi_L\varphi_H D_S - \frac{\omega_{LL}^a}{\omega_{HH}^a} d_{2nL}\varphi_H - \frac{\omega_{HH}^a}{\omega_{LL}^a} d_{1nH}\varphi_L) > 0.
$$

Change in transportation cost The effect of lower transportation costs on the number of firms is given by

$$
\begin{aligned}
\frac{\widehat{n}_L}{\widehat{\lambda}_L} &= \frac{1}{\widetilde{D}_S}\varphi_H D_S \mu_{Lf}\omega_L^a \frac{\rho_{HL}^a}{\rho_L^a} \\
\frac{\widehat{n}_H}{\widehat{\lambda}_L} &= -\frac{1}{\widetilde{D}_S}\varphi_L D_S \mu_{Lf}\omega_H^a \frac{\rho_{HL}^a}{\rho_L^a}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\widehat{n}_L}{\widehat{\lambda}_H} &= -\frac{1}{\widetilde{D}_S}\varphi_H D_S d_{1\lambda_H} \\
\frac{\widehat{n}_H}{\widehat{\lambda}_H} &= \frac{1}{\widetilde{D}_S}\varphi_L D_S d_{2\lambda_H}.
\end{aligned}
$$

The effect of lower trade costs on the product range yields:

$$
\begin{aligned}
\frac{\widehat{m}_L}{\widehat{\lambda}_L} &= \frac{\mu_{Lf}}{\widetilde{D}_S}\frac{\rho_{HL}^a}{\rho_L^a}\omega_L^a D_S\varphi_H \left(\varphi_L - \tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right) \\
\frac{\widehat{m}_H}{\widehat{\lambda}_L} &= -\frac{\mu_{Lf}}{\widetilde{D}_S}\frac{\rho_{HL}^a}{\rho_H^a}\omega_H^a D_S\varphi_L \left(\varphi_H - \tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right) \\
\frac{\widehat{m}_L}{\widehat{\lambda}_H} &= -\frac{\mu_{Hf}}{\widetilde{D}_S}\frac{\rho_{HL}^b}{\rho_L^b}(1-\omega_L^a) D_S\varphi_H \left(\varphi_L - \tfrac{n_L^2(n_L-2)}{(n_L^2-n_L+1)(n_L-1)}\right) \\
\frac{\widehat{m}_H}{\widehat{\lambda}_H} &= \frac{\mu_{Hf}}{\widetilde{D}_S}\frac{\rho_{HL}^b}{\rho_H^b}(1-\omega_H^a) D_S\varphi_L \left(\varphi_H - \tfrac{n_H^2(n_H-2)}{(n_H^2-n_H+1)(n_H-1)}\right).
\end{aligned}
$$