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Research Article

An Impulsive Two-Prey One-Predator System with Seasonal Effects

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In recent years, the impulsive population systems have been studied by many researchers. However, seasonal effects on prey are rarely discussed. Thus, in this paper, the dynamics of the Holling-type IV two-competitive-prey one-predator system with impulsive perturbations and seasonal effects are analyzed using the Floquet theory and comparison techniques. It is assumed that the impulsive perturbations act in a periodic fashion, the proportional impulses (the chemical controls) for all species and the constant impulse (the biological control) for the predator at different fixed time but, the same period. In addition, the intrinsic growth rates of prey population are regarded as a periodically varying function of time due to seasonal variations. Sufficient conditions for the local and global stabilities of the two-prey-free periodic solution are established. It is proven that the system is permanent under some conditions. Moreover, sufficient conditions, under which one of the two preys is extinct and the remaining two species are permanent, are also found. Finally, numerical examples and conclusion are given.

1. Introduction

Recently, it is of great interest to study dynamical properties for impulsive perturbations in population dynamics. Impulsive prey-predator population systems have been discussed by a number of researchers [1–8] and, what is more, there are also many literatures on simple multispecies systems consisting of a three-species food chain with impulsive perturbations [7, 9–18]. Especially, two-prey and one-predator impulsive systems are drawing notice. For examples, Song and Li [13] studied dynamical behavior of a Holling type II two-prey one-predator system with impulsive effect concerning biological control and chemical control strategies at fixed time. Zhang et al. [17, 18] studied a Lotka-Volterra type two-prey one-predator system with impulsive effect on the predator of a fixed moment.

It is necessary and important to consider systems with periodic ecological parameters which might be quite naturally exposed such as those due to seasonal effects of weather or food supply [19]. Indeed, it has been studied that dynamical systems with simple dynamical behavior may display complex dynamical behavior when they have periodic perturbations[20–22]. For this reason, in this paper, we consider the intrinsic growth rates A of prey population as a periodically varying function of time due to seasonal variations. The seasonality is superimposed as follows[19–22]:

$$A_0 = A\big(1 + \epsilon \sin(\omega t)\big), \tag{1.1}$$

where the parameter ϵ $(i = 1,2)$ represents the degree of seasonality, $\lambda = A\epsilon \geq 0$ is the magnitude of the perturbation in A_0, and ω is the angular frequency of the fluctuation caused by seasonality. It is pertinent to point out that the forced ecosystem we are studying in this paper is similar to forced nonlinear oscillators in physics such as the Duffing oscillator.

Thus, we develop the Holling-type IV two-competitive-prey one-predator system with seasonality by introducing a proportional periodic impulsive poisoning(spraying pesticide) for all species and a constant periodic releasing, or immigrating, for the predator at different fixed time as follows:

$$\begin{aligned}
x_1'(t) &= x_1(t)\left(a_1 + \lambda_1 \sin(\omega_1 t) - b_1 x_1(t) - \gamma_1 x_2(t) - \frac{e_1 y(t)}{1 + c_1 x_1^2(t)}\right),\\
x_2'(t) &= x_2(t)\left(a_2 + \lambda_2 \sin(\omega_2 t) - b_2 x_2(t) - \gamma_2 x_1(t) - \frac{e_2 y(t)}{1 + c_2 x_2^2(t)}\right),\\
y'(t) &= y(t)\left(-a_3 + \frac{\beta_1 x_1(t)}{1 + c_1 x_1^2(t)} + \frac{\beta_2 x_2(t)}{1 + c_2 x_2^2(t)}\right),\\
&\qquad t \neq nT, \quad t \neq (n + \tau - 1)T,\\
\Delta x_1(t) &= -p_1 x_1(t),\\
\Delta x_2(t) &= -p_2 x_2(t), \quad t = (n + \tau - 1)T,\\
\Delta y(t) &= -p_3 y(t),\\
\Delta x_1(t) &= 0,\\
\Delta x_2(t) &= 0, \quad t = nT,\\
\Delta y(t) &= q,\\
\big(x_1(0^+), x_2(0^+), y(0^+)\big) &= \big(x_{01}, x_{02}, y_0\big),
\end{aligned} \tag{1.2}$$

where a_i $(i = 1,2)$ are intrinsic rates of increase, b_i $(i = 1,2)$ are the coefficients of intra-specific competition, γ_i $(i = 1,2)$ are parameters representing competitive effects between two preys, e_i $(i = 1,2)$ are the per-capita rates of predation of the predator, c_i $(i = 1,2)$ are the half-saturation constants, $a_3 > 0$ denotes the death rate of the predator, β_i $(i = 1,2)$ are the rates of conversing prey into predator, λ_i $(i = 1,2)$ are the magnitude, ω_i $(i = 1,2)$ are the angular frequency, τ, T are the period of spaying pesticides (harvesting) and the impulsive immigration or stock of the predator, respectively, $0 \leq p_1, p_2, p_3 < 1$ present the fraction of the preys and the predator which die due to the harvesting or pesticides, and q is the size of immigration or stock of the predator.

In Section 2, we give some notations and lemmas. In Section 3, we show the boundedness of the system and take into account the local and global stabilities of two-prey-free periodic solutions by using Floquet theory for the impulsive equation, small amplitude perturbation skills and comparison techniques, and finally, prove that the system is permanent under some conditions. Moreover, we give the sufficient conditions under which one of the two prey extinct and the remaining two species are permanent. Numerical examples are given in Section 4.

2. Preliminaries

Let $\mathbb{R}_+ = [0,\infty)$, $\mathbb{R}_+^* = (0,\infty)$, and $\mathbb{R}_+^3 = \{\mathbf{x} = (x(t), y(t), z(t)) \in \mathbb{R}^3 : x(t), y(t), z(t) \geq 0\}$. Denote $\mathbb{N}$ the set of all of nonnegative integers and $f = (f_1, f_2, f_3)^T$ the right hand of the first three equations in (1.2). Let $V : \mathbb{R}_+ \times \mathbb{R}_+^3 \to \mathbb{R}_+$, then V is said to belong to class V_0 if

(1) V is continuous on $((n-1)T, (n+\tau-1)T] \times \mathbb{R}_+^3 \cup ((n+\tau-1)T, nT] \times \mathbb{R}_+^3$, and $\lim_{(t,y)\to(t_0,\mathbf{x})} V(t,y) = V(t_0,\mathbf{x})$ exists, where $t_0 = (n+\tau-1)T^+$ and nT^+,

(2) V is locally Lipschitzian in $\mathbf{x}$.

Definition 2.1. Let $V \in V_0$. For $(t,x) \in ((n-1)T, (n+\tau-1)T] \times \mathbb{R}_+^3 \cup ((n+\tau-1)T, nT] \times \mathbb{R}_+^3$, the upper right derivative of V with respect to the impulsive differential system (1.2) is defined as

$$D^+V(t,\mathbf{x}) = \limsup_{h\to 0+} \frac{1}{h}\left[V\left(t+h, \mathbf{x}+hf(t,\mathbf{x})\right) - V(t,\mathbf{x})\right]. \tag{2.1}$$

The solution of system (1.2) is a piecewise continuous function $X(t) : \mathbb{R}_+ \to R_+^3$, $X(t)$ is continuous on $((n-1)T, (n+\tau-1)T) \cup ((n+\tau-1)T, nT)$ $(n \in \mathbb{N}, 0 \leq \tau \leq 1)$. Obviously, the smoothness properties of f guarantee the global existence and uniqueness of solutions of system (1.2) [23, 24].

Definition 2.2. The system (1.2) is permanent if there exist $M \geq m > 0$ such that, for any solution $(x_1(t), x_2(t), y(t))$ of system (1.2) with $(x_{01}, x_{02}, y_0) > 0$,

$$\begin{aligned} m &\leq \lim_{t\to\infty} \inf x_1(t) \leq \lim_{t\to\infty} \sup x_1(t) \leq M, \\ m &\leq \lim_{t\to\infty} \inf x_2(t) \leq \lim_{t\to\infty} \sup x_2(t) \leq M, \\ m &\leq \lim_{t\to\infty} \inf y(t) \leq \lim_{t\to\infty} \sup y(t) \leq M. \end{aligned} \tag{2.2}$$

We will use a comparison result of impulsive differential inequalities. Suppose that $g : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ satisfies the following hypotheses.

(H) g is continuous on $((n-1)T, (n+\tau-1)T] \times \mathbb{R}_+ \cup ((n+\tau-1)T, nT] \times \mathbb{R}_+$ and the limit $\lim_{(t,y)\to(t_0,x)} g(t,y) = g(t_0,x)$ exists, where $t_0 = (n+\tau-1)T^+$ and nT^+, and is finite for $x \in \mathbb{R}_+$ and $n \in \mathbb{N}$.

Lemma 2.3 (see [24]). *Suppose $V \in V_0$ and*

$$\begin{aligned} D^+V(t,\mathbf{x}) &\le g(t, V(t,\mathbf{x})), \quad t \ne (n+\tau-1)T, nT, \\ V(t,\mathbf{x}(t^+)) &\le \psi_n^1(V(t,\mathbf{x})), \quad t = (n+\tau-1)T, \\ V(t,\mathbf{x}(t^+)) &\le \psi_n^2(V(t,\mathbf{x})), \quad t = nT, \end{aligned} \tag{2.3}$$

where $g : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ satisfies (H), and $\psi_n^1, \psi_n^2 : \mathbb{R}_+ \to \mathbb{R}_+$ are nondecreasing for all $n \in \mathbb{N}$. Let $r(t)$ be the maximal solution for the impulsive Cauchy problem

$$\begin{gathered} u'(t) = g(t, u(t)), \quad t \ne (n+\tau-1)T, nT, \\ u(t^+) = \psi_n^1(u(t)), \quad t = (n+\tau-1)T, \\ u(t^+) = \psi_n^2(u(t)), \quad t = nT, \\ u(0^+) = u_0 \ge 0, \end{gathered} \tag{2.4}$$

defined on $[0,\infty)$. Then, $V(0^+, \mathbf{x}_0) \le u_0$ implies that $V(t,\mathbf{x}(t)) \le r(t), t \ge 0$, where $\mathbf{x}(t)$ is any solution of (2.3).

We now indicate a special case of Lemma 2.3 which provides estimations for the solution of a system of differential inequalities. For this, we let $\mathrm{PC}(\mathbb{R}_+, \mathbb{R})(\mathrm{PC}^1(\mathbb{R}_+, \mathbb{R}))$ denote the class of real piecewise continuous (real piecewise continuously differentiable) functions defined on $\mathbb{R}_+$.

Lemma 2.4 (see [24]). *Let the function $u(t) \in PC^1(\mathbb{R}^+, \mathbb{R})$ satisfy the inequalities*

$$\begin{gathered} \frac{du}{dt} \le f(t)u(t) + h(t), \quad t \ne \tau_k, \ t > 0, \\ u(\tau_k^+) \le \alpha_k u(\tau_k) + \theta_k, \ k \ge 0, \\ u(0^+) \le u_0, \end{gathered} \tag{2.5}$$

where $f, h \in PC(\mathbb{R}_+, \mathbb{R})$ and $\alpha_k \ge 0$, θ_k and u_0 are constants and $(\tau_k)_{k\ge 0}$ is a strictly increasing sequence of positive real numbers. Then, for $t > 0$,

$$\begin{aligned} u(t) \le{}& u_0\left(\prod_{0<\tau_k<t} \alpha_k\right) \exp\left(\int_0^t f(s)ds\right) + \int_0^t \left(\prod_{s\le\tau_k<t} \alpha_k\right) \exp\left(\int_s^t f(\gamma)d\gamma\right) h(s)ds \\ &+ \sum_{0<\tau_k<t}\left(\prod_{\tau_k<\tau_j<t} \alpha_j\right) \exp\left(\int_{\tau_k}^t f(\gamma)d\gamma\right)\theta_k. \end{aligned} \tag{2.6}$$

Similar result can be obtained when all conditions of the inequalities in the Lemmas 2.3 and 2.4 are reversed. Using Lemma 2.4, it is possible to prove that the solutions of the Cauchy problem (2.4) with strictly positive initial value remain strictly positive.

Lemma 2.5. *The positive octant* $(\mathbb{R}_+^*)^3$ *is an invariant region for system* (1.2).

Proof. Let $(x_1(t), x_2(t), y(t)) : [0, t_0) \to \mathbb{R}^3$ be a solution of system (1.2) with a strictly positive initial value (x_{01}, x_{02}, y_0). By Lemma 2.4, we can obtain that, for $0 \le t < t_0$,

$$\begin{aligned} x_1(t) &\ge x_{01}(1-p_1)^{[t/T]} \exp\left(\int_0^t g_1(s)ds\right), \\ x_2(t) &\ge x_{02}(1-p_2)^{[t/T]} \exp\left(\int_0^t g_2(s)ds\right), \\ y(t) &\ge y_0(1-p_3)^{[t/T]} \exp\left(\int_0^t g_3(s)ds\right), \end{aligned} \tag{2.7}$$

where $g_1(s) = a_1 - \lambda_1 - b_1x_1(s) - \gamma_1x_2(s) - e_1y(s)$, $g_2(s) = a_2 - \lambda_2 - b_2x_2(s) - \gamma_2x_1(s) - e_2y(s)$, and $g_3(s) = -a_3$. Thus, $x_1(t), x_2(t)$ and $y(t)$ remain strictly positive on $[0, t_0)$. □

Now, we give the basic properties of an impulsive differential equation as follows:

$$\begin{aligned} y'(t) &= -a_3y(t), \quad t \ne nT,\ t \ne (n+\tau-1)T, \\ \Delta y(t) &= -p_3y(t), \quad t = (n+\tau-1)T, \\ \Delta y(t) &= q, \quad t = nT. \end{aligned} \tag{2.8}$$

System (2.8) is a periodically forced linear system. It is easy to obtain that

$$y^*(t) = \begin{cases} \dfrac{q\exp\left(-a_3(t-(n-1)T)\right)}{1-(1-p_3)\exp\left(-a_3T\right)}, & (n-1)T < t \le (n+\tau-1)T, \\ \dfrac{q(1-p_3)\exp\left(-a_3(t-(n-1)T)\right)}{1-(1-p_3)\exp\left(-a_3T\right)}, & (n+\tau-1)T < t \le nT, \end{cases} \tag{2.9}$$

$y^*(0^+) = y^*(nT^+) = q/(1-(1-p_3)\exp(-a_3T))$, $y^*((n+\tau-1)T^+) = (q(1-p_3)\exp(-a_3\tau T))/(1-(1-p_3)\exp(-a_3T))$ is a positive periodic solution of (2.8). Moreover, we can obtain that

$$y(t) = \begin{cases} (1-p_3)^{n-1}\left(y(0^+) - \dfrac{q(1-p_3)e^{-T}}{1-(1-p_3)\exp\left(-a_3T\right)}\right)\exp\left(-a_3t\right) + y^*(t), \\ (n-1)T < t \le (n+\tau-1)T, \\ (1-p_3)^{n}\left(y(0^+) - \dfrac{q(1-p_3)e^{-T}}{1-(1-p_3)\exp\left(-a_3T\right)}\right)\exp\left(-a_3t\right) + y^*(t), \\ (n+\tau-1)T < t \le nT \end{cases} \tag{2.10}$$

is a solution of (2.8). From (2.9) and (2.10), we get easily the following result.

Lemma 2.6. $\lim_{t\to\infty}|y(t) - y^*(t)| = 0$ *for all solutions* $y(t)$ *of* (2.8) *with* $y(0^+) \geq 0$.

Therefore, system (1.2) has a two-prey-free periodic solution

$$(0, 0, y^*(t)). \tag{2.11}$$

3. Main Results

Theorem 3.1. *The periodic solution* $(0, 0, y^*(t))$ *of system* (1.2) *is globally asymptotically stable if for* $i = 1, 2$,

$$(a_i + \lambda_i)T - \frac{b_i^2 e_i q \Psi}{b_i^2 + c_i(a_i + \lambda_i)^2} < \ln \frac{1}{1 - p_i}, \tag{3.1}$$

where $\Psi = (1 - (1 - p_3)\exp(-a_3 T) - p_3 \exp(-a_3 \tau T))/a_3(1 - (1 - p_3)\exp(-a_3 T))$.

Proof. First, we will prove the local stability of the periodic solution $(0, 0, y^*(t))$. For this, consider the following impulsive differential equation:

$$\begin{aligned}
x'_{11}(t) &= x_{11}(t)\left(a_1 + \lambda_1 - b_1 x_{11}(t) - \gamma_1 x_{12}(t) - \frac{e_1 y_1(t)}{1 + c_1 x_{11}^2(t)}\right),\\
x'_{12}(t) &= x_{12}(t)\left(a_2 + \lambda_2 - b_2 x_{12}(t) - \gamma_2 x_{11}(t) - \frac{e_2 y_1(t)}{1 + c_2 x_{12}^2(t)}\right),\\
y'_1(t) &= y_1(t)\left(-a_3 + \frac{\beta_1 x_{11}(t)}{1 + c_1 x_{11}^2(t)} + \frac{\beta_2 x_{12}(t)}{1 + c_2 x_{12}^2(t)}\right),\\
&\qquad t \neq nT, \quad t \neq (n + \tau - 1)T,\\
\Delta x_{11}(t) &= -p_1 x_{11}(t),\\
\Delta x_{12}(t) &= -p_2 x_{12}(t), \quad t = (n + \tau - 1)T,\\
\Delta y_1(t) &= -p_3 y_1(t),\\
\Delta x_{11}(t) &= 0,\\
\Delta x_{12}(t) &= 0, \quad t = nT,\\
\Delta y_1(t) &= q,\\
(x_{11}(0^+), x_{12}(0^+), y_1(0^+)) &= (x_{01}, x_{02}, y_0).
\end{aligned} \tag{3.2}$$

Then, $0 \leq x_1(t) \leq x_{11}(t)$, $0 \leq x_2(t) \leq x_{12}(t)$, and $0 \leq y(t) \leq y_1(t)$ by Lemma 2.3, where $(x_1(t), x_2(t), y(t))$ and $(x_{11}(t), x_{12}(t), y_1(t))$ are solutions of systems (1.2) and (3.2), respectively. Thus we will show the local stability of the solution $(0, 0, y_1^*(t))$ of system (3.2), where $y_1^*(t) = y^*(t)$. The local stability of the two-pest-free periodic solution $(0, 0, y_1^*(t))$ may be determined by considering the behavior of small amplitude perturbations of the solution. Let $(x_{11}(t), x_{12}(t), y_1(t))$ be any solution of system (3.2). Define $u(t) = x_{11}(t), v(t) = x_{12}(t), w(t) = y(t) - y_1^*(t)$. Then, they may be written as

$$\begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u(0) \\ v(0) \\ w(0) \end{pmatrix}, \tag{3.3}$$

where $\Phi(t)$ satisfies

$$\frac{d\Phi}{dt} = \begin{pmatrix} a_1 + \lambda_1 - e_1 y_1^*(t) & 0 & 0 \\ 0 & a_2 + \lambda_2 - c_2 y_1^*(t) & 0 \\ \beta_1 y_1^*(t) & \beta_2 y_1^*(t) & -a_3 \end{pmatrix} \Phi(t) \tag{3.4}$$

and $\Phi(0) = I$, the identity matrix. So the fundamental solution matrix is

$$\Phi(t) = \begin{pmatrix} \exp\left(\int_0^t a_1 + \lambda_1 - e_1 y_1^*(s) ds\right) & 0 & 0 \\ 0 & \exp\left(\int_0^t a_2 + \lambda_2 - e_2 y_1^*(s) ds\right) & 0 \\ \exp\left(\int_0^t \beta_1 y_1^*(s) ds\right) & \exp\left(\int_0^t \beta_2 y_1^*(s) ds\right) & \exp\left(\int_0^t - a_3 ds\right) \end{pmatrix}. \tag{3.5}$$

The resetting impulsive conditions of system (3.2) become

$$\begin{gathered} \begin{pmatrix} u((n+\tau-1)T^+) \\ v((n+\tau-1)T^+) \\ u((n+\tau-1)T^+) \end{pmatrix} = \begin{pmatrix} 1-p_1 & 0 & 0 \\ 0 & 1-p_2 & 0 \\ 0 & 0 & 1-p_3 \end{pmatrix} \begin{pmatrix} u((n+\tau-1)T) \\ v((n+\tau-1)T) \\ w((n+\tau-1)T) \end{pmatrix}, \\ \begin{pmatrix} u(nT^+) \\ v(nT^+) \\ w(nT^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u(nT) \\ v(nT) \\ w(nT) \end{pmatrix}. \end{gathered} \tag{3.6}$$

Note that all eigenvalues of

$$S = \begin{pmatrix} 1-p_1 & 0 & 0 \\ 0 & 1-p_2 & 0 \\ 0 & 0 & 1-p_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(T) \tag{3.7}$$

are $\mu_1 = (1-p_1)\exp(\int_0^T a_1+\lambda_1-e_1 y_1^*(t)dt)$, $\mu_2 = (1-p_2)\exp(\int_0^T a_2+\lambda_2-e_2 y_1^*(t)dt)$, and $\mu_3=(1-p_3)\exp(-a_3T)<1$. Since

$$
\begin{gathered}
(a_1+\lambda_1)T - e_1 q\Psi < (a_1+\lambda_1)T - \frac{b_1^2 e_1 q\Psi}{b_1^2+c_1(a_1+\lambda_1)^2}, \\
(a_2+\lambda_2)T - e_2 q\Psi < (a_2+\lambda_2)T - \frac{b_2^2 e_2 q\Psi}{b_2^2+c_2(a_2+\lambda_2)^2}, \\
\int_0^T y_1^*(t)dt = \frac{q(1-(1-p_3)\exp(-a_3T)-p_3\exp(-a_3\tau T))}{a_3(1-(1-p_3)\exp(-a_3T))},
\end{gathered}
\tag{3.8}
$$

we obtain from (3.1) that the conditions $|\mu_1|<1$ and $|\mu_2|<1$ hold. Therefore, from the Floquet theory [23], we obtain $(0,0,y^*(t))$ is locally stable.

Now, to prove the global stability of the two-prey-free periodic solution, let $(x_1(t),x_2(t),y(t))$ be a solution of system (1.2). From (3.1), we can take a sufficiently small number $\epsilon_1>0$ satisfying

$$
\zeta = (1-p_1)\exp\left((a_1+\lambda_1)T + \frac{b_1^2 e_1(\epsilon_1 T - q\Psi)}{b_1^2+c_1((a_1+\lambda_1)^2+2(a_1+\lambda_1)b_1\epsilon_1+b_1^2\epsilon_1^2)}\right)<1. \tag{3.9}
$$

It follows from the first equation in (1.2) that $x_1'(t)\le x_1(t)(a_1+\lambda_1-b_1x_1(t))$ for $t\ne nT$, $t\ne(n+\tau-1)T$ and $x_1(t^+)=(1-p_1)x_1(t)\le x_1(t)$ for $t=(n+\tau-1)T$. Then, from Lemma 2.3, we have $x_1(t)\le u(t)$, where $u(t)$ is a solution of the following impulsive differential equation:

$$
\begin{gathered}
u'(t) = u(t)(a_1+\lambda_1-b_1u(t)), \quad t\ne nT,\ t\ne(n+\tau-1)T, \\
\Delta u(t) = 0, t=nT, \quad t=(n+\tau-1)T, \\
u(0^+) = x_1(0^+).
\end{gathered}
\tag{3.10}
$$

Since $u(t)\to(a_1+\lambda_1)/b_1$ as $t\to\infty$, $x_1(t)\le(a_1+\lambda_1)b_1+\epsilon$ for any $\epsilon>0$ with t large enough. For simplicity, we may assume that $x_1(t)\le(a_1+\lambda_1)/b_1+\epsilon_1$ for all $t>0$. Similarly, we get $x_2(t)\le(a_2+\lambda_2)/b_2+\epsilon_2$ for any $\epsilon_2>0$ and $t>0$. Since $y'(t)\ge -a_3y(t)$ for $t\ne nT$, $(n+\tau-1)T$, we can obtain from Lemmas 2.3 and 2.6 that

$$
y(t) > y^*(t)-\epsilon_1, \tag{3.11}
$$

for t sufficiently large. Without loss of generality, we may suppose that (3.11) holds for all $t\ge 0$. From (1.2), and (3.11) we obtain

$$
\begin{gathered}
x_1'(t) \le x_1(t)\left(a_1+\lambda_1 - \frac{e_1(y^*(t)-\epsilon_1)}{1+c_1(((a_1+\lambda_1)/b_1)+\epsilon_1)^2}\right), \quad t\ne nT,\ t\ne(n+\tau-1)T, \\
\Delta x_1(t) = -p_1x_1(t), \quad t=(n+\tau-1)T, \\
\Delta x_1(t) = 0, \quad t=nT.
\end{gathered}
\tag{3.12}
$$

Integrating (3.12) on $((n+\tau-1)T,(n+\tau)T]$, we get

$$\begin{aligned} x_1((n+\tau)T) &\leq x_1\left((n+\tau-1)T^+\right)\exp\left(\int_{(n+\tau-1)T}^{(n+\tau)T} a_1+\lambda_1-\frac{e_1\left(y^*(t)-\epsilon_1\right)}{1+c_1\left(\left((a_1+\lambda_1)/b_1\right)+\epsilon_1\right)^2}dt\right) \\ &= x_1\left((n+\tau-1)T\right)\zeta, \end{aligned} \tag{3.13}$$

and thus $x_1((n+\tau)T)\leq x_1(\tau T)\zeta^n$ which implies that $x_1((n+\tau)T)\to 0$ as $n\to\infty$. Further, we obtain, for $t\in((n+\tau-1)T,(n+\tau)T]$,

$$\begin{aligned} x_1(t) &\leq x_1\left((n+\tau-1)T+\right)\exp\left(\int_{(n+\tau-1)T}^{t} a_1+\lambda_1-\frac{e_1\left(y^*(t)-\epsilon_1\right)}{1+c_1\left(\left((a_1+\lambda_1)/b_1\right)+\epsilon_1\right)^2}dt\right) \\ &\leq x_1\left((n+\tau-1)T\right)\exp\left((a_1+\lambda_1+e_1\epsilon_1)T\right), \end{aligned} \tag{3.14}$$

which implies that $x_1(t)\to 0$ as $t\to\infty$. Similarly, we obtain $x_2(t)\to 0$ as $t\to\infty$. Now, take sufficiently small positive numbers ϵ_3 and ϵ_4 satisfying $\beta_1\epsilon_3+\beta_2\epsilon_4<a_3$ to prove that $y(t)\to y^*(t)$ as $t\to\infty$. Without loss of generality, we may assume that $x_1(t)\leq\epsilon_3$ and $x_2(t)\leq\epsilon_4$ for all $t\geq 0$. It follows from the third equation in (1.2) that, for $t\neq(n+\tau-1)T$ and $t\neq nT$,

$$y'(t)\leq y(t)\left(-a_3+\beta_1\epsilon_3+\beta_2\epsilon_4\right). \tag{3.15}$$

Thus, by Lemma 2.3, we induce that $y(t)\leq\tilde{y}^*(t)$, where $\tilde{y}^*(t)$ is the solution of (2.8) with a_3 changed into $a_3-\beta_1\epsilon_3-\beta_2\epsilon_4$. Therefore, by taking sufficiently small ϵ_3 and ϵ_4, we obtain from Lemma 2.6 and (3.11) that $y(t)$ tends to $y^*(t)$ as $t\to\infty$. □

Let $F_i(T)=(a_i+\lambda_i)T-(b_i^2e_iq\Psi/(b_i^2+c_i(a_i+\lambda_i)^2))+\ln(1-p_i)$ for $i=1,2$. Then $F_i(0)=\ln(1-p_i)<0$, $F_i(T)\to\infty$, and $F_i''(T)>0$. Thus $F_i(T)$ has a unique positive solution T_i.

Corollary 3.2. *The periodic solution* $(0,0,y^*(t))$ *of system* (1.2) *is globally asymptotically stable if* $T<\min\{T_1,T_2\}$.

From the proof of Theorem 3.1, we can easily get the following corollary.

Corollary 3.3. *The periodic solution* $(0,0,y^*(t))$ *of system* (1.2) *is locally stable if*

$$(a_1+\lambda_1)T-e_1q\Psi<\ln\frac{1}{1-p_1},\qquad (a_2+\lambda_2)T-e_2q\Psi<\ln\frac{1}{1-p_2}. \tag{3.16}$$

Furthermore, the periodic solution of system (1.2) may remain globally stable even if there are no the seasonal effects on system (1.2).

Corollary 3.4. *Suppose that $\lambda_1 = \lambda_2 = 0$. Then, the periodic solution $(0, 0, y^*(t))$ of system* (1.2) *is globally asymptotically stable if*

$$a_1 T - \frac{b_1^2 e_1 q \Psi}{b_1^2 + c_1 a_1^2} < \ln \frac{1}{1-p_1}, \qquad a_2 T - \frac{b_2^2 e_2 q \Psi}{b_2^2 + c_2 a_2^2} < \ln \frac{1}{1-p_2}. \tag{3.17}$$

Now, we show that all solutions of system (1.2) are uniformly bounded.

Theorem 3.5. *There is an $M > 0$ such that $x_1(t) \le M, x_2(t) \le M$, and $y(t) \le M$ for all t large enough, where $(x_1(t), x_2(t), y(t))$ is a solution of system* (1.2).

Proof. Let $(x_1(t), x_2(t), y(t))$ be a solution of (1.2) with $x_{01}, x_{02}, y_0 \ge 0$ and let $F(t) = (\beta_1/e_1)x_1(t) + (\beta_2/e_2)x_2(t) + y(t)$ for $t > 0$. Then, if $t \ne nT$ and $t \ne (n+\tau-1)T$, we obtain that $(dF(t)/dt) + \delta F(t) = -(b_1\beta_1/e_1)x_1^2(t) + (\beta_1/e_1)(a_1 + \lambda_1 \sin(\omega_1 t) + \delta)x_1(t) - (\beta_1\gamma_1/e_1)x_1(t)x_2(t) - (b_2\beta_2/e_2)x_2^2(t) + (\beta_2/e_2)(a_2 + \lambda_2 \sin(\omega_2 t) + \delta)x_2(t) - (\beta_2\gamma_2/e_2)x_1(t)x_2(t) + (\delta - a_3)y(t)$. From choosing $0 < \delta_0 < a_3$, we have, for $t \ne nT, t \ne (n+\tau-1)T$ and $t > 0$,

$$\frac{dF(t)}{dt} + \delta_0 F(t) \le -\frac{b_1\beta_1}{e_1}x_1^2(t) + \frac{\beta_1}{e_1}(a_1 + \lambda_1 + \delta_0)x_1(t) - \frac{b_2\beta_2}{e_2}x_2^2(t) + \frac{\beta_2}{e_2}(a_2 + \lambda_2 + \delta_0)x_2(t). \tag{3.18}$$

As the right-hand side of (3.18) is bounded from above by $M_0 = (\beta_1(a_1 + \lambda_1 + \delta_0)^2/4b_1e_1) + (\beta_2(a_2 + \lambda_2 + \delta_0)^2/4b_2e_2)$, it follows that

$$\frac{dF(t)}{dt} + \delta_0 F(t) \le M_0, \quad t \ne nT,\ n \ne (n+\tau-1)T,\ t > 0. \tag{3.19}$$

If $t = nT$, then $\Delta F(t) = q$ and if $t = (n+\tau-1)T$, then $\Delta F(t) \le -pF(t)$, where $p = \min\{p_1, p_2, p_3\}$. From Lemma 2.4, we get that

$$\begin{aligned} F(t) &\le F_0(1-p)^{[t/kT]} \exp\left(\int_0^t -\delta_0 ds\right) \\ &\quad + \int_0^t (1-p)^{[(t-s)/kT]} \exp\left(\int_s^t -\delta_0 d\gamma\right) M_0 ds \\ &\quad + \sum_{j=1}^{[t/kT]} (1-p)^{[(t-kT)/jT]} \exp\left(\int_{kT}^t -\delta_0 d\gamma\right) q \\ &\le F_0 \exp(-\delta_0 t) + \frac{M_0}{\delta_0}(1 - \exp(-\delta_0 t)) + \frac{q \exp(\delta_0 T)}{\exp(\delta_0 T) - 1}, \end{aligned} \tag{3.20}$$

where $F_0 = (\beta_1/e_1)x_{01} + (\beta_2/e_2)x_{02} + y_0$. Since the limit of the right-hand side of (3.20) as $t \to \infty$ is

$$\frac{M_0}{\delta_0} + \frac{q \exp(\delta_0 T)}{\exp(\delta_0 T) - 1} < \infty, \tag{3.21}$$

it easily follows that $F(t)$ is bounded for sufficiently large t. Therefore, $x_1(t), x_2(t)$ and $y(t)$ are bounded by a constant M for sufficiently large t. □

Theorem 3.6. *System* (1.2) *is permanent if* $a_3 > \max\{((a_1-\lambda_1)\beta_1/b_1), ((a_2-\lambda_2)\beta_2/b_2)\}$,

$$\left(a_1-\lambda_1-\frac{\gamma_1(a_2-\lambda_2)}{b_2}\right)T - e_1 q\Theta_2 > \ln\frac{1}{1-p_1},$$
$$\left(a_2-\lambda_2-\frac{\gamma_2(a_1-\lambda_1)}{b_1}\right)T - e_2 q\Theta_1 > \ln\frac{1}{1-p_2}, \tag{3.22}$$

where

$$\Theta_i = \frac{1-(1-p_3)\exp\left((-a_3+(\beta_i(a_i-\lambda_i)/b_i))T\right) - p_3\exp\left((-a_3+(\beta_i(a_i-\lambda_i)/b_i))\tau T\right)}{(a_3-(\beta_i(a_i-\lambda_i)/b_i))\left(1-(1-p_3)\exp\left((-a_3+(\beta_i(a_i-\lambda_i)/b_i))T\right)\right)}, \quad i=1,2. \tag{3.23}$$

Proof. Let $(x_1(t), x_2(t), y(t))$ be a solution of system (1.2) with $(x_{01}, x_{02}, y_0) > 0$. From Theorem 3.5, we may assume that $x_1(t), x_2(t), y(t) \le M$ and $M > \max\{(a_1/e_1), (a_2/e_2)\}$. Thus, we only need to prove the existence of the lower bound m. For this, we consider the following impulsive differential equation:

$$\begin{aligned}
x_{21}'(t) &= x_{21}(t)\left(a_1-\lambda_1-b_1x_{21}(t)-\gamma_1x_{22}(t)-\frac{e_1y_2(t)}{1+c_1x_{21}^2(t)}\right),\\
x_{22}'(t) &= x_{22}(t)\left(a_2-\lambda_2-b_2x_{22}(t)-\gamma_2x_{21}(t)-\frac{e_2y_2(t)}{1+c_2x_{22}^2(t)}\right),\\
y_2'(t) &= y_2(t)\left(-a_3+\frac{\beta_1x_{21}(t)}{1+c_1x_{11}^2(t)}+\frac{\beta_2x_{22}(t)}{1+c_2x_{22}^2(t)}\right),\\
&\qquad t\neq nT,\quad t\neq(n+\tau-1)T,\\
\Delta x_{21}(t) &= -p_1x_{21}(t),\\
\Delta x_{22}(t) &= -p_2x_{22}(t),\quad t=(n+\tau-1)T,\\
\Delta y_2(t) &= -p_3y_2(t),\\
\Delta x_{21}(t) &= 0,\\
\Delta x_{22}(t) &= 0,\quad t=nT,\\
\Delta y_2(t) &= q,\\
(x_{21}(0^+), x_{22}(0^+), y_2(0^+)) &= (x_{01}, x_{02}, y_0).
\end{aligned} \tag{3.24}$$

Then, $x_1(t) \ge x_{21}(t)$, $x_2(t) \ge x_{22}(t)$ and $y(t) \ge y_2(t)$ by Lemma 2.3, where $(x_1(t), x_2(t), y(t))$, and $(x_{21}(t), x_{22}, y_2(t))$ are solutions of systems (1.2) and (3.24), respectively. So, we will show that $x_{21}(t) \geq m_1,\ x_{22}(t) \geq m_1$, and $y_2(t) \geq m_1$. As in the proof of Theorem 3.1, we can show that $x_{21}(t) \leq ((a_1 - \lambda_1)/b_1) + \epsilon_1$ and $x_{22}(t) \leq ((a_2 - \lambda_2)/b_2) + \epsilon_2$ for $t > 0$. Let $m = (q(1 - p_3)\exp(-a_3 T)/1 - (1 - p_3)\exp(-a_3 T)) - \epsilon$ for $\epsilon > 0$. Since $y_2(t) \geq -a_3 y(t)$ for $t \neq nT,\ t \neq (n + \tau - 1)T$, it follows from Lemmas 2.3 and 2.6 that $y_2(t) > y^*(t) - \epsilon$ and hence $y_2(t) > m$ for sufficiently large t. Thus we only need to find $\overline{m}_1$ and $\overline{m}_2$ such that $x_{21}(t) \geq \overline{m}_1$ and $x_{22}(t) \geq \overline{m}_2$ for t large enough. We will do this in the following two steps.

Step 1. First, take sufficiently small positive numbers m_1 and m_2 such that $m_1 < (1/\beta_1)(a_3 - \beta_2(((a_2-\lambda_2)/b_2)+\epsilon_2))$, $m_2 < (1/\beta_2)(a_3 - \beta_1(((a_1-\lambda_1)/b_1)+\epsilon_1))$ and $\beta_1 m_1 + \beta_2 m_2 < a_3$. We will prove, there exist $t_1, t_2 \in (0, \infty)$ such that $x_{21}(t_1) \geq m_1$ and $x_{22}(t_2) \geq m_2$. Suppose not. Then that, we have only the following three cases:

(i) there exists a $t_2 > 0$ such that $x_{22}(t_2) \geq m_2$, but $x_{21}(t) < m_1$, for all $t > 0$;

(ii) there exists a $t_1 > 0$ such that $x_{21}(t_1) \geq m_1$, but $x_{22}(t) < m_2$, for all $t > 0$;

(iii) $x_{21}(t) < m_1$ and $x_{22}(t) < m_2$ for all $t > 0$.

Case (i): from (3.22) we can take $\eta_1 > 0$ small enough such that

$$\phi_1 = (1 - p_1)\exp\left(\left(a_1 - \lambda_1 - b_1 m_1 - \gamma_1\left(\frac{a_2 - \lambda_2}{b_2} + \epsilon_2\right) - e_1\eta_1\right)T - e_1 q\Theta_2\right) > 1. \tag{3.25}$$

We obtain from the condition of case (i) that $y'(t) \leq y_2(t)(-a_3+\beta_1 x_{21}(t)+\beta_2 x_{22}(t)) \leq y_2(t)(-a_3 + \beta_1 m_1 + \beta_2(((a_2 - \lambda_2)/b_2) + \epsilon_2)) \equiv E_1 y_2(t)$ for $t \neq nT,\ t \neq (n + \tau - 1)T$, where $E_1 = -a_3 + \beta_1 m_1 + \beta_2(((a_2 - \lambda_2)/b_2) + \epsilon_2) < 0$. Thus we have $y_2(t) \leq u(t)$ and $u(t) \to u^*(t)$ as $t \to \infty$, where $u(t)$ is a solution of system

$$\begin{gathered}
u'(t) = E_1 u(t), \quad t \neq nT,\ t \neq (n + \tau - 1)T, \\
\Delta u(t) = -p_3 u(t), \quad t = (n + \tau - 1)T, \\
\Delta u(t) = q, \quad t = nT, \\
u(0^+) = y(0^+), \\
u^*(t) = \begin{cases} \dfrac{q\exp\left(E_1(t - (n-1)T)\right)}{1 - (1 - p_3)\exp\left(E_1 T\right)}, & (n-1)T < t \leq (n + \tau - 1)T, \\[2ex] \dfrac{q(1 - p_3)\exp\left(E_1(t - (n-1)T)\right)}{1 - (1 - p_3)\exp\left(E_1 T\right)}, & (n + \tau - 1)T < t \leq nT. \end{cases}
\end{gathered} \tag{3.26}$$

Therefore, we can take a $T_1 > 0$ such that $y(t) \leq u(t) < u^*(t) + \eta_1$ for $t > T_1$. Thus we get

$$\begin{aligned}
x'_{21}(t) &\geq x_{21}(t)\left(a_1 - \lambda_1 - b_1 m_1 - \gamma_1\left(\frac{a_2 - \lambda_2}{b_2} + \epsilon_2\right) - e_1\left(u^*(t) + \eta_1\right)\right), \\
&\qquad\qquad t \neq nT,\ t \neq (n + \tau - 1)T, \\
\Delta x_{21}(t) &= -p_1 x_{21}(t), \quad t = (n + \tau - 1)T, \\
\Delta x_{21}(t) &= 0, \quad t = nT,
\end{aligned} \tag{3.27}$$

for $t > T_1$. Let $N_1 \in \mathbb{N}$ be such that $(N_1 + \tau - 1)T \geq T_1$. Integrating (3.17) on $((n+\tau-1)T, (n+\tau)T]$, $n \geq N_1$, we can obtain that $x_{21}((n+\tau)T) \geq x_{21}((n+\tau-1)T)(1-p_1)\exp(\int_{(n+\tau-1)T}^{(n+\tau)T} a_1 - \lambda_1 - b_1 m_1 - \gamma_1(((a_2-\lambda_2)/b_2)+\epsilon_2) - e_1(v^*(t)+\eta_1)dt = x_{21}((n+\tau-1)T)\phi_1$. Thus $x_{21}((N_1+k+\tau)T) \geq x_{21}((N_1+\tau)T)\phi_1^k \to \infty$ as $k \to \infty$, which is a contradiction to the boundedness of $x_{21}(t)$.

Case (ii): the same argument as the case (i) can be applied. So we omit it.

Case (iii): we choose $\eta_2 > 0$ sufficiently small so that

$$\phi_2 = (1-p_1)\exp\left((a_1 - \lambda_1 - b_1 m_1 - \gamma_1 m_2)T - e_1(q\Psi + \eta_2 T)\right) > 1. \tag{3.28}$$

Then we obtain $y_2'(t) \leq y_2(t)(-a_3 + \beta_1 m_1 + \beta_2 m_2) \equiv E_2 y_2(t)$ for $t \neq nT, t \neq (n+\tau-1)T$, where $E_2 = -a_3 + \beta_1 m_1 + \beta_2 m_2 < 0$. It follows from Lemmas 2.3 and 2.6 that $y_2(t) \leq w(t)$ and $w(t) \to w^*(t)$ as $t \to \infty$, where $w(t)$ is a solution of the following system:

$$\begin{gathered}
w'(t) = E_2 w(t), \quad t \neq nT,\ t \neq (n+\tau-1)T, \\
\Delta w(t) = -p_3 w(t), \quad t = (n+\tau-1)T, \\
\Delta w(t) = q, \quad t = nT, \\
w(0^+) = y(0^+), \\
w^*(t) = \begin{cases} \dfrac{q\exp\left(E_2(t-(n-1)T)\right)}{1-(1-p_3)\exp\left(E_2 T\right)}, & (n-1)T < t \leq (n+\tau-1)T, \\[2ex] \dfrac{q(1-p_3)\exp\left(E_2(t-(n-1)T)\right)}{1-(1-p_3)\exp\left(E_2 T\right)}, & (n+\tau-1)T < t \leq nT. \end{cases}
\end{gathered} \tag{3.29}$$

Thus there exists a $T_2 > 0$ such that $y_2(t) \leq w(t) < w^*(t) + \eta_2$ for $t > T_2$ and

$$\begin{aligned}
x_{21}'(t) &\geq x_{21}(t)\left(a_1 - \lambda_1 - b_1 m_1 - \gamma_1 m_2 - e_1(w^*(t) + \eta_2)\right), \\
&\qquad\qquad t \neq nT,\ t \neq (n+\tau-1)T, \\
\Delta x_{21}(t) &= -p_1 x_{21}(t), \quad t = (n+\tau-1)T, \\
\Delta x_{21}(t) &= 0, \quad t = nT,
\end{aligned} \tag{3.30}$$

for $t > T_2$. Let $N_2 \in \mathbb{N}$ be such that $(N_2 + \tau - 1)T \geq T_2$. Integrating (3.30) on $((n+\tau-1)T, (n+\tau)T]$, $n \geq N_2$, we can obtain that $x_{21}((n+\tau)T) \geq x_{21}((n+\tau-1)T)(1-p_1)\exp(\int_{(n+\tau-1)T}^{(n+\tau)} a_1 - \lambda_1 - b_1 m_1 - \gamma_1 m_2 - e_1(w^*(t)+\eta_2)dt) = x_{21}((n+\tau-1)T)\phi_2$. Similarly, we have $x_{21}((N_2+k+\tau)T) \geq x_{21}((N_2+\tau)T)\phi_2^k \to \infty$ as $k \to \infty$, which is a contradiction to the boundedness of $x_{21}(t)$. Therefore, there exist $t_1 > 0$ and $t_2 > 0$ such that $x_{21}(t_1) \geq m_1$ and $x_{22}(t_2) \geq m_2$.

Step 2. If $x_{21}(t) \geq m_1$ for all $t \geq t_1$, then we are done. If not, we may let $t^* = \inf_{t > t_1}\{x_{21}(t) < m_1\}$. Then, $x_{21}(t) \geq m_1$ for $t \in [t_1, t^*]$ and, by the continuity of $x_{21}(t)$, we have $x_{21}(t^*) = m_1$. In this step, we have only to consider two possible cases.

(i) Suppose that $t^* = (n_1 + \tau - 1)T$ for some $n_1 \in \mathbb{N}$. Then, $(1 - p_1)m_1 \leq x_{21}(t^{*+}) = (1 - p_1)x_{21}(t^*) < m_1$. Select $n_2, n_3 \in \mathbb{N}$ such that $(n_2 - 1)T > \ln(\eta_1/(M + q))/E_1$ and $(1 - p_1)^{n_2}\phi_1^{n_3}\exp(n_2\sigma T) > (1 - p_1)^{n_2}\phi_1^{n_3}\exp((n_2 + 1)\sigma T) > 1$, where $\sigma = a_1 - \lambda_1 - b_1 m_1 - \gamma_1(((a_2 - \lambda_2)/b_2) + \epsilon_2) - e_1 M < 0$. Let $T' = n_2 T + n_3 T$. In this case, we will show that there exists $t_3 \in (t^*, t^* + T']$ such that $x_{21}(t_3) \geq m_1$. Otherwise, by (2.10) and (3.26) with $v(n_1 T^+) = y(n_1 T^+)$, we have

$$v(t) = \begin{cases} (1 - p_3)^{n-(n_1+1)}\left(v(n_1 T^+) - \dfrac{q(1 - p_3)\exp(-T)}{1 - (1 - p_3)\exp(E_1 T)}\right) \\ \exp(E_1(t - n_1 T)) + v^*(t), \quad (n - 1)T < t \leq (n + \tau - 1)T, \\ (1 - p_3)^{(n-n_1)}\left(v(n_1 T^+) - \dfrac{q(1 - p_3)\exp(-T)}{1 - (1 - p_3)\exp(E_1 T)}\right) \\ \exp(E_1(t - n_1 T)) + v^*(t), \quad (n + \tau - 1)T < t \leq nT, \end{cases} \tag{3.31}$$

and $n_1 + 1 \leq n \leq n_1 + 1 + n_2 + n_3$. So we get $|v(t) - v^*(t)| \leq (M + q)\exp(E_1(t - n_1 T)) < \eta_1$ and $y_2(t) \leq v(t) \leq v^*(t) + \eta_1$ for $n_1 T + (n_2 - 1)T \leq t \leq t^* + T'$, which implies that (3.27) holds for $t \in [t^* + n_2 T, t^* + T']$. As in step 1, we have

$$x_{21}(t^* + T') \geq x_{21}(t^* + n_2 T)\phi_1^{n_3}. \tag{3.32}$$

Since $y_2(t) \leq M$, we have

$$\begin{aligned} x_{21}'(t) &\geq x_{21}(t)\left(a_1 - \lambda_1 - b_1 m_1 - \gamma_1\left(\frac{a_2 - \lambda_2}{b_2} + \epsilon_2\right) - e_1 M\right) = \sigma x_{21}(t), \\ &\qquad t \neq nT,\ t \neq (n + \tau - 1)T, \\ \Delta x_{21}(t) &= -p_1 x_{21}(t), \quad t = (n + \tau - 1)T, \\ \Delta x_{21}(t) &= 0, \quad t = nT, \end{aligned} \tag{3.33}$$

for $t \in [t^*, t^* + n_2 T]$. Integrating (3.33) on $[t^*, t^* + n_2 T]$, we have

$$\begin{aligned} x_{21}((t^* + n_2 T)) &\geq m_1 \exp(\sigma n_2 T) \\ &> m_1. \end{aligned} \tag{3.34}$$

Thus $x_{21}(t^* + T') \geq m_1(1 - p_1)^{n_2}\exp(\sigma n_2 T)\phi_1^{n_3} > m_1$ which is a contradiction. Now, let $\bar{t} = \inf_{t > t^*}\{x_{21}(t) \geq m_1\}$. Then, $x_{21}(t) \leq m_1$ for $t^* \leq t < \bar{t}$ and $x_{21}(\bar{t}) = m_1$. So, we have, for $t \in [t^*, \bar{t})$, $x_{21}(t) \geq m_1(1 - p_1)^{n_2+n_3}\exp(\sigma(n_2 + n_3)T) \equiv \overline{m}_1$. For $t > t^*$ the same argument can be continued since $x_{21}(\bar{t}) \geq m_1$. Hence $x_{21}(t) \geq \overline{m}_1$ for all $t > t_1$.

(ii) $t^* \neq (n + \tau - 1)T,\ n \in \mathbb{N}$. Suppose that $t^* \in ((n_1' + \tau - 1)T, (n_1' + \tau)T)$ for some $n_1' \in \mathbb{N}$. There are two possible cases for $t \in (t^*, (n_1' + \tau)T)$. Firstly, if $x_{21}(t) \leq m_1$ for all $t \in (t^*, (n_1' + \tau)T)$, similar to case (i), we can prove there must be a $t_3' \in [(n_1' + \tau)T, (n_1' + \tau)T + T']$ such that $x_{21}(t_3') \geq m_1$. Here we omit it. Let $\hat{t} = \inf_{t > t^*}\{x_{21}(t) \geq m_1\}$. Then, $x_{21}(t) \leq m_1$ for $t \in (t^*, \hat{t})$

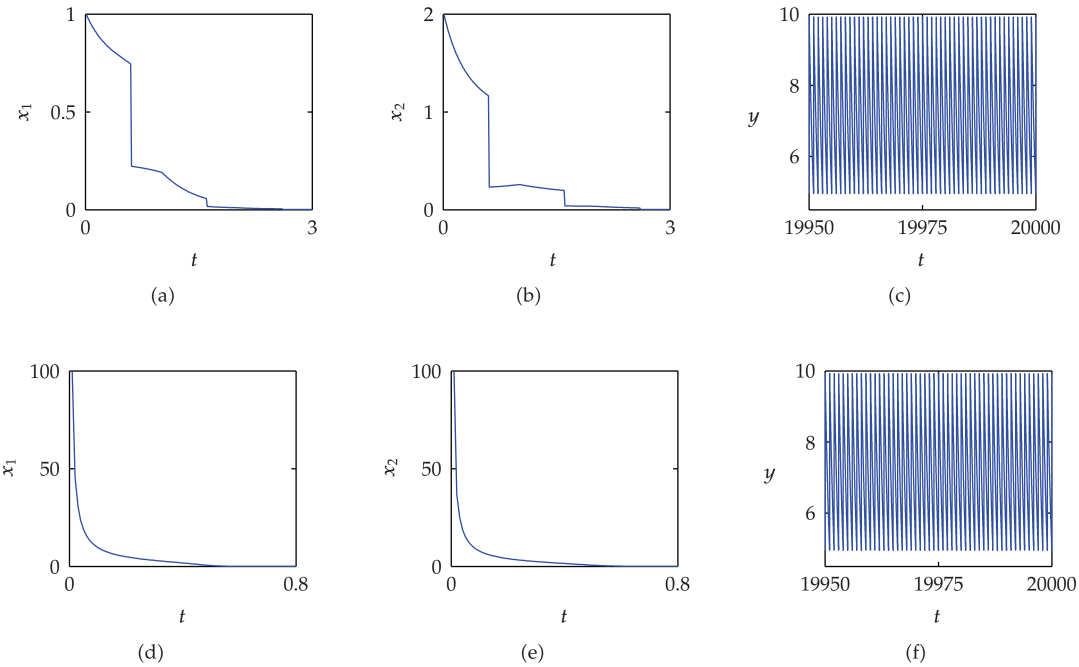

Figure 1: (a)–(c) Time series of system (1.2) with an initial value $(2,3,1)$, (d)–(f) time series of system (1.2) with an initial value $(100,100,100)$.

and $x_{21}(\hat{t}) = m_1$. For $t \in (t^*, \hat{t})$, we have $x_{21}(t) \geq m_1(1-p_1)^{n_2+n_3}\exp(\sigma(n_2+n_3+1)T) = \overline{m}_1$. So, $m_1 < \overline{m}_1$ and $x_{21}(t) \geq m_1$ for $t \in (t^*, \hat{t})$. For $t > t^*$ the same argument can be continued since $x_{21}(\hat{t}) \geq m_1$. Hence $x_{21}(t) \geq \overline{m}_1$ for all $t > t_1$. Secondly, if there exists a $t \in (t^*, (n_1' + \tau)T)$ such that $x_{21}(t) \geq m_1$. Let $\check{t} = \inf_{t>t^*}\{x_{21}(t) \geq m_1\}$. Then $x_{21}(t) \leq m_1$ for $t \in (t^*, \check{t})$ and $x_{21}(\check{t}) = m_1$. For $t \in (t^*, \check{t})$, we have $x_{21}(t) \geq x_{21}(t^*)\exp(\sigma(t-t^*)) \geq m_1\exp(\sigma T) > m_1$. This process can be continued since $x_{21}(\check{t}) \geq m_1$, and have $x_{21}(t) \geq \overline{m}_1$ for all $t > t_1$. Similarly, we can show that $x_{22}(t) \geq \overline{m}_2$ for all $t > t_2$. This completes the proof. □

Corollary 3.7. *Suppose that* $\lambda_1 = \lambda_2 = 0$. *Then, system* (1.2) *is permanent if* $a_3 > \max\{(a_1\beta_1/b_1), (a_2\beta_2/b_2)\}$, $(a_1 - (\gamma_1 a_2/b_2))T - e_1 q\Theta_2 > \ln\ 1/(1-p_1)$, *and* $(a_2 - (\gamma_2 a_1/b_1))T - e_2 q\Theta_1 > \ln\ 1/(1-p_2)$.

It follows from Theorems 3.1 and 3.6 that one of the two prey extinct and the remaining two species are permanent under some conditions.

Corollary 3.8. *Let* $(x_1(t), x_2(t), y(t))$ *be any solution of system* (1.2). *Then,* x_1 *and* $y(t)$ *are permanent, and* $x_2(t) \to 0$ *as* $t \to \infty$ *provided that* $a_3 > (a_2 - \lambda_2)\beta_2/b_2$, $(a_1 - \lambda_1 - (\gamma_1(a_2 - \lambda_2)/b_2))T - e_1 q\Theta_2 > \ln(1/(1-p_1))$, *and* $(a_2+\lambda_2)T - (b_2^2 e_2 q\Psi/(b_2^2 + c_2(a_2+\lambda_2)^2)) < \ln(1/(1-p_2))$.

Corollary 3.9. *Let* $(x_1(t), x_2(t), y(t))$ *be any solution of system* (1.2). *Then,* x_2 *and* $y(t)$ *are permanent, and* $x_1(t) \to 0$ *as* $t \to \infty$ *provided that* $a_3 > (a_1 - \lambda_1)\beta_1/b_1$, $(a_1+\lambda_1)T - (b_1^2 e_1 q\Psi/(b_1^2 + c_1(a_1+\lambda_1)^2)) < \ln\ 1/(1-p_1)$ *and* $(a_2 - \lambda_2 - (\gamma_2(a_1-\lambda_1)/b_1))T - e_2 q\Theta_1 > \ln\ 1/(1-p_2)$.

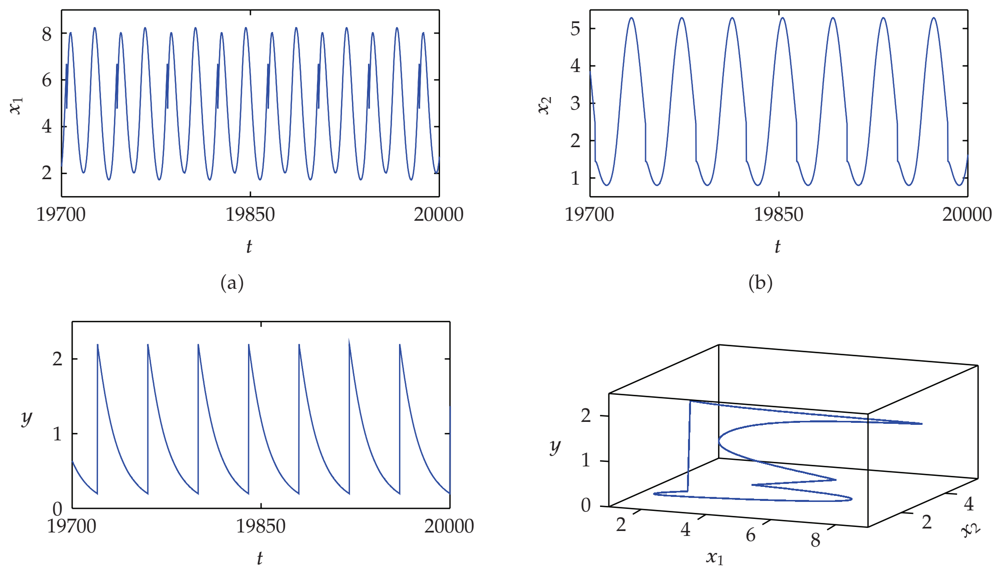

Figure 2: (a)–(c) Time series. (d) The trajectory of system (1.2) with an initial value $(2, 3, 1)$.

4. Numerical Examples

In this section, we will give some examples.

Case 1. If we take $a_1 = 1.1$, $a_2 = 1.0$, $a_3 = 0.7$, $b_1 = 1.0$, $b_2 = 1.2$, $c_1 = 0.9$, $c_2 = 1.0$, $e_1 = 0.3$, $e_2 = 0.2$, $\gamma_1 = 0.1$, $\gamma_2 = 0.1$, $\beta_1 = 0.8$, $\beta_2 = 1.0$, $p_1 = 0.7$, $p_2 = 0.8$, $p_3 = 0.001$, $\tau = 0.6$, $T = 1$, $q = 5$, $\lambda_1 = 0.5$, and $\lambda_2 = 0.6$, then these parameters satisfy the condition of Theorem 3.1. Thus the periodic solution $(0, 0, y^*(t))$ is globally asymptotically stable. (See Figure 1.)

Case 2. Let $a_1 = 5$, $a_2 = 4$, $a_3 = 0.7$, $b_1 = 1$, $b_2 = 1.2$, $c_1 = 0.9$, $c_2 = 1$, $e_1 = 0.3$, $e_2 = 0.2$, $\gamma_1 = 0.1$, $\gamma_2 = 0.1$, $\beta_1 = 0.1$, $\beta_2 = 0.2$, $p_1 = 0.3$, $p_2 = 0.4$, $p_3 = 0.001$, $\tau = 0.6$, $T = 4$, $q = 2$, $\lambda_1 = 4.0$ and $\lambda_2 = 3$. Then, from Theorem 3.6, we know that system (1.2) is permanent, (see Figure 2).

Case 3. From Corollary 3.8, we obtain that x_1 and $y(t)$ are permanent, and $x_2(t) \to 0$ as $t \to \infty$, where $(x_1(t), x_2(t), y(t))$ is a solution of system (1.2) with $a_1 = 5$, $a_2 = 1.3$, $a_3 = 0.7$, $b_1 = 1$, $b_2 = 0.8$, $c_1 = 0.9$, $c_2 = 1$, $e_1 = 0.3$, $e_2 = 1$, $\gamma_1 = 0.1$, $\gamma_2 = 0.1$, $\beta 1 = 0.8$, $\beta_2 = 0.9$, $p_1 = 0.5$, $p_2 = 0.8$, $p_3 = 0.001$, $\tau = 0.6$, $\lambda_1 = 1$, $\lambda 2 = 0.5$, $T = 1$, and $q = 2$. Figure 3 exhibits this phenomenon.

Case 4. It follows from Corollary 3.9 that x_2 and $y(t)$ are permanent, and $x_1(t) \to 0$ as $t \to \infty$, where $(x_1(t), x_2(t), y(t))$ is a solution of system (1.2) with $a_1 = 1$, $a_2 = 5$, $a_3 = 0.8$, $b_1 = 0.8$, $b_2 = 0.8$, $c_1 = 0.8$, $c_2 = 1$, $e_1 = 1$, $e_2 = 0.2$, $\gamma_1 = 0.1$, $\gamma_2 = 0.1$, $\beta_1 = 0.9$, $\beta_2 = 0.8$, $p_1 = 0.8$, $p_2 = 0.5$, $p_3 = 0.001$, $\tau = 0.6$, $T = 1$, $q = 3$, $\lambda_1 = 0.5$ and $\lambda_2 = 1$. (see Figure 4.)

5. Conclusion

In this paper, based on a Holling-type IV two-prey one-predator system, we established an impulsive differential equation to model the seasonal effects, the process of a proportional

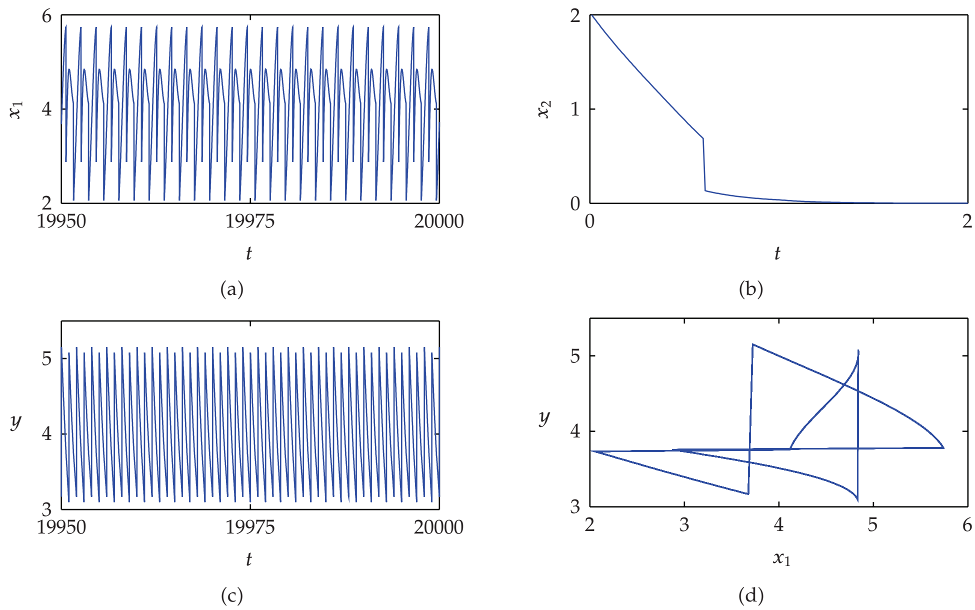

Figure 3: (a)–(c) Time series. (d) The trajectory of system (1.2) with an initial value $(2, 3, 1)$.

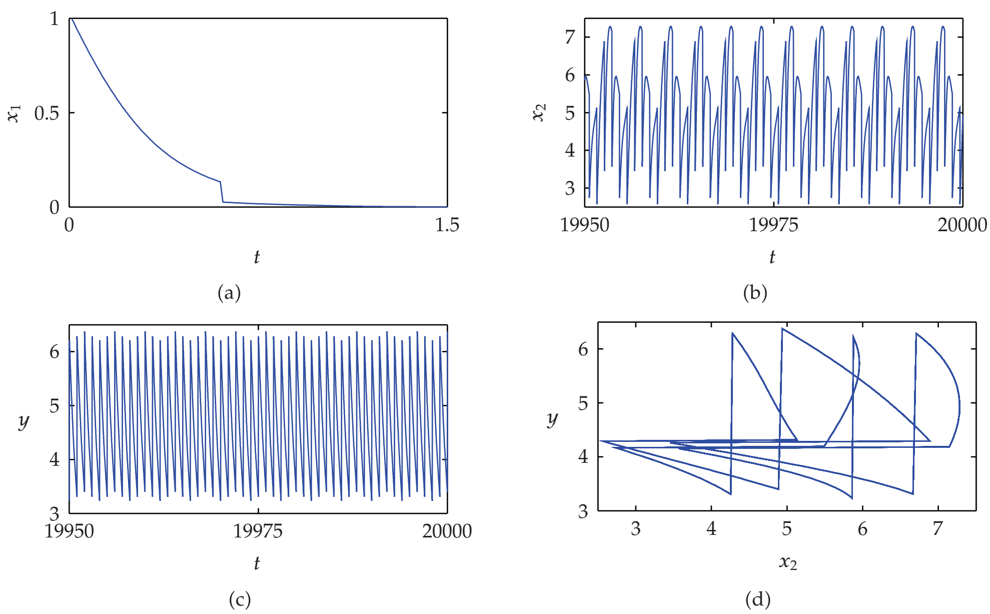

Figure 4: (a)–(c) Time series. (d) The trajectory of system (1.2) with an initial value $(2, 3, 1)$.

periodic impulsive harvesting, and a constant periodic releasing of the predator at different fixed time. Using the Floquet theory of impulsive differential equation and small amplitude perturbation skills, we proved that there exists a globally (locally) stable two-prey-free periodic solution if the conditions of Theorem 3.1 (Corollary 3.3) are satisfied. Also, we established permanence conditions of system (1.2) via the method of comparison including multiple Liapunov functions when the conditions of Theorem 3.6 are satisfied. From main Theorems, we easily obtained the sufficient conditions under which one of the two preys is extinct and whic of the remaining two species are permanent. In addition, we gave some examples by using numerical simulations.

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