Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 429101, 9 pages
http://dx.doi.org/10.1155/2013/429101

Research Article

A Second-Order Sliding Mode Controller Design for Spacecraft Tracking Control

Yu-Xin Zhao, Tian Wu, and Gang Li

College of Automation, Harbin Engineering University, Harbin 150001, China

Correspondence should be addressed to Yu-Xin Zhao; zhaoyuxin@hrbeu.edu.cn

Received 21 July 2013; Accepted 19 August 2013

Academic Editor: Rongni Yang

For spacecraft attitude tracking system, there exists the chattering phenomenon. In this paper, the spacecraft motion is decomposed into three-channel subsystems, and a second-order sliding mode control is proposed. This method has been proved to have good convergence and robustness. Combined with the proposed sliding surface, the three-channel controllers are designed. The control performance is confirmed by the simulation results, the approaching process is improved effectively, and a smooth transition is achieved without overshoot and buffeting.

1. Introduction

Sliding mode variable structure control (SMC) is very suitable for spacecraft control system design because of its unique advantages, such as the simple algorithm, no need of the precise mathematical model for the controlled object, and the invariability of the control system when the parameter perturbation exists and the external disturbance conforms exactly to the matching condition [1].

But classical variable structure control system has a fatal flaw, namely "chattering" phenomenon, which is the main obstacle for SMC to promote in the engineering field [2]. In order to eliminate or suppress the chattering, scholars have conducted a wide-ranging and in-depth study [3–5]. In the past, the solution was to use the continuous function instead of the discontinuous parts, which had played a certain role in weakening buffeting, but it also greatly reduced the control performance [6]. A variable structure terminal guidance law with adaptive quick-reaching law and a filter approach based on sliding mode observer was developed [7], and the simulation results demonstrated higher accuracy, better robustness, and easy realization of the designed guidance law. For the guidance of attacking target on the ground, a new robust nonlinear sliding mode guidance law with terminal angle constraint was proposed [8]. Wu et al. introduced some slack matrices and a delay-dependent sufficient condition to guarantee the sliding mode dynamics is generalized, quadratically stable, and robustly passive [9, 10]. And Wu et al. designed an integral sliding surface function and an observer to estimate the system states, which was proved to be attained in a finite time [11, 12]. An integral-type sliding surface function is designed for establishing a sliding mode dynamics, which can be formulated by a switched stochastic system with an external disturbance/uncertainty [13]. But the control methods used are based on a general variable structure control theory, which is very unfavorable for the normal operation of spacecraft control system due to the existence of the chattering phenomenon; it could easily lead to system instability. And in the application of general variable structure control theory, the controlled object must be a relative degree 1. When the controlled object relative degree is greater than 1, this theory cannot be used.

During the flight, the aerodynamic parameters change intensely, and the coupling between three channels is very serious, which is caused by the flight speed, dramatically changed air density, and other disturbances. It brings about great difficulties because the entire spacecraft control system presents a strong multivariable coupling, time-varying, nonlinearity and uncertainty. The traditional method of spacecraft control system design is based on the assumption of small perturbation and the coefficient of frozen [14, 15]. However, the large angle of aircraft maneuvering is inevitable. Therefore, it is difficult to reconcile three-channel controller design based on the classical control theory. BTT control technology is a future trend in this field, but it would make the coupling aggravate [16–18]. In the design of spacecraft control system, even if the spacecraft is regarded as a particle, the relative motion between spacecraft and target is strongly nonlinear [19, 20].

Due to the reasons above, it will encounter enormous difficulties to use classical control theory to design the spacecraft control system in future [21, 22]. Therefore, it is necessary to search a high-level sliding variable structure control method for the spacecraft control system, which cannot only keep strong robustness but also can achieve continuous control [4, 23–26]. Shtessel et al. proposed the second-order sliding mode variable structure control algorithm, which was proved to be good robustness and to eliminate effectively the chattering [27–29]. A second-order sliding mode controller is designed [30] for the aerodynamic missile, which was simple and effective. The results of the simulation indicated that this second-order sliding mode controller is robust for the systems with uncertainties and can effectively reduce the chatting.

However, there are certain limitations to the high-order sliding mode control algorithm, such as the demanding controlled object and the complexity of algorithm. The paper conducts a new research on the second-order sliding mode variable structure control system, and simulation results show that the method we proposed can effectively eliminate the chattering phenomenon and have good robustness in the spacecraft control systems. The research results have injected new idea for the sliding mode variable structure control algorithm and also have provided a new solution for the spacecraft control system design. Therefore, the study of this subject has not only theoretical significance but also has a very important application reference value.

In this paper, we are concerned with the chattering phenomenon in the spacecraft attitude tracking system. The spacecraft motion is decomposed into three-channel subsystems, and a second-order sliding mode control is proposed, which has good convergence and robustness. Combined with the proposed sliding surface, the three-channel controllers are designed. Furthermore, the control performance is confirmed by the simulation results, the approaching process is improved effectively, and a smooth transition is achieved without overshoot and buffeting. The rest of this paper consists of five sections about reaching condition and controller design. The spacecraft model is described in Section 2, and a three-channel decomposition model is proposed. Section 3 proposes a second-order sliding mode. Section 4 designs the three-channel controller. Section 5 validates control performance. Finally, we conclude this paper in Section 6.

2. Spacecraft Model

The spacecraft attitude motion equations introduced in [31] is considered here. The model are described as follows:

$$\begin{aligned}
J_x \frac{d\omega_x}{dt} &= M_x - \left(J_z - J_y\right)\omega_z\omega_y,\\
J_y \frac{d\omega_y}{dt} &= M_y - \left(J_x - J_z\right)\omega_z\omega_x,\\
J_z \frac{d\omega_z}{dt} &= M_z - \left(J_y - J_x\right)\omega_x\omega_y,\\
\dot{\vartheta} &= \omega_y \sin\gamma + \omega_z \cos\gamma,\\
\dot{\psi} &= \frac{1}{\cos\vartheta}\left(\omega_y \cos\gamma - \omega_z \sin\gamma\right),\\
\dot{\gamma} &= \omega_x - \tan\vartheta\left(\omega_y \cos\gamma - \omega_z \sin\gamma\right),
\end{aligned} \tag{1}$$

where the pitch angle ϑ, the yaw angle ψ, and the roll angle γ are referred to the posture angle. ω_x, ω_y, and ω_z are the angular velocity of the spacecraft around the shaft Ox, Oy, and Oz. J_x, J_y, and J_z are the moments of inertia, M_x, M_y, and M_z are the external torques on the body frame.

From (1), we can see that the spacecraft attitude motion equation is nonlinear. And because of the existence of the large attitude angle, the coupling between three channels cannot be ignored.

2.1. Decomposition of Mathematical Model. In order to make the process of decomposition clear, (1) can be broken down into three subsystems.

(i) Channel Subsystem of Pitch Angle. The state function for the channel subsystem of pitch angle can be obtained by Figure 1.

From Figure 1, we can obtain the following state function:

$$\begin{bmatrix}\dot{x}_{1z}\\ \dot{x}_{2z}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_{1z}\\ x_{2z}\end{bmatrix} + \begin{bmatrix}0\\ \dfrac{\cos\gamma}{J_z}\end{bmatrix}u_z + \begin{bmatrix}0\\ f_z\end{bmatrix}, \tag{2}$$

where $u_z = M_z$, which is the controller of pitch channel subsystem.

And $f_z = \dot{\omega}_y \sin\gamma + \omega_y\dot{\gamma}\cos\gamma - (\cos\gamma/J_z)(J_y - J_x)\omega_x\omega_y - \omega_z\dot{\gamma}\sin\gamma$ is the coupling between the channels, which can be regarded as the external interference.

(ii) Channel Subsystem of Yaw Angle. The state function for the channel subsystem of yaw angle can be obtained by Figure 2.

From Figure 2, we can obtain the following state function:

$$\begin{bmatrix}\dot{x}_{1y}\\ \dot{x}_{2y}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_{1y}\\ x_{2y}\end{bmatrix} + \begin{bmatrix}0\\ \dfrac{\cos\gamma}{\cos\vartheta J_y}\end{bmatrix}u_y + \begin{bmatrix}0\\ f_y\end{bmatrix}, \tag{3}$$

where $u_y = M_y$ is the controller of yaw channel subsystem, and

$$\begin{aligned}
f_y = {} & \frac{\cos\gamma}{\cos\vartheta J_y}\left(J_z - J_x\right)\omega_x\omega_z + \frac{\cos\gamma\sin\vartheta\dot{\vartheta} - \sin\gamma\dot{\gamma}\cos\vartheta}{\cos^2\vartheta}\omega_y\\
& - \dot{\omega}_z\frac{\sin\gamma}{\cos\vartheta} - \frac{\cos\gamma\cos\vartheta\dot{\gamma} + \sin\gamma\dot{\vartheta}\sin\vartheta}{\cos^2\vartheta}\omega_z
\end{aligned} \tag{4}$$

can be regarded as the external interference.

(iii) Channel Subsystem of Roll Angle. The state function for the channel subsystem of roll angle can be obtained by Figure 3.

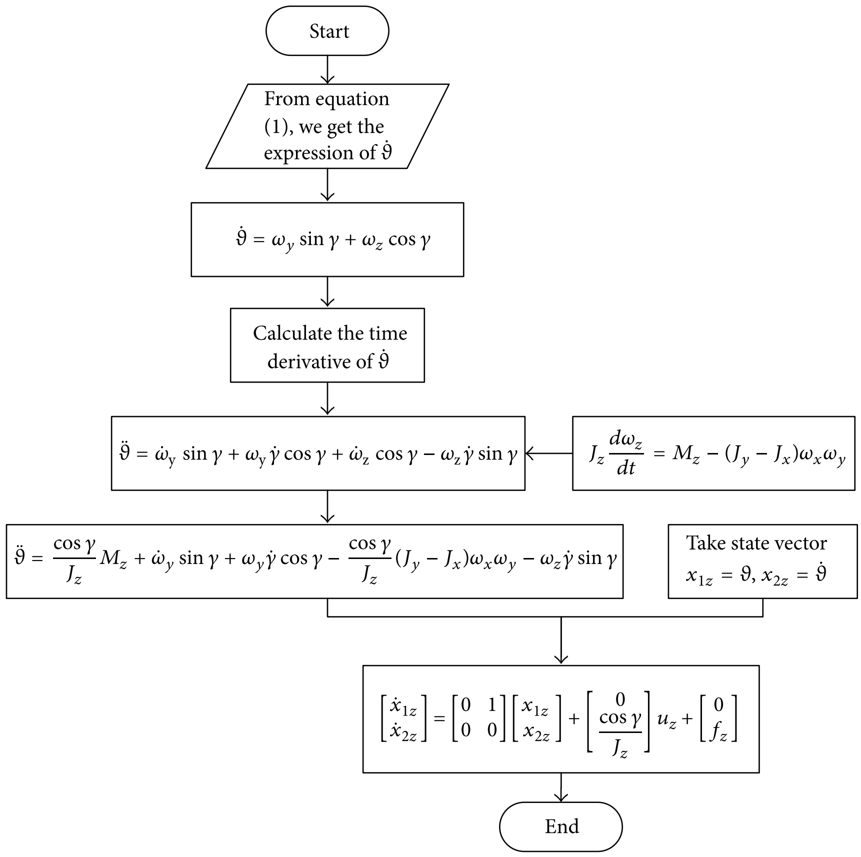

FIGURE 1: The state function for the channel subsystem of pitch angle.

From Figure 3, we can obtain the following state function:

$$\begin{bmatrix} \dot{x}_{1x} \\ \dot{x}_{2x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1x} \\ x_{2x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_x} \end{bmatrix} u_x + \begin{bmatrix} 0 \\ f_x \end{bmatrix}, \tag{5}$$

where $u_x = M_x$ is the controller of roll channel subsystem, and

$$f_x = \frac{J_y - J_z}{J_x} \omega_z \omega_y - \sec^2\vartheta \cdot \dot{\vartheta} \left(\omega_y \cos\gamma - \omega_z \sin\gamma \right) - \tan\vartheta \left(\dot{\omega}_y \cos\gamma - \omega_y \dot{\gamma} \sin\gamma - \dot{\omega}_z \sin\gamma - \omega_z \dot{\gamma} \cos\gamma \right) \tag{6}$$

can be regarded as the external interference.

The process of deduction regards the coupling between three channels as the external interference. When the spacecraft carries a maneuver of large angle attitude, the coupling will be exacerbated, which will affect the work of three subsystems seriously. Therefore, in the design of controller, we should make the control system have better robustness.

3. Reaching Condition Design and Proof

The system (2) can be expressed as:

$$\dot{x}(t) = A(x) + B(x)u + f(x), \tag{7}$$

where $x \in R^n$, $u \in R^m$, $A \in R^n$, $B \in R^{n\times m}$, and $B(x) = (b_1, b_2, \ldots, b_m)$; each component of the vector function $A(x)$ and b_i $(i = 1, \ldots, m)$ are sufficiently smooth scalar functions. $f(x)$ is the external interference. The phase trajectory of the system (2) will be significantly affected due to the existence of $f(x)$.

3.1. Model System Description and Simplification. Assume that $f(x) = 0$, and the system description can be simplified as

$$\dot{x}(t) = A(x) + B(x)u. \tag{8}$$

Designing the sliding mode surface $\sigma = x_1$, due to the theory [32] of second-order sliding mode, we can get the following function:

$$\begin{aligned} \dot{\sigma} &= x_2, \\ \ddot{\sigma} &= a(x) + b(x)u. \end{aligned} \tag{9}$$

3.2. Reaching Conditions. Define the following reaching condition:

$$\ddot{s} = -k_1\dot{s} - k_2 s - \varepsilon |s|^{\alpha} \operatorname{sat}(s), \tag{10}$$

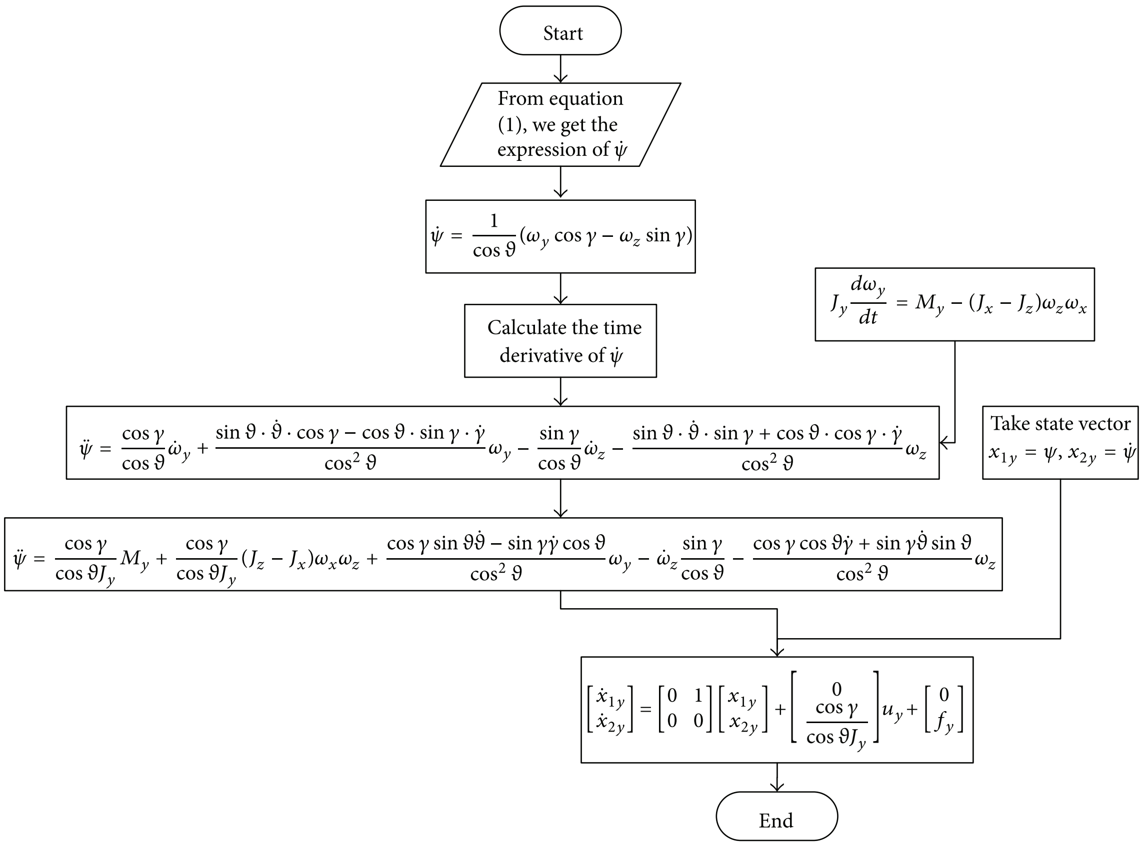

FIGURE 2: The state function for the channel subsystem of yaw angle.

where $k_1 > 0$, $k_2 > 0$, $\varepsilon > 0$, $\alpha > 1$, and $\text{sat}(\cdot)$ indicates the saturation function, which is used to eliminate chattering

$$\text{sat}(s) = \begin{cases} 1 & s > \Delta \\ ks & |s| \le \Delta \\ -1 & s < -\Delta \end{cases} \quad k = \frac{1}{\Delta}. \tag{11}$$

Theorem 1. *By satisfying the reaching condition* (10)*, the phase trajectory of the system* (8) *can reach the set points* $\Omega = \{x \mid \dot{s}(x) = s(x) = 0\}$ *at a limited time and stay at this state.*

Proof. Define that $h(s) = k_2 s + \varepsilon|s|^\alpha \text{sat}(s)$, and we can get $sh(s) > 0$. Here, we select the Lyapunov function below

$$V = 0.5\dot{s}^2 + \int_0^s h(y)\,dy \tag{12}$$

and $V > 0$. The time derivative of formula (12) can be obtained as

$$\dot{V} = \dot{s}\left(-k_1\dot{s} - k_2 s - \varepsilon|s|^\alpha \text{sat}(s)\right) + h(s)\,\dot{s}. \tag{13}$$

By substituting $h(s) = k_2 s + \varepsilon|s|^\alpha \text{sat}(s)$ into (13), $\dot{V}$ can hold as

$$\dot{V} = -k_1\dot{s}^2. \tag{14}$$

Next, we will make some analysis about $\dot{V}$.

(i) When $\dot{s} \ne 0$, $\dot{V} < 0$; that is to say that $V \to 0$. And in order to make $\lim_{|s|\to+\infty}\int_0^s h(y)dy = 0$, there must be $\lim_{t\to+\infty} s = 0$ for an arbitrary initial position of the system.

(ii) When $\dot{s} = 0$, $\ddot{s} = 0$. At this time, $h(s) = k_2 s + \varepsilon|s|^\alpha \text{sat}(s) = 0$. Because of $sh(s) > 0$, $s = 0$ is the only zero for this function. Therefore, $s \equiv 0$.

According to (i) (ii) above, s will be zero in the end, and the phase trajectory of system (8) will reach the set points $\Omega = \{x \mid \dot{s}(x) = s(x) = 0\}$ asymptotically and stay at this state. □

Explanation. In the reaching condition $\ddot{s} = -k_1\dot{s} - k_2 s - \varepsilon|s|^\alpha \text{sat}(s)$ above, the reaching speed will become faster when $k_2 s+\varepsilon|s|^\alpha \text{sat}(s)$ gets bigger, but the phase trajectory will resuilt in larger chattering in neighborhood of $s = 0$. However, if we make $k_2 s + \varepsilon|s|^\alpha \text{sat}(s)$ smaller, the reaching time will become longer with the decrease of speed. Besides, the reaching speed will slow down if the coefficient k_1 is too large, and the chattering in neighborhood of $s = 0$ will be weakened in this condition. On the contrary, if the coefficient k_1 is too small, the reaching speed will enlarge and the phase trajectory will result in larger chattering in neighborhood of $s = 0$.

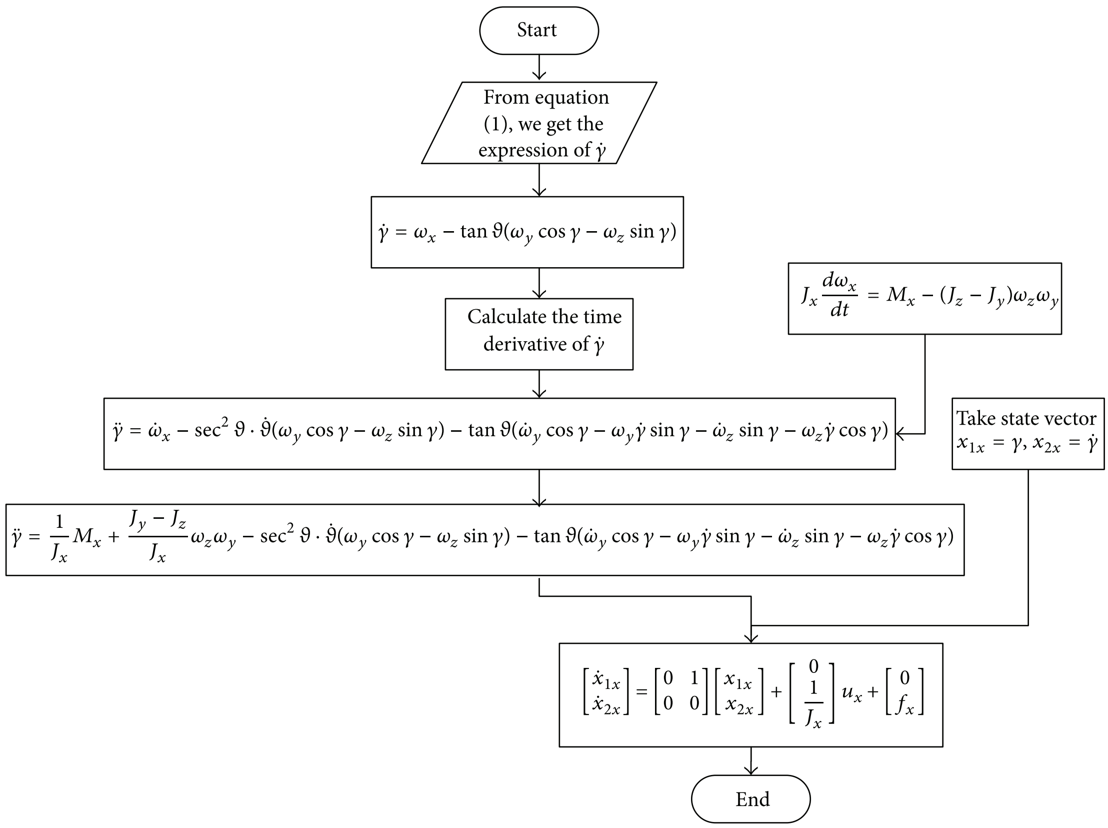

Figure 3: The state function for the channel subsystem of roll angle.

3.3. Robustness Prove. Combining (9) with (10), we can get $a(x) + b(x)u = -k_1\dot{s} - k_2 s - \varepsilon|s|^{\alpha}\,\mathrm{sat}(s)$, and the expression of controller can hold as

$$u = b^{-1}(x)\left[-k_1\dot{s} - k_2 s - \varepsilon|s|^{\alpha}\,\mathrm{sat}(s) - a(x)\right]. \quad (15)$$

Here, the equivalent external interference is considered, namely, $f(x) \neq 0$. Then, we can get

$$\dot{\sigma} = x_2, \qquad \ddot{\sigma} = a(x) + b(x)u + f(x). \quad (16)$$

Thus, by substituting (15) into (16), the sliding mode surface can be formulated as

$$\ddot{s} = -k_1\dot{s} - k_2 s - \varepsilon|s|^{\alpha}\,\mathrm{sat}(s) + f(x). \quad (17)$$

Assumption 2. $f(x)$ is bounded, there exists $|f| \le M$, $i = 1, \dots, m$.

Theorem 3. *By satisfying the Assumption 2 and $\varepsilon > M$ ($i = 1, 2, \dots, m$), the phase trajectory of the system* (7) *can reach the set points $\Omega = \{x \mid \dot{s}(x) = s(x) = 0\}$ under the control law* (15) *and stay at this state.*

Proof. Define that $h(s) = k_2 s + \varepsilon|s|^{\alpha}\,\mathrm{sat}(s) - f$, and we can get

$$sh(s) = k_2 s^2 + \varepsilon|s|^{\alpha+1} - fs \ge k_2 s^2 + \varepsilon|s|^{\alpha+1} - Ms = k_2 s^2 + \left(\varepsilon|s|^{\alpha} - M\right)s. \quad (18)$$

(i) When $|s| > \sqrt[\alpha]{M/\varepsilon}$, $sh(s) \ge 0$.

(ii) When $|s| < \sqrt[\alpha]{M/\varepsilon}$, $|s|^{\alpha+1} \to 0$, $\alpha \to 1$, $k_2 > \varepsilon$.

$$sh(s) \ge k_2 s^2 - Ms = k_2\left(\frac{M}{\varepsilon}\right) - M > 0. \quad (19)$$

Here, we select the Lyapunov function below

$$V = 0.5\dot{s}^2 + \int_0^s h(y)\,dy. \quad (20)$$

The process of proof is the same as Theorem 1. Then, we can get the conclusions of $\dot{V} = -k_1\dot{s}^2$ and $\lim_{t\to+\infty} s = 0$. □

From Theorems 1 and 3, formula (17) can be regarded as the system controller regardless of the equivalent external interference if only it meets Assumption 2. So, the robustness of the controller can be reflected. Theoretically, we can make k_1 and ε large enough to achieve the better robustness and dynamic performance. However, in practical system, we will encounter limited energy of the controller; thus, it is impossible to make k_1 and ε infinite.

4. Controller Design

Combined with the proposed second order sliding mode control law (10), we can get the following controllers.

4.1. Controller Design of Pitch Channel. Assume that pitch angle ϑ tracks a given angle ϑ_d and ϑ_d is second-order derivative. Meanwhile, the equivalent external interference f_z is bounded.

Let $x_z = [x_{1z} \;\; x_{2z}]^T$, and in order to use the sliding mode variable structure control of second-order conveniently, we select $s_z = e = x_{1z} - \vartheta_d$. And then, we can get the following functions:

$$\dot{s}_z = x_{2z} - \dot{\vartheta}_d, \tag{21}$$

$$\ddot{s}_z = \dot{x}_{2z} - \ddot{\vartheta}_d = \frac{\cos\gamma}{J_z} u_z - \ddot{\vartheta}_d. \tag{22}$$

And if s_z is regarded as the output of system (2), its relative order is 2, which satisfies the condition of Theorem 1. Then, we select the reaching condition as follows:

$$\ddot{s}_z = -k_{1z}\dot{s}_z - k_{2z}s_z - \varepsilon_z |s|^\alpha \operatorname{sat}(s_z), \tag{23}$$

where $k_{1z} > 0$, $k_{2z} > 0$, and $\varepsilon_z > 0$.

Combining formula (22) with formula (23), we can get

$$-k_{1z}\dot{s}_z - k_{2z}s_z - \varepsilon_z |s|^\alpha \operatorname{sat}(s_z) = \frac{\cos\gamma}{J_z} u_z - \ddot{\vartheta}_d. \tag{24}$$

Therefore,

$$u_z = \frac{J_z}{\cos\gamma}\left(\ddot{\vartheta}_d - k_{1z}\dot{s}_z - k_{2z}s_z - \varepsilon_z |s|^\alpha \operatorname{sat}(s_z)\right). \tag{25}$$

From the deduction above, we know that s_z will converge to zero in a finite time. That is to say that the pitch angle ϑ will be on track for the given angle ϑ_d in a limited time.

4.2. Controller Design of Yaw Channel. The process of deduction is the same as pitch channel. For the system (3), assume that the yaw angle ψ tracks a given angle ψ_d and ψ_d is second-order derivative. Meanwhile, the equivalent external interference f_y is bounded.

Let $x_y = [x_{1y} \;\; x_{2y}]^T$, and we select $s_y = e = x_{1y} - \psi_d$. And then, we can get the following functions:

$$\dot{s}_y = x_{2y} - \dot{\psi}_d, \tag{26}$$

$$\ddot{s}_y = \dot{x}_{2y} - \ddot{\psi}_d = \frac{\cos\gamma}{\cos\vartheta J_y} u_y - \ddot{\psi}_d. \tag{27}$$

Regarding s_y as the output of system (3), we select the reaching condition as follows:

$$\ddot{s}_y = -k_{1y}\dot{s}_y - k_{2y}s_y - \varepsilon_y |s|^\alpha \operatorname{sat}(s_y), \tag{28}$$

where $k_{1y} > 0$, $k_{2y} > 0$, and $\varepsilon_y > 0$.

Combining formula (27) with formula (28), we can get

$$u_y = \frac{\cos\vartheta J_y}{\cos\gamma}\left(\ddot{\vartheta}_d - k_{1y}\dot{s}_y - k_{2y}s_y - \varepsilon_y |s|^\alpha \operatorname{sat}(s_y)\right). \tag{29}$$

Thus, s_y will converge to zero in a finite time. That is to say that the yaw angle ψ will be on track for the given angle ψ_d in a limited time.

4.3. Controller Design of Roll Channel. For the system (5), assume that the roll angle γ tracks a given angle γ_d and γ_d is second-order derivative. Meanwhile, the external outside interference f_x is bounded.

Let $x_x = [x_{1x} \;\; x_{2x}]^T$, and we select $s_x = e = x_{1x} - \gamma_d$. Then, we can get the following functions:

$$\begin{aligned} \dot{s}_x &= x_{2x} - \dot{\gamma}_d, \\ \ddot{s}_x &= \dot{x}_{2x} - \ddot{\gamma}_d = \frac{1}{J_x} u_x - \ddot{\gamma}_d. \end{aligned} \tag{30}$$

Regarding s_x as the output of system (5), we select the reaching condition as follows:

$$\ddot{s}_x = -k_{1x}\dot{s}_x - k_{2x}s_x - \varepsilon_x |s|^\alpha \operatorname{sat}(s_x), \tag{31}$$

where $k_{1x} > 0$, $k_{2x} > 0$, and $\varepsilon_x > 0$.

And we can get

$$u_x = J_x\left(\ddot{\gamma}_d - k_{1x}\dot{s}_x - k_{2x}s_x - \varepsilon_x |s|^\alpha \operatorname{sat}(s_x)\right). \tag{32}$$

Thus, s_x will converge to zero in a finite time. That is to say that the yaw angle γ will be on track for the given angle γ_d in a limited time.

In the spacecraft control system, we want roll angle to calm to zero, and in order to achieve target tracking, generally, we let the pitch angle and yaw angle track a given angle.

5. Simulation Results

In this section, experimental simulations will be carried out to evaluate the effectiveness of the proposed sliding mode controller of second order:

$$\begin{bmatrix} \dot{x}_{1z} \\ \dot{x}_{2z} \\ \dot{x}_{1y} \\ \dot{x}_{2y} \\ \dot{x}_{1x} \\ \dot{x}_{2x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1z} \\ x_{2z} \\ x_{1y} \\ x_{2y} \\ x_{1x} \\ x_{2x} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ \dfrac{\cos x_{1x}}{J_z} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \dfrac{\cos x_{1x}}{\cos x_{1z} J_y} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{J_x} \end{bmatrix} \begin{bmatrix} u_z \\ u_y \\ u_x \end{bmatrix} + \begin{bmatrix} 0 \\ f_z \\ 0 \\ f_y \\ 0 \\ f_x \end{bmatrix}. \tag{33}$$

Detailed parameters of this control system are presented as follows. The initial angles are $[\vartheta \;\; \psi \;\; \gamma] = [\pi/6 \;\; \pi/3 \;\; \pi/4]$; the angular accelerations are $[\dot{\vartheta} \;\; \dot{\psi} \;\; \dot{\gamma}] = [0.2 \;\; 0.4 \;\; 0.5]$. And the desired tracking angles are $[\vartheta_d \;\; \psi_d \;\; \gamma_d] = [\pi/4 \;\; \pi/6 \;\; 0]$. The related parameters of system are $J_x = 224\,\mathrm{kg\cdot m^2}$, $J_y = 284\,\mathrm{kg\cdot m^2}$, and $J_z = 324\,\mathrm{kg\cdot m^2}$. The equivalent interference

$f_z = f_y = f_x = 3\sin t$. Then, formula (33) can be reduced as follows:

$$\begin{bmatrix} \dot{x}_{1z} \\ \dot{x}_{2z} \\ \dot{x}_{1y} \\ \dot{x}_{2y} \\ \dot{x}_{1x} \\ \dot{x}_{2x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1z} \\ x_{2z} \\ x_{1y} \\ x_{2y} \\ x_{1x} \\ x_{2x} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ \dfrac{\cos x_{1x}}{324} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \dfrac{\cos x_{1x}}{284\cos x_{1z}} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{224} \end{bmatrix} \begin{bmatrix} u_z \\ u_y \\ u_x \end{bmatrix} + \begin{bmatrix} 0 \\ 3\sin t \\ 0 \\ 3\sin t \\ 0 \\ 3\sin t \end{bmatrix}. \quad (34)$$

The three sliding surface are selected as $s_z = x_{1z} - \pi/4$, $s_y = x_{1y} - \pi/6$, and $s_x = x_{1x}$. Firstly, we assume that $f_z = f_y = f_x = 0$, and we can get the following functions:

$$\dot{s}_z = x_{2z}, \quad (35)$$

$$\ddot{s}_z = \dot{x}_{2z} = \frac{\cos x_{1x}}{324} u_z, \quad (36)$$

$$\dot{s}_y = x_{2z}, \quad (37)$$

$$\ddot{s}_y = \dot{x}_{2z} = \frac{\cos x_{1x}}{284\cos x_{1z}} u_y, \quad (38)$$

$$\dot{s}_x = x_{2x}, \quad (39)$$

$$\ddot{s}_x = \dot{x}_{2x} = \frac{1}{224} u_x. \quad (40)$$

And we select the same reaching conditions:

$$\begin{aligned} \ddot{s}_z &= -8\dot{s}_z - 5s_z - 5|s_z|^{\alpha} \operatorname{sat}(s_z), \\ \ddot{s}_y &= -8\dot{s}_y - 5s_y - 5|s_y|^{\alpha} \operatorname{sat}(s_y), \\ \ddot{s}_x &= -8\dot{s}_x - 5s_x - |s_x|^{\alpha} \operatorname{sat}(s_x). \end{aligned} \quad (41)$$

By taking formulas (36), (38), (40), and (41), the control law of each channel can hold as

$$\begin{aligned} u_z &= \frac{324}{\cos x_{1x}} \left(-8\dot{s}_z - 5s_z - 5|s_z|^{\alpha} \operatorname{sat}(s_z)\right), \\ u_y &= \frac{284\cos x_{1z}}{\cos x_{1x}} \left(-8\dot{s}_y - 5s_y - 5|s_y|^{\alpha} \operatorname{sat}(s_y)\right), \\ u_x &= 224\left(-8\dot{s}_x - 5s_x - |s_x|^{\alpha} \operatorname{sat}(s_x)\right). \end{aligned} \quad (42)$$

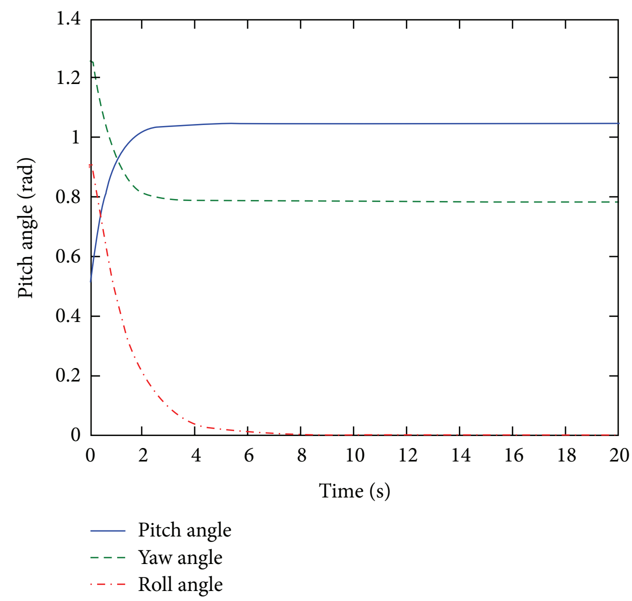

FIGURE 4: The angle tracking of the three subsystems.

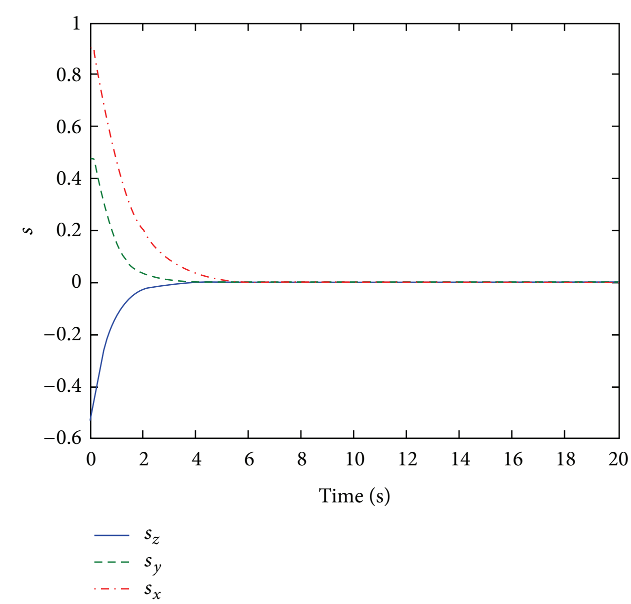

FIGURE 5: The switching function curve of the three subsystems.

Under the control law (42), the phase trajectory of three-channel will reach the sliding mode in a finite time. Simulation diagrams are as shown in Figures 4, 5, and 6.

From Figure 4, we can see that the proposed control law achieves a better angle tracking, and no overshoot occurs. From Figure 5, it can be seen that the proposed sliding surface achieves a smooth transition in the whole process of approaching, and there is no chattering. By Figure 6, we can see that chattering of the controllers reduces significantly during the approaching process under a certain external interference.

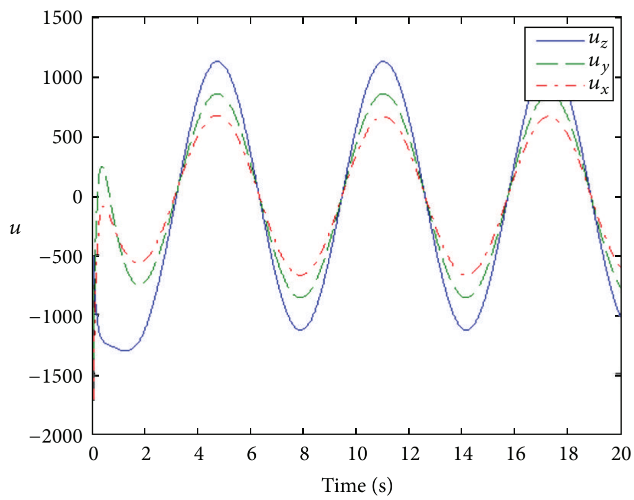

Figure 6: The control output of the three subsystems.

6. Conclusion

A three-channel model of spacecraft has been analysed in this paper. By simplifying the model, a second-order sliding surface was proposed, and its conditions of convergence and robustness were analyzed. Combined with the proposed sliding surface, three-channel controllers were designed, respectively. The control performance was confirmed by the simulation results, the approaching process was improved effectively, and a smooth transition was achieved without overshoot and buffeting. The coupling between the three channels was fully taken into account in the design of the controller, and it was equivalent to external interference, which provided a high practical value.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (nos. 60904087 and 51109045), and the Fundamental Research Funds for the Central Universities of China (nos. HEUCF110419 and HEUCFX41302).

References

[1] W. B. Gao, *Variable Structure Control Theory and Design Methods*, Science Press, 1996.

[2] K. Y. Zhuang, *Issues on variable structure control theory and application [Ph.D. thesis]*, Zhejiang University, 2002.

[3] X. Yu and O. Kaynak, "Sliding-mode control with soft computing: a survey," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 9, pp. 3275–3285, 2009.

[4] K.-W. Tong, X. Zhang, Y. Zhang, Z. Xie, and R.-X. Cao, "Sliding mode variable structure control of permanent magnet synchronous machine based on a novel reaching law," *Proceedings of the Chinese Society of Electrical Engineering*, vol. 28, no. 21, pp. 102–106, 2008.

[5] H. F. Ho, Y. K. Wong, and A. B. Rad, "Adaptive fuzzy sliding mode control with chattering elimination for nonlinear SISO systems," *Simulation Modelling Practice and Theory*, vol. 17, no. 7, pp. 1199–1210, 2009.

[6] J. S. Fan, H. X. Zhang, and X. G. Wang, "A survey of higher order sliding mode control and its applications in flight control," *Control Technology of Tactical Missile*, vol. 28, no. 1, pp. 12–15, 2011.

[7] Y. Song and W. Chen, "High-accuracy quick-reaching sliding mode variable structure terminal guidance law," *Journal of Beijing University of Aeronautics and Astronautics*, vol. 38, no. 3, pp. 319–323, 2012.

[8] Z. P. Li, J. G. Guo, and J. Zhou, "Sliding mode guidance law design based on terminal angle constraint," *Flight Dynamics*, vol. 30, no. 4, pp. 345–348, 2012.

[9] L. Wu and W. X. Zheng, "Passivity-based sliding mode control of uncertain singular time-delay systems," *Automatica*, vol. 45, no. 9, pp. 2120–2127, 2009.

[10] L. Wu and D. W. C. Ho, "Sliding mode control of singular stochastic hybrid systems," *Automatica*, vol. 46, no. 4, pp. 779–783, 2010.

[11] L. Wu, H. Gao, and C. Wang, "Quasi sliding mode control of differential linear repetitive processes with unknown input disturbance," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 7, pp. 3059–3068, 2011.

[12] L. Wu, P. Shi, and H. Gao, "State estimation and sliding-mode control of Markovian jump singular systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 5, pp. 1213–1219, 2010.

[13] L. G. Wu, W. X. Zheng, and H. J. Gao, "Dissipativity-based sliding mode control of switched stochastic systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 3, pp. 785–793, 2013.

[14] Y. B. Shtessel, I. A. Shkolnikov, and A. Levant, "Smooth second-order sliding modes: missile guidance application," *Automatica*, vol. 43, no. 8, pp. 1470–1476, 2007.

[15] X. F. Qian, R. X. Lin, and Y. N. Zhao, *Missile Flight Mechanics*, Press of Beijing Technology University, 2000.

[16] R. Sun and Q. Wang, "Reentry control using sliding modes for BTT missile," *Electronics Optics & Control*, vol. 15, no. 9, pp. 77–83, 2008.

[17] Y. A. Zhang, X. Yang, P. Y. Cui, and D. Yang, "Research of guidance and control for BTT winged missile," *Journal of Astronautics*, vol. 21, no. 4, pp. 71–75, 2000.

[18] Y. A. Zhang, Y. A. Hu, and S. B. Su, "Robust dynamic inversion control for BTT missile," *Journal of Astronautics*, vol. 23, no. 2, pp. 89–91, 2002.

[19] Y. Yao, B. Yang, F. He, Y. Qiao, and D. Cheng, "Attitude control of missile via fliess expansion," *IEEE Transactions on Control Systems Technology*, vol. 16, no. 5, pp. 959–970, 2008.

[20] M. Hou and G. Duan, "Integrated guidance and control of homing missiles against ground fixed targets," *Chinese Journal of Aeronautics*, vol. 21, no. 2, pp. 162–168, 2008.

[21] L. Duke, "Controller design challenge proposal," in *Proceeding of the AIAA Guidance Navigation and Control Conference*, pp. 256–263, The American Institute of Aeronautics and Astronautics, Washington, DC, USA, 1990.

[22] L. Sonneveldt, Q. P. Chu, and J. A. Mulder, "Nonlinear flight control design using constrained adaptive backstepping," *Journal of Guidance, Control, and Dynamics*, vol. 30, no. 2, pp. 322–336, 2007.

[23] S. Laghrouche, F. Plestan, and A. Glumineau, "Higher-order sliding mode control based on integral sliding mode," *Automatica*, vol. 43, no. 3, pp. 531–537, 2007.

[24] A. Levant, "Principles of 2-sliding mode design," *Automatica*, vol. 43, no. 4, pp. 576–586, 2007.

[25] G. Bartolini, A. Ferrara, E. Usai, and V. I. Utkin, "On multi-input chattering-free second-order sliding mode control," *IEEE Transactions on Automatic Control*, vol. 45, no. 9, pp. 1711–1717, 2000.

[26] Y. M. Hu, H. M. Chao, C. K. Lee, and T. P. Leung, "High-order sliding mode control of nonlinear affine control systems," *Acta Automatica Sinica*, vol. 28, no. 2, pp. 284–289, 2002.

[27] Y. B. Shtessel and C. H. Tournes, "Integrated higher-order sliding mode guidance and autopilot for dual-control missiles," *Journal of Guidance, Control, and Dynamics*, vol. 32, no. 1, pp. 79–94, 2009.

[28] Y. B. Shtessel, I. A. Shkolnikov, and A. Levant, "Smooth second-order sliding modes: missile guidance application," *Automatica*, vol. 43, no. 8, pp. 1470–1476, 2007.

[29] D. Galzi and Y. Shtessel, "UAV formations control using high order sliding modes," in *Proceedings of the American Control Conference (ACC '06)*, pp. 4249–4254, Minneapolis, Minn, USA, June 2006.

[30] J.-Y. Yu, W.-J. Gu, and H.-C. Zhao, "Second order sliding-mode controller design for the longitudinal control of the aerodynamical missile," *Journal of System Simulation*, vol. 15, no. 9, pp. 1319–1321, 2003.

[31] S. Bi, *Muti-Rigid-Body Modeling and Attitude Control of Variable Structure Aircraft*, University of Science and Technology of China, 2010.

[32] R.-B. Dou and K. Zhang, "Research on terminal guidance for re-entry vehicle based on second-order sliding mode control," *Journal of Astronautics*, vol. 32, no. 10, pp. 2109–2114, 2011.