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Research Article

Risk Measurement for Portfolio Credit Risk Based on a Mixed Poisson Model

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Experiences manifest the importance of comovement and communicable characters among the risks of financial assets. Therefore, the portfolio view considering dependence relationship among credit entities is at the heart of risk measurement. This paper introduces a mixed Poisson model assuming default probabilities of obligors depending on a set of common economic factors to construct the dependence structure of obligors. Further, we apply mixed Poisson model into an empirical study with data of four industry portfolios in the financial market of China. In the process of model construction, the classical structural approach and option pricing formula contribute to estimate dynamic default probabilities of single obligor, which helps to obtain the dynamic Poisson intensities under the model assumption. Finally, given the values of coefficients in this model calculated by a nonlinear estimation, Monte Carlo technique simulates the progress of loss occurrence. Relationship between default probability and loss level reflected through the MC simulation has practical features. This study illustrates the practical value and effectiveness of mixed Poisson model in risk measurement for credit portfolio.

1. Introduction

Financial system is the core of modern economy and the risk in it has a huge impact on economic development. Two main components of financial risk are market risk and credit risk. Whereas market risk is the risk of losses in positions arising from movements in market prices, credit risk refers to the risk that a borrower will default on any type of debt by failing to make contractual payments.

Giesecke [1] proposes that there are two main quantitative approaches to analyze how to measure portfolio credit risk. In the structural approach, a firm defaults if its assets are insufficient according to some measure and then the probability of default can be deduced by tracing the dynamics of a firm's intrinsic value. The basic application of structural approach goes back to Black and Scholes [2] and Merton [3]. In recent years, structural approach is still in widespread use; see Chan et al. [4] and Schäfer and Koivusalo [5]. The other one, reduced-form approach, is silent about why a firm defaults because the dynamics of default are exogenously given through a default probability. Thus this paper applies the structural approach to measure default risk of a single firm.

The financial crisis we experienced these years tells that the financial health of a firm varies with randomly fluctuating macroeconomic factors. Therefore, different firms are affected by common macroeconomic factors; there is dependence between defaults. The portfolio view considering dependence relationship among credit entities first introduced by CreditMetrics [6] is the most important feature of modern credit risk management. In the consideration of the integrated risk of a portfolio, we can classify credit risk models into two categories: bottom-up and top-down; see Gordy [7]. In a bottom-up model, the portfolio default intensity is an aggregate of the constituents. The approach proposed by Barnhill and Maxwell [8] relates financial market volatility to firm specific credit risk and integrates interest rate, interest rate spread, and foreign exchange rate risk into one overall fixed income portfolio risk assessment. References [9, 10] study the motion features of risk factors. In a top-down model, the portfolio intensity is specified without reference to the constituents. Instead, dependence is introduced through a set of "risk factors" and defaults become independent conditional on the risk factors. Here, copula functions originally associated with J.P. Morgan's CreditMetrics system [6] are now widely employed for linking the marginal distributions of losses resulting from different risk factors to obtain the distribution of aggregate loss; see Wen and Liu [11], Dimakos and Aas [12], and Rosenberg and Schuermann [13]. And Liang et al. [14] present a factor copula model for the integration of Chinese commercial bank's credit risk and market risk.

However, it is quite difficult to choose a correct copula function, especially because the relative scarcity of data on credit losses. Frey and McNeil [15] showed that the aggregate portfolio loss distribution is often very sensitive to the choice of copula and the estimation of parameters. For large portfolios of tens of thousands of obligors there remains considerable model risk. Therefore, Glasserman and Li [16] propose another top-down model, a mixed Poisson mechanism, originally associated with CreditRisk+ [17], to capture the dependence between risk factors. This paper introduces this model and applies it into empirical study with data in financial market of China for the reason that the mixed Poisson model has less model risk, because the loss distribution in mixed Poisson is the aggregate of all units whose model risk can be offset by each other. Further, it is more convenient for statistical fitting and simulation purposes in empirical study.

The paper is organized as follows. We review the structural approach and get the formula of individual default probability in Section 2. In Section 3 we introduce the mixed Poisson model. Section 4 brings the empirical study of our model. Summary and conclusion are given in Section 5.

2. Structural Approach

Since the 1990s of the last century, quantitative analysis has been blended into models of credit risk measurement. The structural model is based on the option pricing theory of Merton who indicated that equity is a kind of call option with the strike price of corporate liability. This structural model first estimates the market value of corporate equity and also its volatility and then it obtains the default distance and the default probability under the relevant corporate liability.

We will be confronted with two fundamental questions when measuring the credit risk of portfolio. One is how to describe the relationship among the default probabilities of obligors and the other one is how to link credit risks of obligors with the economic environment they face. These two questions can be solved by mixed Poisson model in this paper, but structural approach must be used first to quantize the default probability of a single obligor.

2.1. Classical Approach. Consider a firm with intrinsic value V, which represents the expected future cash flows of the firm. The firm is financed by equity and a zero coupon bond with face value K and maturity date T. The firm has to repay the amount K to the bond investors at time T or its bond holders have the right to take over this firm. Hence the default time τ is a discrete random variable given by

$$\tau = \begin{cases} T & \text{if } V_T < K \\ \infty & \text{otherwise.} \end{cases} \quad (1)$$

Meanwhile, we make assumptions that the evolution of asset prices over time follows geometric Brownian motion:

$$\frac{dV_t}{V_t} = \mu dt + \sigma dW_t, \quad (2)$$

where $\mu \in R$ is a drift parameter, $\sigma > 0$ is a volatility parameter, and W is a standard Brownian motion. Setting $m = \mu - (1/2)\sigma^2$, Ito's lemma implies that

$$V_t = V_0 e^{mt+\sigma W_t}. \quad (3)$$

Since W_T is normally distributed with mean zero and variance T, default probabilities $p(T)$ are given by

$$\begin{aligned} p(T) &= P(V_T < K) \\ &= P(\sigma W_T < \log L - mT) = \Phi\left(\frac{\log L - mT}{\sigma T}\right), \end{aligned} \quad (4)$$

where $L = (K/V_0)$ is the initial leverage ratio and Φ is the standard normal distribution function.

2.2. Theoretical Solution of Model. Given the equity value E_t and its volatility σ^E, Jones et al. [18] suggested that a firm's intrinsic value V_t and its volatility σ can be obtained through the option pricing formula. Generally, intrinsic value as well as its volatility of a public corporation can be estimated through the market value of its shares, the volatility of its stock price, and the book value of its debt. Because the market value of a company's shares reflects investors' expectations about the future value of the company, the equity of corporation can be viewed as a European call option on the assets of the firm with strike price D and maturity T. The value of the equity at time 0 is given by

$$E_0 = e^{-rT}\widehat{E}\left[\max(V_T - D, 0)\right] \quad (5)$$

which is equivalent to the payoff of a European call. r is the risk free rate and V_T is hold-to-maturity price of the underlying assets.

Pricing equity can be transformed to pricing European options. Equity value applied with the classical Black-Scholes formula is given by

$$E_0 = e^{-rT}\left[V_0 N(d_1) e^{rT} - DN(d_2)\right], \quad (6)$$

where

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}. \quad (7)$$

Since $E_t = f(V_t, t)$, Ito's lemma implies that

$$df(V_t,t) = (f_x(V_t,t)uV_t) + \frac{1}{2}f_{xx}(V_t,t)\sigma^2V_t^2 + f_t(V_t,t)dt + \sigma f_x(V_t,t)V_t dW_t. \tag{8}$$

Meanwhile, we have $dE_t = u^E E_t dt + \sigma^E E_t dW_t$ so we can obtain

$$E_t = \frac{\sigma}{\sigma^E} f_x(V_t,t) V_t. \tag{9}$$

The combination of (6) and (9) gives the value of V_T. Further, because the firm's value V_T follows geometric Brownian motion of (2), we can get parameters of μ and σ through (10). Consider the following:

$$\mu = \frac{\left(\overline{\Delta V} + S_{\Delta V}^2/2\right)}{\Delta t}, \qquad \sigma = \frac{S_{\Delta V}}{\sqrt{\Delta t}}, \tag{10}$$

where $\overline{\Delta V} = (\sum_{t=1}^{n} \Delta V_t/n)$, $S_{\Delta V}^2 = (\sum_{t=1}^{n}(\Delta V_t - \overline{\Delta V})^2/(n-1))$, $\Delta V_t = \ln(V_{t+1}) - \ln(V_t)$, and $t = 1, \ldots, n$.

Eventually, we put parameters of μ, σ, and L into (4) to obtain default probability of every single obligor at time T.

3. Portfolio Credit Risk

Practical experience manifests the importance of comovement and communicable characters among the risks of financial assets. It is not sufficient to study the credit risk of some assets independently. Therefore, portfolio view is at the heart of the field about credit risk measurement. Generally speaking, the credit portfolio can be classified into two species: one is homogeneous portfolio and the other one is the heterogeneous. The latter is what we study in this paper. Next we carry out two methods which can be applied to describe the dependence structure of heterogeneous portfolio. The following notations are needed:

m: number of obligors in the portfolio that have the probability to default,

c_i: default risk exposure of ith obligor,

s_i: loss given default of ith obligor,

L: gross loss of a credit portfolio.

3.1. Portfolio Credit Risk in the Normal Copula Model. Here, we introduce the widely used normal copula model to describe the dependence among lots of obligors. To specify the distribution of losses from default of this heterogeneous portfolio over a fixed horizon, the following notations are additionally needed:

Y_i: default indicator for ith obligor; if ith obligor defaults, $Y_i = 1$, otherwise 0;

p_i: marginal probability that ith obligor defaults.

If there are m obligors in a portfolio, the gross loss is

$$L = \sum_{i=1}^{m} L_i = \sum_{i=1}^{m} c_i * s_i * Y_i. \tag{11}$$

The goal is to estimate tail probabilities $P(L > x)$ to measure credit risk of the whole portfolio. To model the dependence structure among obligors, we need to introduce dependence among the default indicator Y_i. In the normal copula model, dependence is introduced through a multivariate normal vector $W = (W_1, \ldots, W_m)$. Consider the following:

$$W_i^T = \frac{\log\left(V_T^i/V_0^i\right) - m_i T}{\sigma_i} \text{ is the standardized return,}$$
$$B_i = \frac{\log(L_i) - m_i T}{\sigma_i} \text{ is the standardized book value.} \tag{12}$$

Each default indicator is represented as

$$Y_i = 1\{W_i < B_i\}, \quad i = 1, \ldots, m. \tag{13}$$

There is

$$P(Y_i = 1) = P(W_i < B_i) = p_i. \tag{14}$$

That is to say, obligor i defaults if $W_i \le \Phi^{-1}(p_i)$. Through this construction, the correlations among W_i determine the dependence among Y_i. The underlying correlations are specified through a factor model of the form

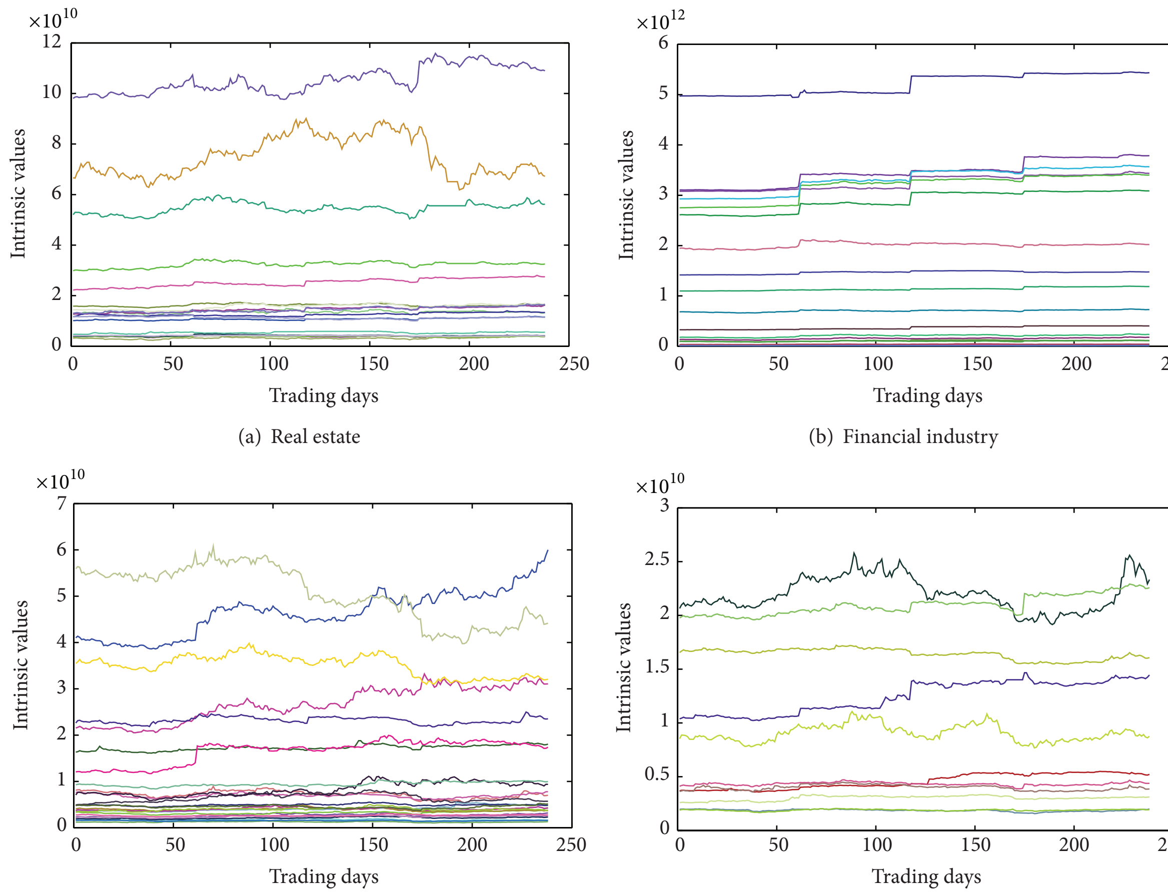
$$W_i = \sum_{k=1}^{n} a_{ik} Z_k + b_i \varepsilon_i \tag{15}$$

for some $n < m$, a n-dimensional random vector $Z \sim N_n(0, \Omega)$, and independent standard normally distributed random variables $\varepsilon_1, \ldots, \varepsilon_m$, which are also independent of Z. In practice $Z_1, \ldots, Z_n$ are systematic risk factors representing macroeconomic effects such as country and industry factors and ε_i is the specific factor associated with the ith obligor. $a_{i1}, \ldots, a_{in}$ are the factor loadings for the ith obligor and $b_i = \sqrt{1 - (a_{i1}^2 + \cdots + a_{in}^2)}$.

Denote by $F_i(x) = P(W_i < B_i)$ the marginal distribution functions of W and default probability of company i is given by $p_i = F_i(B_i)$. To determine the multivariate distribution of W most of the researchers use normal copula C^N for W, so that

$$P(W_1 < B_1, \ldots, W_m < B_m) = C^N(F_1(B_1), \ldots, F_m(B_m)). \tag{16}$$

While most credit portfolio models prevailing in this field are based on the normal copula, there is no reason why we have to assume a normal copula. Alternative copulas can lead to very different credit loss distributions and it is obvious from (16) that the copula crucially determines joint default probabilities of groups of obligors and thus the tendency of the model to produce many joint defaults.



(a) Real estate

(b) Financial industry

(c) Machinery manufacturing

(d) Transportation

FIGURE 1: Paths for intrinsic values of industries.

3.2. Portfolio Credit Risk in the Mixed Poisson Model. As mentioned above, the aggregate portfolio loss distribution is often very sensitive to the choice of copula and the estimation of parameters. This paper introduces the mixed Poisson model to describe the dynamics of default occurrence for its less model risk. For large portfolios of tens of thousands of obligors, the change of several individuals will not affect the whole loss distribution, because the loss distribution in mixed Poisson is the aggregate of all units whose model risk can be offset by each other. And the other advantages of this method are as follows:

(i) mixed Poisson models are easy to simulate in Monte Carlo risk analyses;

(ii) mixed models are more convenient for statistical fitting purposes.

This model makes no assumptions about the causes of default-credit defaults which occur as a sequence of events in such a way that it is not possible to forecast neither the exact time of occurrence of any default nor the exact total number of defaults. There is an exposure to default losses from a large number of obligors and the probability of default by any particular obligor is small. This situation is well represented by the Poisson distribution.

In the mixed Poisson model, the portfolio loss over the horizon is still

$$L = \sum_{i=1}^{m} c_i * s_i * Y_i \tag{17}$$

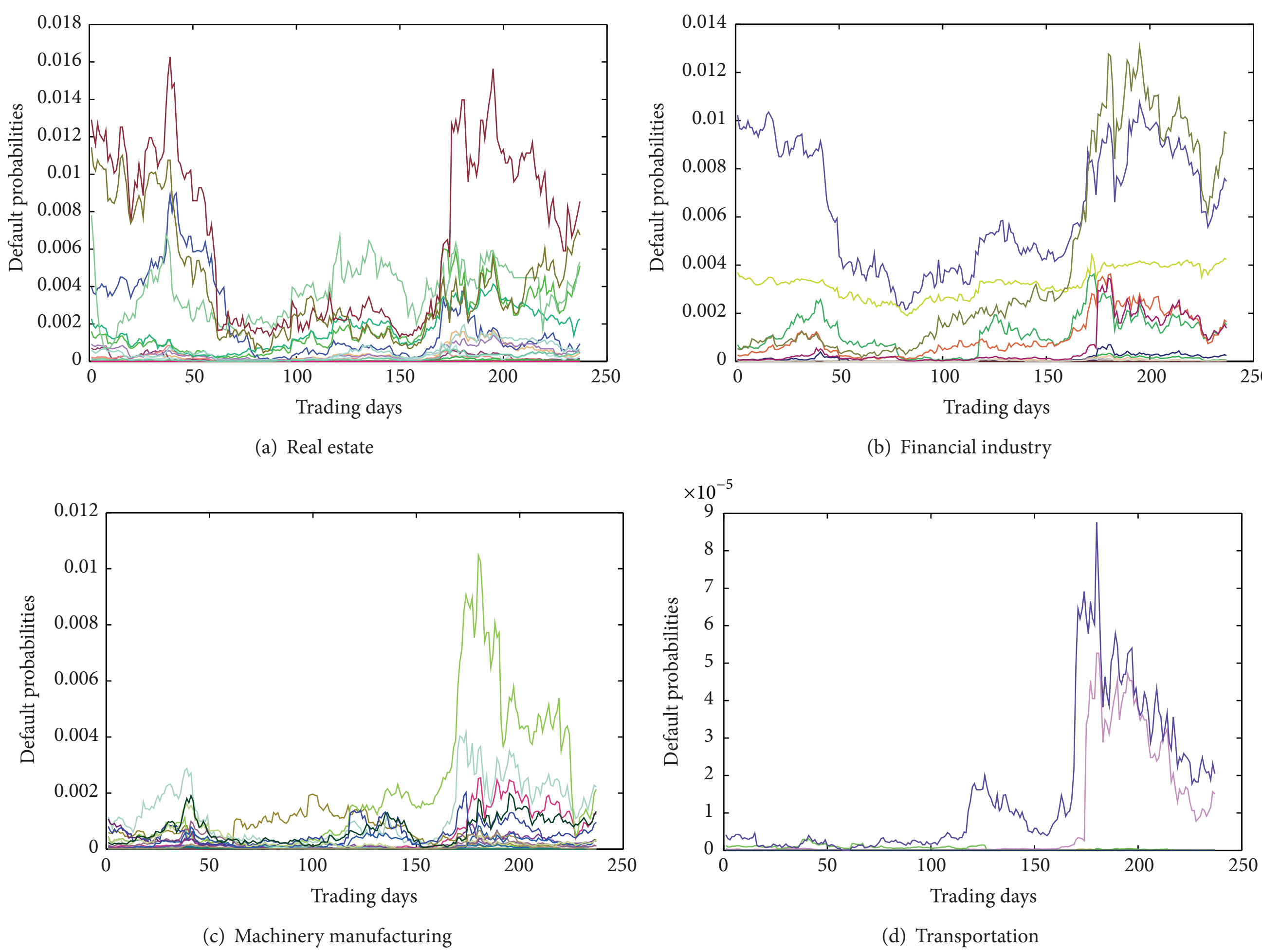
but the default indicator Y_i is generated from a Poisson distribution instead of being generated by a variable W_i falling below some threshold. Consider the following:

$$Y_i \sim \text{Poisson}\left(R_i\right). \tag{18}$$

A Poisson random variable with a very small intensity has a very small probability of taking a value which is larger than 1. Although this assumption makes it possible to default more than once, a realistic model calibration generally ensures that the probability of this happening is little.

Mixed Poisson is also a top-down model whose default probability of an obligor is virtually assumed to depend on a set of common economic factors X_j, $j = 1, 2, \ldots, k$. This mechanism is realized through the intensity of Poisson distribution. Conditional on these factors, each Y_i has a Poisson distribution with intensity R_i,

$$R_i = a_{i0} + a_{i1}X_1 + \cdots + a_{ik}X_k \tag{19}$$



(a) Real estate

(b) Financial industry

(c) Machinery manufacturing

(d) Transportation

Figure 2: Dynamic default probabilities of four industries.

for some positive coefficients $a_{i0}, \ldots, a_{id}$. Thus each Y_i may be viewed as a Poisson dynamic variable with a dynamic intensity—a mixed Poisson dynamic variable.

In the mixed Poisson model, Y_i, we talked about what follows the Poisson distribution. Because a Poisson random variable with a very small intensity has a trifling probability of taking a value which is larger than 1, Y_i can be applied as the default indicator in (17). We can achieve the results of default probabilities for each company at each trading day as illustrated in Section 2.1, which can help to bring out the Poisson intensity of Y_i.

Poisson distribution usually shows approximates with binominal distribution when $n \geq 10$, $p \leq 0.1$, so an obligor default is equal to $Y_i = 1$.

Given $P(X = k) = (e^{-\lambda}\lambda^k / k!)$ in Poisson, the intensity R_i can be estimated by

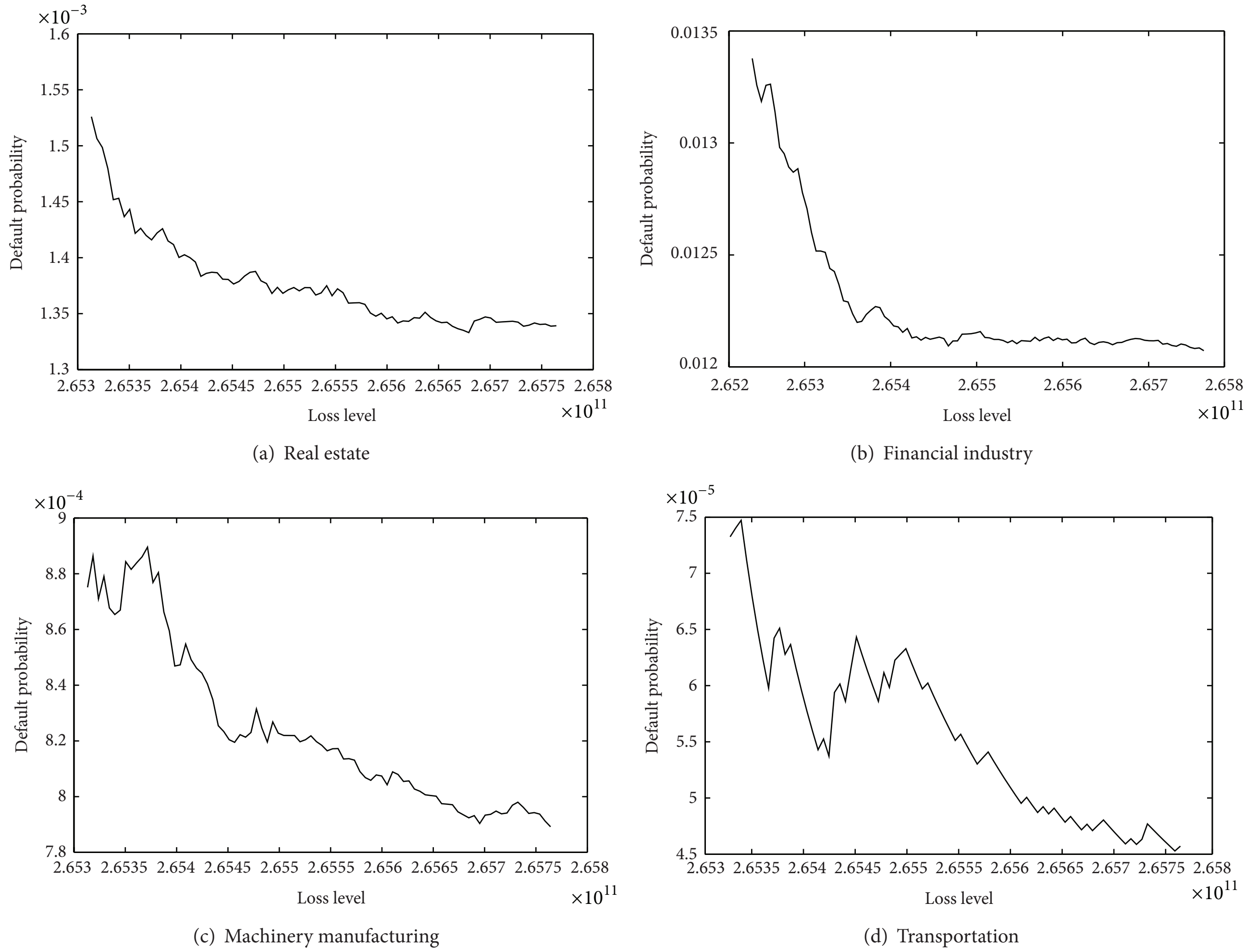
$$P_{\text{default}} = e^{-R_i} \cdot R_i. \tag{20}$$

4. Empirical Study

This paper uses mixed Poisson model to study the portfolio credit risk. Mixed Poisson is the structure describing dependence relationship among obligors and those macroeconomic factors essentially lead to the source of portfolio credit risk. It remains to find suitable factor loadings to construct a complete model. These factor loadings need to be derived from a historical default probability of single obligor.

4.1. Default Probability of Single Obligor. As mentioned in Section 2.2, public companies are the study objects of this paper. To obtain the historical default probability of single obligor, we need to know the market value of its shares, the volatility of its stock price, and the book value of its debt. Because the mixed Poisson model assumes that portfolio credit risk depends on a set of common economic factors, obviously the weight on each common factor varies with the industry characteristics. We select data of market value E_t, debt D_t, and closing price P_t for stocks in four industries of Shanghai Stock Exchange. The time horizon is from 2012.10.01 to 2013.09.30 which contains 238 trading days. Finally, we choose 20 companies from real estate, 25 from machinery manufacturing, 20 from financial industry, and 12 from transportation.

The history volatility σ^E in this study is the standard deviation of logarithmic change of closing prices.



(a) Real estate

(b) Financial industry

(c) Machinery manufacturing

(d) Transportation

Figure 3: Loss distributions of each sample portfolio under Monte Carlo simulation.

We use formulas of (5) and (9) to obtain the intrinsic values of these companies at every present moment. Further, the trajectories of V_t are shown in Figure 1.

So far, V_t is already known and we put parameters of μ, σ produced by (10), and L into (4) to get historical default probability of every single company. Part of the result can be seen in Table 1. Here, an overview of dynamic default probabilities on each industry is given in Figure 2.

From Figure 2, we can find that the dynamic patterns of default curve in (a) and (b) are similar and the patterns in (c) and (d) are also similar. As a matter of fact, real estate in (a) and financial industry in (b) do have relatively strong relationship these years. And by this analogy, in the other two industries this phenomenon also exists, where it has been assumed in the mixed Poisson model that dynamic changes of default probabilities are driven by a series of common macroeconomic factors.

4.2. Mixed Poisson Model. We use default probabilities obtained before to estimate the value of Poisson intensity of each company as illustrated in (20), and the result is shown in Table 1 (as space is limited, we list part of the result for simplicity).

This model assumes that the intensity of Poisson variable is driven dynamically by several common factors. This paper chooses index prices of real estate index, infrastructure index, transportation index, and finance index to be the macroeconomic factors in this model. We need to obtain the factor loadings $a_{i0}, a_{i1}, \ldots, a_{ik}$ for each company. Since these indexes are negatively correlated with the intensity of default, letting e^{-X_j} be factors in (19) is suitable, where X_j, $j = 1, 2, 3, 4$ are the macroeconomic factors after standardization. Consider the following:

$$R_i = a_{i0} + a_{i1} \cdot e^{-X_1} + a_{i2} \cdot e^{-X_2} + a_{i3} \cdot e^{-X_3} + a_{i4} \cdot e^{-X_4}. \quad (21)$$

With the application of nonlinear estimation on this modified Poisson intensity equation, coefficients of the factors are shown in Table 2 (part of the result for simplicity).

With the combination of coefficients and common macroeconomic factors, the intensity of each Poisson variable Y_i in our model can be generated. Then we turn to formula (17) to get the loss distribution of sample credit portfolio.

Table 1: Dynamic default probabilities and Poisson intensity.

Stock code		Ordinal number of each trading day						
		1	2	3	⋯	236	237	238
Real Estate								
600048.SH	P^*	$1.604E-03$	$1.297E-03$	$1.243E-03$	⋯	$1.604E-03$	$1.297E-03$	$1.243E-03$
	R^*	$1.607E-03$	$1.299E-03$	$1.244E-03$		$4.411E-03$	$4.656E-03$	$5.134E-03$
600077.SH	P	$6.251E-03$	$5.358E-03$	$4.638E-03$		$9.777E-03$	$1.230E-02$	$1.241E-02$
	R	$6.290E-03$	$5.387E-03$	$4.660E-03$		$9.874E-03$	$1.245E-02$	$1.256E-02$
600162.SH	P	$3.985E-03$	$3.655E-03$	$3.585E-03$		$5.782E-04$	$7.794E-04$	$9.436E-04$
	R	$4.001E-03$	$3.668E-03$	$3.598E-03$		$5.785E-04$	$7.800E-04$	$9.445E-04$
Machinery								
600150.SH	P	$2.326E-02$	$2.148E-02$	$2.213E-02$	⋯	$2.326E-02$	$2.148E-02$	$2.213E-02$
	R	$2.382E-02$	$2.196E-02$	$2.264E-02$		$3.336E-03$	$5.137E-03$	$5.000E-03$
600166.SH	P	$1.156E-03$	$9.690E-04$	$7.421E-04$		$2.018E-03$	$2.283E-03$	$2.214E-03$
	R	$1.158E-03$	$9.700E-04$	$7.427E-04$		$2.023E-03$	$2.289E-03$	$2.219E-03$
600169.SH	P	$4.647E-04$	$2.570E-04$	$2.359E-04$		$1.387E-03$	$1.991E-03$	$2.138E-03$
	R	$4.649E-04$	$2.570E-04$	$2.360E-04$		$1.389E-03$	$1.995E-03$	$2.143E-03$
Financial Industry								
600000.SH	P	$3.869E-02$	$3.784E-02$	$3.791E-02$	⋯	$2.539E-02$	$2.797E-02$	$2.766E-02$
	R	$3.722E-02$	$3.644E-02$	$3.650E-02$		$2.475E-02$	$2.720E-02$	$2.691E-02$
600015.SH	P	$2.794E-02$	$2.641E-02$	$2.650E-02$		$1.737E-02$	$1.892E-02$	$1.854E-02$
	R	$2.717E-02$	$2.572E-02$	$2.580E-02$		$1.707E-02$	$1.857E-02$	$1.820E-02$
600016.SH	P	$3.719E-02$	$3.659E-02$	$3.674E-02$		$2.705E-02$	$2.758E-02$	$2.761E-02$
	R	$3.583E-02$	$3.527E-02$	$3.541E-02$		$2.633E-02$	$2.683E-02$	$2.686E-02$

[*] (1) P represents the default probability of each obligor.
(2) R represents the estimation of Poisson intensity.

Table 2: Coefficients of the factors in mixed Poisson model.

Stock code	Common factors				Specific factor
	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i0}
Real Estate					
600048.SH	$5.024E-03$	$1.455E-02$	$6.538E-02$	$-4.537E-02$	$-1.272E-02$
600077.SH	$-4.543E-02$	$1.013E-01$	$5.354E-02$	$-2.203E-02$	$-2.747E-02$
600162.SH	$-2.875E-02$	$4.103E-02$	$-3.844E-02$	$6.869E-02$	$-1.359E-02$
Machinery					
600150.SH	$1.060E-01$	$-1.010E-01$	$-2.355E-01$	$3.381E-01$	$-2.440E-02$
600166.SH	$-1.511E-03$	$2.158E-02$	$3.428E-02$	$-1.519E-02$	$-1.318E-02$
600169.SH	$1.956E-02$	$8.639E-03$	$1.278E-01$	$-8.848E-02$	$-2.282E-02$
Financial Industry					
600000.SH	$3.760E-02$	$-1.212E-01$	$1.499E-01$	$1.111E-01$	$-3.520E-02$
600015.SH	$2.131E-02$	$-1.426E-02$	$7.727E-02$	$8.240E-02$	$-4.316E-02$
600016.SH	$-4.171E-02$	$7.909E-02$	$-3.450E-02$	$1.204E-01$	$-1.721E-02$

Since the relevant data about recovery rate of default is extremely rare, we assume $s_i = 1$, which means that the default exposure is just the loss if any defaults. We now use historical data of the four macroeconomic indexes to run the Monte Carlo simulation. This is easily implemented through the following algorithm:

(1) input the relevant macroeconomic factors X_j into this model;

(2) compute R_i, $i = 1, 2, \ldots, m$, from (21);

(3) generate $Y_i \sim \text{Poisson}(R_i)$, $i = 1, 2, \ldots, m$;

(4) calculate portfolio loss L from (17);

(5) return to step (1).

The loss distribution of each sample portfolio is shown in Figure 3. Each point in it is based on 10,000 simulations. And the specific loss percentage and default probability of several points are listed in Table 3 (transportation portfolio is not listed because of its low default probabilities) which also gives the standard deviation of them.

Table 3: Standard deviations of the default probabilities.

Portfolio		Loss level (percentage)				
		0.05%	0.09%	0.13%	0.23%	0.50%
Real Estate	P^*	0.153%	0.141%	0.133%	0.120%	1.163E-004
	Std.*	$3.5E-004$	$3.1E-004$	$2.3E-004$	$1.7E-004$	$1.05E-005$
Financial Industry	P	1.32%	1.28%	1.21%	0.71%	0.14%
	Std.	$6.81E-003$	$6.77E-003$	$5.47E-003$	$7.6E-004$	$2.21E-005$
Machinery	P	$8.73E-004$	$8.69E-004$	$8.04E-004$	$6.27E-004$	$3.245E-005$
	Std.	$2.17E-004$	$3.16E-004$	$2.81E-004$	$2.11E-004$	$6.22E-006$

* (1) P represents the default probability of each portfolio.
(2) Std. represents the standard deviation of each calculation.

From Figure 3, we can observe that the default probability of each industry portfolio decreases with the increase of loss level. And the default probability in financial industry is the largest, which also does meet the fact that financial companies such as banks are usually highly leveraged. Further, the standard deviation of each calculation in Table 3 represents the reliability of MC simulation and indicates that we can accept these results. This Monte Carlo simulation illustrates the practical value and effectiveness of mixed Poisson model in risk measurement for credit portfolio.

5. Conclusions

Mixed Poisson model is introduced in this paper to replace the widely used copula model. To apply the mixed Poisson theory to practical study, we bring the structural approach into the calculation of single obligor's default probability, which helps to estimate the parameters of mixed Poisson model. Finally, Monte Carlo simulation drives out the curve about default probabilities and loss levels, which is in accordance with the practical rules. This study illustrates the practical value and effectiveness of mixed Poisson model in risk measurement for credit portfolio.

Because good data on credit losses is extremely rare in financial market of China, we use the data in stock market for substitution based on the assumptions of structural approach. If there are enough default data in a sound financial market, estimation of model parameters can be more accurate. And the number of obligors in our sample portfolio is relatively small. We believe a Monte Carlo simulation of a larger sample will be much more stable.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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