http://www.ijmp.jor.br
v. 8, n. 2, April - June 2017
ISSN: 2236-269X
DOI: 10.14807/ijmp.v8i2.535

A FUZZY INVENTORY MODEL (EOQ MODEL) WITH UNIT PRODUCTION COST,TIME DEPENDED HOLDING COST,WITH-OUT SHORTAGES UNDER A SPACE CONSTRAINT: A FUZZY PARAMETRIC GEOMETRIC PROGRAMMING (FPGP) APPROACH

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Submission: 02/09/2016
Accept: 13/10/2016

ABSTRACT

In this paper, an Inventory model with unit production cost, time depended holding cost, with-out shortages is formulated and solved. We have considered here a single objective inventory model. In most real world situation, the objective and constraint function of the decision makers are imprecise in nature, hence the coefficients, indices, the objective function and constraint goals are imposed here in fuzzy environment. Geometric programming provides a powerful tool for solving a variety of imprecise optimization problem. Here we have used nearest interval approximation method to convert a triangular fuzzy number to an interval number then transform this interval number to a parametric interval-valued functional form and solve the parametric problem by geometric programming technique. Here two necessary theorems have been derived. Numerical example is given to illustrate the model through this Fuzzy Parametric Geometric-Programming (FPGP) method.

Keywords: Inventory model, Fuzzy number, Space constraint, Geometric Programming, Interval-valued function

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1. INTRODUCTION

An inventory deals with decision that minimize the total average cost or maximize the total average profit. For this purpose the task is to construct a mathematical model of the real life Inventory system, such a mathematical model is based on various assumptions and approximations. In ordinary inventory model it consider all parameter like set-up cost, holding cost, interest cost a fixed. But in real life situation it will have some little fluctuations. So consideration of fuzzy variables is more realistic.

Geometric Programming (GP) method is an effective method used to solve a non-linear programming problem like structural problem. It has certain advantages over the other optimization methods. Here, the advantage is that it is usually much simpler to work with the dual than the primal one. Solving a non-linear programming problem by GP method with degree of difficulty (DD) plays essential role. (It is defined as DD = total number of terms in objective function and constraints – total number of decision variables – 1). Since late 1960's, Geometric Programming (GP) used in various field (like OR, Engineering science etc.). Geometric Programming (GP) is one of the effective methods to solve a particular type of Non linear programming problem. The theory of Geometric Programming (GP) first emerged in 1961 by Duffin and Zener. The first publication on GP was published by Duffin and Zener on (1967). There are many references on applications and methods of GP in the survey paper by Ecker. They describe GP with positive or zero degree of difficulty. But there may be some problems on GP with negative degree of difficulty. Sinha et al. proposed it theoretically. Abot-El-Ata and his group applied modified form of GP in inventory modelsPark and Wang studied shortages and partial backlogging of items. Friedman (1978) presented continuous time inventory model with time varying demand. Ritchie (1984) studied in inventory model with linear increasing demand. Goswami, Chaudhuri (1991) discussed an inventory model with shortage. Gen et. Al. (1997) considered classical inventory model with Triangular fuzzy number. Yao and Lee (1998) considered an economic production quantity model in the fuzzy sense. Kumar, Kundu and Goswami (2003) presented an economic production quantity inventory model involving fuzzy demand rate. Syde and Aziz (2007) applied sign distance method to fuzzy inventory model without shortage . D.Datta and Pravin Kumar published several paper of fuzzy inventory

http://www.ijmp.jor.br v. 8, n. 2, April - June 2017
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with or without shortage. Islam and Roy (2006) presented a fuzzy EPQ model with flexibility and reliability consideration and demand depended unit Production cost under a space constraint .A solution method of posynomial geometric programming with interval exponents and coefficients was developed by Liu (2008). Kotb, Fergancy (2011), presented Multi-item EOQ model with both demand-depended unit cost and varying Lead time via Geometric Programming. Dey and Roy (2015) presented Optimum shape design of structural model with imprecise coefficient by parametric geometric programming.

In this paper we first considered crisp inventory model. There after it transformed to fuzzy inventory mode and developed. Here two necessary theorems have been derived. At last it made an example and solved it by Parametric Geometric-Programming Technique.

2. MATHEMATICAL MODEL

An Inventory model is developed under the following notations and assumptions.

2.1. Notations

$I(t)$: Inventory level at any time, $t \geq 0$.

D: Demand per unit time, which is constant.

T: Cycle of length.

S: Set-up cost per unit time.

H: Holding cost per unit item, which is time depended.

P: Unit demand and set-up cost dependent production cost.

q: Production quantity per batch.

$f(D,S)$: Unit production cost per cycle.

$TAC(D,S,q)$: Total average cost per unit time.

w_0: Space area per unit quantity.

W: Total storage space area.

2.2. Assumptions

a) The inventory system involves only one item.

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b) The replenishment occur instantaneously at infinite rate.

c) The lead time is negligible.

d) Demand rate is constant.

e) The unit production cost is continuous function of demand and Set-up cost and take the following form:

P = $\theta D^{-x} S^{-1}$, $\theta, x \in \mathbb{R}$ (>0).

f) Holding cost is time depended, as "at".

2.3. Crisp model

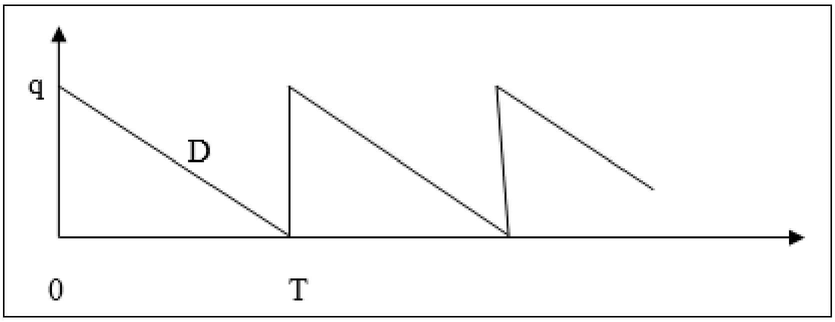

Figure1: Inventory Model

The differential equation describing I(t) as follows

$$\frac{dI(t)}{dt} = -D \quad , \qquad 0 \le t \le T \qquad (2.3.1)$$

With the boundary condition I(0) = q, I(T) = 0.

The solution of (2.3.1) is obtained as

$$I(t) = q - Dt \qquad (2.3.2)$$

Also there are T= q/D.

Here inventory holding cost = $H\int_0^T at.I(t)dt = \frac{aHq^3}{6D^2}$. (2.3.3)

Total inventory related cost per cycle = set-up cost + holding cost + production cost

$$= S + \frac{aHq^3}{6D^2} + Pq \qquad (2.3.4)$$

So total average cost per cycle is given by

$$TAC(D,S,q) = \frac{1}{T}\left(S + \frac{aHq^3}{6D^2} + Pq\right)$$

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$$= \frac{SD}{q} + \frac{aHq^2}{6D} + \theta D^{1-x} S^{-1}. \quad (2.3.5)$$

And storage area = w_0q.

So the inventory model can be written as

$$\text{Min TAC}(D,S,q) = \frac{SD}{q} + \frac{aHq^2}{6D} + \theta D^{1-x} S^{-1} \quad (2.3.6)$$

subject to $w_0q \le W$, $D,S,q > 0$.

2.4. Fuzzy model

When the objective and constraint goals, coefficients and exponents become fuzzy sets and fuzzy Numbers respectively, the crisp model (2.3.6) written to be a fuzzy model, as

$$\widetilde{Min} \quad \text{TAC}(D,S,q) = \frac{SD}{q} + \frac{\tilde{a}\tilde{H}q^2}{6D} + \tilde{\theta} D^{1-x} S^{-1}$$

subject to $w_0 q \lesssim \widetilde{W}$, $D,S,q > 0$. (2.4.1)

3. GEOMETRIC PROGRAMMING (GP) PROBLEM

3.1. Primal program

Primal Geometric Programming (PGP) problem is:

$$\text{Minimize } g_0(t) = \sum_{k=1}^{T_0} C_{0k} \prod_{j=1}^{m} t_j^{\alpha_{0kj}} \quad (3.1)$$

$$\text{subject to } g_i(t) = \sum_{k=1+T_{r-1}}^{T_0} C_{rk} \prod_{j=1}^{m} t_j^{\alpha_{rkj}} \le 1 \quad (r=1,2,\ldots,l)$$

$t_j > 0$, $(j=1,2,\ldots,m)$.

Where $C_{0k}(>0)$ $(k=1,2,\ldots,T_0)$, $C_{rk}(>0)$ and α_{rkj} $(k=1,2,\ldots,1+T_{r-1},\ldots,T_r; r = 0,1,2,\ldots,l;$ $j = 1,2,\ldots,m)$ are real numbers. It is constrained polynomial PGP problem. The number of term each polynomial constrained functions varies and it is denoted by T_r for each $r = 0,1,2,\ldots,l$. Let $T = T_0 + T_1 + T_2 + \ldots T_l$ be the total number of terms in the primal program. The Degree of Difficulty is (DD) = T – (m+1).

3.2. Dual program:

Dual programming (DP) problem of (3.1) is:

$$\text{Maximize } d(\delta) = \prod_{r=0}^{l} \prod_{k=1}^{T_r} \left(\frac{C_{rk}}{\delta_{rk}}\right)^{\delta_{rk}} \left(\sum_{s=1+T_{r-1}}^{T_r} \delta_{rs}\right)^{\delta_{rk}} \quad (3.2)$$

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Subject to $\sum_{k=1}^{T_0} \delta_{0k} = 1$ (Normality condition)

$\sum_{r=0}^{l} \sum_{k=1}^{T_r} \alpha_{rkj} \delta_{rk} = 0$, (Orthogonal conditions)

$\delta_{rk} > 0$, $(k=1,2,\ldots\ldots,T_r)$ (Positivity constant)

- Case-1 : For $T_0 \geq M+1$, the dual program presents a system of linear equations for the dual variables, where the number of linear equations is either less than or equal to dual variables. More or unique solutions exist for the dual vectors.
- Case-2 : For $T_0 < M+1$, the dual program presents a system of linear equations for the dual variables, where the number of linear equations is greater than the number of dual variables. In this case generally no solution vector exists for the dual variables. However one can get an approximate solution vector for the system using either the Latest Square(SQ) or Max-Min(MN) method.

These are applied to solve such a system of linear equations. Ones optimal dual variable vector δ^* are known, the corresponding values of the primal variable vector x is found from the following relations:

$$c_{0k} \prod_{j=1}^{m} x_j^{\alpha_{kj}} = \delta_{0k}^{*} v^{*}(\delta^{*}) \quad \text{if } i = 0,$$

and $c_{ik} \prod_{j=1}^{m} x_j^{\alpha_{kj}} = \frac{\delta_{rk}^{*}}{\sum \delta_{rk}^{*}}$ if $i \geq 1$, $(k=1,2,\ldots\ldots,T_0)$.

3.3. Solution procedure of crisp model by Geometric Programming (G.P) technique:

Here the primal problem is

$$\text{Min} \quad TAC(D,S,q) = \frac{SD}{q} + \frac{\tilde{a}\tilde{H}q^2}{6D} + \theta D^{1-x} S^{-1} \tag{3.1.1}$$

subject to $w_0 q \leq W$, $D, S, q > 0$.

Corresponding dual form of (3.1.1) is given by

$$\text{Max } d(\delta) = \left(\frac{1}{\delta_1}\right)^{\delta_1} \left(\frac{aH)}{6\delta_2}\right)^{\delta_2} \left(\frac{\theta}{\delta_3}\right)^{\delta_3} \left(\frac{w_0}{W\delta_{01}}\right)^{\delta_{01}} \delta_{01}^{\delta_{01}}$$

subject to $\delta_1 + \delta_2 + \delta_3 = 1$ (3.1.2)

$\delta_1 - \delta_3 = 0$

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$$\delta_1 - \delta + (1\text{-}x)\delta_3 = 0$$

$$-\delta_1 + 2\delta_2 + \delta_{01} = 0$$

$$\delta_1, \delta_2, \delta_3, \delta_{01} > 0.$$

From (3.1.2) we get $\delta_1 = \frac{1}{4-x}, \delta_2 = \frac{2-x}{4-x}, \ \delta_3 = \frac{1}{4-x}, and\ \delta_{01} = \frac{2x-3}{4-x}$.

Putting the values in (3.1.2) we get the optimal solution of dual problem. The values of D, S, q is obtained by using the primal dual relation as follows.

From primal dual relation we get

$$\frac{SD}{q} = \delta_1{}^* \times d^*(\delta),$$

$$\frac{aHq^2}{6D} = \delta_2{}^* \times d^*(\delta),$$

$$\theta D^{1-x} S^{-1} = \delta_3{}^* \times d^*(\delta),$$

$$\frac{w_0\, q}{W} = 1.$$

The optimal solution of the model through the parametric approach is given by

$$d^*(\delta) = (4-x)^{\frac{1}{4-x}} \left(\frac{aH(4-x)}{(2-x)6}\right)^{\frac{2-x}{4-x}} \left(\theta(4-x)\right)^{\frac{1}{4-x}} \times \left(\frac{w_0\,(4-x)}{W(2x-3)}\right)^{\frac{2x-3}{4-x}} \left(\frac{2x-3}{4-x}\right)^{\frac{2x-3}{4-x}}$$

and $$S^* = \frac{6\delta_1{}^*\delta_2{}^* d^*(\delta)^2}{aH},$$

$$D^* = \frac{aHq^2}{6\delta_2{}^* d^*(\delta)},$$

$$q^* = \frac{W}{w_0}.$$

4. FUZZY NUMBER AND ITS NEAREST INTERVAL APPROXIMATION:

4.1. Fuzzy number:

A real number $\tilde{A}$ described as fuzzy subset on the real line $\mathcal{R}$ whose membership function $\mu_{\tilde{A}}(x)$ has the following characteristics with $-\propto < a_1 \le a_2 \le a_3 < \propto$

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{A}}{}^L(x) & if \quad a_1 \le x \le a_2 \\ \mu_{\tilde{A}}{}^R(x) & if \quad a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

Where

ISSN: 2236-269X
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$\mu_{\tilde{A}}{}^{L}(x): [a_1, a_2] \to [0,1]$ is continuous and strictly increasing and
$\mu_{\tilde{A}}{}^{R}(x): [a_2, a_3] \to [0,1]$ is continuous and strictly decreasing.

α-cut of $\tilde{A}$: The α-cut of $\tilde{A}$, is defined by A_α={x: μ_A(x)≥α.}

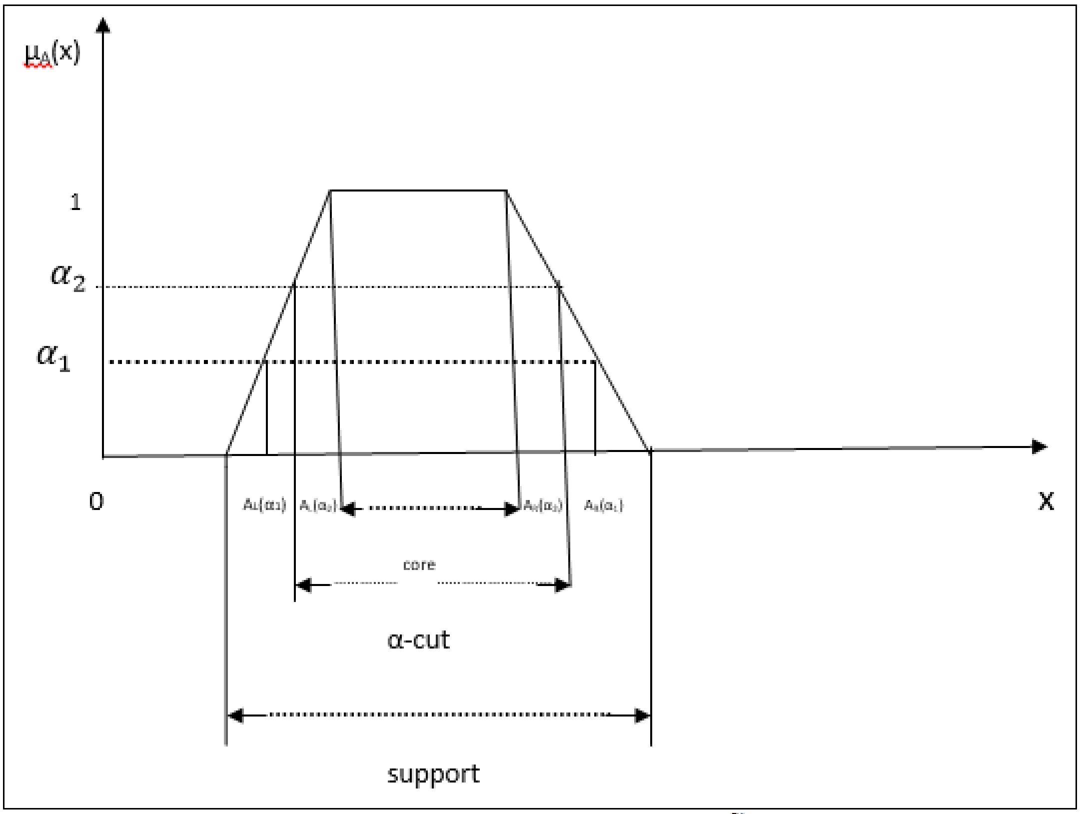

Figure2: Trapezoidal fuzzy number of $\tilde{A}$ with α-cut.

A_α is a non-empty bounded closed interval in X and it can be denoted by A_α = [A_L(α), A_R(α)]. Where A_L(α) and A_R(α) are the lower and upper bounds of the closed interval respectively.

Figure 2 shows a fuzzy number $\tilde{A}$ with α-cuts $A_{\alpha1}$ = [$A_L(\alpha_1)$, $A_R(\alpha_1)$], $A_{\alpha2}$ = [$A_L(\alpha_2)$, $A_R(\alpha_2)$]. It Seen that if $\alpha_2 \geq \alpha_1$ then $A_L(\alpha_2) \geq A_L(\alpha_1)$ and $A_R(\alpha_1) \geq A_R(\alpha_2)$.

4.2. Interval number

An interval number A is defined by an ordered pair of real numbers as follows A = $[a_L, a_R] = \{x: a_L \leq x \leq a_R\}, x \in \mathcal{R}\}$ where and a_L are the left and a_R right bounds of interval A, respectively. The interval A, is also defined by center (a_c) and half-width (a_w) as follows

A = (a_c, a_W) = {x: $a_c - a_w \leq x \leq a_c + a_w, x \in \mathcal{R}\}$ where $a_c = \frac{a_R + a_L}{2}$ is the center and $a_w = \frac{a_R - a_L}{2}$ is the half-width of A.

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4.3. Nearest interval approximation

Here we want to approximate a fuzzy number by a crisp model. Suppose $\tilde{A}$ and $\tilde{B}$ are two fuzzy numbers with α-cuts are $[A_L(\alpha), A_R(\alpha)]$ and $[B_L(\alpha), B_R(\alpha)]$ respectively. Then the distance between $\tilde{A}$ and $\tilde{B}$ is

$$d(\tilde{A},\tilde{B}) = \sqrt{\int_0^1 (A_L(\alpha) - B_L(\alpha))^2 + \int_0^1 (A_R(\alpha) - B_R(\alpha))^2 d\alpha}.$$

Given$\hat{A}$ is a fuzzy number. We have to find a closed interval $C_D(\tilde{A})$, which is the nearest to with respect to metric. We can do it since each interval is also a fuzzy number with constant α-cut for all α ∈ [0,1]. Hence $(C_D(\tilde{A}))\alpha = [C_L, C_R]$. Now we have to minimize

$$d(\tilde{A}, C_D(\tilde{A})) = \sqrt{\int_0^1 (A_L(\alpha) - C_L)^2 + \int_0^1 (A_R(\alpha) - C_R)^2 d\alpha}$$

with respect to $C_L and\ C_R$.

In order to minimize $d(\tilde{A}, C_D(\tilde{A}))$, it is sufficient to minimize the function

$$D(C_L, C_R) = (d^2(\tilde{A}, C_D(\tilde{A}))).$$

The first partial derivatives are

$$\frac{\partial}{\partial C_L} D(C_L, C_R) = -2\int_0^1 A_L(\alpha)d\alpha + 2\,C_L.$$

And

$$\frac{\partial}{\partial C_R} D(C_L, C_R) = -2\int_0^1 A_R(\alpha)d\alpha + 2\,C_R.$$

Solving $\frac{\partial}{\partial C_L} D(C_L, C_R) = 0$ and $\frac{\partial}{\partial C_R} D(C_L, C_R) = 0$, we get

$C_L = \int_0^1 A_L(\alpha)d\alpha$ and $C_R = \int_0^1 A_R(\alpha)d\alpha$.

Again since $\frac{\partial^2}{\partial {C_L}^2}(D(C_L^*, C_R^*)) = 2 > 0$, $\frac{\partial^2}{\partial {C_R}^2}(D(C_L^*, C_R^*)) = 2 > 0$ and

$H(C_L^*, C_R^*) = \frac{\partial^2}{\partial {C_L}^2}(D(C_L^*, C_R^*)) . \frac{\partial^2}{\partial {C_R}^2}(D(C_L^*, C_R^*)) - (\frac{\partial^2}{\partial C_L^* C_L^*}(D(C_L^*, C_R^*)))^2 = 4 > 0.$

So $D(C_L^*, C_R^*)$ i.e. $d(\tilde{A}, C_D(\tilde{A}))$ is global minimum. Therefore, the interval

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$C_d(\tilde{A}) = [\int_0^1 A_L(\alpha)d\alpha,\ \int_0^1 A_R(\alpha)d\alpha]$ is the nearest interval approximation of fuzzy number $\tilde{A}$ with respect to the metric d.

Let $\tilde{A} = (a_1,a_2,a_3)$ be a triangular fuzzy number. The α-cut interval of $\tilde{A}$ is defined as

$A_\alpha = [A_L(\alpha), A_R(\alpha)]$ where $A_L(\alpha) = a_1+\alpha(a_2 - a_1)$ and $A_R(\alpha) = a_3 - \alpha(a_3 - a_2)$. By nearest interval approximation method,

the lower limit of the interval is

$$C_L = \int_0^1 A_L(\alpha)d\alpha = \int_0^1 [a_1 + \alpha(a_2 - a_1)]d\alpha = \frac{a_1+a_2}{2},$$

and the upper limit of the interval is

$$C_R = \int_0^1 A_R(\alpha)d\alpha = \int_0^1 [a_3 - \alpha(a_3 - a_2)]d\alpha = \frac{a_3+a_2}{2}.$$

Therefore, the interval number corresponding to a given fuzzy number $\tilde{A}$ is $[\frac{a_1+a_2}{2}, \frac{a_3+a_2}{2}] = [m,n]$. In the centre and half–width form the interval number of $\tilde{A}$ is defined as $\langle \frac{1}{4}(a_1 + 2a_2 + a_3), \frac{1}{4}(a_3 - a_1)\rangle$.

4.4. Parametric Interval-valued function:

Let [m, n] be an interval, where m > 0, n > 0.. From analytical geometry point of view, any real number can be represented on a line. Similarly; we can express an interval by a function. The parametric interval-valued function for the interval [m, n] can be taken as g(s) = $m^{1-s}n^s$ for s ∊ [0,1], which is strictly monotone, continuous function and its inverse exits. Let ψ be the inverse of g(s), then

$$s = \frac{\log\psi - \log m}{\log n - \log m}.$$

5. GEOMETRIC PROGRAMMING WITH FUZZY COEFFICIENT:

When all coefficients of Eq. (6) are triangular fuzzy number, then the geometric programming problem is of the form

$$\text{Min} \quad \tilde{g}_0(x) \tag{5.1}$$

$$\text{subject to } \tilde{g}_i(x) \lesssim 1 \quad (1 \le i \le n)$$

$$x > 0.$$

Its objective functions

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$\tilde{g}_0$(x) = $\sum_{k=1}^{T_i} \tilde{c}_{0k} \prod_{j=1}^{m} x_j^{\alpha_{okj}}$

and constraints of the form

$\tilde{g}_i$ (x) = $\sum_{k=1}^{T_i} \tilde{c}_{ik} \prod_{j=1}^{m} x_j^{\alpha_{ikj}}$ (0≤i≤n)

are all posynomials of x in which coefficients $\tilde{c}_{0k}$ and indexes $\tilde{c}_{ik}$ are fuzzy numbers.

Where $\tilde{c}_{0k} = (c^1{}_{0k}, c^2{}_{0k}, c^3{}_{0k})$ and $\tilde{c}_{ik} = (c^1{}_{ik}, c^2{}_{ik}, c^3{}_{ik})$. Using nearest interval approximation method, we transform all triangular fuzzy number into interval number i.e. $[c_{0k}{}^L, c_{0k}{}^U]$ and $[c_{ik}{}^L, c_{ik}{}^U]$. The geometric programming problem with imprecise parameters is of the following form

Min $\hat{g}_0$(x) (5.2)

subject to $\hat{g}_i$(x) $\lesssim$ 1 (1≤i≤n)

x>0,

Its objective function is

$\hat{g}_0$(x) = $\sum_{k=1}^{T_i} \hat{c}_{0k} \prod_{j=1}^{m} x_j^{\alpha_{okj}}$,

and constraints of the form is

$\hat{g}_i$(x) = $\sum_{k=1}^{T_i} \hat{c}_{ik} \prod_{j=1}^{m} x_j^{\alpha_{ikj}}$, (1≤i≤n).

Where $\hat{c}_{0k}$ *and* $\hat{c}_{ik}$ denote the interval counterparts i.e. $\hat{c}_{0k} \in [c_{0k}{}^L, c_{0k}{}^U]$ and $\hat{c}_{ik} \in [c_{ik}{}^L, c_{ik}{}^U]$. $c_{0k}{}^L > 0, c_{ik}{}^L > 0$ for all i and k. Using parametric interval-valued functional form, the problem (5.2) reduces to

Min g_0(x,s) = $\sum_{k=1}^{T_0} (c_{0k}{}^L)^{1-s} (c_{0k}{}^U)^s \prod_{j=1}^{m} x_j^{\alpha_{okj}}$ (5.3)

Subject to g_i(x,s) = $\sum_{k=1}^{T_i} (c_{ik}{}^L)^{1-s} (c_{ik}{}^U)^s \prod_{j=1}^{m} x_j^{\alpha_{ikj}} \le 1$

$x_j > 0$ for i = 1,2,…….n, j = 1,2,……..m.

This is a parametric geometric programming problem. We get different solutions of this problem for different value of the parameter *s.*

the dual programming of (5.3) is as follows:

Max d(δ,s) = $\prod_{i=0}^{n} \prod_{k=0}^{T_i} \left(\frac{(c_{ik}{}^L)^{1-s}(c_{ik}{}^U)^s}{\delta_{ik}}\right)^{\delta_{ik}} \left(\sum_{s=1}^{T_i} \delta_{is}\right)^{(\sum_{s=1}^{T_i} \delta_{is})}$ (5.4)

Subject to

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$\sum_{k=1}^{T_0} \delta_{ok} = 1,$

$\sum_{i=0}^{n} \sum_{k=1}^{T_0} \alpha_{ikj} \delta_{ik} = 0,$

$\delta_{ik} > 0.$

5.1. Theorem 5.1.

If x is a feasible vector for the constraints PGP and if δ is a feasible vector for the corresponding DP, then g_0(x,s) ≥ d(δ,s) (Primal- Dual Inequality).

5.2. Proof.

The expression for g_0(x,s) can be written as

$$g_0(x,s) = \sum_{k=1}^{T_0} \delta_{0k} \left(\frac{{c_{0k}^L}^{1-s} {c_{0k}^U}^{s} \prod_{j=1}^{m} x_j^{\alpha_{0kj}}}{\delta_{0k}}\right).$$

Here the weights are $\delta_{01}, \delta_{02}, \ldots\ldots\ldots, \delta_{0T_0}$ and positive terms are $\frac{{c_{01}^L}^{1-s} {c_{01}^U}^{s} \prod_{j=1}^{m} x_j^{\alpha_{01j}}}{\delta_{01}}, \frac{{c_{02}^L}^{1-s} {c_{02}^U}^{s} \prod_{j=1}^{m} x_j^{\alpha_{02j}}}{\delta_{02}}, \ldots\ldots\ldots, \frac{{c_{0T_0}^L}^{1-s} {c_{0T_0}^U}^{s} \prod_{j=1}^{m} x_j^{\alpha_{0T_0j}}}{\delta_{0T_0}}$.

Now applying A.M.-.G.M inequality, we get

$$\left(\frac{({c_{01}}^L)^{1-s}({c_{01}}^U)^s \prod_{j=1}^{m} x_j^{\alpha_{01j}} + ({c_{02}}^L)^{1-s}({c_{02}}^U)^s \prod_{j=1}^{m} x_j^{\alpha_{02j}} + \ldots + ({c_{0T_0}}^L)^{1-s}({c_{0T_0}}^U)^s \prod_{j=1}^{m} x_j^{\alpha_{0T_0j}}}{(\delta_{01}+\delta_{02}+\cdots+\delta_{0T_0})}\right)^{(\delta_{01}+\delta_{02}+\cdots+\delta_{0T_0})}$$

$$\geq$$

$$\left(\left(\frac{{c_{01}^L}^{1-s} {c_{01}^U}^{s} \prod_{j=1}^{m} x_j^{\alpha_{01j}}}{\delta_{01}}\right)^{\delta_{01}} \left(\frac{{c_{02}^L}^{1-s} {c_{02}^U}^{s} \prod_{j=1}^{m} x_j^{\alpha_{02j}}}{\delta_{02}}\right)^{\delta_{01}} \ldots \left(\frac{{c_{0T_0}^L}^{1-s} {c_{0T_0}^U}^{s} \prod_{j=1}^{m} x_j^{\alpha_{0T_0j}}}{\delta_{0T_0}}\right)^{\delta_{0T_0}}\right)$$

Or $\left(\frac{g_0(x,s)}{\sum_{k=1}^{T_0} \delta_{ik}}\right)^{\sum_{k=1}^{T_0} \delta_{0k}} \geq \prod_{k=1}^{T_0}\left(\frac{{c_{0k}^L}^{1-s} {c_{0k}^U}^{s} \prod_{j=1}^{m} x_j^{\alpha_{0kj}}}{\delta_{0k}}\right)^{\delta_{0k}}$ $[as \sum_{k=1}^{T_0} \delta_{0k} = 1]$

Or $g_0(x,s) \geq \left(\frac{{c_{0k}^L}^{1-s} {c_{0k}^U}^{s}}{\delta_{0k}}\right)^{\sum_{k=1}^{T_0} \delta_{ok}} \prod_{j=1}^{m} x_j^{\sum_{k=1}^{T_0} \alpha_{0kj} \delta_{ok}}$

Or $g_0(x,s) \geq \prod_{k=1}^{T_i}\left(\frac{({c_{ik}}^L)^{1-s}({c_{ik}}^U)^s}{\delta_{ik}}\right)^{\delta_{ik}} \prod_{j=1}^{m} x_j^{\sum_{k=1}^{T_0} \alpha_{0kj} \delta_{ok}}$ (5.1.1)

Again g_i(x,s) can be written as

$$g_i(x,s) = \sum_{k=1}^{T_I} \delta_{ik} \left(\frac{{c_{ik}^L}^{1-s} {c_{ik}^U}^{s} \prod_{j=1}^{m} x_j^{\alpha_{ikj}}}{\delta_{ik}}\right).$$

Now applying A.M.-.G.M inequality, we get

ISSN: 2236-269X
DOI: 10.14807/ijmp.v8i2.535

Or $\left(\frac{g_i(x,s)}{\sum_{k=1}^{T_i}\delta_{ik}}\right)^{\sum_{k=1}^{T_i}\delta_{ik}} \geq \prod_{k=1}^{T_i}\left(\frac{(c_{ik}{}^{L})^{1-s}(c_{ik}{}^{U})^{s}\prod_{j=1}^{m}x_j{}^{\alpha_{ikj}}}{\delta_{ik}}\right)^{\delta_{ik}}$

$$g_i(x,s)^{\sum_{k=1}^{T_i}\delta_{ik}} \geq \prod_{k=1}^{T_i}\left(\frac{(c_{ik}{}^{L})^{1-s}(c_{ik}{}^{U})^{s}}{\delta_{ik}}\right)^{\delta_{ik}} \prod_{j=1}^{m}x_j{}^{\sum_{k=1}^{T_i}\alpha_{ikj}\delta_{ik}}\left(\sum_{k=1}^{T_i}\delta_{ik}\right)^{(\sum_{k=1}^{T_i}\delta_{ik})} \quad (5.1.2)$$

Using $g_i(x,s)^{\sum_{k=1}^{T_i}\delta_{ik}} \leq 1$, [as g_i(x,s) $\leq$ 1]

We have

$$1 \geq \prod_{k=1}^{T_i}\frac{(c_{ik}{}^{L})^{1-s}(c_{ik}{}^{U})^{s}}{\delta_{ik}})^{\delta_{ik}} \prod_{j=1}^{m}x_j{}^{\sum_{k=1}^{T_i}\alpha_{ikj}\delta_{ik}}\left(\sum_{k=1}^{T_i}\delta_{is}\right)^{\sum_{k=1}^{T_i}\delta_{ik}} \quad (5.1.3)$$

Multiplying (5.1.1) and (5.1.3) we get

$$g_0(x,s) \geq \prod_{i=0}^{n}\prod_{k=0}^{T_i}\left(\frac{(c_{ik}{}^{L})^{1-s}(c_{ik}{}^{U})^{s}}{\delta_{ik}}\right)^{\delta_{ik}}\left(\sum_{k=1}^{T_i}\delta_{ik}\right)^{(\sum_{k=1}^{T_i}\delta_{ik})}\prod_{j=1}^{m}x_j{}^{\sum_{i=1}^{n}\sum_{k=1}^{T_0}\alpha_{ikj}\delta_{ik}} \quad (5.1.4)$$

Using orthogonal condition the inequality (5.1.4) becomes

$$g_0(x,s) \geq \prod_{i=0}^{n}\prod_{k=0}^{T_i}\left(\frac{(c_{ik}{}^{L})^{1-s}(c_{ik}{}^{U})^{s}}{\delta_{ik}}\right)^{\delta_{ik}}\left(\sum_{k=1}^{T_i}\delta_{ik}\right)^{(\sum_{k=1}^{T_i}\delta_{ik})} = d(\delta,s) \quad (5.1.5)$$

i.e., g_0(x,s) ≥ d(δ,s) . (Proof).

5.3. Theorem 5.2.

δ is a feasible vector for the dual programming (DP) problem , then d(δ,1) ≥ d(δ,0).

5.4. Proof:

We have $c_{ik}{}^{U} \geq c_{ik}{}^{L}$, for all k, (k=1,2,…….,T_0).

Or $(c_{ik}{}^{L})^{1-1}(c_{ik}{}^{U})^{1} \geq (c_{ik}{}^{L})^{1-0}(c_{ik}{}^{U})^{0}$

Or $\frac{(c_{ik}{}^{L})^{1-1}{}_{ik}{}^{U})^{1}}{\delta_{ik}} \geq \frac{(c_{ik}{}^{L})^{1-0}(c_{ik}{}^{U})^{0}}{\delta_{ik}}$

Or $\left(\frac{(c_{ik}{}^{L})^{1-1}(c_{ik}{}^{U})^{1}}{\delta_{ik}}\right)^{\delta_{ik}} \geq \left(\frac{(c_{ik}{}^{L})^{1-0}(c_{ik}{}^{U})^{0}}{\delta_{ik}}\right)^{\delta_{ik}}$

Or $\prod_{k=0}^{T_i}\left(\frac{(c_{ik}{}^{L})^{1-1}(c_{ik}{}^{U})^{1}}{\delta_{ik}}\right)^{\delta_{ik}} \geq \prod_{k=0}^{T_0}\left(\frac{(c_{ik}{}^{L})^{1-0}(c_{ik}{}^{U})^{0}}{\delta_{ik}}\right)^{\delta_{ik}}$

Or $\prod_{i=1}^{n}\prod_{k=0}^{T_i}\left(\frac{(c_{ik}{}^{L})^{1-1}(c_{ik}{}^{U})^{1}}{\delta_{ik}}\right)^{\delta_{ik}}\left(\sum_{k=1}^{T_i}\delta_{ik}\right)^{(\sum_{k=1}^{T_i}\delta_{ik})}$

$\geq \prod_{i=1}^{n}\prod_{k=0}^{T_0}\left(\frac{(c_{ik}{}^{L})^{1-0}(c_{ik}{}^{U})^{0}}{\delta_{ik}}\right)^{\delta_{ik}}\left(\sum_{k=1}^{T_i}\delta_{ik}\right)^{(\sum_{k=1}^{T_i}\delta_{ik})}$

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i.e., d(δ,1) ≥ d(δ,0). (proof)

5.5. Solution procedure of fuzzy model by Geometric Programming (G.P) technique:

When $\tilde{a} = (a_1, a_2, a_3)$, $\widetilde{H} = (H_1, H_2, H_3)$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3)$ and $\widetilde{W} = (W_1, W_2, W_3)$ are triangular fuzzy number .then the fuzzy model is

$$\widetilde{Min}\ TAC(D,S,q) = \frac{SD}{q} + \frac{\tilde{a}\widetilde{H}q^2}{6D} + \tilde{\theta}D^{1-x}S^{-1} \qquad (5.3.1)$$

subject to $w_0 q \lesssim \widetilde{W}$, D, S, q > 0.

Using nearest interval approximation method, the interval number corresponding triangular number $\tilde{a} = (a_1, a_2, a_3)$ is $[\frac{a_1+a_2}{2}, \frac{a_3+a_2}{2}] = [a_L, a_U]$. Similarly interval number corresponding $\widetilde{H}$, $\tilde{\theta}$ and $\widetilde{W}$ are $[\frac{H_1+H_2}{2}, \frac{H_3+H_2}{2}] = [H_L, H_U]$, $[\frac{\theta_1+\theta_2}{2}, \frac{\theta_3+\theta_2}{2}] = [\theta_L, \theta_U]$ and $[\frac{W_1+W_2}{2}, \frac{W_3+W_2}{2}] = [W_L, W_U]$ respectively. The problem (5.3.1) reduces to

$$\text{Min } TAC(D,S,q) = \frac{SD}{q} + \frac{[a_L,a_U][H_L,H_U]q^2}{6D} + [\theta_L, \theta_U]D^{1-x}S^{-1} \qquad (5.3.2)$$

subject to $w_0 q \leq [W_L, W_U]$,D, S, q > 0.

Which is equivalent to

$$\text{Min } TAC(D,S,q) = \frac{SD}{q} + \frac{\hat{a}\widehat{H}q^2}{6D} + \hat{\theta}D^{1-x}S^{-1} \qquad (5.3.3)$$

subject to $w_0 q \lesssim \widehat{W}$, D, S, q > 0,

where $\hat{a} \in [a_L, a_U]$, $\widehat{H} \in [H_L, H_U]$, $\hat{\theta} \in [\theta_L, \theta_U]$ and $\widehat{W} \in [W_L, W_U]$.

According to section 4.4, the fuzzy model (5.2.3) reduces to a parametric programming by replacing

$\hat{a} = {a_L}^{1-s}{a_U}^{s}$, $\widehat{H} = {H_L}^{1-s}{H_U}^{s}$, $\hat{\theta} = {\theta_L}^{1-s}{\theta_U}^{s}$ and $\widehat{W} = {W_L}^{1-s}{W_U}^{s}$ where s ∈ [0,1].

The model takes the reduces form as follows

$$\text{Min } TAC(D,S,q) = \frac{SD}{q} + \frac{({a_L}^{1-s}{a_U}^{s})({H_L}^{1-s}{H_U}^{s})q^2}{6D} + ({\theta_L}^{1-s}{\theta_U}^{s})D^{1-x}S^{-1} \qquad (5.3.4)$$

subject to $w_0 q \lesssim ({W_L}^{1-s}{W_U}^{s})$, D,S,q > 0

Corresponding dual form of (5.3.4) is given by

INDEPENDENT JOURNAL OF MANAGEMENT & PRODUCTION (IJM&P)
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Max $d(\delta, s) = (\frac{1}{\delta_1})^{\delta_1}(\frac{(a_L{}^{1-s}a_U{}^{s})(H_L{}^{1-s}H_U{}^{s})}{6\delta_2})^{\delta_2}(\frac{(\theta_L{}^{1-s}\theta_U{}^{s})}{\delta_3})^{\delta_3}(\frac{w_0}{(W_L{}^{1-s}W_U{}^{s})\delta_{01}})^{\delta_{01}}\delta_{01}{}^{\delta_{01}}$

subject to $\delta_1 + \delta_2 + \delta_3 = 1$ (5.3.5)

$\delta_1 - \delta_3 = 0$

$\delta_1 - \delta + (1\text{-}x)\delta_3 = 0$

$-\delta_1 + 2\delta_2 + \delta_{01} = 0$

$\delta_1, \delta_2, \delta_3, \delta_{01} \geq 0.$

From (5.3.5) we get $\delta_1 = \frac{1}{4-x}, \delta_2 = \frac{2-x}{4-x}, \delta_3 = \frac{1}{4-x}, and\ \delta_{01} = \frac{2x-3}{4-x}$.

Putting the values in (5.3.5) we get the optimal solution of dual problem. The values of D, S, q is obtained by using the primal dual relation as follows:

From primal dual relation we get

$$\frac{SD}{q} = \delta_1{}^* \times d^*(\delta),$$

$$\frac{(a_L{}^{1-s}a_U{}^{s})(H_L{}^{1-s}H_U{}^{s})q^2}{6D} = \delta_2{}^* \times d^*(\delta),$$

$$(\theta_L{}^{1-s}\theta_U{}^{s})D^{1-x}S^{-1} = \delta_3{}^* \times d^*(\delta),$$

$$\frac{w_0\, q}{(W_L{}^{1-s}W_U{}^{s})} = 1.$$

The optimal solution of the model through the parametric approach is given by

$$d^*(\delta, s) = (4-x)^{\frac{1}{4-x}}\left(\frac{(a_L{}^{1-s}a_U{}^{s})(H_L{}^{1-s}H_U{}^{s})(4-x)}{(2-x)6}\right)^{\frac{2-x}{4-x}}\left((\theta_L{}^{1-s}\theta_U{}^{s})(4-x)\right)^{\frac{1}{4-x}}$$

$$\times\left(\frac{w_0(4-x)}{(W_L{}^{1-s}W_U{}^{s})(2x-3)}\right)^{\frac{2x-3}{4-x}}\left(\frac{2x-3}{4-x}\right)^{\frac{2x-3}{4-x}}$$

and

$$S^* = \frac{6\omega_1{}^*\omega_2{}^*d^*(\delta,s)^2}{(a_L{}^{1-s}a_U{}^{s})(H_L{}^{1-s}H_U{}^{s})},$$

$$D^* = \frac{(a_L{}^{1-s}a_U{}^{s})(H_L{}^{1-s}H_U{}^{s})q^2}{6\omega_2{}^*d^*(\delta,s)},$$

$$q^* = \frac{(W_L{}^{1-s}W_U{}^{s})}{w_0}.$$

6. NUMERICAL EXAMPLE AND SOLUTION:

ISSN: 2236-269X
DOI: 10.14807/ijmp.v8i2.535

A manufacturing company produces a machine. It is given that the inventory carrying cost of the machine is \$15 per unit per year. The production cost of the machine varies inversely with the demand and set-up cost. From the past experience, the production cost of the machine is $120D^{-3}S^{-1}$ where D is the demand rate and S is set-up cost. Storage space area per unit time (w_0) and total storage space area (W) are 100 sq. ft. and 2000 sq. ft. respectively. Determine the demand rate (D), set-up cost (S), production quantity (q), and optimum total average cost (TAC) of the production system.

Then the input value of the model (2.3.6) is

Table-1(Input values)

a	H	x	θ	w_0	W
7	15	1.75	120	100	2000

Then the model is of the form

$$\text{Min TAC}(D,S,q) = \frac{SD}{q} + \frac{105\,q^2}{6D} + 120D^{-0.75}S^{-1}$$

subject to 100q ≤ 2000, D, S, q > 0. (6.1)

Table 2: Optimal solution of (2.3.6) for crisp model

Crisp model	S^*	D^*	q^*	$TAC^*(S^*,D^*,q^*)\$$
G.P	0.684	4048	20	140.517
N.L.P	0.685	4047	20	140.685

When the input data of inventory model is taken as triangular fuzzy number i.e.. $\tilde{a}$ = (5,7,9), $\tilde{H}$ = (13,15,17), $\tilde{\theta}$ = (116,120,124) and $\tilde{W}$ = (1800,2000,2200). Using nearest interval approximation method, we get the corresponding interval number and interval-valued function i.e.

$\tilde{a} \approx [6,8]$, $\Rightarrow$ $\hat{a} = (6)^{1-s}(8)^{s} \in [6,8]$,

$\tilde{H} \approx [14,16]$, $\Rightarrow$ $\hat{H} = (14)^{1-s}(16)^{s} \in [14,16]$,

$\tilde{\theta} \approx [118,122]$, $\Rightarrow$ $\hat{\theta} = (118)^{1-s}(122)^{s} \in [118,122]$,

$\tilde{W} \approx [1900,2100]$, $\Rightarrow$ $\hat{W} = (1900)^{1-s}(2100)^{s} \in [1900,2100]$, where $s \in [0,1]$.

The optimal solution of the fuzzy model by interval-valued parametric geometric programming is presented in Table

Table 3: Optimal Solution for Fuzzy Inventory Model

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s	S^*	D^*	q^*	$TAC^*(S^*,D^*,q^*)\$$
0.0	0.820	2983.86	21.00	119.801
0.1	0.786	3175.16	20.79	123.060
0.2	0.753	3378.71	20.58	126.396
0.3	0.722	3595.32	20.38	129.924
0.4	0.693	3825.81	20.18	133.735
0.5	0.664	4071.08	19.97	137.533
0.6	0.637	4332.07	19.78	141.519
0.7	0.610	4609.80	19.58	145.473
0.8	0.585	4905.33	19.38	149.793
0.9	0.561	5219.81	19.19	154.196
1.0	0.538	5554.45	19.00	158.767

For s=0, the lower bound of the interval value of the parameter is used to find the optimal solution. For s=1, the upper bound of interval value of the parameter is used for the optimal solution. These results yield the lower and upper bounds of the optimal solution. The main advantage of the proposed technique is that one can get the intermediate optimal result using proper value s.

Here we have given a rough graph, which shown how change the value of TAC*(S*,D*,q*) for difference values of s.

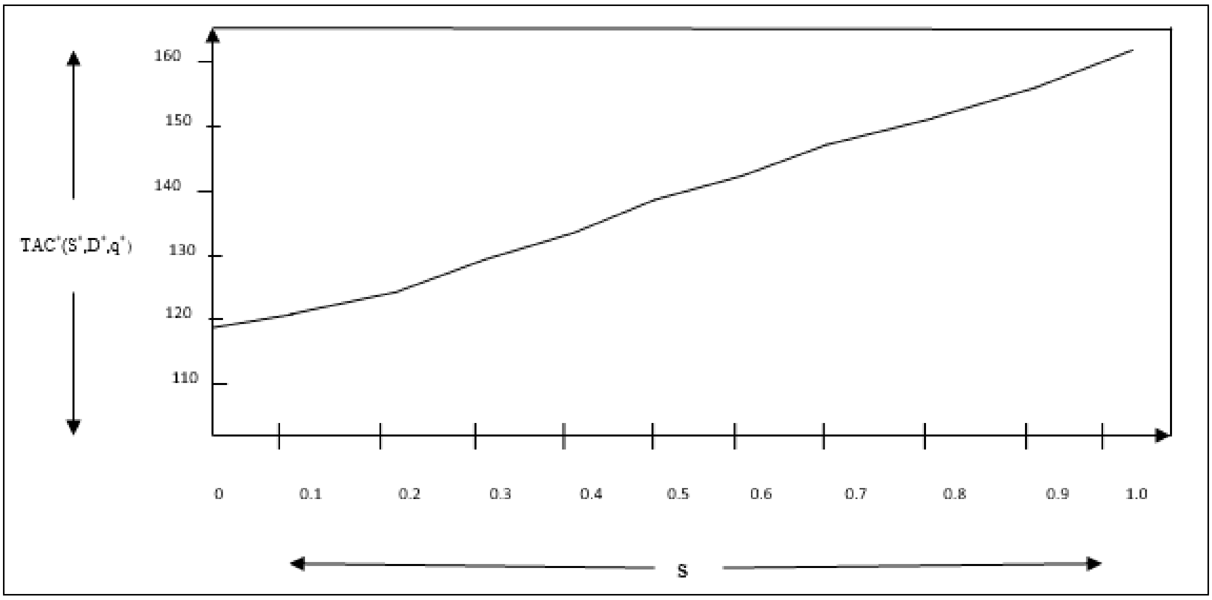

Figure 3: Change of the value of objective function for change of s, by Fuzzy Geometric Programming Technique.

7. SENSITIVITY ANALYSIS:

Effect, for increment the parameter “s”.

(1) For increasing of “s”, set-up cost S^* is decreasing.

ISSN: 2236-269X
DOI: 10.14807/ijmp.v8i2.535

(2) For increasing of "s", demand rate D^* is increasing.

(3) For increasing of "s , Production quantity q^* is decreasing.

(4) For increasing of "s", Total average cost $TAC^*(S^*,D^*,q^*)$ is increasing.

8. CONCLUSION:

In this paper, we have proposed a real life inventory problem in a fuzzy environment and presented solution along with sensitivity analysis approach. The inventory model developed with unit production cost, time depended holding cost, with-out shortages. This model has been developed for single item.

In this paper, we first create a crisp model then it transformed to fuzzy model and solved by parametric Geometric-Programming technique. Here decision maker may obtain the optimal results according to his expectation .In fuzzy we have considered triangular fuzzy number(T.F.N) In future, the other type of membership functions such as piecewise linear hyperbolic, L-R fuzzy number, Trapezoidal Fuzzy Number (TrFN), Parabolic flat Fuzzy Number (PfFN), Parabolic Fuzzy Number (pFN), pentagonal fuzzy number etc can be considered to construct the membership function and then model can be easily solved.

ACKNOWLEDGEMENTS

The authors are thankful to University of Kalyani for providing financial assistance through DST-PURSE Programme. The authors are grateful to the reviewers for their comments and suggestions.

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