Hindawi Publishing Corporation
Journal of Industrial Engineering
Volume 2014, Article ID 429836, 11 pages
http://dx.doi.org/10.1155/2014/429836

Research Article

Optimal Pricing and Ordering Policy for Two Echelon Varying Production Inventory System

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Received 8 June 2014; Accepted 5 August 2014; Published 11 September 2014

Academic Editor: Leopoldo Eduardo Cardenas-Barron

The paper proposes a two-stage supply chain model for price sensitive demand in imperfect production system while manufacturer and supplier are the members of the chain. The supplier screens the raw materials first and supplies good materials to the manufacturer at a constant rate. The production rate varies randomly within a finite interval. The inventory cycle of the manufacturer starts with shortages and production and it finishes with shortages again, in which shortages are partially backlogged. We consider a mixture of LIFO (last in, first out) and FIFO (first in, first out) dispatching policies to fill the backlogged demand. Thus, the objective of the proposed paper is to determine the optimal ordering lot-size and selling price of the manufacturer such that the per unit average integrated expected profit of the supply chain model is maximized. A numerical example is provided to analyze and illustrate the behavior and application of the model. Finally, sensitivity analysis of the key parameters are presented to test feasibility of the model.

1. Introduction

Nowadays, the production inventory management has absorbed a lot of interest, among the researchers as well as practitioners, because production process is one of the most important parts to the companies for their business strategy. In reality, manufacturing companies have been facing big challenges to adjust and control the proper production rate, considering the limitations of labor, machines, power, technology, raw materials, environment, and so forth. Careful production planning is necessary to enrich any businesses, otherwise companies may face overages or shortages of the products. In the increased globalization and competitive marketing system, it is main concern for the industries to find out marketing strategies in production process. Now, our aim is to study a supply chain model consisting of supplier and manufacturer incorporating the important factors: production, pricing, partial backlogging, dispatching policy, defective items, and so forth.

Joint pricing and replenishment policy for deteriorating inventory, where demand decreases linearly with time and cost of items, was developed by Wee [1]. Abad [2] studied an optimal pricing and lot-sizing model under conditions of perishability and partial backordering. Salameh and Jaber [3] introduced a modified inventory model for imperfect quality items when using the EPQ/EOQ formulae. Hayek and Salameh [4] addressed a production inventory model where the production system was not perfect. They studied the effect of imperfect quality items on the finite production model. Yang [5] analyzed an integrated buyer-vendor mathematical model for deteriorating item with quantity discount and found out optimal replenishment and pricing policy for price sensitive demand. Papachristos and Skouri [6] reviewed the papers of Salameh and Jaber [3] and discussed the issue of no shortages in inventory models with imperfect quality and especially in models with proportional imperfect quality. Sarker and Al Kindi [7] studied an EOQ model with optimal ordering policies during the sale period for different scenarios where the optimal ordering policies were compared for the different discount scenarios to study the effect of the discounted price and the length of the sale period. Wee et al. [8] extended the model of Salameh and Jaber [3] and developed an optimal inventory model for items with imperfect quality and shortage backordering. Lo et al. [9] developed an integrated production and inventory model considering the factors varying rate of deterioration, partial backordering, inflation, imperfect production processes, and multiple deliveries from the perspectives of both the manufacturer and the retailer. Cárdenas-Barrón [10] corrected the derivation of the optimal ordering policies, mathematical expressions of the model of Sarker and Kindi [7]. He also derived the closed forms for the optimal total gain costs for each case. Sana [11] studied a production-inventory model in an imperfect production process over a finite planning horizon where the production rate was a dynamic variable and demand rate was time sensitive. An EOQ (economic order quantity) model of deteriorating item over finite time horizon was addressed by Sana [12, 13]. He considered that the demand rate was quadratic decreasing function of price and the time horizon divided into n equal periods with n different prices. Khanra et al. [14] analyzed an EOQ model taking time varying quadratic demand for a deteriorating item having time dependent demand when delay in payment was permissible. Zhang et al. [15] studied a two-item EOQ model with partial backordering in consideration of the correlated demand caused by cross-selling. They considered that the sales of the minor item can either be promoted by the successful sales or be pulled down by the lost sales of the major item where the demand of the major item was independent, but the demand of the minor item was partially correlated with the sales of the major item. Toews et al. [16] addressed a deterministic EOQ and EPQ (economic production quantity) model with partial backordering at a rate that is linearly dependent on time to delivery. Sana [12] presented a three-layer imperfect production inventory model where replenishment lot-size of supplier and production rate of manufacturer were decisions variables and production cost per unit item was dependent on production rate. Sarkar and Moon [17] studied a production inventory model for stochastic demand with the effect of inflation. Pal et al. [18] extended the model of Sana [12] introducing product reliability, different holding cost for good and defective items, and reworking of defective items in the environment of supply chain management. Glock [19] reviewed lot-size models focusing on coordinated inventory replenishment decisions between buyer and vendor and their impact on the performance of the supply chain. Pal et al. [20] analyzed a multi-item EOQ inventory model considering price and level of price breaks dependent demand rate. They also assumed that vendor offered discount on selling price to the customers according to level of price breaks. Sicilia et al. [21] developed an inventory system with a mixture of backorders and lost sales, where the backordered demand rate was an exponential function of time, the customers waiting time before receiving the item. Wee and Widyadana [22] studied an integrated single-vendor single-buyer inventory model with multiple deliveries considering stochastic machine unavailability time. Pal et al. [23] analyzed an imperfect EPQ price dependent inventory model over two types of cycles where the retailer sells only good product with actual price in the first cycle and, in the second, he sells the products with a discount price. Chang [24] developed an integrated production-inventory problem for deteriorating items in a two-echelon supply chain assuming that the deterioration rate of the item was a constant or follows a continuous probability distribution function. Lee and Kim [25] addressed an integrated production-distribution model to determine an optimal policy with both deteriorating and defective items under a single-vendor single-buyer system. San-José et al. [26] studied a production-inventory model for a single product with shortages and lost sale where both the backorder cost and the lost sale cost depended on a fixed cost and a cost proportional to the shortage time. They applied a mixture between the dispatching policies known as LIFO (last in, first out) and FIFO (first in, first out) in the discipline of service to fill the backlogged demand. Many researchers [6, 27–39] and (Pentico and Drake [40] studied production inventory model, considering many issues such as pricing, imperfect production, and backlogging of products.

In this paper, a two-level supply chain of supplier and manufacturer has been addressed in an imperfect production system while production rate varies randomly within finite limits. The supplier supplies good raw material, after screening the whole lot, to the manufacturer and the defective items are sold at lower price to outsider at a single lot. The market demand of the products is considered as selling price sensitive. The inventory cycle of the manufacturer starts and ends with shortages where shortages are partially backlogged. When production is running but inventory level remains into shortages, then the manufacturer applies mixture of LIFO and FIFO dispatching policies to fill up backlogged demand. Now, the objective of our paper is to find out the optimal inventory lot-size and optimal selling price so that the integrated expected per unit average profit of the chain is maximized.

The rest of the paper is organized as follows. Section 2 illustrates fundamental assumptions and notations. Formulation of the model is discussed in Section 3. Section 4 analyzes Numerical analysis. Sensitivity analysis is illustrated in Section 5 and finally conclusion of the paper is provided in Section 6.

2. Fundamental Assumptions and Notations

2.1. Assumptions. The following assumptions are adopted to develop the model.

(i) Model is developed for single item over infinite planning horizon.

(ii) Replenishment rate of supplier is instantaneously infinite, but its size is finite; that means replacement of lot-size is sufficiently large at any number, if needed.

(iii) Demand rate of the customers is dependent on selling price.

(iv) Production rate of the inventory system is not fixed.

(v) Production system is not perfect; it produces good as well as defective items.

(vi) Production cost per unit item depends on production lot-size.

(vii) Shortages are allowed and partially backlogged.

(viii) When production is running but inventory level remains into shortages, the manufacturer considers the mixture between the LIFO and FIFO dispatching policies to satisfy the customer service.

2.2. Notations. The following notations are used throughout the paper.

Q: Replenishment lot-size of supplier

$D(p)$: Demand rate of the customers

α: Fraction of defective raw materials at supplier level

$\beta D(0)$: Production rate of the defective items

λ: Rate of fraction of backorder shortages

A_s: Setup cost of the supplier

h_s: Holding cost (\$) per unit per unit time of the supplier

h_m: Holding cost (\$) per unit per unit time for good items of the manufacturer

h'_m: Holding cost (\$) per unit per unit time for defective items of the manufacturer

h''_m: Holding cost (\$) per unit per unit time for stored raw material of the manufacturer

b_c: The backlogging cost (\$) per unit per unit time due to shortages

l_c: The lost sale cost (\$) per unit per unit time

w_s: Selling price (\$) per unit of good raw materials at supplier level

w'_s: Selling price (\$) per unit of defective raw materials at supplier level

p': Selling price (\$) per unit of defective products at manufacturer level

C_r: Purchasing cost (\$) per unit item

$C_p(Q)$: Per unit production cost (\$)

t_1: Time length of delivering raw material by the supplier

t_r: Time length of backlogging during production run

t_p: Production run-time of the manufacturer

$t_s - t_p$: Time required for selling the items which are stored during production run

$T - t_s$: Time cycle of shortages when production is stopped

T: Cycle time of the supply chain.

3. Formulation of the Model

In the model, we formulate a two-echelon production inventory model (see Figure 1) considering customers' demand $D(p)$ of the products is price sensitive where $D'(p) = dD(p)/dp < 0$. The supplier delivers the screened good raw materials to the manufacturer for production. The manufacturer production system is not perfect and the production rate

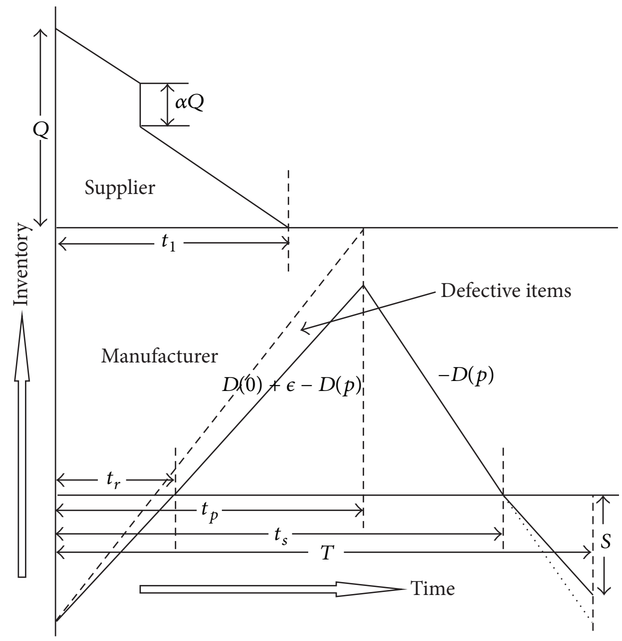

Figure 1: Logistic diagram of the production inventory model.

is also not fixed. It produces defective products at a rate $\beta D(0)$ where $(1 + \beta)D(0) + \epsilon$ is the production rate of the system. We assume that ϵ is a random variable whose variation of range is not so high. Here, we consider that the range of ϵ is $-3\mu/2 \le \epsilon \le \mu$ where μ is a positive real constant greater than 1 but very small compared to $D(0)$. The production rate of the system is not generally fixed due to quality of raw material, labour performance, weather, and so forth. The situation of lesser production from target rate is generally more incidence than the situation of higher production from target rate. So, we consider that the addition part (ϵ) of production rate is random variable and its range is $-3\mu/2 \le \epsilon \le \mu$. The defective items are sold in a lot to the other markets after completing the production. We develop the model considering that the inventory cycle of the manufacturer starts with shortages and production. In the time interval, when production and shortages are both running in the system, two cases may occur. If the new demand is satisfied before backlogged then the system is in LIFO (last in, first out). In the other case, when backlogged order is filled up before new demand, then the system is in FIFO (first in, first out). In that time interval, we formulate the model considering the mixture of LIFO and FIFO dispatching policies. Recovering from backlogged situation, the manufacturing system produces products up to time t_p and then he sells the stored items to the customers up to time t_s. The system falls again in the shortages after time t_s continues up to end of running cycle (T). The individual cases for the supplier and the manufacturer are discussed in the following subsection.

3.1. Suppliers Individual Profit. In the study, the supplier screens the raw material at the rate S_r and supplies the good raw material at a rate $\{(1 + \beta)D(0) + \mu\}$ to the manufacturer. The manufacturer orders to supply materials at rate $\{(1+\beta)D(0)+\mu\}$ to avoid production disruption during production run-time because the production rate is $(\{1+\beta\}D(0)+\epsilon)$ where ϵ is a random variable with range $-3\mu/2 \le \epsilon \le \mu$. So, maximum production rate may be $\{(1+\beta)D(0)+\mu\}$. The total defective materials at supplier level are sent back with sale price w_s' per unit item to the outside.

The governing differential equation, in this stage, is

$$\frac{dI_s(t)}{dt} = -\{(1+\beta)D(0)+\mu\}, \quad \text{with } I_s(0) = (1-\alpha)Q, \quad I_s(t_1)=0, \quad 0 \le t \le t_1. \tag{1}$$

Using the above boundary conditions, we have, from (1),

$$I_s(t) = Q(1-\alpha) - \{(1+\beta)D(0)+\mu\}t, \quad 0 \le t \le t_1. \tag{2}$$

Now,

$$I_s(t_1) = 0 \Longrightarrow t_1 = \frac{Q(1-\alpha)}{\{(1+\beta)D(0)+\mu\}}. \tag{3}$$

Therefore, using (2) and (3), the holding cost for good material is

$$\mathrm{IC}_s = h_s \int_0^{t_1} I_s(t)\,dt = h_s \frac{Q^2(1-\alpha)^2}{2\{(1+\beta)D(0)+\mu\}}. \tag{4}$$

The holding cost for damaged/imperfect material is

$$\mathrm{IC}_{s_d} = h_s \alpha Q \frac{Q}{S_r} = \frac{h_s \alpha Q^2}{S_r}. \tag{5}$$

The average profit of supplier, using (4) and (5), is

$$\begin{aligned} E\pi_s(Q) &= \frac{1}{Q}[\text{Total revenue from sales} - \text{Total inventory cost} \\ &\quad - \text{Screening cost} - \text{Total setup cost}] \\ &= \frac{1}{Q}\left[\{(1-\alpha)w_s + \alpha w_s' - C_s\}Q \right. \\ &\quad \left. - h_s\left(\frac{Q^2(1-\alpha)^2}{2\{(1+\beta)D(0)+\mu\}} + \frac{\alpha Q^2}{S_r}\right) - A_s\right] \\ &= \{(1-\alpha)w_s + \alpha w_s' - C_s\} \\ &\quad - h_s\left(\frac{(1-\alpha)^2}{2\{(1+\beta)D(0)+\mu\}} + \frac{\alpha}{S_r}\right)Q - \frac{A_s}{Q}. \end{aligned} \tag{6}$$

3.2. Manufacturers Individual Profit. In the proposed study, the manufacturer starts production for a new cycle when the backlogged level of products touches the level S. Collecting the raw materials at rate $(1+\beta)D(0)+\mu$ from the supplier, the manufacturer produces the products at varying production rate $(1+\beta)D(0)+\epsilon$ where ϵ is a random variable with range $-3\mu/2 \le \epsilon \le \mu$. As the maximum production rate may be $(1+\beta)D(0)+\mu$, the manufacturer stores the extra raw materials in a temporary warehouse at a rate $[\{(1+\beta)D(0)+\mu\} - \{(1+\beta)D(0)+\epsilon\}] = (\mu-\epsilon)$ up to time t_1 and then the stored materials are used for the rest of production run-time $(t_p - t_1)$. The manufacturing system is not totaly perfect. It produces defective items at constant rate $\beta D(0)$ which are sold in a lot after the production run with less selling price. As the manufacturer faces both backlogged demand and the new customers' demand in the beginning of production, he uses the mixture of LIFO and FIFO dispatching policies. In the time interval when production and shortages are both running, a fraction (θ) of new demands is only filled up according to the priority of the customers demand $(D(p))$ with backlogged demand. In that time, the rest of the new customers $((1-\theta)D(p))$ have to wait for their demand, but a fraction $(1-\lambda)$ of that customers leave the system for their busy schedule or other reasons and others $(\lambda(1-\theta)D(p))$ are waiting for their demand products. Recovering from backlogged situation, that is, after time t_r, the production of the manufacturing system is continuing up to time t_p and the finished products are piled up at rate $[\{(1+\beta)D(0)+\epsilon\} - D(p)]$ through the time $(t_p - t_r)$. After the production run, these finished stored products satisfied the customers' demand $(D(p))$ up to time t_s. Again, the system falls again into the shortages after time t_s continues up to end of running cycle (T), that is, when the shortages level of products touches the level S. Now, the on-hand inventory $I_{m_i}(t)$, $(i = 1,2,3,4)$ for good items, I_{m_d} for defective products, and $I_{w_i}(t)$, $(i = 1,2)$ for stored raw materials in temporary warehouse at time t at the manufacturer can be represented by the following differential equations considering $\psi = \theta + \lambda(1-\theta) = \theta + \lambda - \lambda\theta$:

$$\begin{aligned} &\frac{dI_{m_1}(t)}{dt} = \{(D(0)+\epsilon) - \psi D(p)\} - S, \\ &\text{with } I_{m_1}(t_r) = 0, \quad I_{m_1}(0) = -S, \quad 0 \le t \le t_r. \\ &\frac{dI_{m_2}(t)}{dt} = (D(0)+\epsilon) - D(p), \\ &\text{with } I_{m_2}(t_r) = 0, \quad t_r \le t \le t_p. \\ &\frac{dI_{m_3}(t)}{dt} = -D(p), \\ &\text{with } I_{m_3}(t_p) = I_{m_2}(t_p), \quad I_{m_3}(t_s) = 0, \quad t_p \le t \le t_s, \\ &\frac{dI_{m_4}(t)}{dt} = -\lambda D(p), \quad \text{with } I_{m_4}(t_s) = 0, \quad t_s \le t \le T, \\ &\frac{dI_{m_d}(t)}{dt} = \beta D(0), \quad \text{with } I_{m_d}(0) = 0, \quad 0 \le t \le t_p, \\ &\frac{dI_{m_{w_1}}(t)}{dt} = (\mu - \epsilon), \quad \text{with } I_{m_{w_1}}(0) = 0, \quad 0 \le t \le t_1, \\ &\frac{dI_{m_{w_2}}(t)}{dt} = (\mu-\epsilon)t_1 - \{(1+\beta)D(0)+\epsilon\}, \\ &\text{with } I_{m_{w_2}}(t_p) = 0, \quad t_1 \le t \le t_p. \end{aligned} \tag{7}$$

Using the above boundary conditions, we have, from (7),

$$I_{m_1}(t) = (t - t_r)(D(0) + \epsilon - D(p)\psi - S), \quad 0 \le t \le t_r,$$
$$I_{m_2}(t) = (t - t_r)(D(0) + \epsilon - D(p)), \quad t_r \le t \le t_p,$$
$$I_{m_3}(t) = (t_p - t_r)(D(0) + \epsilon) - D(p)(t - t_r), \quad t_p \le t \le t_s,$$
$$I_{m_4}(t) = \lambda D(p)(t_s - t), \quad t_s \le t \le T,$$
$$I_{m_d}(t) = D(0)\beta t, \quad 0 \le t \le t_p,$$
$$I_{m_{w_1}}(t) = (\mu - \epsilon)t, \quad 0 \le t \le t_1,$$
$$I_{m_{w_2}}(t) = (t - t_p)\{t_1(\mu - \epsilon) - (D(0)(1 + \beta) + \epsilon)\}, \quad t_1 \le t \le t_p. \tag{8}$$

Again

$$I_{m_1}(0) = -S \Longrightarrow t_r = \frac{S}{D(0) + \epsilon - D(p)\psi - S},$$
$$t_p = \frac{\text{Production lot-size}}{\text{Production rate}} = \frac{Q(1 - \alpha)}{D(0)(1 + \beta) + \epsilon},$$
$$I_{m_3}(t_s) = 0 \Longrightarrow t_s = \frac{1}{D(p)}\left[\frac{Q(1 - \alpha)(D(0) + \epsilon)}{D(0)(1 + \beta) + \epsilon} - \frac{S(D(0) + \epsilon - D(p))}{D(0) - S + \epsilon - D(p)\psi}\right],$$
$$I_{m_4}(T) = -S \Longrightarrow T = \frac{1}{D(p)\lambda} \times \left[S + \left(\frac{Q(1 - \alpha)(D(0) + \epsilon)}{D(0)(1 + \beta) + \epsilon} - \frac{S(D(0) + \epsilon - D(p))}{D(0) - S + \epsilon - D(p)\psi}\right)\lambda\right]. \tag{9}$$

The inventory cost for good items is

$$\begin{aligned}\mathrm{IC}_{m_g} &= h_m\left[\int_0^{t_r} I_{m_1}(t)\,dt + \int_{t_r}^{t_p} I_{m_2}(t)\,dt + \int_{t_p}^{t_s} I_{m_3}(t)\,dt + \int_{t_s}^{T} I_{m_4}(t)\,dt\right] \\ &= \Big(\{D(0) + \epsilon\}\{D(0) - D(p) + \epsilon\} \\ &\quad \times [D(0)S(1 + \beta) + S\epsilon - Q(1 - \alpha) \\ &\quad \times \{D(0) - S + \epsilon - \psi D(p)\}]^2\Big) \\ &\quad \times \left(2D(p)\{D(0)(1 + \beta) + \epsilon\}^2 \times \{D(0) - S + \epsilon - \psi D(p)\}^2\right)^{-1}.\end{aligned} \tag{10}$$

The inventory cost for defective items is

$$\mathrm{IC}_{m_d} = h'_c \int_0^{t_p} I_{m_d}(t)\,dt = \frac{h'_c D(0) Q^2 (1 - \alpha)^2 \beta}{2\{D(0)(1 + \beta) + \epsilon\}^2}. \tag{11}$$

The inventory cost for stored raw materials in temporary warehouse is

$$\begin{aligned}\mathrm{IC}_{m_w} &= h''_c \int_0^{t_1} I_{w_1}(t)\,dt + \int_{t_1}^{t_p} I_{w_2}(t)\,dt \\ &= \frac{h''_c Q^2 (1 - \alpha)^2 (\mu - \epsilon)}{2((1 + \beta)D(0) + \epsilon)^2((1 + \beta)D(0) + \mu)^3} \\ &\quad \times \left[(D(0))^3(1 + \beta)^3 - Q(1 - \alpha)(\mu - \epsilon)^2 + \epsilon\mu^2 + D(0) \times (1 + \beta)\mu(2\epsilon + \mu) + D^2(0)(1 + \beta)^2(\epsilon + 2\mu)\right].\end{aligned} \tag{12}$$

The total amount of backlogged cost at the end of cycle is

$$\begin{aligned}\mathrm{BC}_m &= \int_0^{t_r} (1 - \theta)\lambda D(p)\,dt + \int_{t_s}^{T} \lambda D(p)\,dt \\ &= S + \frac{D(p) S (1 - \theta)\lambda}{D(0) - S + \epsilon - \psi D(p)}.\end{aligned} \tag{13}$$

The total amount of lost sale cost during shortages time is

$$\begin{aligned}\mathrm{LC}_m &= l_c\left((1 - \theta)\int_0^{t_r} (1 - \lambda) D(p)\,dt + \int_{t_s}^{T} (1 - \lambda) D(p)\,dt\right) \\ &= l_c\left[\frac{D(p) S (1 - \theta)(1 - \lambda)}{D(0) - S + \epsilon - D(p)\psi} + \frac{S(1 - \lambda)}{\lambda}\right].\end{aligned} \tag{14}$$

In this case, total expected average per unit quantity profit of the manufacturer is

$$\begin{aligned}E\pi_m(Q, p) &= \frac{1}{Q}[\text{Total expected revenue from sales} \\ &\quad - \text{Total expected production cost} \\ &\quad - \text{Total expected inventory cost} \\ &\quad - \text{Total expected backlogging cost}\end{aligned}$$

$$- \text{Total expected lost sale cost}]$$
$$= \frac{1}{Q} E\Big[(D(0)+\epsilon)\, p t_p + p'\beta D(0)\, t_p - (1-\alpha) Q\left(w_s + C_p\right) - \mathrm{IC}_{m_g} - \mathrm{IC}_{m_d} - \mathrm{IC}_{m_w} - \mathrm{BC}_m - \mathrm{LC}_m\Big]. \tag{15}$$

3.3. Integrated Profit. Hence, from (6) and (15), the integrated expected average profit per unit per quantity is

$$E\pi(Q,p) = E\pi_s(Q) + E\pi_m(Q,p) = Z_1(p) + QZ_2(p) + Q^2 Z_3(p) + \frac{Z_4(p)}{Q}, \tag{16}$$

$$Z_1(p) = \frac{2h_m S(1-\alpha)}{5(A-Bp^2)(S-Bp^2\psi + A(\beta+\psi))\mu} \times \Bigg[A\beta\left(A-Bp^2+A\beta\right)\operatorname{Log}\left[\frac{2A(1+\beta)-3\mu}{A(1+\beta)+\mu}\right] - \left(S^2 - \left(A-Bp^2\right)^2(1-\psi)\psi - \left(A-Bp^2\right)S(1-2\psi)\right) \times \operatorname{Log}\left[\frac{2\left(A(1-\psi)-S+Bp^2\psi\right)-3\mu}{A(1-\psi)-S+Bp^2\psi+\mu}\right]\Bigg] + (1-\alpha)(p-w_s) - 2A\left(p-p'\right)(1-\alpha)\beta \times \operatorname{Log}\left[\frac{A(1+\beta)+\mu}{A(1+\beta)-3\mu/2}\right](5\mu)^{-1} - \frac{h_m S(1-\alpha)((A\beta - S)\operatorname{Log}[4] - 5\mu)}{5(A-Bp^2)\mu} - \frac{2h_m S(1-\alpha)(1-\psi)\operatorname{Log}[2]}{5\mu},$$

$$Z_2(p) = -\frac{1}{10}(1-\alpha)^2 \times \Bigg[\frac{10Ah_m\beta\left(A-Bp^2+A\beta\right)}{(A-Bp^2)(2A(1+\beta)-3\mu)(A(1+\beta)+\mu)} + \frac{10Ah'_m\beta}{(2A(1+\beta)-3\mu)(A+A\beta+\mu)} + \frac{2}{\mu}\operatorname{Log}\left[\frac{2A(1+\beta)-3\mu}{A(1+\beta)+\mu}\right] \times \left(\frac{h_m\left(A-Bp^2+2A\beta\right)}{(A-Bp^2)} - h''_m\right) + \frac{h''_m(A(1+\beta)\operatorname{Log}[4] - \mu(5-\operatorname{Log}[4]))}{\mu(A+A\beta+\mu)} + \frac{5h_m}{A-Bp^2} - \frac{h_m\operatorname{Log}[4]}{\mu}\left(1+\frac{2A\beta}{A-Bp^2}\right)\Bigg],$$

$$Z_3(p) = \frac{h''_m(1-\alpha)^3}{5\mu(A(1+\beta)+\mu)^3} \times \Bigg[\{A(1+\beta)+\mu\}^2 \times \left\{3\operatorname{Log}\left[\frac{2A(1+\beta)-3\mu}{A(1+\beta)+\mu}\right] - 1 - 3\operatorname{Log}[2]\right\} + \left(2A(1+\beta)\left\{8A^2(1+\beta)^2 + 64A(1+\beta)\mu + 29\mu^2\right\} - 179\mu^3\right)(8(2A(1+\beta)-3\mu))^{-1}\Bigg],$$

$$Z_4(p) = -\Bigg[h_m S^2\left(S^2 + Bp^2 S(1-2\psi) - A^2(1-\psi)\psi - B^2p^4(1-\psi)\psi + A\left(2Bp^2(1-\psi)\psi - S(1-2\psi)\right)\right) \times \left(\left(A-Bp^2\right)\left(A(1-\psi)+Bp^2\psi+\mu-S\right) \times \left(2\left(A(1-\psi)+Bp^2\psi-S\right)-3\mu\right)\right)^{-1} + \frac{h_m S^2\left(2S - (1-2\psi)\left(A-Bp^2\right)\right)}{5(A-Bp^2)\mu} \times \Bigg[\operatorname{Log}[2] - \operatorname{Log}\left[\frac{2\left(A(1-\psi)+Bp^2\psi-S\right)-3\mu}{A(1-\psi)+Bp^2\psi-S+\mu}\right]\Bigg] + l_c\Bigg[\frac{S(1-\lambda)}{\lambda} + \frac{2\left(A-Bp^2\right)S(1-\theta)(1-\lambda)}{5\mu} \times \operatorname{Log}\left[\frac{A-S-\left(A-Bp^2\right)\psi+\mu}{A-S-(A-Bp^2)\psi-3\mu/2}\right]\Bigg] + \frac{h_m S^2}{2(A-Bp^2)} + L(1-\alpha)$$

$$+ b_c \left(S + \frac{2\left(A - Bp^2\right) S (1-\theta) \lambda}{5\mu} \times \operatorname{Log}\left[\frac{A - S - \left(A - Bp^2\right)\psi + \mu}{A - S - \left(A - Bp^2\right)\psi - 3\mu/2} \right] \right) \bigg] . \quad (17)$$

Proposition 1. $Z_4(p) < 0$ *for all* $0 < p < \sqrt{A/B}$.

Proof. In the expression $Z_4(p)$, we see that all the terms are individual positive within the third bracket except the first two terms for all $0 < p < \sqrt{A/B}$. First term is very small, it does not affect very much. In the second term, we see that $[\operatorname{Log}[2] - \operatorname{Log}[(2(A(1-\psi) + Bp^2\psi - S) - 3\mu)/(A(1-\psi) + Bp^2\psi - S + \mu)]] = [\operatorname{Log}[2] - \operatorname{Log}[2] + \operatorname{Log}[(A(1-\psi) + Bp^2\psi - S + \mu)/((A(1-\psi) + Bp^2\psi - S) - 3\mu/2)]] = \operatorname{Log}[(A(1-\psi) + Bp^2\psi - S + \mu)/((A(1-\psi) + Bp^2\psi - S) - 3\mu/2)] > 0$ as $(A(1-\psi) + Bp^2\psi - S + \mu)/((A(1-\psi) + Bp^2\psi - S) - 3\mu/2) > 1$. Hence, the expression $Z_4(p)$ is negative for all $0 < p < \sqrt{A/B}$. □

Partial differentiations of the profit function (16) with respect to Q and p are

$$\frac{\partial E\pi(Q,p)}{\partial Q} = Z_2(p) + 2Z_2(p)Q - \frac{Z_4(p)}{Q^2}, \quad (18)$$

$$\frac{\partial E\pi(Q,p)}{\partial p} = \frac{dZ_1(p)}{dp} + Q\frac{dZ_2(p)}{dp} + \frac{1}{Q}\frac{dZ_4(p)}{dp}, \quad (19)$$

$$\frac{\partial^2 E\pi(Q,p)}{\partial Q^2} = 2Z_3(p) + \frac{2Z_4(p)}{Q^3}, \quad (20)$$

$$\frac{\partial^2 E\pi(Q,p)}{\partial p^2} = \frac{d^2Z_1(p)}{dp^2} + Q\frac{d^2Z_2(p)}{dp^2} + \frac{1}{Q}\frac{d^2Z_4(p)}{dp^2}, \quad (21)$$

$$\frac{\partial^2 E\pi(Q,p)}{\partial Q\partial p} = \frac{dZ_2(p)}{dp} - \frac{1}{Q^2}\frac{dZ_4(p)}{dp}. \quad (22)$$

Proposition 2. $\partial^2 E\pi(Q,p)/\partial Q^2 < 0$ *for all* $0 < p < \sqrt{A/B}$.

Proof. As $\partial^2 E\pi(Q,p)/\partial Q^2 = 2Z_3(p) + 2Z_4(p)/Q^3$ where $Z_4(p) < 0$ (see Proposition 1), we can say $\partial^2 E\pi(Q,p)/\partial Q^2 < 0$ if $|2Z_3(p)| < |2Z_4(p)/Q^3|$. We see from expression $Z_3(p)$ that $Z_3(p)$ does not depend on p; that is, it is constant expression. Also, it is clear that the value of the term $Z_3(p)$ is very small. But $Q^3Z_3(p)$ may not be so small such that $|Z_4(p)| > |Q^3Z_3(p)|$ when Q is very large. Hence, $\partial^2 E\pi(Q,p)/\partial Q^2 < 0$ for all $0 < p < \sqrt{A/B}$ if $|Z_4(p)| > |Q^3Z_3(p)|$. □

Proposition 3. *The solutions* (Q^*, p^*) *of the equations* $\partial E\pi(Q,p)/\partial Q = 0$ *and* $\partial E\pi(Q,p)/\partial p = 0$ *are optimal if* $|d^2Z_1(p)/dp^2| < |Q(d^2Z_2(p)/dp^2) + (1/Q)(d^2Z_3(p)/dp^2)|$ *and* $2(Z_3(p) + Z_4(p)/Q^3)(d^2Z_1(p)/dp^2 + Q(d^2Z_2(p)/dp^2) + (1/Q)(d^2Z_4(p)/dp^2)) > (dZ_2(p)/dp - (1/Q^2)(dZ_4(p)/dp))^2$ *are satisfied at the point* (Q^*, p^*).

Proof. The solutions (Q^*, p^*) of the equations $\partial E\pi(Q,p)/\partial Q = 0$ and $\partial E\pi(Q,p)/\partial p = 0$ are optimal if the Hessian matrix of per unit expected integrated profit function (16) is negative definite at that solution point.

Hessian matrix of the per unit expected profit function is

$$H = \begin{pmatrix} \dfrac{\partial^2 E\pi(Q,p)}{\partial Q^2} & \dfrac{\partial^2 E\pi(Q,p)}{\partial Q\partial p} \\ \dfrac{\partial^2 E\pi(Q,p)}{\partial Q\partial p} & \dfrac{\partial^2 E\pi(Q,p)}{\partial p^2} \end{pmatrix}_{(Q^*,p^*)} . \quad (23)$$

Therefore

$$|H| = \left[2\left(Z_3(p) + \frac{Z_4(p)}{Q^3}\right) \times \left(\frac{d^2Z_1(p)}{dp^2} + Q\frac{d^2Z_2(p)}{dp^2} + \frac{1}{Q}\frac{d^2Z_4(p)}{dp^2}\right) - \left(\frac{dZ_2(p)}{dp} - \frac{1}{Q^2}\frac{dZ_4(p)}{dp}\right)^2 \right]_{(Q^*,p^*)} . \quad (24)$$

Now, the matrix H will be negative definite if $\partial^2 E\pi(Q,p)/\partial Q^2 < 0$, $\partial^2 E\pi(Q,p)/\partial p^2 < 0$, and $|H| > 0$ at (Q^*, p^*). From Proposition 2, we have $\partial^2 E\pi(Q,p)/\partial Q^2 < 0$. The expression $\partial^2 E\pi(Q,p)/\partial p^2$ will be negative when $|d^2Z_1(p)/dp^2| < |Q(d^2Z_2(p)/dp^2) + (1/Q)(d^2Z_3(p)/dp^2)|$ (see (20)). Now, the determinant value of the matrix H will be positive if $\partial^2 E\pi(Q,p)/\partial Q^2 < 0$, $\partial^2 E\pi(Q,p)/\partial p^2 < 0$, and $2(Z_3(p) + Z_4(p)/Q^3)(d^2Z_1(p)/dp^2 + Q(d^2Z_2(p)/dp^2) + (1/Q)(d^2Z_4(p)/dp^2)) > (dZ_2(p)/dp - (1/Q^2)(dZ_4(p)/dp))^2$.

Hence the proof. □

4. Numerical Example

Here, we illustrate our model numerically to gain the insight behavior of the model. We consider the demand function $D(p)$ as $D(p) = A - Bp^2$; that is, the demand rate decreases quadratically with respect to the increasing price, where the parameters of demand function are assumed to satisfy $A >> B$, as demand rate can never be negative; that is, selling price will never be greater than $\sqrt{A/B}$; otherwise negative demand will appear.

We consider the values of the parameters in appropriate units as follows: $h_s = \$1.5$ per unit per unit time, $h_m = \$3$ per unit per unit time, $h'_m = \$2.5$ per unit per unit time, $h''_m = \$2$ per unit per unit time, $b_c = \$5$ per unit per unit time, $\alpha = 0.05$, $\beta = 0.2$, $\mu = 10$, $A = 1000$, $B = 0.12$, $S = 100$ unit, $\theta = 0.2$, $\lambda = 0.75$, $l_c = \$2$ per unit per unit time, $p' = p/3$, $w_s = \$40$ per unit, $L = 4000$ $w'_s = \$15$ per unit, $C_s = \$25$ per unit, $S_r = 10000$, $S_c = \$0.05$ per unit, and $A_s = \$400$.

Then, the optimal lot-size is $Q^* = 1373.77$ unit, optimal selling price is $p^* = \$85.63$, and optimal per unit expected integrated profit is $E\pi(Q^*, p^*) = \$36.45$. We also have supplier individual per unit expected average profit $E\pi_s(Q^*, p^*) = \$12.95$, manufacturer individual per unit expected average profit $E\pi_m(Q^*, p^*) = \$23.50$, optimal production run-time $t_p^* = 1.09$ unit, optimal cycle time $T^* = 9.26$, optimal time length of delivering raw material by the supplier $t_1^* = 1.08$ unit, optimal time length of backlogging during production run $t_r^* = 0.20$ unit, optimal time required for selling the items which are stored during production run $(t_s^* - t_p^*) = 6.63$ unit, and optimal time cycle of shortages when production is stopped $(T^* - t_s^*) = 1.55$ unit.

The above results are optimal as $\partial^2 E\pi(Q^*, p^*)/\partial p^2 = -0.31 \leq 0$, $\partial^2 E\pi(Q^*, p^*)/\partial Q^2 = -7.37 \times 10^{-6} \leq 0$, and $((\partial^2 E\pi(Q^*, p^*)/\partial Q^2)(\partial^2 E\pi(Q^*, p^*)/\partial p^2) - (\partial^2 E\pi(Q^*, p^*)/\partial Q\partial p) - (\partial^2 E\pi(Q^*, p^*)/\partial p\partial Q)) = 1.47 \times 10^{-6} \geq 0$ at the value of $Q^* = 1373.77$ unit and $p^* = \$85.63$. It is clear from Figure 2 that the objective function $E\pi$ is concave and unimodal function.

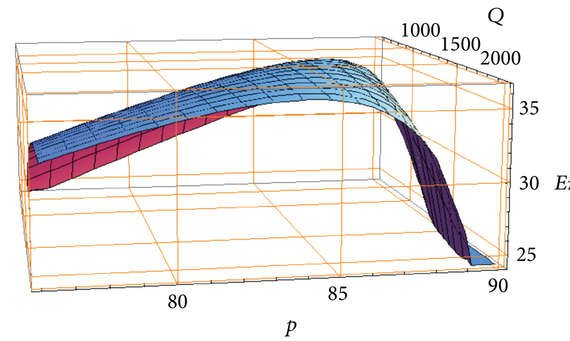

FIGURE 2: Expected per unit integrated profit versus lot-size and price.

5. Sensitivity Analysis

From Table 1, we observe the sensitivity of the parameters which help the decision makers to take appropriate decisions on their marketing strategy. The following features and managerial insights are observed.

(i) With the increasing value of the demand function parameter A (Figure 3), the optimal lot-size, optimal selling price, optimal per unit expected integrated profit, and optimal per unit expected profit of the supplier and manufacturer increase but the production run-time (t_p) and cycle time (T) decrease.

(ii) The optimal ordering size and production run-time (t_p) increase but optimal selling price per unit expected integrated profit and per unit expected profit of the supplier and manufacturer decreases with the increasing value of demand function parameter B (Figure 4). The cycle time T also decreases with higher value of B for both cases.

(iii) When the value of the backlogged level S (Figure 5) is increased, the optimal ordering size and production run-time (t_p) decrease but optimal selling price increases. The cycle time (T), the optimal per unit expected integrated profit, and optimal per unit expected profit of the supplier and manufacturer increase with the higher value of S.

(iv) The optimal ordering size, the optimal selling price, t_p, T, optimal per unit expected integrated profit, and optimal per unit expected profit of the supplier and manufacturer decrease with the higher value of per unit per unit time holding cost (h_m, Figure 6) of the good products of manufacturer.

(v) When the parameter λ (Figure 7) is increased, the optimal ordering size, optimal production run-time (t_p), and optimal cycle time (T) decrease. But, optimal selling price, optimal per unit expected integrated profit, and optimal per unit expected profit of the supplier and manufacturer increase with the higher value of λ.

(vi) With the increasing value of parameter β (Figure 8), the optimal lot-size, optimal selling price, optimal cycle time (T), and optimal per unit expected profit of the supplier increase but optimal production run-time (t_p), optimal per unit expected integrated profit, and optimal per unit expected profit of the manufacturer decrease.

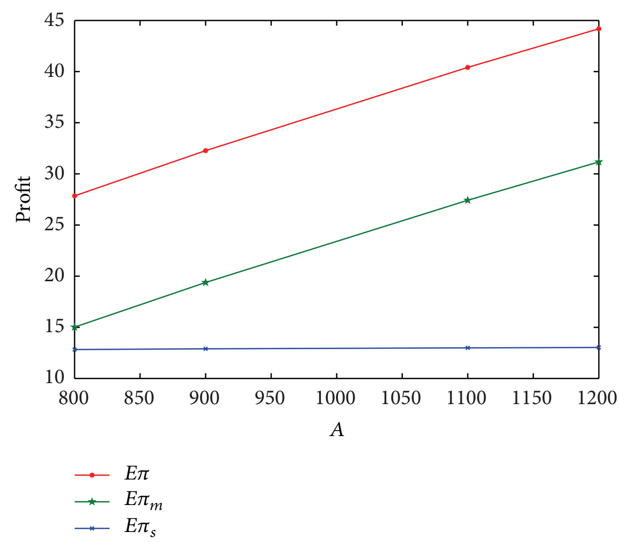

FIGURE 3: Profit versus A.

6. Conclusion

In markets, a manufacturer plays important and critical role for any business organization. Controlling and adjusting manufacturing rates according to the demand in the market are big challenges to the manufacturers because they face the constraints of labor, machines, power, technology, raw materials, environment, and so forth. Another important factors are adjusting proper delivery time of the products to the customers and tying up the customers during shortages of the products. Keeping in mind the above factors, the authors formulate and analyze a joint pricing and ordering policy for two-echelon production inventory model. In the model, after screening, supplier delivers good raw material to the manufacturer for production and sells the rest of materials to the outside. The production rate of the manufacturer

TABLE 1: Sensitivity Analysis of Numerical Example.

Parameter values	Optimal Solutions						
	Q	p	t_p	T	$E\pi_s$	$E\pi_m$	$E\pi$
A							
800	1313.53	75.93	1.30	9.75	12.83	15.02	27.85
900	1345.23	80.91	1.19	9.49	12.90	19.38	32.27
1100	1399.77	90.13	1.01	9.07	12.99	27.42	40.41
1200	1423.67	94.42	0.94	8.89	13.03	31.17	44.19
B							
0.10	1328.42	94.21	1.05	9.58	12.95	30.41	43.36
0.11	1351.96	89.63	1.07	9.41	12.95	26.71	39.67
0.13	1394.12	82.11	1.11	9.13	12.94	20.67	33.61
0.14	1413.20	78.97	1.12	9.01	12.94	18.15	31.09
S							
100	1381.15	85.18	1.09	8.64	12.94	23.10	36.04
125	1378.07	85.40	1.09	8.94	12.95	23.29	36.24
175	1368.24	85.88	1.08	9.61	12.95	23.73	36.68
200	1361.45	86.15	1.08	9.99	12.95	23.99	36.94
h_m							
1.0	1673.39	86.71	1.33	13.80	12.89	26.24	39.12
1.5	1493.60	86.09	1.18	10.92	12.93	24.69	37.62
2.5	1285.55	85.25	1.02	8.16	12.96	22.52	35.48
3.0	1216.61	84.94	0.97	7.36	12.97	21.69	34.66
λ							
0.50	1458.71	85.40	1.16	10.29	12.93	22.89	35.82
0.65	1399.93	85.56	1.11	9.59	12.94	23.30	36.24
0.80	1363.19	85.66	1.08	9.12	12.95	23.59	36.53
0.95	1338.38	85.72	1.06	8.79	12.95	23.80	36.75
β							
0.10	1289.67	85.45	1.12	9.19	12.92	26.71	39.63
0.15	1332.11	85.54	1.10	9.23	12.93	25.04	37.97
0.25	1414.71	85.72	1.08	9.30	12.96	22.09	35.05
0.30	1454.94	85.81	1.06	9.33	12.97	20.79	33.76

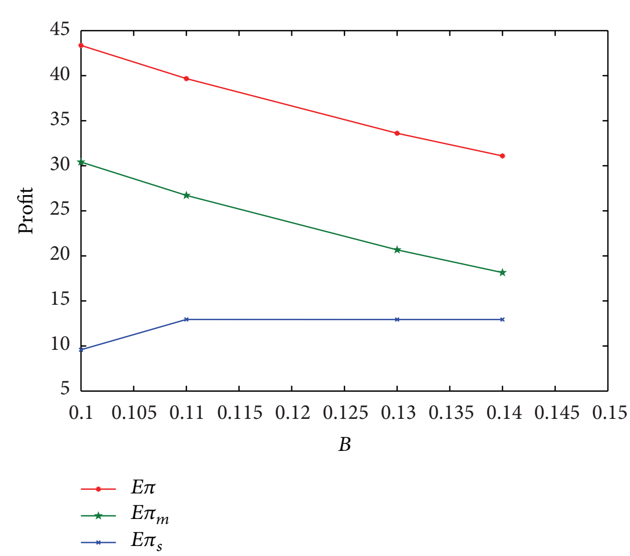

FIGURE 4: Profit versus B.

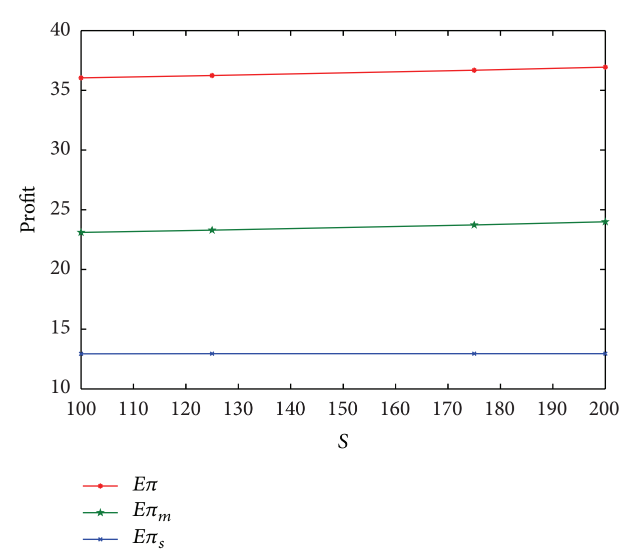

FIGURE 5: Profit versus S.

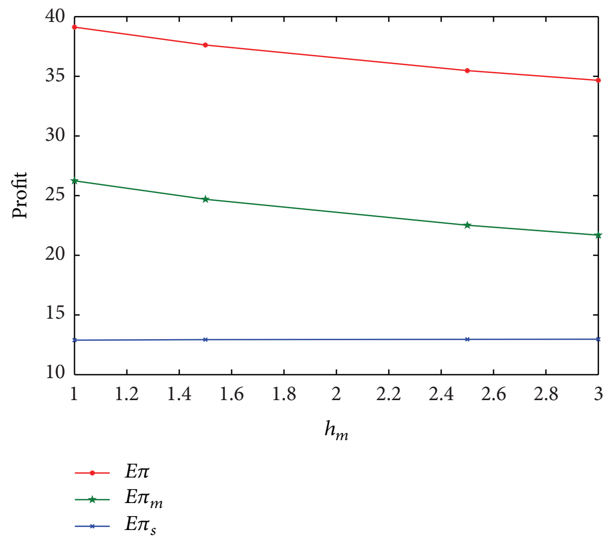

Figure 6: Profit versus h_m.

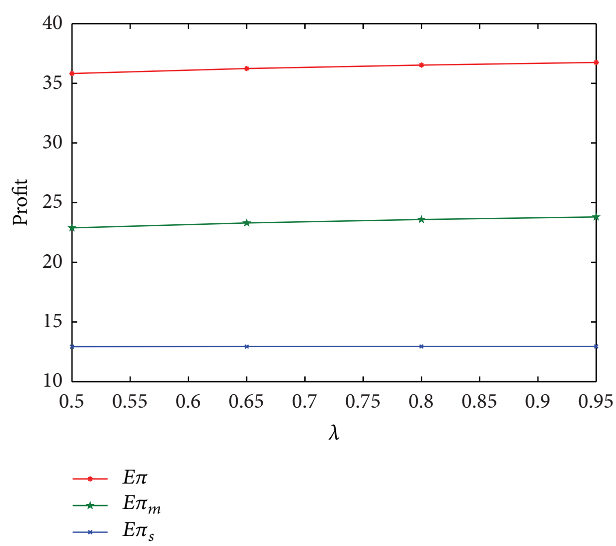

Figure 7: Profit versus λ.

is not fixed, a portion of the production rate is varying randomly. The production process of the manufacturer is not perfect, it produces good as well as defective products. After completion of the production, manufacturer sells them outside the market in a lot. We develop the model considering that the inventory cycle of the manufacturer starts and ends with shortages where shortages are partially backlogged. A mixture of the LIFO and FIFO dispatching policies are applied to fill up the backlogged demand. An integrated two-echelon supply chain model over price sensitive market demand is analyzed with respect to the ordering lot-size of raw material and selling price of the manufacturer such that the expected per unit profit of the integrated model is maximum. We also study the model through some numerical examples and discuss the sensitivity of the main parameters.

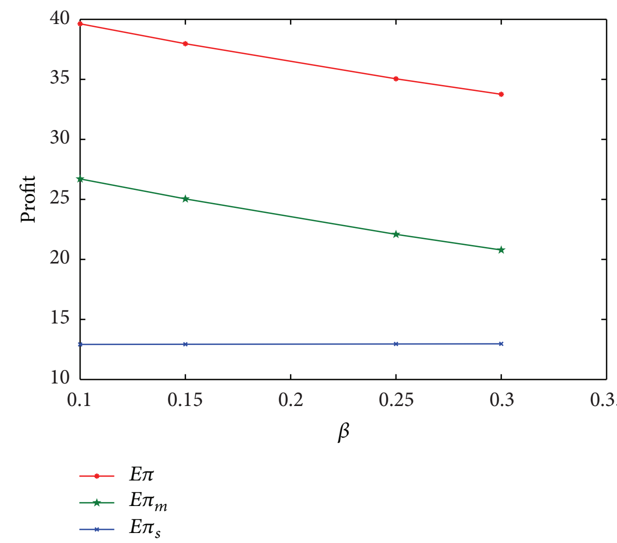

Figure 8: Profit versus β.

The major contribution of the model is to study a supplier-manufacturer imperfect production inventory model with varying production rate where the market demand of the products is considered as selling price sensitive. The mixture of LIFO and FIFO dispatching policies are considered to fill up backlogged demand.

In future, the present model can be extended considering stochastic types of competitive market demand. The present model could also be extended combining financial strategies such as quantity discount, cash discount, and others into the model. We may also introduce some restriction like storage, capital, and so forth to our model.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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